Comparison of the confined β-soft Rotor Model and a microscopic collective Hamiltonian based on the relativistic mean field model in ^{150,152}Nd *

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- Motivation
- Theoretical framework
- Comparison of the model calculations
- Summary

SFB 634

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Motivation

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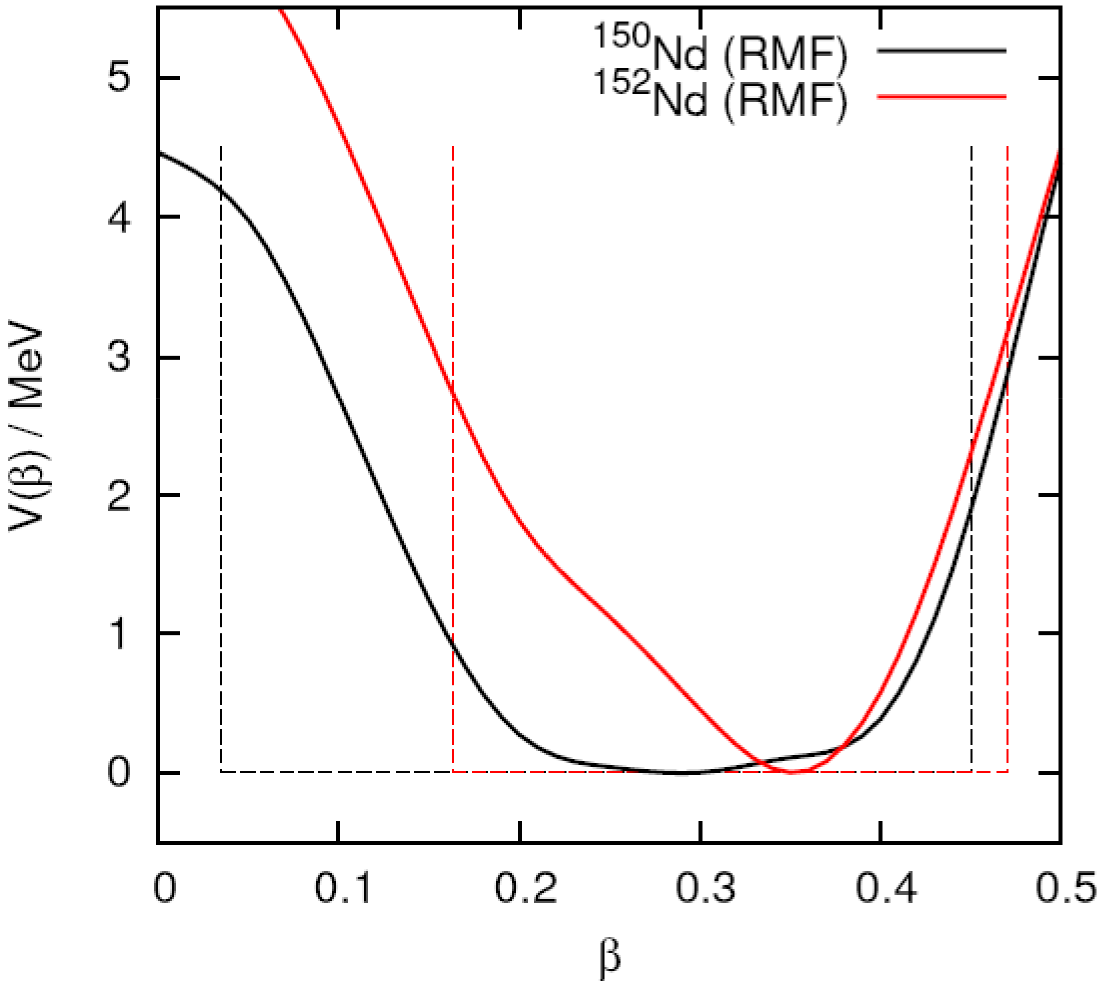
- Description of nuclei in the A=150 phase shape transitional region
- CBS model has a very simple macroscopic potential ansatz for solving the Bohr Hamiltonian (square well potential)
- Has the CBS model a microscopic justification?
- In $^{150,152}Nd$ microscopic calculations based on Relativistic Mean Field approach are available

→ Comparison of both models

Model Potentials



- Study impact on the evolution of nuclear structure originating in the shift of the inner potential wall

- To what extend the use of square well potentials is microscopically justified?

