



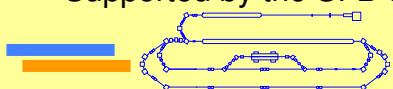
# Giant resonances, fine structure, wavelets and spin- and parity-resolved level densities

- Motivation
- Experimental data
- Fluctuation analysis
- Discrete wavelet transform and determination of background
- Results and test of models
- Summary and outlook

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S-DALINAC / iThemba LABS / RCNP / KVI

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## Level densities: Recent results

- Back-shifted Fermi gas model\*  
semiempirical approach, shell and pairing effects
  
- Many-body density of states\*\*  
two-component Fermi gas, shell effects, deformations, periodic orbits
  
- HF-BCS\*\*\*  
microscopic statistical model, MSk7 force, shell effects, pairing correlations, deformation effects, collective excitations

\* T. Rauscher, F.-K. Thielemann, and K.-L. Kratz, Phys. Rev. C56 (1997) 1613

T. von Egidy and D. Bucurescu, Phys. Rev. C72 (2005) 044311; Phys. Rev. C73 (2006) 049901(E)

\*\* P. Leboeuf and J. Roccia, Phys. Rev. Lett. 97 (2006) 010401

\*\*\* P. Demetriou and S. Goriely, Nucl. Phys. A695 (2001) 95

## Level densities: Recent results

- HFB\*

microscopic combinatorial model, MSk13 force, shell effects,  
pairing correlations, deformation effects, collective excitations

- Large-scale prediction of the parity distribution in the level density\*\*

macroscopic-microscopic approach, deformed Wood-Saxon potential,  
BCS occupation numbers, back-shifted Fermi Gas model

- Monte-Carlo shell model\*\*\*

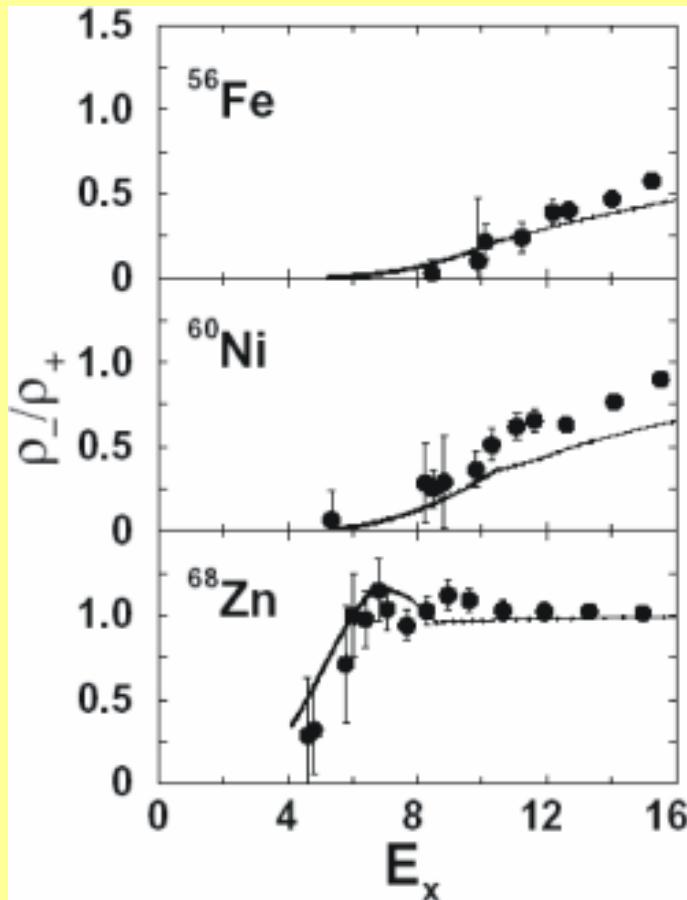
microscopic model, large model space, pairing+quadrupole force

\* S. Hilaire and S. Goriely, Nucl. Phys. A779 (2006) 63

\*\* D. Mocelj et al., Phys. Rev. C75 (2007) 045805

\*\*\* C. Özen, K. Langanke, G. Martinez-Pinedo, and D.J. Dean, nucl-th/0703084 (2007)

## Monte-Carlo shell model predictions: pf + g<sub>9/2</sub> shell



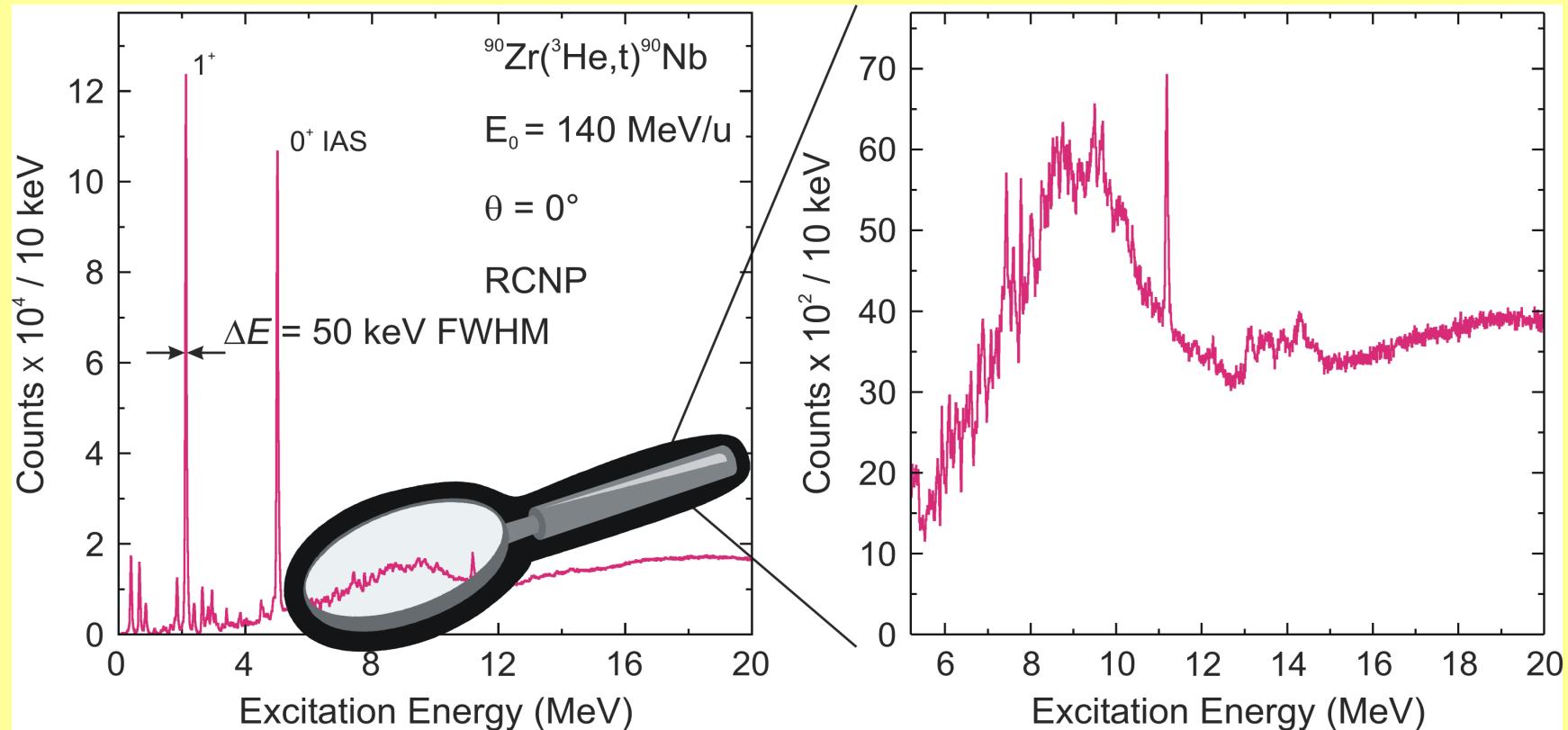
$\Delta_{pf-g_{9/2}}$  small  
 $\rho_-$  important at low energies

