Electron scattering on the Hoyle state and carbon production in stars

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Abstract. High-resolution inelastic electron scattering experiments were performed at the S-DALINAC for precise determination of the partial pair width Γ_π of the second $J^\pi = 0^+$ state, the so-called Hoyle state, in ^{12}C. Results for the monopole matrix element (directly related to Γ_π) from a nearly model-independent analysis based on an extrapolation of low-q data to zero momentum transfer are presented. Additionally, a Fourier-Bessel analysis of the transition form factor is discussed. The combined result of both methods leads to a partial pair width Γ_π = 62.2(10) μeV.

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INTRODUCTION

The production of the element carbon is a key reaction of stellar nucleosynthesis. The creation of its most abundant isotope ^{12}C is the result of the triple-alpha process [1], which passes through the $J^\pi = 0^+$ resonance at an excitation energy E_x = 7.65 MeV in ^{12}C. The state is called Hoyle state as it was postulated [2] by the astrophysicist Fred Hoyle to explain the observed abundance of carbon in the Universe. The Hoyle state has unusual structure considered as a pure example of an α cluster condensate [3, 4].

Despite its astrophysical relevance, the carbon production rate through the triple-alpha process is known with insufficient precision only [5, 6]. For temperatures above 10^8 K the reaction rate $r_{3\alpha}$ can be written as

$$r_{3\alpha} \propto \Gamma_{rad} \exp\left(-\frac{Q_{3\alpha}}{kT}\right), \tag{1}$$

where Γ_{rad} is the radiative width of the Hoyle state, k the Boltzmann constant, T the temperature, and $Q_{3\alpha}$ the energy released in the decay of the Hoyle state into three alpha particles.

As seen from Eq. (1), for a precise determination of $r_{3\alpha}$ only Γ_{rad} and $Q_{3\alpha}$ need to be known. The released energy $Q_{3\alpha}$ = 379.38 keV is measured with a precision $\pm$0.20 keV (see Ref. [7] and references therein). The uncertainty of the 3α process due to this factor is very small and corresponds to only $\pm$1.2% at 2×10^8 K, decreasing as $1/T$ with increasing temperature. Consequently, the main uncertainty in $r_{3\alpha}$ is due to the uncertainty in Γ_{rad} which, however, cannot be measured directly. It is determined as a product of three independently measured quantities

$$\Gamma_{rad} = \Gamma_\gamma + \Gamma_\pi = \frac{\Gamma_\gamma + \Gamma_\pi}{\Gamma} \cdot \frac{\Gamma}{\Gamma_\pi} \cdot \Gamma_\pi, \tag{2}$$

where $\Gamma = \Gamma_\alpha + \Gamma_\gamma + \Gamma_\pi$ is the total decay width taking into account α, γ and $e^\pm$ decay of the Hoyle state. At present these three quantities are known with an accuracy (left to right) of ±2.7% (see Ref. [8] and references therein), ±9.2% (see Ref. [9] and references therein), and ±6.4% (from combination of values quoted in Refs. [10, 11]), leading to a total $r_{3\alpha}$ uncertainty of ±11.6%. A new experiment for a precise measurement of the second quantity is discussed in Refs. [5, 12]. Additionally, a new result for the third quantity, *i.e.* the pair width Γ_π, with a quoted accuracy of ±2.7% is given by Crannell *et al*. [13]. It would reduce the total uncertainty of $r_{3\alpha}$ to ±10.0%. Unfortunately, the new value is inconsistent within error bars with the earlier values of Γ_π [10, 11].

A main purpose of the present work is to resolve the existing inconsistency in the partial pair width Γ_π of the Hoyle state using high-resolution electron scattering measurements at low momentum transfers.

EXPERIMENTAL DETAILS

The experiment was performed at the 169° magnetic spectrometer of the S-DALINAC. Excitation energy spectra were taken at initial electron energies between 29.3 MeV and 78.3 MeV and scattering angles between 69° and 141°. This corresponds to the momentum transfer range for the Hoyle state between 0.21 fm^{-1} and 0.66 fm^{-1}. A self-supporting carbon target with natural isotopic content and an areal density of 6.4 mg/cm^2 was used. Typical beam currents were about 1 μA. In the energy-loss mode an energy resolution ΔE ≈ 28 keV (full width at half maximum, FWHM) was achieved.

