## Experiment 2.8-B:

## Basics of positron emission tomography (PET)

Positron emission tomography is used in medicine for detecting physiological processes. You will investigate the position-dependent behaviour of a radioactive source. This will be done using $e^{+} e^{-}$annihilation, in which two collinear photons are created. The process will be measured by a pair of detectors at the same time (coincidence) after the reaction. In this experiment, you will get familiar with the experimental setup for coincidence measurement. You will learn about the BGO detectors used in the experiment with photo-multipliers (PMTs), look at the output of individual electronic components of the setup using an oscilloscope, and measure and interpret the spectrum of ${ }^{22} \mathrm{Na}$ source. The goal of the experiment is to use PET to locate radioactive sources in a sealed container. In addition, you can determine the relative intensity (activity) of the sources.

Keywords: $\beta^{+}$-decay, positrons in matter, interaction of $\gamma$-radiation with matter, $\gamma$-spectroscopy, principle of scintillation counters, detection of coincident events, multi-channel analyzer, energy calibration, medical applications of nuclear physics (especially PET), imaging procedures.

## 1 Introduction

In medical diagnostics it is essential to "see inside" the patient's body for early detection of changes or damages to the tissue. General well-known methods are X-ray and also computer-assisted tomographic (e.g. nuclear spin tomography and scintography). These methods almost always only give a picture of the attenuation of the radiation used by the examined tissue again. This is linked to the density and therefore the structure of the fabric, such that one can recognizes in the pictures the corresponding anatomical changes.

However, some clinical pictures (e.g. the early stages of tumor formation) before that are already recognizable by changes in metabolism. This is where the positron emission tomography (PET) enters: the patients are given radioactively marked substances which, depending on the metabolic activity, enter the cells to varying degrees to be built in. To determine the spatial and quantity of this enrichment, a coincidence measurement is used.


Figure 1: Principle of a PET measurement. A detector ring consists of up to 512 BGO crystals. Up to eight rings are arranged one above the other, separated by lead septa, in order to create faster full-body photos.

To do this, one uses the fact that positrons combine with the electrons of matter to form two $\gamma$ quanta of the same energy, flying apart at an angle of $180^{\circ}$ (collinearity). If the $\gamma$ 's are found in two different detectors of a ring array at the same time (coincident) the events are registered, and it is assumed that they are on the connecting line (Line of Response: LOR) of the detectors from the positron emitter. From the large number of linear combinations that can be obtained by such an arrangement the activity distribution of the plane can be reconstructed. The images obtained in this way represent the so-called metabolic spaces. With suitable reconstruction methods and measurement conditions, one can make statements about the metabolic activity of the tissue and any early changes can be recognized. Comparing to anatomical images, the accuracy of diagnosis is further increased.

A problem with PET is the relatively high cost of building one measurement setup. The selection of the sources used also needs to be carefully considered. Fortunately, ${ }^{11} \mathrm{C},{ }^{13} \mathrm{~N},{ }^{15} \mathrm{O}$ and ${ }^{18} \mathrm{~F}$ are (relatively) easy to produce and manageable isotopes, when are incorporated into organic molecules such as glucose. The spatial resolution of PET is physically limited, since the positrons do not annihilate at the location of their emission, but depending on the isotope, a range of up to two millimeters in the possess tissue. In addition, so-called false coincidences influence the measurement (absorbed, scattered, random and discarded coincidences). Also, the reconstruction algorithms do not always provide unambiguous results: it can lead to artifacts, which include misclassified activities etc..

Despite these disadvantages, PET is an increasingly widespread diagnostic method in medicine. The significance of PET in the case of clinical process of existing findings is established more and more in everyday life. Newer application of PET can be found in process engineering. Since the range of the positrons depends heavily on the density of the medium surrounding them, PET can also be used to determine the density of so-called 'two-phase flows'.

## Suggestions and questions

- What properties should an isotope have in terms of lifetime, production, decay in the ground state and chemical handling possess in order to be used for medical PET measurement?
- Look for the isotopes mentioned on the nuclide chart [HZK1995]. How can one produce them and to which isotopes they decay into?
- What do you think can be false coincidences? Try to clarify ideas using a sketch for a pair of detectors.