- Total level density (not spin projected) shows strong parity dependence\*
- Questioned by recent experiments ( $^{45}\text{Sc}$ )\*\*

\* Y. Alhassid, G.F. Bertsch, S. Liu, and H. Nakada, Phys. Rev. Lett. 84 (2000) 4313

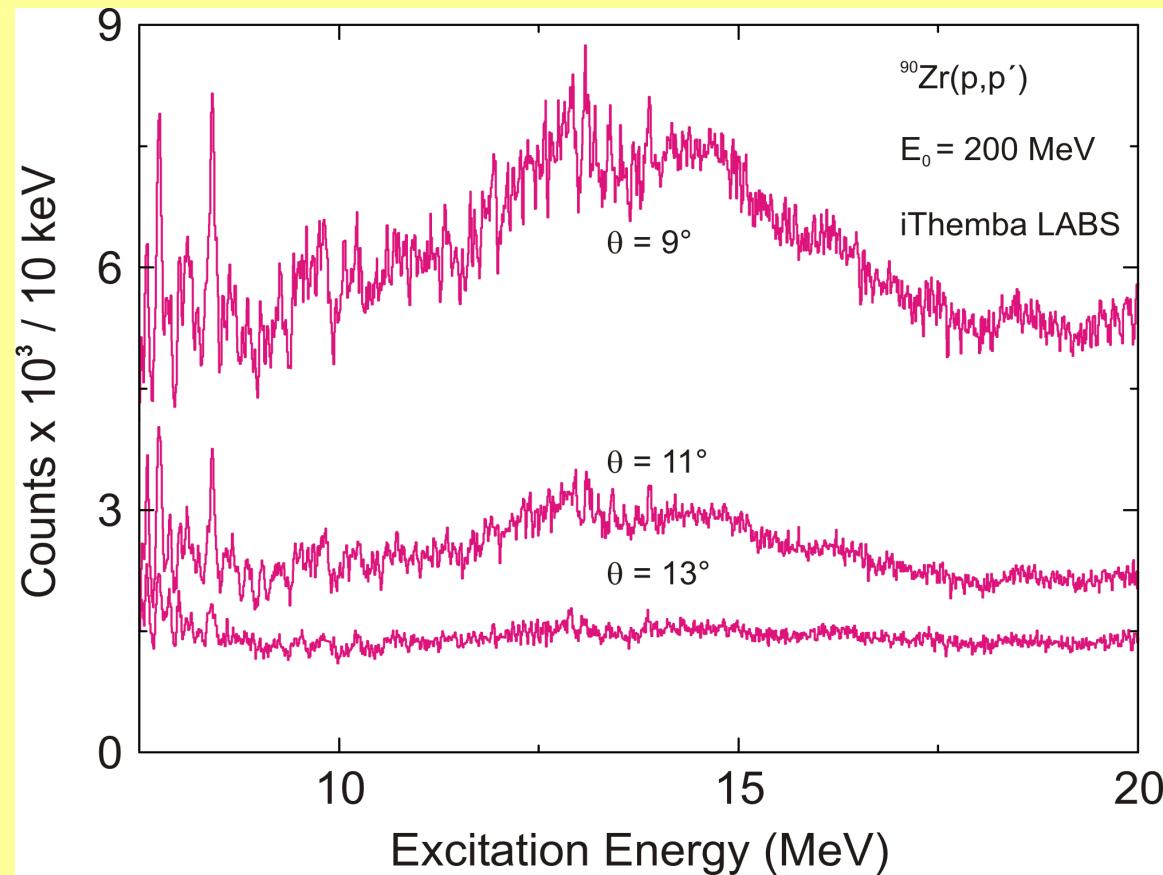
\*\* S.J. Lokitz, G.E. Mitchell, and J.F. Shriner, Jr., Phys. Rev. C71 (2005) 064315

# Fine structure of the spin-flip GTR: A = 90



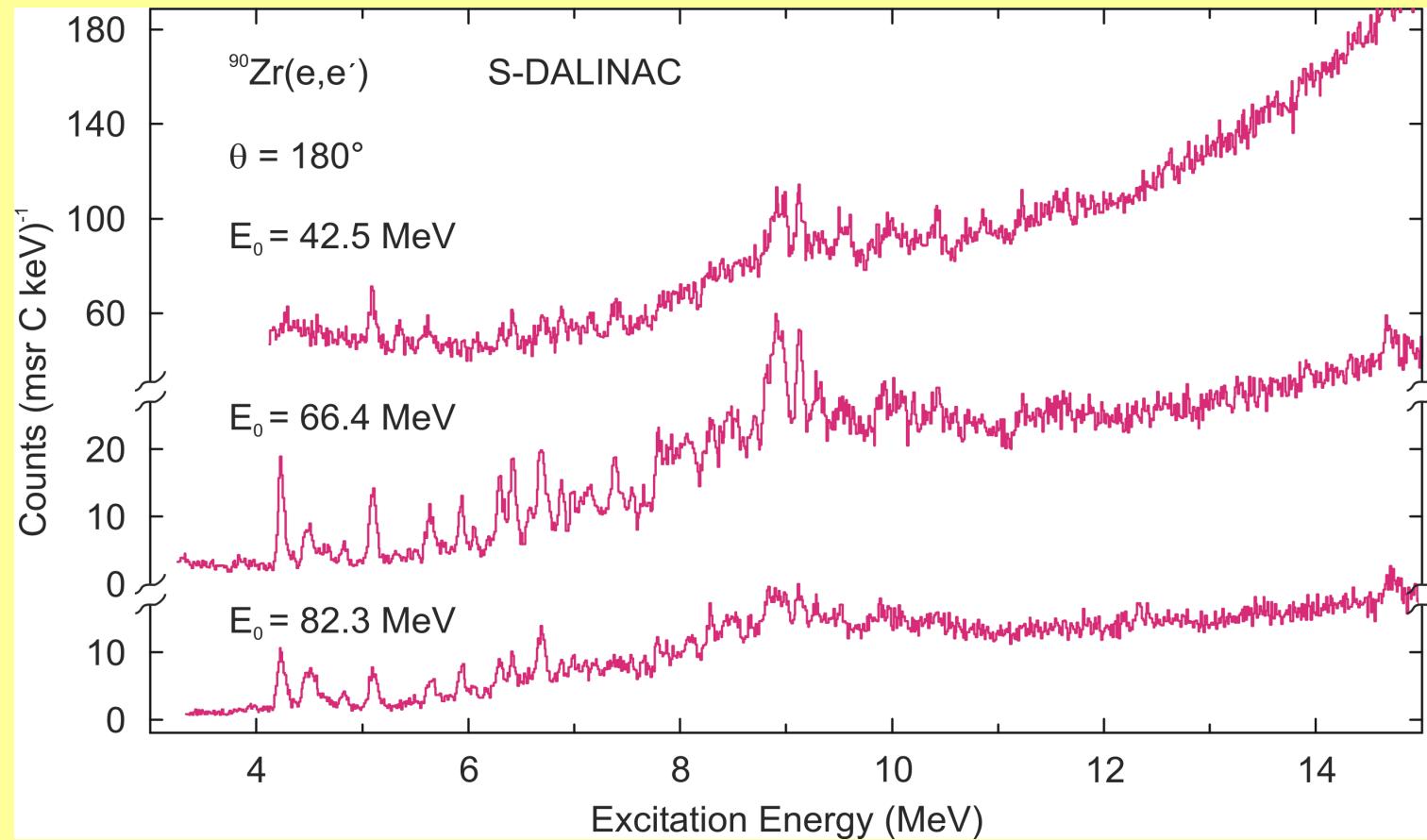
- Selective excitation of  $1^+$  states

## Fine structure of the ISGQR: A = 90



- Selective excitation of  $2^+$  states

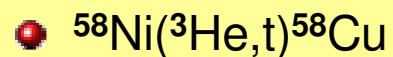
## Fine structure of the M2 resonance: A = 90