The collective Hamiltonian

- Very general form: $\hat{H} = \hat{T}_{\text{vib}} + \hat{T}_{\text{rot}} + V_{\text{coll}}$

- Vibrational kinetic energy:

$$\hat{T}_{\text{vib}} = -\frac{\hbar^2}{2\sqrt{wr}}\left\{\frac{1}{\beta^4}\left[\frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^4 B_{\gamma\gamma}\frac{\partial}{\partial\beta} - \frac{\partial}{\partial\beta}\sqrt{\frac{r}{w}}\beta^3 B_{\beta\gamma}\frac{\partial}{\partial\gamma}\right] + \frac{1}{\beta\sin 3\gamma}\left[-\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\frac{\partial}{\partial\beta} + \frac{1}{\beta}\frac{\partial}{\partial\gamma}\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\frac{\partial}{\partial\gamma}\right]\right\}$$

- Rotational kinetic energy:

$$\hat{T}_{\text{rot}} = \frac{1}{2}\sum_{k=1}^{3}\frac{\hat{J}_k^2}{\mathcal{I}_k}$$

- Discuss two different approaches for V_{coll}

Confined β-Soft (CBS) Rotor Model

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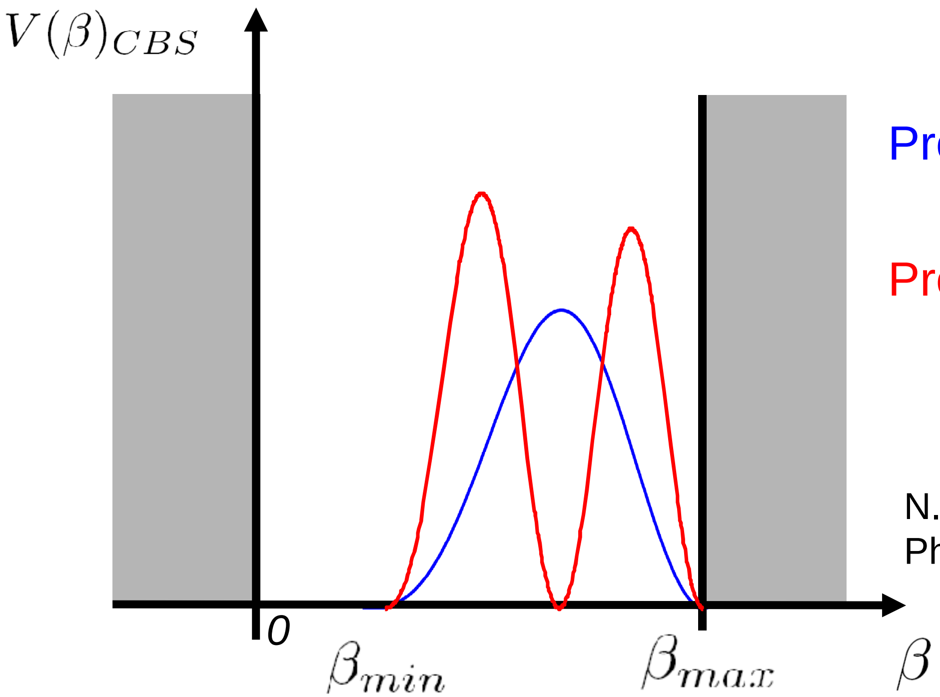


Probability density distribution of g.s.

Probability density distribution of 0^+_2 state

N. Pietralla and O. Gorbachenko, Phys. Rev. C **70**, 011304(R) (2004)

- Analytical wave functions in deformation coordinate β
- Very good description of B(E2) strengths & g.s. band energies in transitional nuclei
- Prediction for E0 transition strengths   J. Bonnet et al. Phys. Rev. C **79**, 034307 (2009)