## 2 Goal

The aim of the experiment is to explain the physical and technical basics of PET and possible problems in the procedure. As an example, the radioactive sources are hidden in a container and their activity can be compared. The knowledge required to set up a measurement for coincident events should be conveyed. This includes, on the one hand, the physical basics (e.g. annihilation of positrons, behaviour of $\gamma$-radiation in matter) and on the other hand the detectors used, the electronics and the analysis software. In order to put the knowledge into practice, the following studies are carried out:

- the output signals of all electronic components used are displayed on the oscilloscope, observed and interpreted.
- The $\gamma$-spectrun of ${ }^{22} \mathrm{Na}$ source is recorded and interpreted.
- The spatial resolution of the coincidence measurement arrangement is determined in order to work as effectively as possible in the container analysis.
- Finally, the container is "scanned" and from the data obtained the two-dimensional activity distribution in the container is determined.


## 3 Physical basics

## $3.1 \beta^{+}$decay

$\beta$-decays include all decay processes in which the atomic number $Z$ changes by one $\left(Z_{\text {daughter }}=Z_{\text {mother }} \pm 1\right)$ and the mass number $A$ is conserved ( $A_{\text {daughter }}=A_{\text {mother }}$ ). For the positron emission tomography described in Sec. 1, a socalled positron emitter is required. These are isotopes that present a radioactive decay. $\beta^{+}$decay: the conversion of a proton into a neutron by a weak interaction produces a positron and a neutrino. In general one can describe the $\beta^{+}$as follows:

$$
\begin{equation*}
{ }_{Z}^{A} X^{N} \rightarrow{ }_{Z-1}^{A} Y^{N+1}+e^{+}+v_{e} \tag{1}
\end{equation*}
$$

The equivalent process to this is the so-called electron capture, in which however, no positrons are formed:

$$
\begin{equation*}
{ }_{Z}^{A} X^{N}+e^{-} \rightarrow{ }_{Z-1}^{A} Y^{N+1}+v_{e} . \tag{2}
\end{equation*}
$$

Since the $\beta^{+}$decay is a three-particle decay, the positrons have a continuous energy spectrum. The maximum positron energy $E_{e^{+} \max }$, corresponds exactly to the $Q$-value of the decay. In order for a $\beta^{+}$decay to take place at all, the mass of the daughter nucleus must change at least two electron rest masses ( $m_{0}=511 \mathrm{keV} / c^{2}$ ) smaller than that of the mother nucleus. From Eq. 1 it can be seen the only proton-rich can be accounted for this type of decay.

## Suggestions and questions

- In the nuclear chart [FZK1995], where do you find isotopes decay by positron emission?
- In what process are positrons also created? Why it is not used for PET?


### 3.2 Annihilation of positrons

When an anti-matter particle, which itself may be stable, collides with its companion matter particle, both particles are annihilated in pairs, with the emission of electromagnetic radiation (annihilation). The energy of the radiation quanta sets from the rest energies of the particles involved and their total kinetic energy together at the time of annihilation.


Figure 2: Positron decay and annihilation with typical range of positrons in body tissue

Since positrons are the antiparticles of electrons, they find very quickly in reality a partner for annihilation. However, the cross section for the annihilation is highly energy-dependent: only when the relative velocity of particles is low, there is a high probability that the process will take place. Positrons from $\beta^{+}$decay typically have energies in the MeV range, so they first have to be slowed down in the matter. The loss of energy occurs primarily by collisions with the atomic electrons of the surrounding matter and obeys the modified Bethe-Bloch formula. It only plays at very high energies (greater than 10 MeV ). Energy loss due to bremsstrahlung plays a role. Since the deviations from collinearity of the two annihilation quanta is very small, one assumes that the residual energies of positron and electron are in the range of a few electron volts.

## Suggestions and questions

- What is the range of positrons in air, water and plastic? Appreciate them with the help of Fig. 2 (body tissue consists essentially of water).
- What is the energy of the annihilation quanta from an $e^{+} e-$ annihilation? What results under otherwise identical conditions for a proton-antiproton annihilation?
- With the help of a sketch, make clear the influence of the residual energies of the electron and positron on the properties of the annihilation quanta.


### 3.3 Scintillation counter

Scintillation counters essentially consist of a scintillation crystal to which a photomultiplier (PMT) for electron multiplication is connected. Scintillation crystals are characterized by the fact that their atoms or molecules are excited by
high-energy radiation with the emission of light flashed in the ground state. This light is collected at a photocathode and converted into a few electrons. The electrons are generated via a dynode system accelerated so that about $10^{7}$ secondary electrons are generated per electron. These can then be registered and evaluated with the help of suitable electronics. From a certain minimum energy, the luminous is proportional to the deposited energy in the crystal so that the electrical signal is also proportional to it when the PMT is set correctly. This gives rise to the possibility of energy spectroscopy. Due to the short reaction time of the crystal, there are short dead-times of the system, so that even very high count rates can be processed well.