- Selective excitation of  $2^-$  states

# Summary of experiments

A = 58



$J^\pi = 1^+$

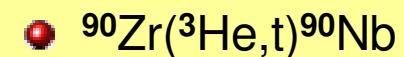


$J^\pi = 2^-$



$J^\pi = 2^+$

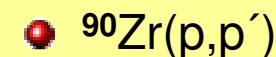
A = 90



$J^\pi = 1^+$



$J^\pi = 2^-$



$J^\pi = 2^+$

# Experimental techniques

- Selectivity

- hadron scattering at extremely forward angles and intermediate energies
  - electron scattering at  $180^\circ$  and low momentum transfers

- High resolution

- lateral and angular dispersion matching
  - faint beam method\*

- Level density

- fluctuation analysis\*\*

- Background

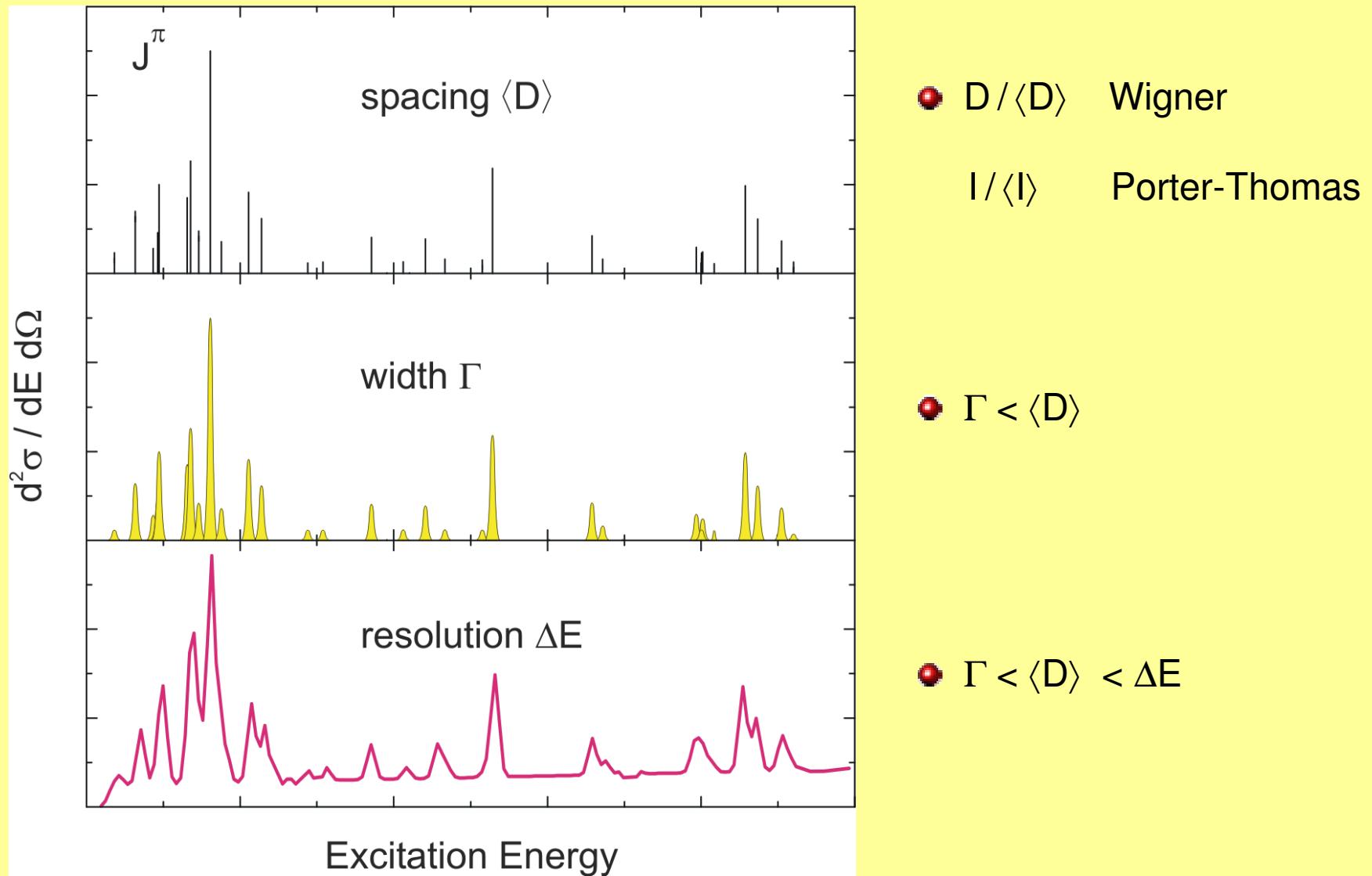
- discrete wavelet transform\*\*\*

\* H. Fujita et al., Nucl. Instr. and Meth. A484 (2002) 17

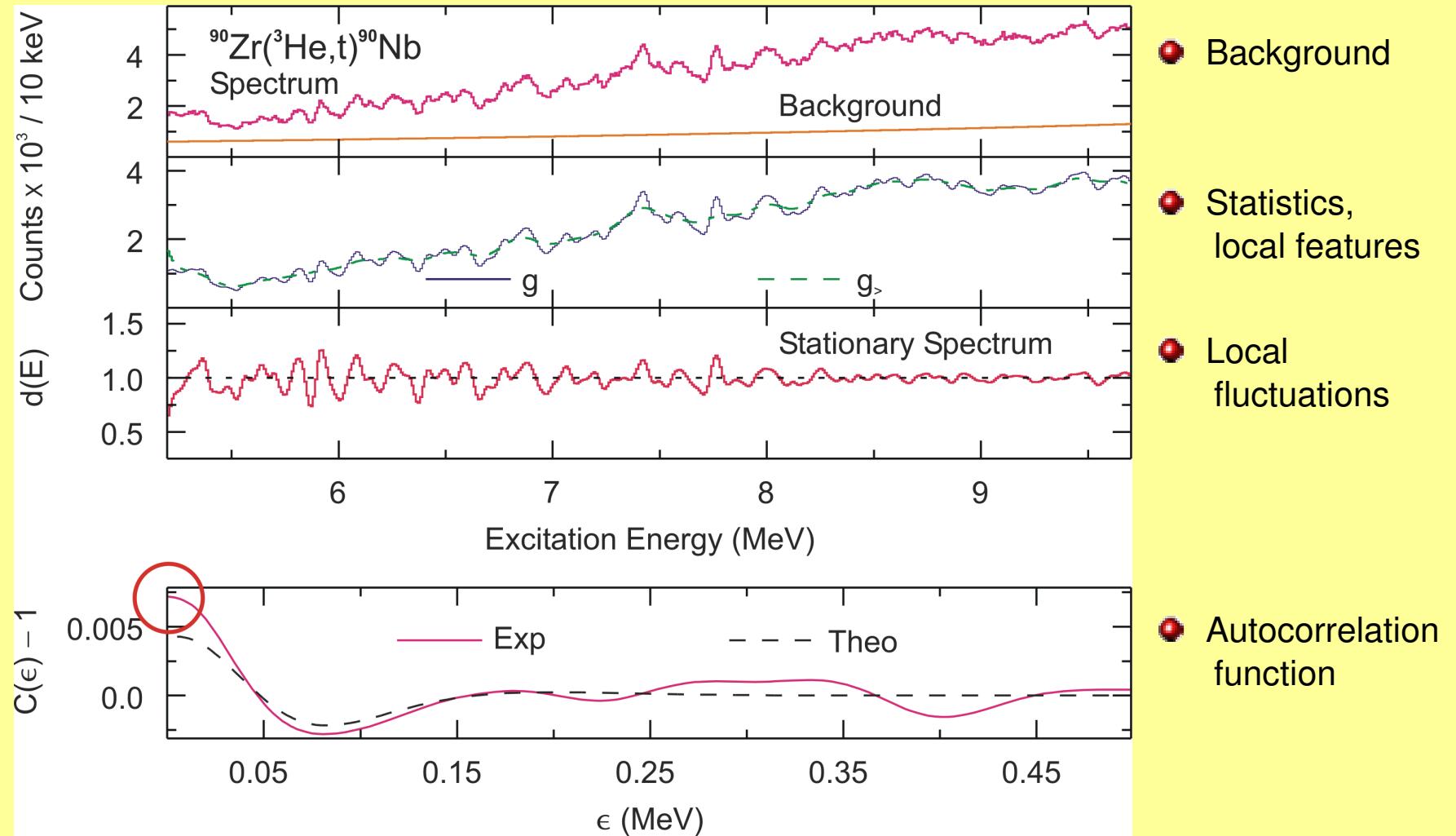
\*\* P.G. Hansen, B. Jonson, and A. Richter, Nucl. Phys. A518 (1990) 13