Typical spectra measured at a beam energy 73 MeV are presented in Fig. 1. The prominent peaks correspond to the elastic line, the 2_1^+ state at E_x = 4.44 MeV and the

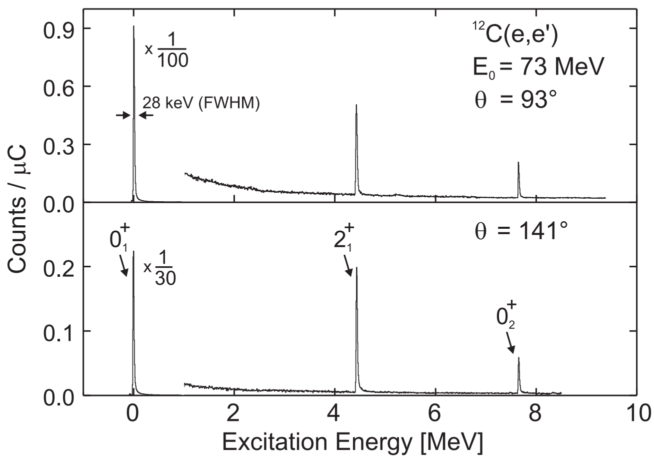

FIGURE 1. Spectra of the ^{12}C(e,e′) reaction measured at a beam energy E_0 = 73 MeV and scattering angles θ = 93° (top) and θ = 141° (bottom).

Hoyle state at E_x = 7.65 MeV in ^{12}C. For an energy calibration of the measured spectra electron scattering on ^{9}Be, ^{28}Si and ^{94}Mo was performed under the same kinematics.

PWBA ANALYSIS

In the plane wave Born approximation (PWBA) the monopole matrix element *ME* for the transition to the Hoyle state can be extracted in a nearly model-independent way [14] by a least squares fit of the reduced transition probabilities using the equation

$$\sqrt{4\pi B(C0,q)} = \frac{q^2}{6}(ME)\left[1 - \frac{q^2}{20}R_{tr}^2 + \cdots\right], \tag{3}$$

where $B(C0,q)$ is the reduced transition probability, *ME* the monopole matrix element, R_{tr} the transition radius, and q the momentum transfer. The outcome of the fit procedure

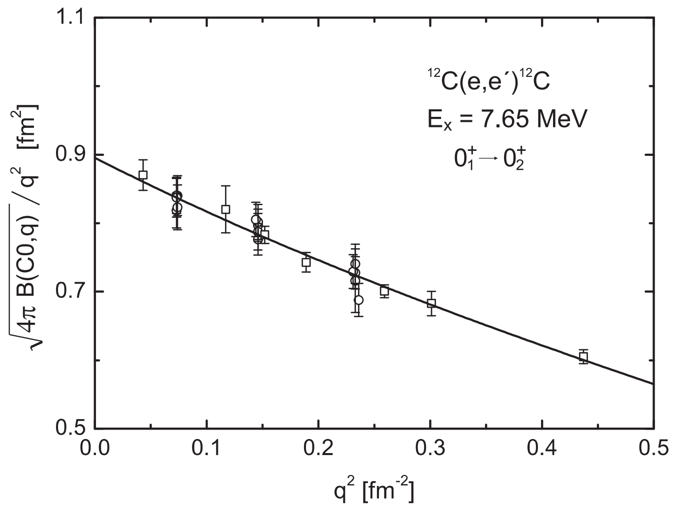

FIGURE 2. Extraction of the monopole matrix element by extrapolation of the reduced transition probability to zero momentum transfer. Data are from Ref. [11] (circles) and the present work (squares).

is presented in Fig. 2, where the experimental values $\sqrt{4\pi B(C0,q)}/q^2$ and the fit function are plotted as a function of q^2. Data measured in the present work (squares) are shown together with older data (circles) from Ref. [11]. The solid curve represents the fit to the data resulting in *ME* = 5.37(7) fm^2 and a transition radius R_{tr} = 4.30(12) fm. In the analysis the data points were weighted by their error bars and terms up to order q^6 were considered. The resulting partial pair width Γ_π = 59.6(16) μeV is in a good agreement with the values extracted by Crannell *et al*. [10] and Strehl [11], but with a significantly reduced uncertainty.

FOURIER-BESSEL ANALYSIS

An alternative method to determine Γ_π is a reconstruction of the $0_1^+ \rightarrow 0_2^+$ transition density by means of a Fourier-Bessel analysis, which then allows to extract the transition charge density and the matrix element *ME*. For an accurate extraction of the transition density, data should be available over a broad momentum transfer range. This is the case for the Hoyle state in a region $q = 0.2 - 3.1$ fm^{-1} (Refs. [11, 15] and the present work), as shown in Fig. 3.