Figure 3: Principle of a scintillation counter (from [LEO1987], p. 157 Fig. 7.1).


Figure 4: Section through a PMT with a so-called box and grid arrangement of the dynodes (from [LEO1987], p. 182 Fig. 8.4b).

## Suggestions and questions

- Not every scintillator material is suitable for building detectors. What properties must be fulfilled?
- At which points in the detector system can there be a violation of the proportionality to the deposited energy?


## $3.4 \gamma$-spectroscopy

In order to understand a $\gamma$ spectrum, one must know how $\gamma$ radiation behaves in matter. The three essential processes (which will not be explained here in details) are: photoelectric effect, Compton effect and pair formation.

- Photoelectric effect: the $\gamma$-quantum transfers all of its energy to an atomic electron, which is thereby knocked out of the nuclear network. Its energy can then be measured in the detector. The photo effect dominates above all at small $\gamma$ energies $E_{\gamma}<100 \mathrm{keV}$.
- Compton effect: the $\gamma$-quantum is scattered by a (quasi) free electron. It then has less energy and a momentum directed in a different direction. The energy of the scattered electron is again measured in the detector. The Compton effect occurs above all at medium $\gamma$ energies $E_{\gamma} \simeq 1 \mathrm{MeV}$.
- Pair formation: in the electric field of an atomic nucleus (rarely also a electron) the $\gamma$-quantum forms an $e^{+} e^{-}$pair. One assigns the kinetic energy of electron and positron in the detector. In addition, the positron also annihilates with an electron into two $\gamma$-quanta with $E_{\gamma}=511 \mathrm{keV}$. Because of the rest masses pair formation electron and positron is only possible at $E_{\gamma} \geq 1022 \mathrm{keV}$, and occurs mainly at high $\gamma$ energies $E_{\gamma} \geq 5 \mathrm{MeV}$.


## Suggestions and questions

- Sketch the $\gamma$ spectrum of ${ }^{22} \mathrm{Na}$ source at a $2 \pi$ geometry of a BGO detector. Note the term scheme for ${ }^{22} \mathrm{Na}$ (App. Fig. 8) and the cross sections of photons in BGO (App. Fig. 9).
- ${ }^{22} \mathrm{Na}$ meets the requirements for a PET (see suggestions and questions about Sec. 1 )?


### 3.5 Detection efficiency

The detection efficiency of a detector results from the quotient of the detected and the total emitted quanta or particles. Below is the geometric response and the intrinsic response of a detector, which have to fulfilled. The geometric efficiency $\varepsilon_{g e o m}$ results from the detector surface which covers a solid angle range $d \Omega$. For a distance $a$ to the detector with an effective surface $A$ :

$$
\begin{equation*}
\varepsilon_{\mathrm{geom}}=\frac{d \Omega}{\Omega_{\mathrm{ges}}}=\frac{\frac{A}{a^{2}}}{4 \pi}=\frac{A}{4 \pi \cdot a^{2}} . \tag{3}
\end{equation*}
$$

The intrinsic efficiency $\varepsilon_{i n t}$ can be found in tables. It results from the quotient of the number of detected and number of quanta of particles reaching the detectors. It can be calculated from the distance traveled in the detector $x$ and the mean free path $\lambda$ :

$$
\begin{equation*}
\varepsilon_{i n t}=1-\exp \left(\frac{-x}{\lambda}\right) \tag{4}
\end{equation*}
$$

For a homogeneous detector material, the detection efficiency can be approximated as

$$
\begin{equation*}
\varepsilon=\varepsilon_{\text {geom }} \cdot \varepsilon_{i n t} . \tag{5}
\end{equation*}
$$

In the case of coincidence measurements, it should be noted that a signal is registered only if both detectors obtained a signal. The probability of a coincidence measurement is:

$$
\begin{equation*}
\varepsilon_{k o i n z}=\varepsilon_{g e o m, 1} \cdot \varepsilon_{i n t, 1} \cdot \varepsilon_{g e o m, 2} \cdot \varepsilon_{i n t, 2} \tag{6}
\end{equation*}
$$

## Suggestions and questions

- What can you change to halve or triple the geometric response in Eq. 3?
- Simplify Eq. 6 under the assumption that with two identical detectors $\gamma$-quanta from a positron annihilation should be registered in coincidence.
- Consider the Line of Response (LOR) of a detector pair with a given distance $R$. At what point of the LOR you need to put a positron emitter to get the maximum coincidence count rate?