Relativistic mean field Hamiltonian

- Moments of inertia
- Mass Parameters
- collective potential

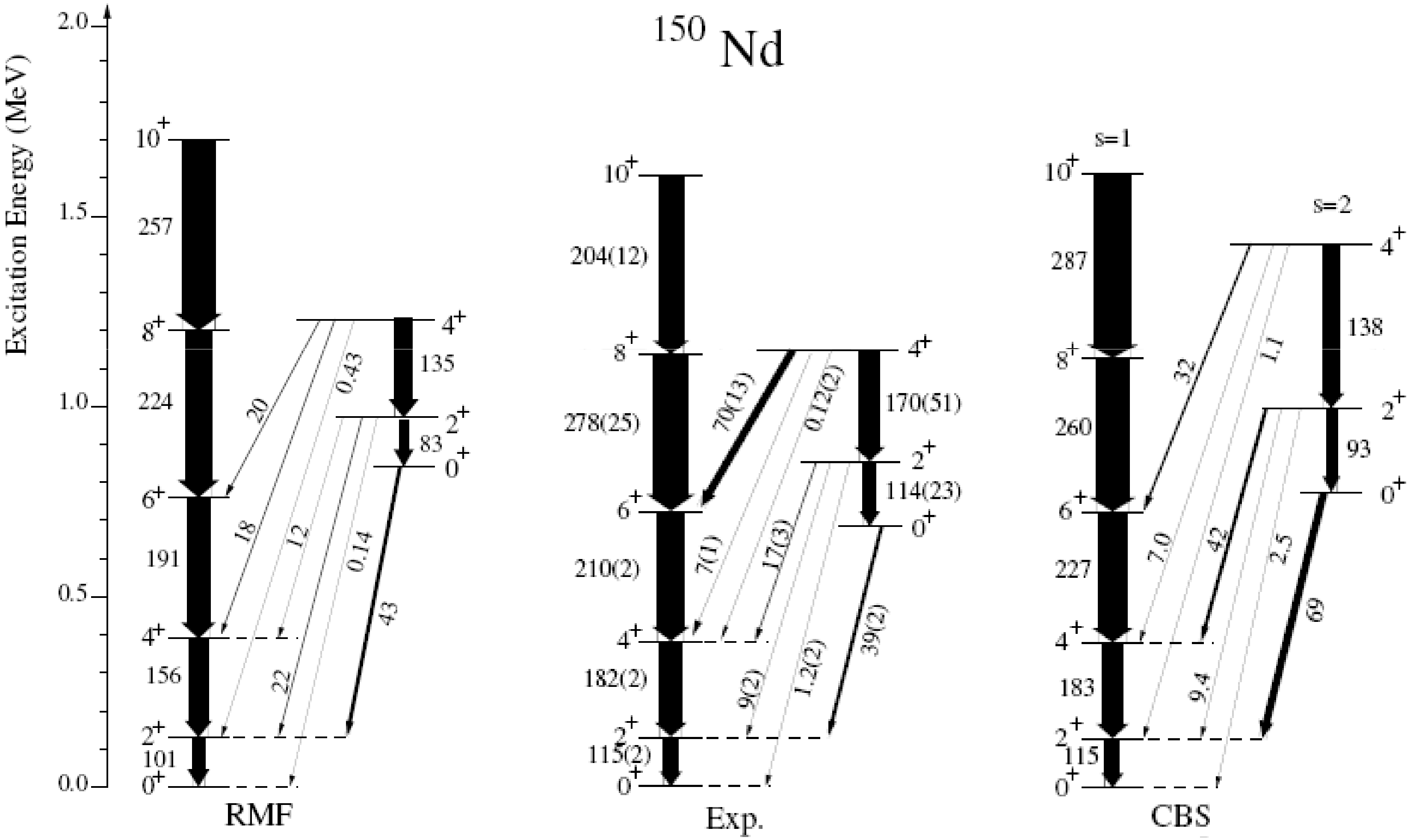
} microscopic nuclear energy-density functional

- Potential V_{coll}:

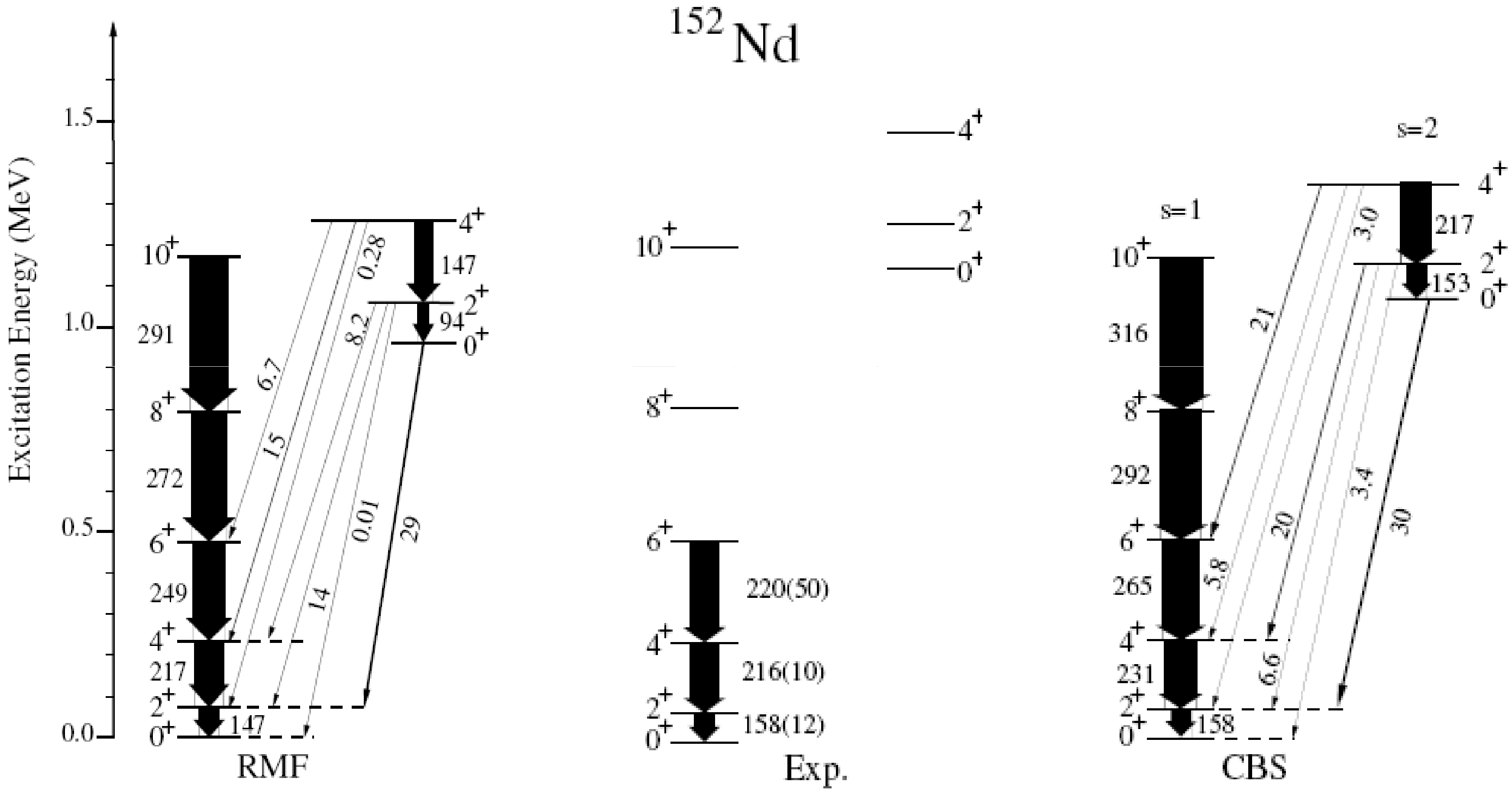
  Subtracting zero-point energy corrections from total energy (corresponds to solution of constrained RMF+BCS equations at each point of the triaxial deformation plane)

Nikšić T, Li Z P, Vretenar D, Prochniak L, Meng J and Ring P 2009 Phys. Rev. C 79, 034303