\*\*\* Y. Kalmykov et al., Phys. Rev. Lett. 96 (2006) 012502

# Fluctuations and level densities



# Fluctuation analysis



## Autocorrelation function and mean level spacing

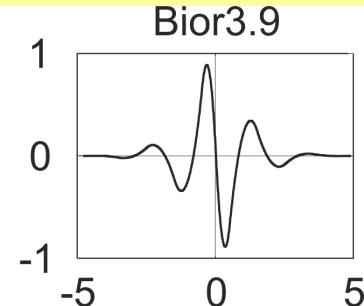
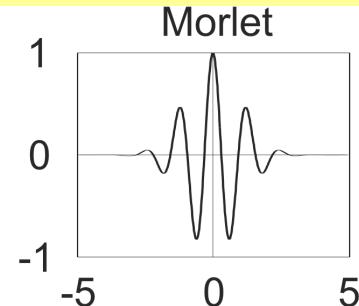
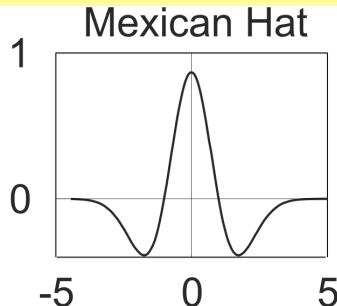
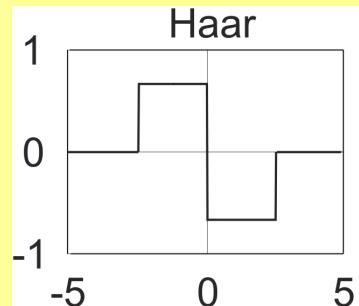
- $C(\varepsilon) = \frac{\langle d(E_X) d(E_X + \varepsilon) \rangle}{\langle d(E_X) \rangle \langle d(E_X + \varepsilon) \rangle}$  autocorrelation function
- $C(\varepsilon = 0) - 1 = \frac{\langle d^2(E_X) \rangle - \langle d(E_X) \rangle^2}{\langle d(E_X) \rangle^2}$  variance
- $C(\varepsilon) - 1 = \frac{\alpha \langle D \rangle}{2\sigma\sqrt{\pi}} \times f(\sigma, \varepsilon)$  level spacing  $\langle D \rangle$
- $\alpha = \alpha_{PT} + \alpha_W$  selectivity
- $\sigma$  resolution

S. Müller, F. Beck, D. Meuer, and A. Richter, Phys. Lett. 113B (1982) 362

P.G. Hansen, B. Jonson, and A. Richter, Nucl. Phys. A518 (1990) 13

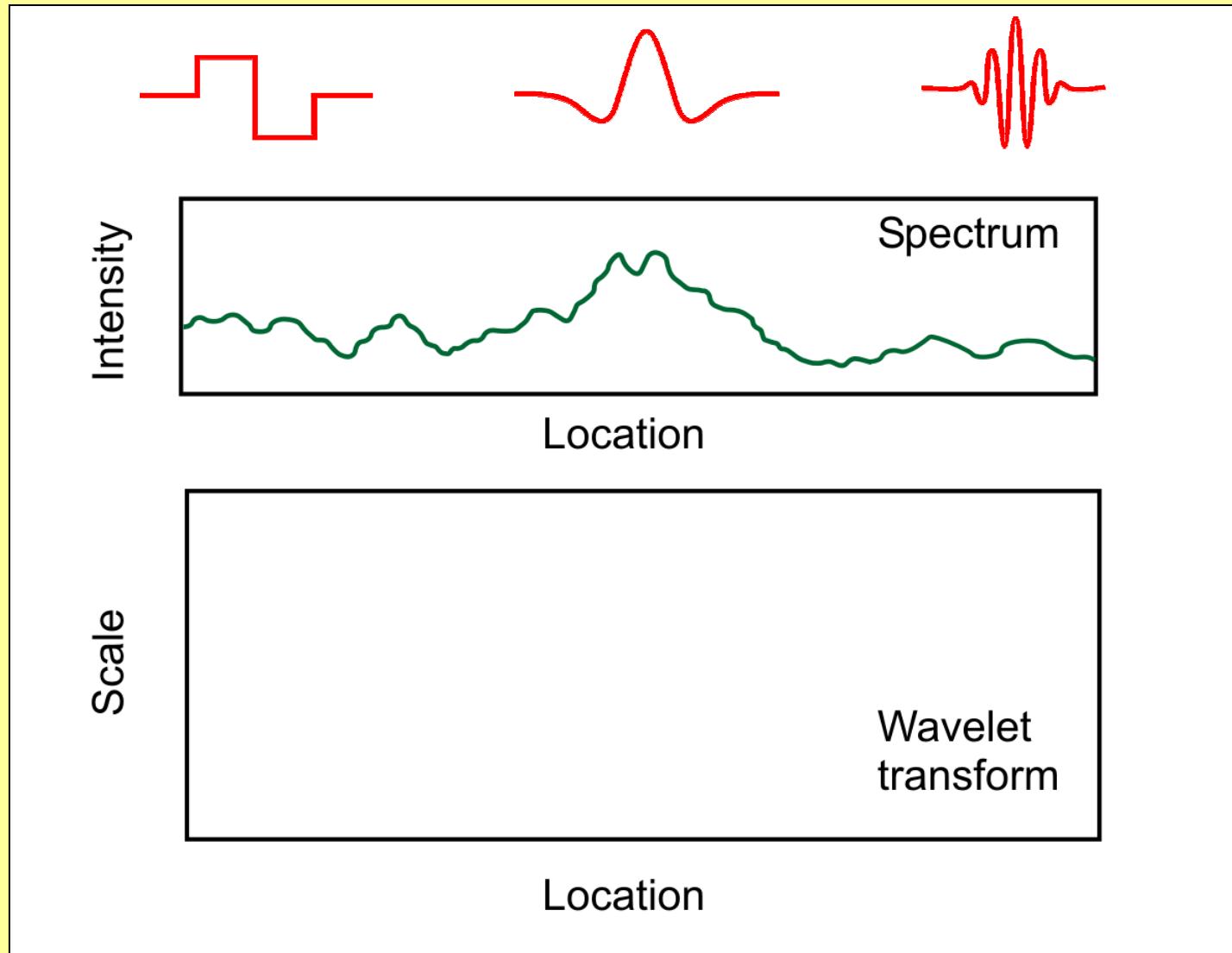
**How to determine  
the background in the spectra?**