A detailed description of the fit procedure can be found in Ref. [16]. In the present analysis a bias tail starting from the radius R_t = 5 fm with a weight P = 20 was used. Due to program limitations only 100 data points could be used in the analysis. These were chosen to be 22 Darmstadt low-q data points, 13 high-q points from HEPL and the rest was taken randomly from the Bates and NIKHEF data. Comparison of different data sets in regions where they overlap has shown that the HEPL data were systematically 15% higher than the data from Bates and NIKHEF, the latter being in good agreement with each other. The used HEPL data points were reduced by a factor of 1.15.

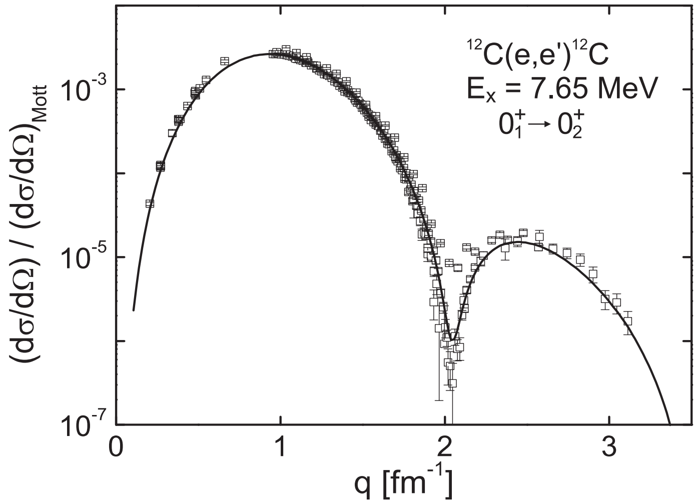

FIGURE 3. Transition form factor for the transition from the ground state in ^{12}C to the Hoyle state extracted by Fourier-Bessel analysis in comparison with the experimental data.

Figure 3 shows the transition form factor to the Hoyle state obtained by the fit procedure in comparison with the experimental data. The corresponding transition density yields a monopole matrix element of the transition from the ground state in ^{12}C to the Hoyle state *ME* = 5.55(5) fm^2. This leads to a partial pair width Γ_π = 63.7(12) μeV in disagreement with Ref. [13].

Additional details concerning both analyses are contained in Ref. [17].

EXTRACTION OF THE PAIR WIDTH

As the low-q extrapolation and the Fourier-Bessel analysis can be considered independent of each other, the obtained results are averaged. This leads to a weighted mean of the partial pair width for the transition from the ground state to the Hoyle state in ^{12}C

$$\Gamma_\pi = 62.2(10)\ \mu\text{eV}, \tag{4}$$

the most precise result to date.

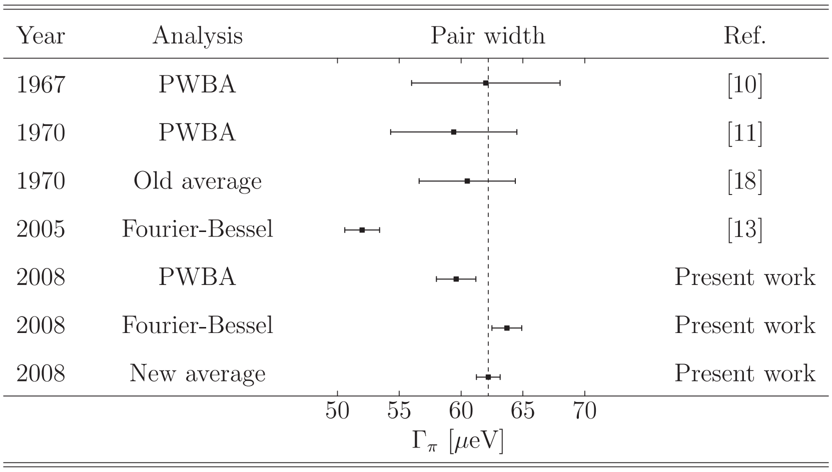

FIGURE 4. Summary of partial pair widths derived over the last 40 years for the Hoyle state in ^{12}C.

Figure 4 shows a comparison of the results for the partial pair width of the Hoyle state obtained in the present work with the data available in the literature. The newly obtained partial pair width agrees well with older data – except the one from Ref. [13] – but has a significantly reduced uncertainty.

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