Level Scheme in ^{150}Nd



Level Scheme in ^{152}Nd



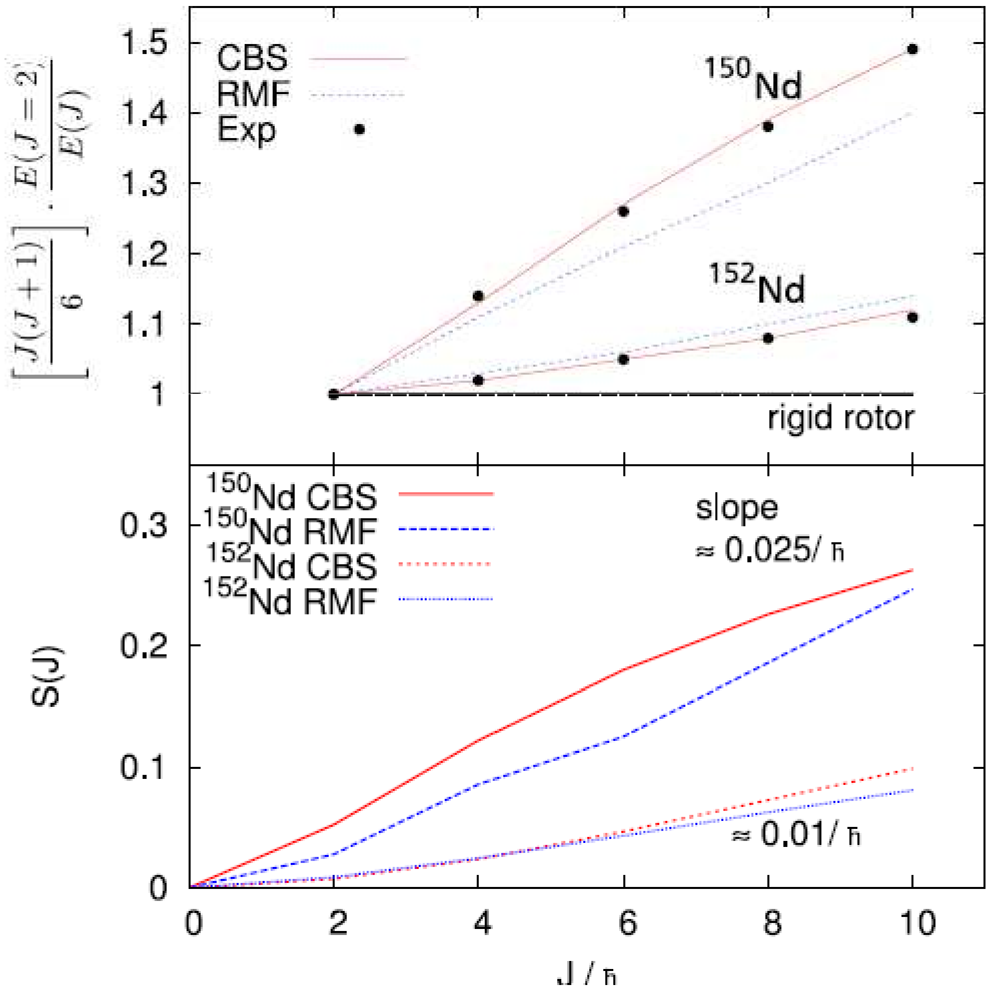
Centrifugal stretching

- Relative dynamic moment of inertia:

$$\frac{\theta(J)}{\theta(2)} = \left[\frac{J(J+1)}{6}\right] \frac{E(J=2)}{E(J)}$$

- Stretching parameter:

$$S(J) = \frac{\langle\beta\rangle_{J_i} - \langle\beta\rangle_{0_1^+}}{\langle\beta\rangle_{0_1^+}}$$



Wave functions of the g.s. band

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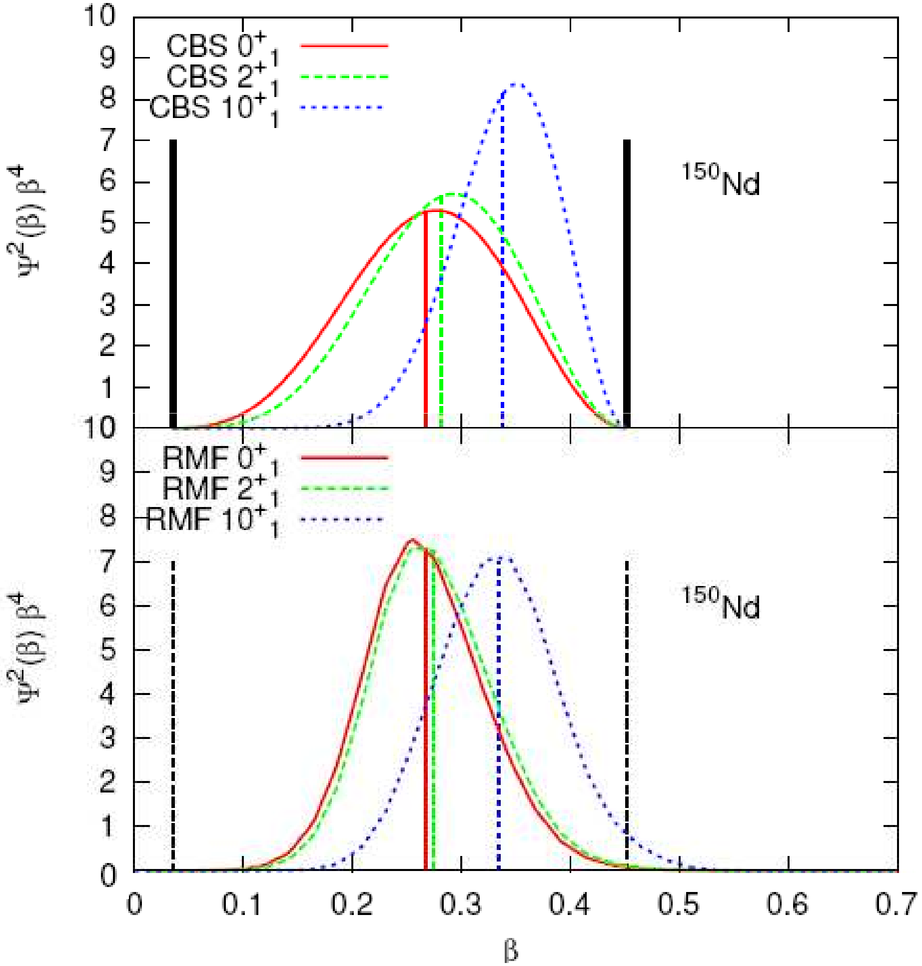