# Wavelets and wavelet transform

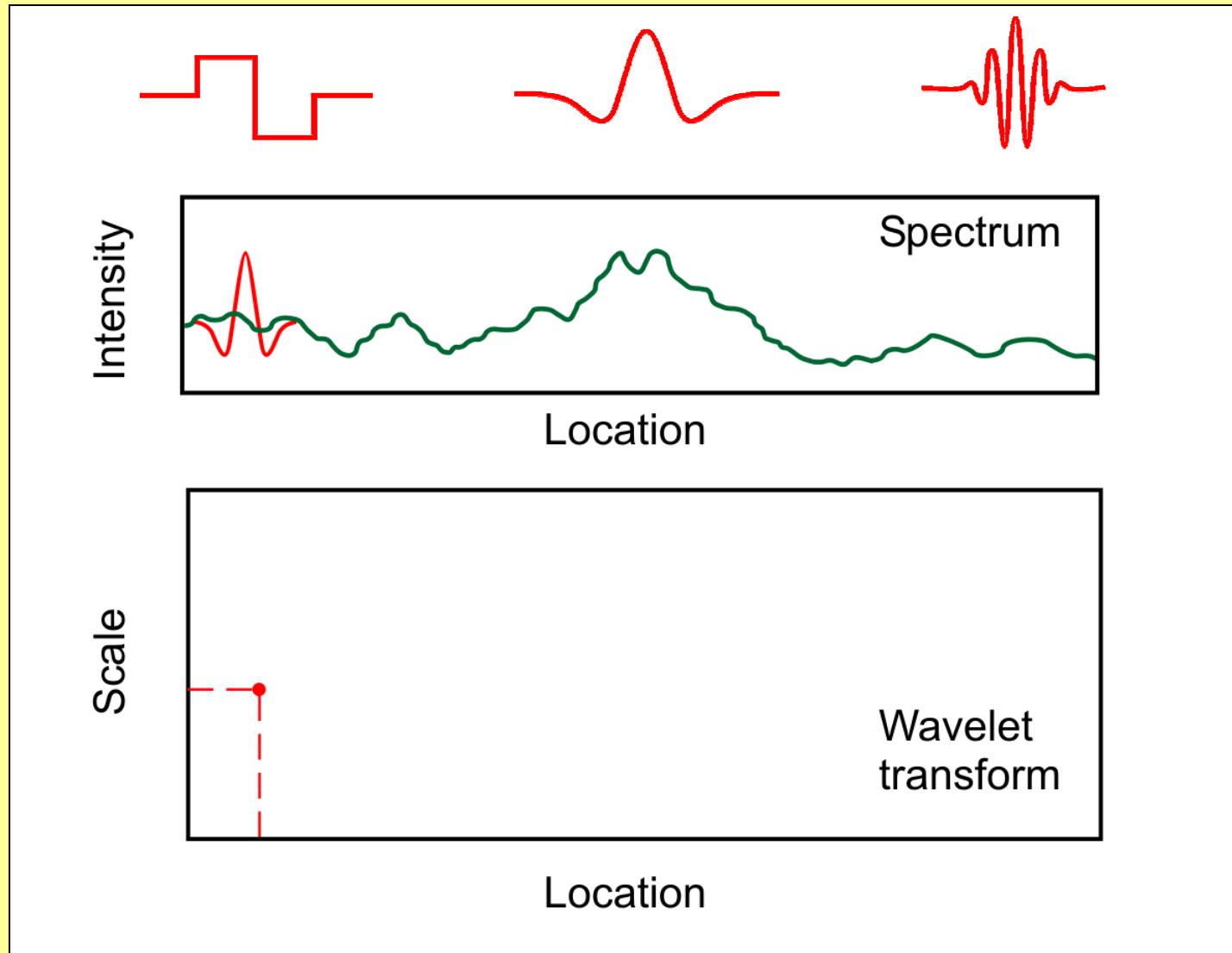


- $\int_{-\infty}^{+\infty} \Psi^*(E) dE = 0$       wavelet
- $\int_{-\infty}^{+\infty} |\Psi^*(E)|^2 dE < \infty$       finite support (square integrable)
- $C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int_{-\infty}^{+\infty} \sigma(E) \Psi^* \left( \frac{E_x - E}{\delta E} \right) dE$       wavelet coefficients