### 3.6 Peculiarities of PET

The crucial feature for the coincidence measurement in PET is the collinearity of the two annihilation quanta: one can assume that the geometric response for the second detector is $100 \%$ if an annihilation quantum was registered in the first detector. The detectors that measure the coincident events in pairs are identical, i.e. they are made of the same material and have the same dimensions. Therefore, in Eq. 6, the intrinsic responses are identical.

Since the spatial resolution of the PET is already physically limited (see Sec. 1), the best possible resolution can be achieved by using solid angles small as possible: the detector surface is reduced by collimators made of lead, but have to accept a lower count rate. In applications, on the other hand, it has proven to be advantageous to use the smallest possible ones. This results in a large number of detectors in for a system (see Fig. 1) and thus high costs of the measurement array.

## 4 Technical basics

The electronic components required for this experiment and their function are briefly described. More detailed information and technical data can be found in the listed references and literature.

### 4.1 Amplifier

An amplifier has the task of amplifying an incoming signal, i.e. increase its amplitude. A distinction is made between preamplifiers and main amplifiers. A preamplifier is connected directly behind a detector and amplifies the signal so that the subsequent losses in the cable are as low as possible. However, this is not necessary for the BGO detectors. A main amplifier should either provide a signal with the shortest possible rising edge produce (Timing Filter Amplifier TFA) or the signal as proportionally as possible to amplify the input signal (spectroscopy amplifier). It is not usually possible, but also not necessary in most applications, to achieve both the shortest rising edges and the most proportional gain possible. Both options are available to you during the experiment.

### 4.2 Discriminator

Discriminators only work when the input signal exceeds a predetermined pulse height. If this is the case, a logical signal of certain height and width issued. The three parameters can be selected individually. If the output begins exactly when the threshold is exceeded (leading edge discrimination), then signals with the same rising edge but different amplitude registered at different times. There is a time offset for same time incoming signals. To prevent this, the incoming signal is first processed, as shown in Fig. 5. Then the logic signal is modified at the zero crossing of this input signal (constant fraction discriminator).


Figure 5: How a constant fraction discriminator works: the input signal is first determined by the time $\tau_{d}$ (pulse $V_{d}$ ). It is also inverted and the amplitude compressed by a factor $k$ (pulse $V_{c}$ ). The zero crossing of the sum pulse from $V_{d}$ and $V_{c}$ starts the logical output signal (from [LEO1987], p. 327 Fig. 17.14 and p. 328, Fig. 17.5).

### 4.3 Time-to-amplitude-converter

A time-to-amplitude-converter (TAC) has two inputs: one for the "start" and one for the "stop" signal. The output delivers one pulse, the height of which is proportional to the time difference between the arrival of "start" and "stop" signals. This is usually done by constantly discharging a capacitor during this time. The discharge speed and thus the pulse height can also be selected.

### 4.4 Delay

There is always a time delay between the "stop" signal on the TAC and the "start" signal, necessary if the experiment does not determine which detector's signal is activated first. In order not to lose coincident events, you postpone its "stop" signal by a certain time (delay). If this time is in the range of a few 100 ns , it simply becomes the cable length for the "stop" signal, increased with the help of a variable delay box. For time delays greater than 1 us an electrical circuit can be used to avoid losses.

### 4.5 Analog-to-digital-converter

The job of an analog-to-digital-converter (ADC) is converting an analog signal into a digital one, so that it can be read by a computer, for example and further processed. It is important that the assigned channel is proportional to the amplitude of the signal (integral linearity) and that the channels all have the same width (differential linearity). There are two basic methods of converting the signal. In the Wilkinson method, it's activated when the input signal exceeds a threshold capacitor charged. If the threshold is undercut, the discharging of the capacitor begins (run down). During the discharging process, the pulses from a frequency generator are counted and used as a digital signal. This method is used in the lab. The approximation method, on the other hand, compares the pulse height of the input signal successively with predetermined values resulting from previous comparison. Depending on whether they are larger or smaller, a ' 1 ' or ' 0 ' is attached to the appropriate one digit, written in binary code.

### 4.6 Multi-channel-analyzed

A multi-channel-analyzer (MCA) sorts incoming signals into a memory channel according to their pulse height and returns the number of signals in each channel in tabular or graphical form. In the present experiment, this is done by "MAESTRO", a program for recording, display and analysis of $\gamma$ spectra.