- Wave functions are very similar
- Centroids at the same deformation β
- Similar amount of centrifugal stretching in the two models
- Potential stiffens with increasing neutron number

Wave functions of the g.s. band

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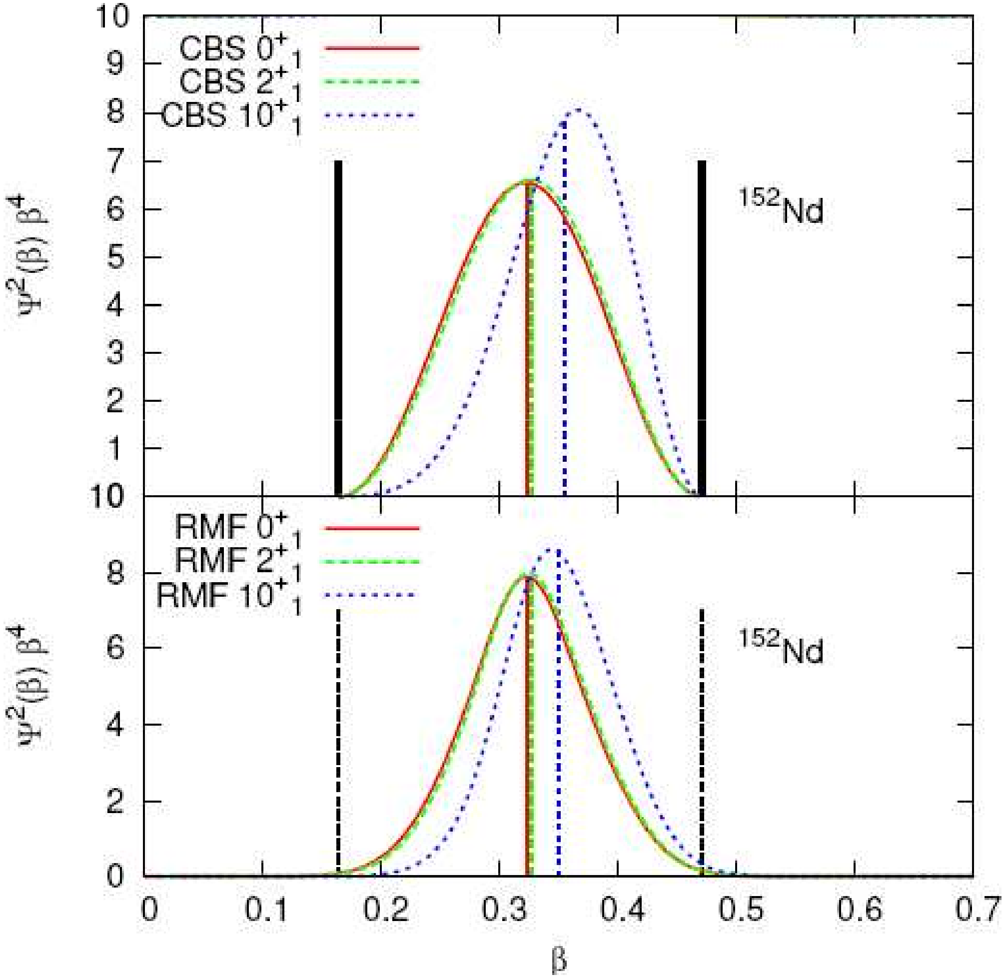
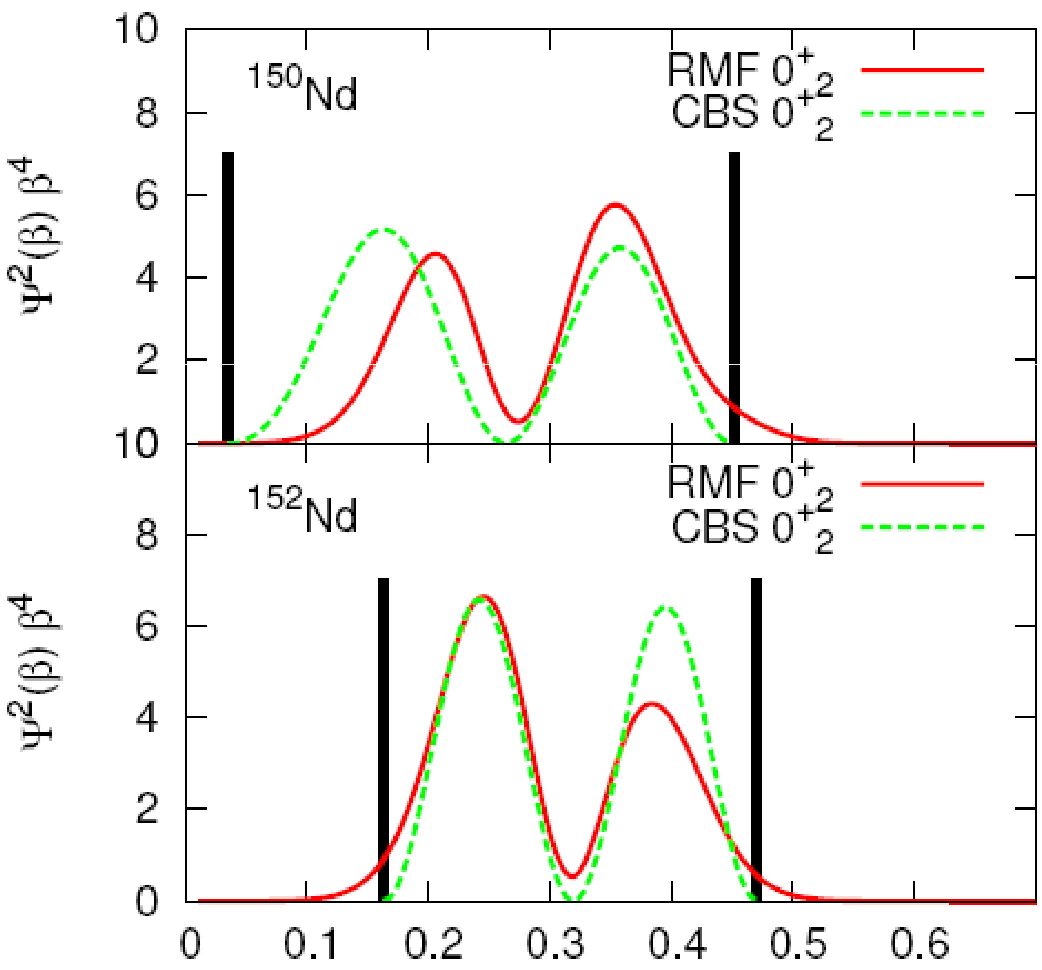


- Wave functions are very similar
- Centroids at the same deformation β
- Similar amount of centrifugal stretching in the two models
- Potential stiffens with increasing neutron number

Wave functions of the β-band



- Large overlap between both models
- Centroids coincide for ^{152}Nd
- For ^{150}Nd RMF potential more repulsive for small deformations

Summary

- Two solutions of the collective Hamiltonian have been tested and compared in the nuclei ^{150,152}Nd
- Remarkable similarity of the two models in: energies, intra- and interband B(E2)-values and centrifugal stretching
- Centroids of the wave functions almost exactly coincide
- Ansatz in CBS:
  - outer potential wall stays almost constant with varying number of valence neutrons
  - inner wall that shifts to higher deformations

→ Microscopically justified for ^{150,152}Nd

Thank you for your attention!