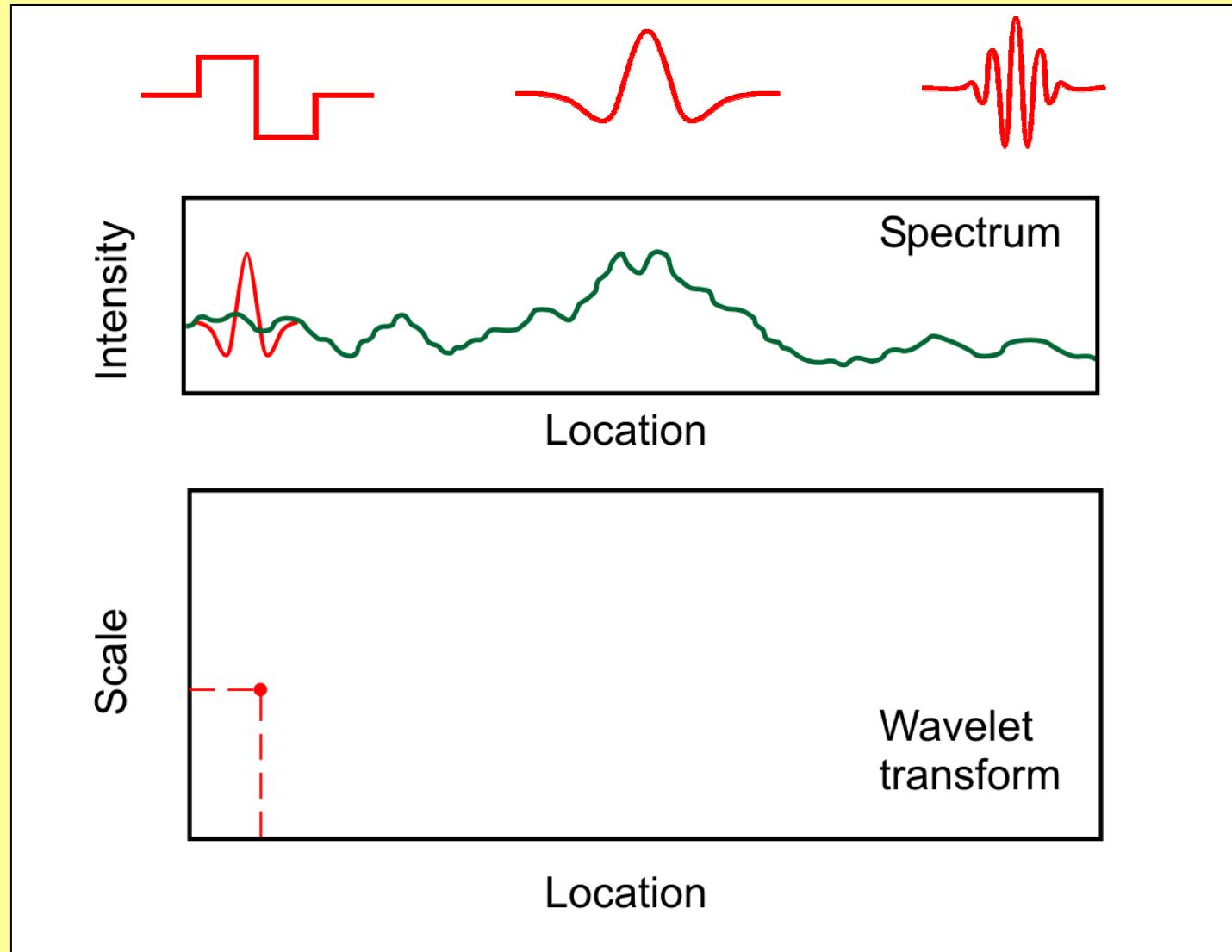
# Wavelet analysis



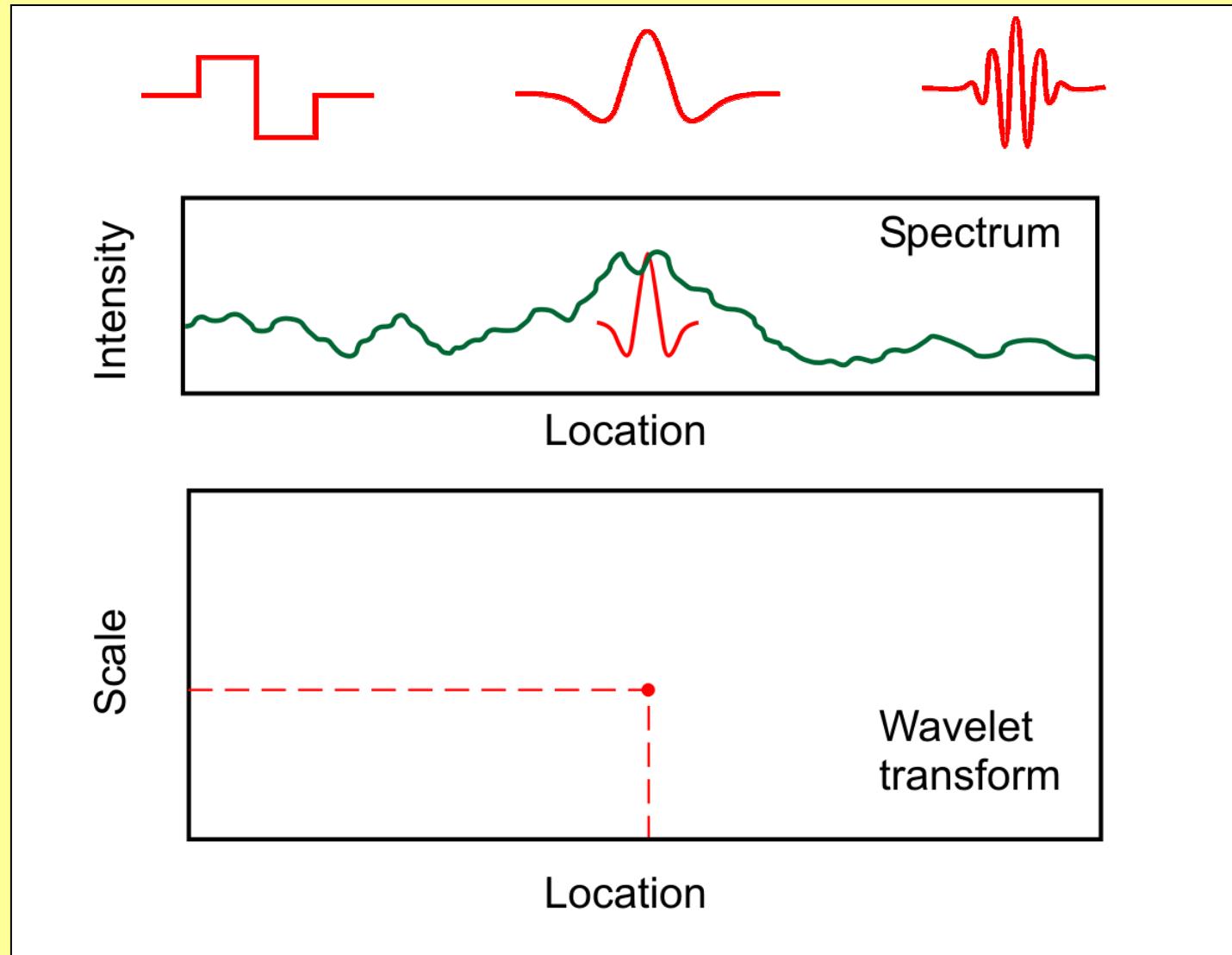
# Wavelet analysis



# Wavelet analysis



# Wavelet analysis



## Discrete wavelet transform

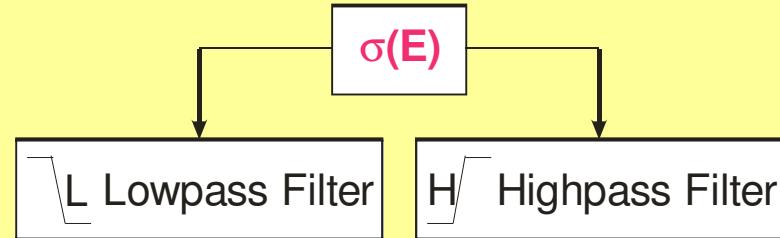
- $C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int_{-\infty}^{+\infty} \sigma(E) \Psi * \left( \frac{E_x - E}{\delta E} \right) dE$  wavelet coefficients

- Discrete wavelet transform\*
  - $\delta E = 2^j$  and  $E_x = k \cdot \delta E$  with  $j, k = 1, 2, 3, \dots$
  - exact reconstruction is possible
  - is fast

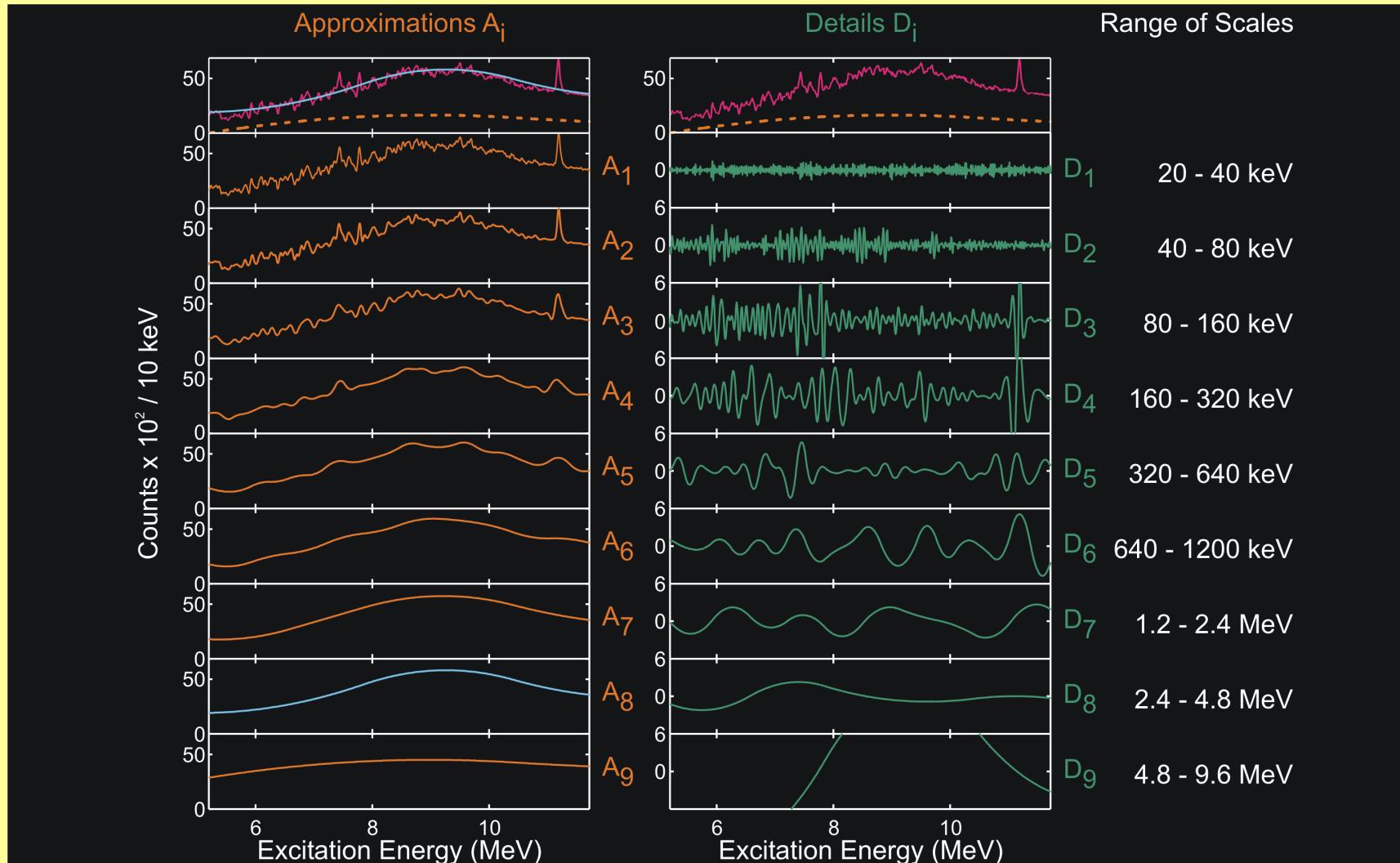
- $\int_{-\infty}^{+\infty} E^n \Psi * \left( \frac{E_x - E}{\delta E} \right) dE = 0, \quad n = 0, 1, \dots, m-1$  vanishing moments