## Suggestions and questions

- Which electronic components are necessary to create an energy spectrum and to record a coincidence measurement? Outline the structure of each measuring arrangement.
- What happens to the peak in the spectrum of a coincidence measurement when you use another delay?


## 5 Measurements

### 5.1 Setup of the experiment

Figure 6 shows a sketch of the setup. Before you start the measurement, make sure that the experimental setup is properly adjusted. If necessary, please adjust the structure under the instructions of the supervisor. Your supervisor will provide you with the cables and plugs you will need later on request.

### 5.2 Detector signal on the oscilloscope

Connect the two BGO detectors to the high voltage power supply. Connect the detector output of one of the detectors to one of the channels of the oscilloscope.

Once your supervisor has checked the wiring, first turn on the NIM crate. Then fix the high voltage slowly up to 700 V : increase first the low scale to $100 \mathrm{~V}, 200 \mathrm{~V}$, and then the large scale by additional 500 V . All further wiring is also done with supervision.

Place the source in the holder and make sure that it's approximately in the middle of the detector surface. The handling of the radioactive source is done only by the supervisor or under the instructions of the supervisor. The sources may only be touched with tweezers.


Figure 6: A sketch of the essential components for the experiment. The individual cabling required for the measurement is not shown. The following electronic components are located in the NIM crate: 1/3-Amplifier, 2/4-TFA, 5-Discriminator, 6-Delay, 7-High voltage power supply for the detectors, 8-TAC, and 9-ADC.

Once the source is in the holder, start with the following settings on the oscilloscope: voltage: 50 mV , time: 10 us, and tune them to see a clear signal. Make sure the negative signal triggering is set correctly (Flanke: negativ). Repeat for the second detector output and take pictures of the signals.

### 5.3 Amplifier signal on the oscilloscope

Connect the output of the BGO detector to an amplifier and set an appropriate reinforcement on. Take the signal from the unipolar output and connect it to the oscilloscope, start with the following settings: voltage: 1 V , time: 2.5 us. Consider the signal with both positive and negative slope triggering. Repeat for the second detector output and take pictures of the signals.

### 5.4 Spectrum of the ${ }^{22} \mathrm{Na}$ source

Start the program "MAESTRO" on the PC. The operation of the program is Windows-compliant (for further questions a manual exists in the lab). Create a sub-directory in the folder /FP_PET_2022 in which you can put your measured spectra. Define this directory as the working directory of the program (File $\rightarrow$ Settings $\rightarrow$ Directories $\rightarrow$ Spectra). You should adjust the length of the spectrum according to the setting $2^{3}-2^{12}$ on the ADC. Set 1024 channels (Acquire $\rightarrow$ MCB Properties $\rightarrow$ Conversion Gain). For this measurement the GATE should be off: Acquire $\rightarrow$ MCB Properties $\rightarrow$ $A D C \rightarrow$ Gate $\rightarrow$ Off. Measure the spectrum for about two and a half minutes (Acquire $\rightarrow$ MCB Properties $\rightarrow$ Presets $\rightarrow$ Live Time). Determine the measurement time such that the uncertainty of the smallest peak is less than one percent (how many counts is that?). Save that spectrum from the ADC (File $\rightarrow$ Save As $\rightarrow$ Dateityp : ASCII SPE).

### 5.5 TFA-signal on the oscilloscope

You now need both BGO detectors and the two TFAs. Make sure first that both detectors have the same distance from the source. Observe the signals of both TFAs one after the other on the oscilloscope, start with the following settings: voltage: 0.5 V , time: 500 ns , and negative edge. Compare both signals and take a picture.

### 5.6 Discriminator signal on the oscilloscope

Connect the outputs of the TFAs to an input on the discriminator. Look at the output signals one after the other on the oscilloscope, start with the settings: voltage: 1 V , time: 250 ns , and negative edge. Compare both signals and take a
picture.

### 5.7 TAC-signal on the oscilloscope

Connect one of the discriminator outputs to the TACs "start" input, connect the other discriminator output to the "stop" input via a delay. You can observe the output signal of the TAC on the oscilloscope, start with the settings: voltage: 0.5 V , time: 5 us, and a positive edge. Take a picture of the signal too. Connect in addition an amplifier signal for one of the detectors to the oscilloscope. Tune the parameters of the amplified signal and the range of the TAC. Then, connect the amplified signal to ADC IN and the TAC output to GATE. In MAESTRO, change the GATE to 'Coincidence' and start a measurement. Now you should see only the coincidence region. To see information for a specific region in the program, choose it with the mouse, then right click and choose 'Mark RIO', right click again and 'Peak Info'.