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- Collaborators:

IKP Darmstadt:

- Peter von Neumann-Cosel

- Norbert Pietralla

Southwest University Chonquing:

- Zhipan P. Li

Peking University:

- Jie Meng

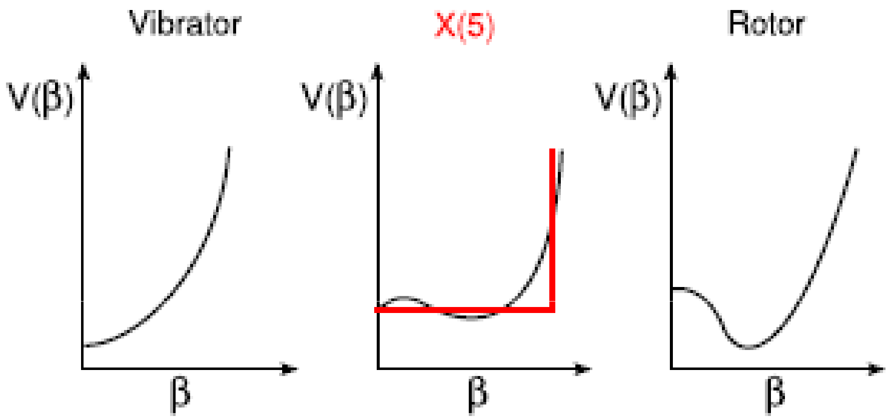
University of Zagreb:

- Dario Vretenar

- Results submitted to Journal of Physics G.

Backup 1: CBS Model

- With increasing A, heavy nuclei can undergo rapid quantum phase transitions



- Geometrical collective solution of the Bohr Hamiltonian:

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2 \sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right] + V(\beta,\gamma)$$

- Parameter free square well potential
- Analytical wave functions (Bessel functions)

Backup 1: CBS operators

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E0-transition operator

$$\hat{T}(E0)_{\mathrm{tr}} = \frac{3}{4\pi} ZeR^2\, \hat{\beta}^2$$

E0-transition operator

$$\hat{T}(E2)_{\Delta K=0} = \frac{3}{4\pi} ZeR^2 \hat{\beta}$$

Backup 2: RMF Hamiltonian

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$$r = B_1 B_2 B_3 \qquad w = B_{\beta\beta} B_{\gamma\gamma} - B_{\beta\gamma}^2$$

Moments of inertia calculated microscopically from Inglis-Belyaev

$$\mathcal{I}_k = \sum_{i,j} \frac{(u_i v_j - v_i u_j)^2}{E_i + E_j} \langle i | \hat{J}_k | j \rangle |^2 \quad k = 1, 2, 3,$$

Mass parameters are calculated in the cranking approximation

$$B_{\mu\nu}(q_0, q_2) = \frac{\hbar^2}{2} \left[\mathcal{M}_{(1)}^{-1} \mathcal{M}_{(3)} \mathcal{M}_{(1)}^{-1} \right]_{\mu\nu} \qquad q_0 = \langle \hat{Q}_{20} \rangle \text{ and } q_2 = \langle \hat{Q}_{22} \rangle$$

$$\mathcal{M}_{(n),\mu\nu}(q_0, q_2) = \sum_{i,j} \frac{\langle i | \hat{Q}_{2\mu} | j \rangle \langle j | \hat{Q}_{2\nu} | i \rangle}{(E_i + E_j)^n} (u_i v_j + v_i u_j)^2$$

Backup 3: RMF Projections

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Probability density distribution in the (β,γ)-plane

$$\rho_{I\alpha}(\beta,\gamma) = \sum_{K\in\Delta I} |\psi^{I}_{\alpha K}(\beta,\gamma)|^2 \beta^3 |\sin 3\gamma|$$

Normalization:

$$\int_0^\infty \beta \mathrm{d}\beta \int_0^{2\pi} \mathrm{d}\gamma \ \rho_{I\alpha}(\beta,\gamma) = 1$$

Projection to the (β)-plane for comparing with CBS:

$$\rho'_{I\alpha}(\beta) = \sum_{K\in\Delta I} \int_0^{2\pi} |\psi^{I}_{\alpha K}(\beta,\gamma)|^2 \beta^4 |\sin 3\gamma| \mathrm{d}\gamma$$