this defines the shape and magnitude of the background

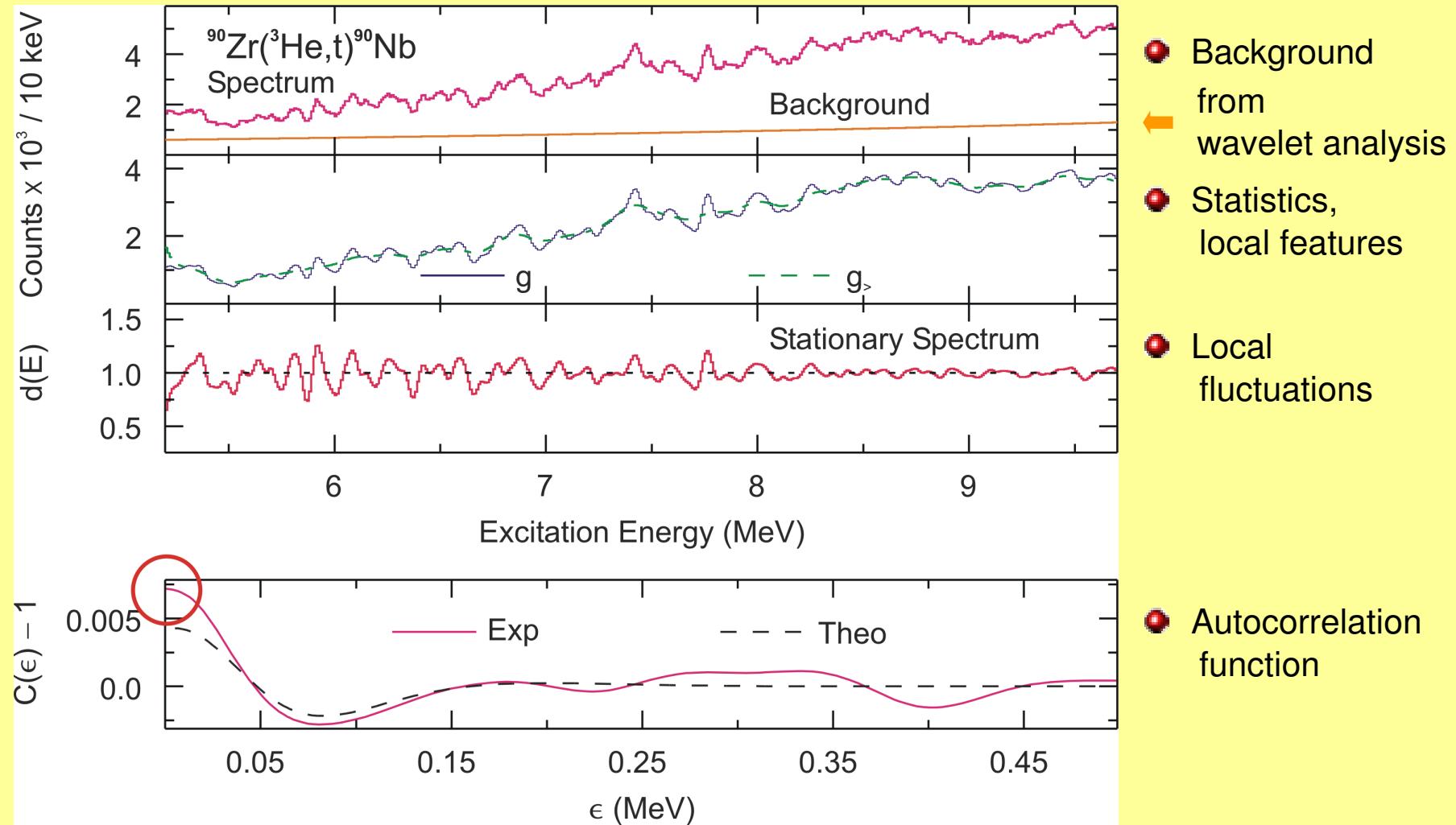
# Decomposition of spectra



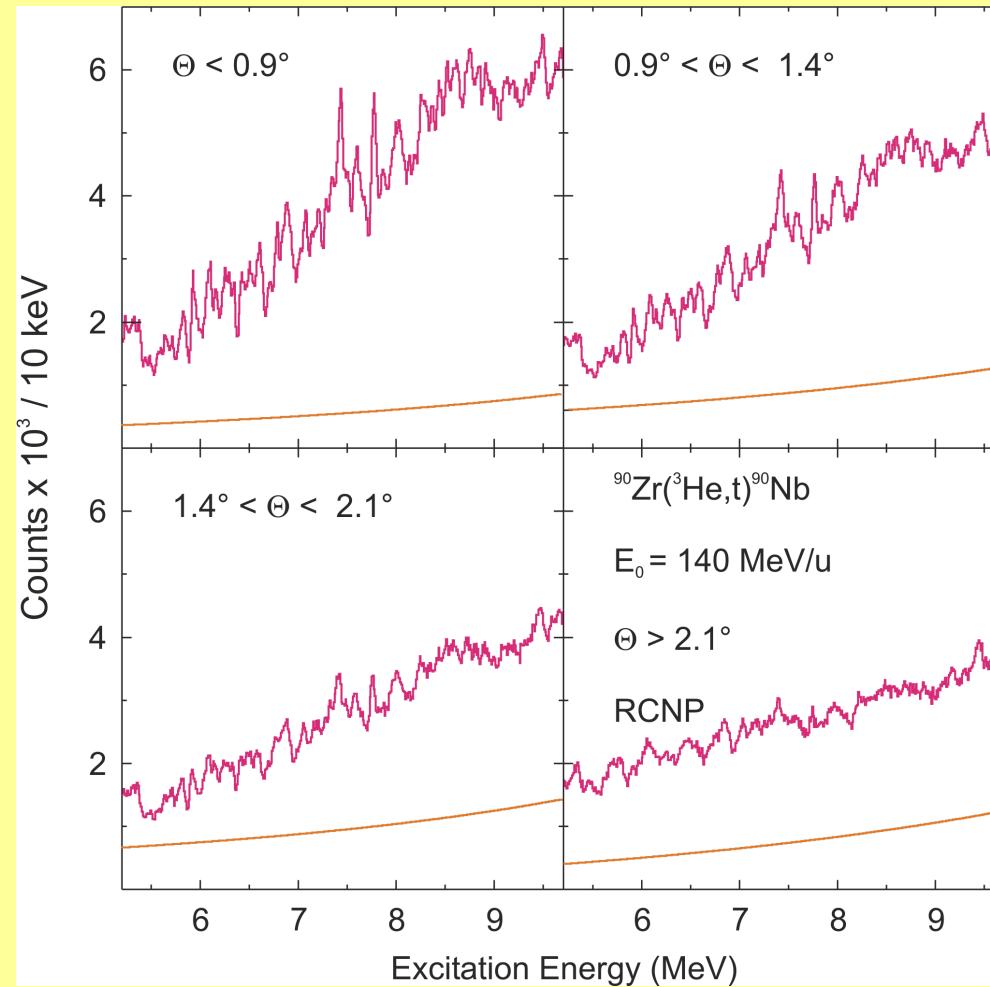
# Application: Decomposition of $^{90}\text{Zr}({}^3\text{He},\text{t})^{90}\text{Nb}$ spectrum