### 5.8 Spatial resolution of the coincidence measurement

The aim of this task is to determine the influence of the location of the source, the LOR of the two detectors on the coincidence count rate. Besides that you should define the integration limits for the rest of the experiment. To do this, push the cart with the source through the relevant axis with a fixed increment. Measure for two minutes at a time the spectrum and save it, specifying the position of the carriage. Set the integration limits in the most appropriate spectrum (which one is that?). Try an even smaller increment in the most relevant area. The total number of measurements should bot be greater than ten.

### 5.9 PET analysis of the 'treasure box"

You will now receive a locked box from your supervisor, inside which there are two different radioactive ${ }^{22} \mathrm{Na}$ sources. By several coincidence measurements you should determine the location of the sources inside the container and roughly the activity ratio of the sources. For the first measurement (both in $x-$ and $y$-direction) please note the location of the edge of the container with respect to the LOR of the detectors. Then measure at different positions with a fixed time of 120 s . Think of a sensible increment (the test should remain finite..). If you have measured the $x$-direction, proceed in the same way in the $y$-direction, by removing the container from the trolley and inserting it again rotated by $90^{\circ}$. Finally, carry out the measurements in the diagonal. Think about the position of the edge relative to the LOR and the increment based on your previous measurements. Figure 7 in Sec. 6.4 provides an overview of the geometric situation. Save the spectra for later analysis.

### 5.10 End of experiment

When you have finished the measurement, please inform your supervisor. Copy all spectra that you need for further analysis onto a USB-stick. Slowly regulate the high voltage down and then turn them off. Turn off the NIM crate too.

## 6 Analysis of the measurements

### 6.1 Analysis of various signals on the oscilloscope

Present the pictures for the signals observed on the oscilloscope. Give also the respective settings of the examined component, and describe the meaning of signal height and width. Which components supply logical signals? How one recognizes that?

### 6.2 Calibration and interpretation of the ${ }^{22} \mathrm{Na}$ spectrum

First perform an energy calibration of the spectrum using the two known $\gamma$-lines. To get the exact channel number of the peaks integrate them. After the calibration you should look at the spectrum in a logarithmic scale, to see even small effects. Interpret all peaks and find their associated Compton edges. Present one of the spectra. What problems could arise with the coincidence measurement?

### 6.3 Analysis of the spatial resolution

Load your spectra one by one and integrate them within your defined limits (channel numbers are more suitable). Which of the calculated values are a measure of the number of coincident events and thus of the activity on the LOR? Display this graphically with uncertainty information. Can you determine the spatial resolution in relation to the LOR? What half-width does your peak have?

### 6.4 PET analysis

The aim of the analysis is to determine the location of the sources in the tank and their activity ratio. To do this, all spectra must be in the range specified in Sec. 6.3 (done already during the measurement). Write down the measured values for the number of coincident events and thus for the activity on the respective LOR. In order to be able to draw conclusions about the 2D activity distribution, create a matrix of these values by multiplying each $x$-value by each $y$-value. Match the diagonal measurements to the correct intersections of vertical and horizontal measurements (compare Fig. 7). What measurement results do you expect for the individual measurements in the three directions, if there is a strong source at position $(x 2, y 4)$ in Fig. 7? Then how do you see the associated matrices?


Figure 7: By multiplying five measurements in the $x$ and $y$ directions, you get 25 measurement points. The measurements in the $q$-direction are multiplied in a suitable way in order to eliminate errors in the back calculation.

Now set the values of you matrices, i.e., the distribution of the coincidence count rate of your measurement in 2D by giving different counts different colors. With the help of the information about the geometry of the container and the position of the carriage in relation to the LOR, you can determine the position of the sources in the container. Arrange the sources relative to the strongest activity. Note that for each point in the matrix, you have results of two independent measurements multiplied together.

## References

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## Basic Literature

In the following references you will find the answers to the Suggestions and questions from Secs. 1, 3, and 4, as well as many other useful information about the experiment.

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## Appendix



Figure 8: Decay scheme for ${ }^{22} \mathrm{Na}$. Note the branching ratios. Source: [FIRESTONE1996], p. 43.


Figure 9: Energy dependence of photoelectric effect, Compton effect, and pairing in a BGO crystal $\mathrm{Bi}_{4} \mathrm{Ge}_{3} 0_{12}$ from keV to 1000 MeV . Note the double logarithmic scale. Source: [DATABASE BNL], based on the compilation of [CULLEN89].