# Fluctuation analysis

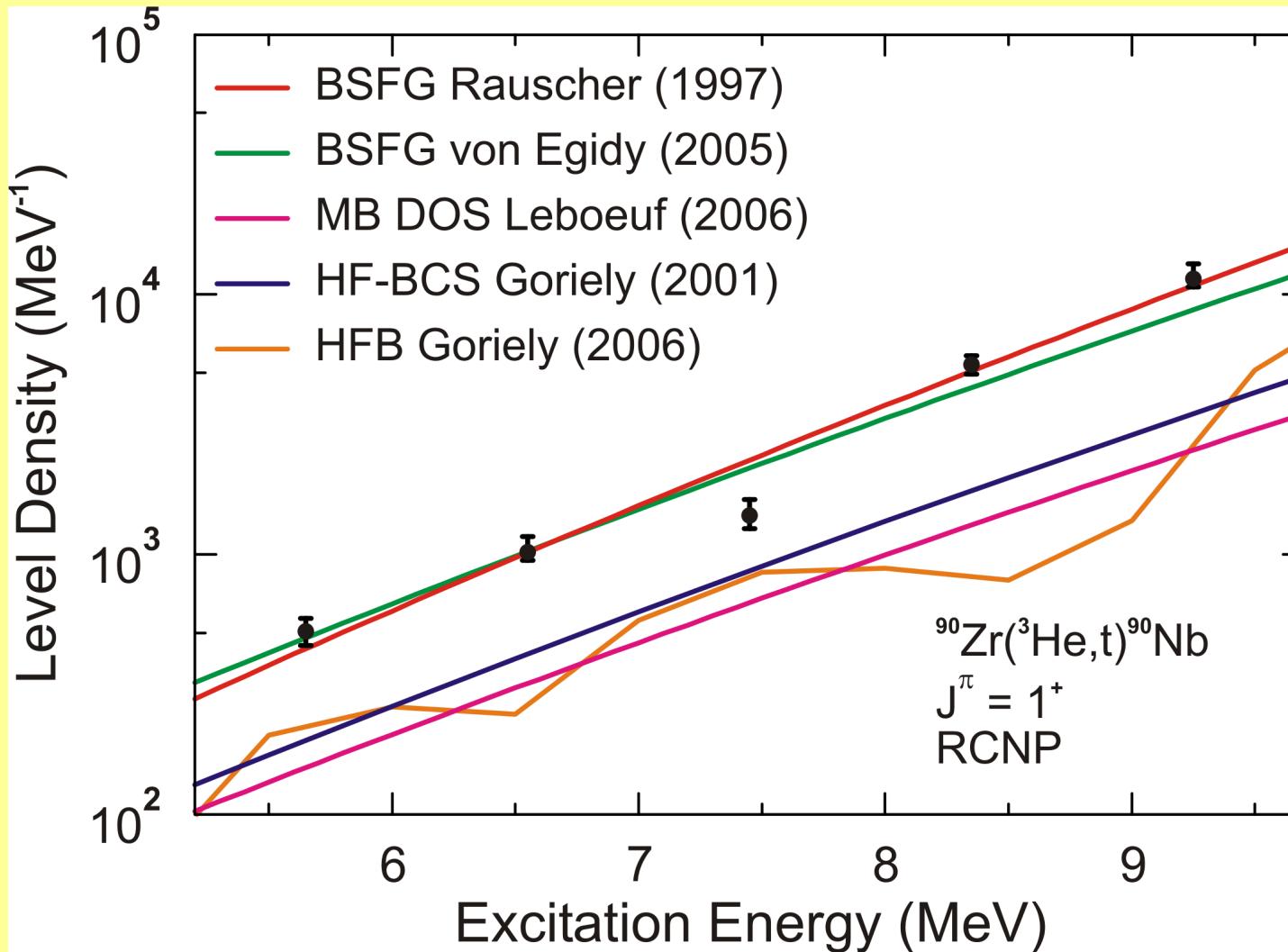


## Angular distribution: $^{90}\text{Zr}({}^3\text{He},t){}^{90}\text{Nb}$

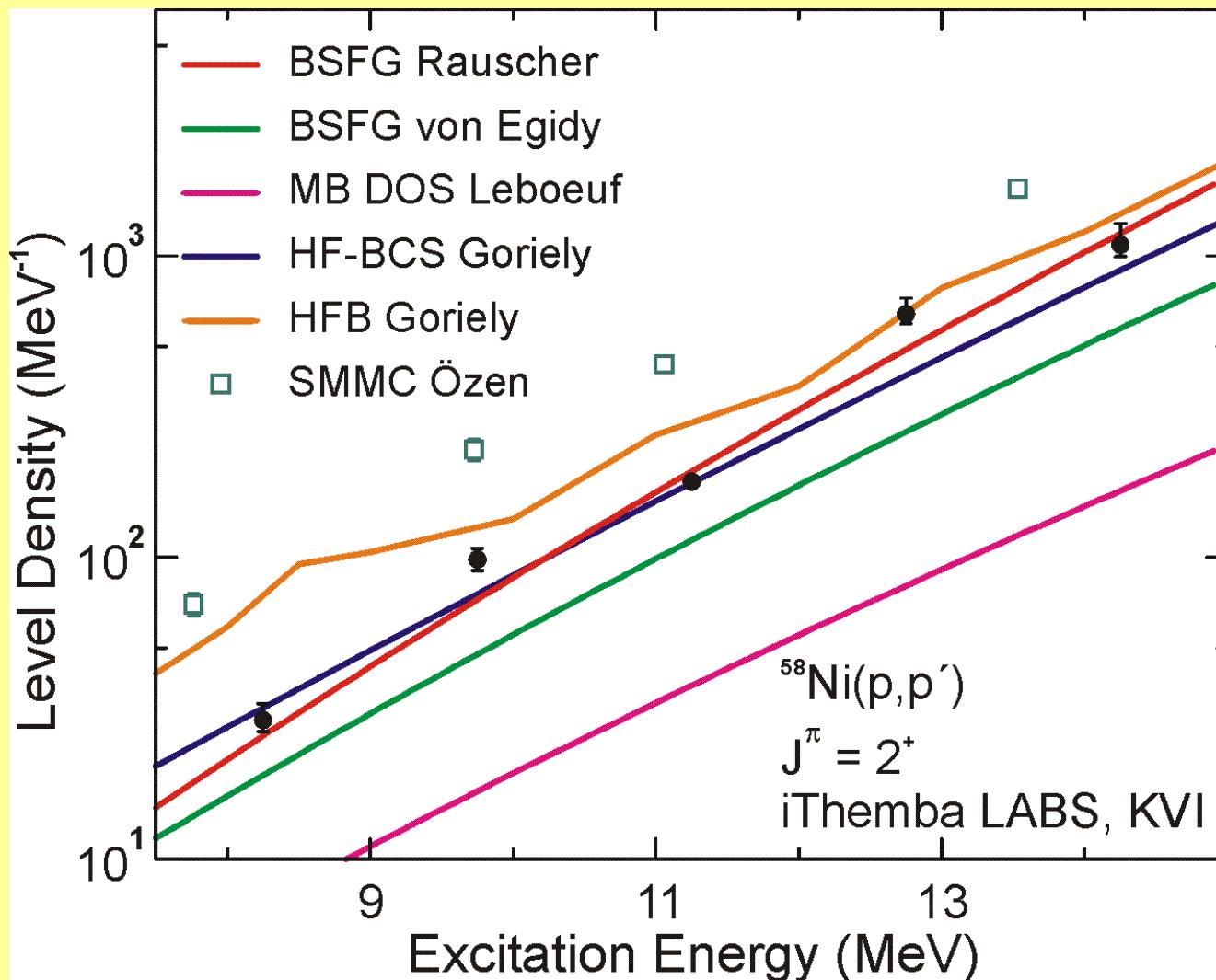


- The requirement of a constant level density in all spectra is a constraint in the analysis

## Results and model predictions: $A = 90$ , $J^\pi = 1^+$



## Results and model predictions: $A = 58$ , $J^\pi = 2^+$



# Phenomenological and microscopic models

- Different quality of model predictions
- BSFG, MB DOS
  - parameters fitted to experimental data
  - no distinction of parity
- HF-BCS
  - microscopic
  - no distinction of parity
- HFB, SMMC
  - fully microscopic calculation of levels
  - with spin and parity
- HFB
  - fine structure of level densities



## Ingredients of HFB

- Nuclear structure: HFB calculation with a conventional Skyrme force
  - single particle energies
  - pairing strength for each level
  - quadrupole deformation parameter
  - deformation energy
- Collective effects
  - rotational enhancement
  - vibrational enhancement
  - disappearance of deformation at high energies

## Ingredients of SMMC

- Partition function of many-body states with good  $J^\pi$

$$Z_J^\pi(\beta) = \text{Tr}_{J,\pi} e^{-\beta H}$$

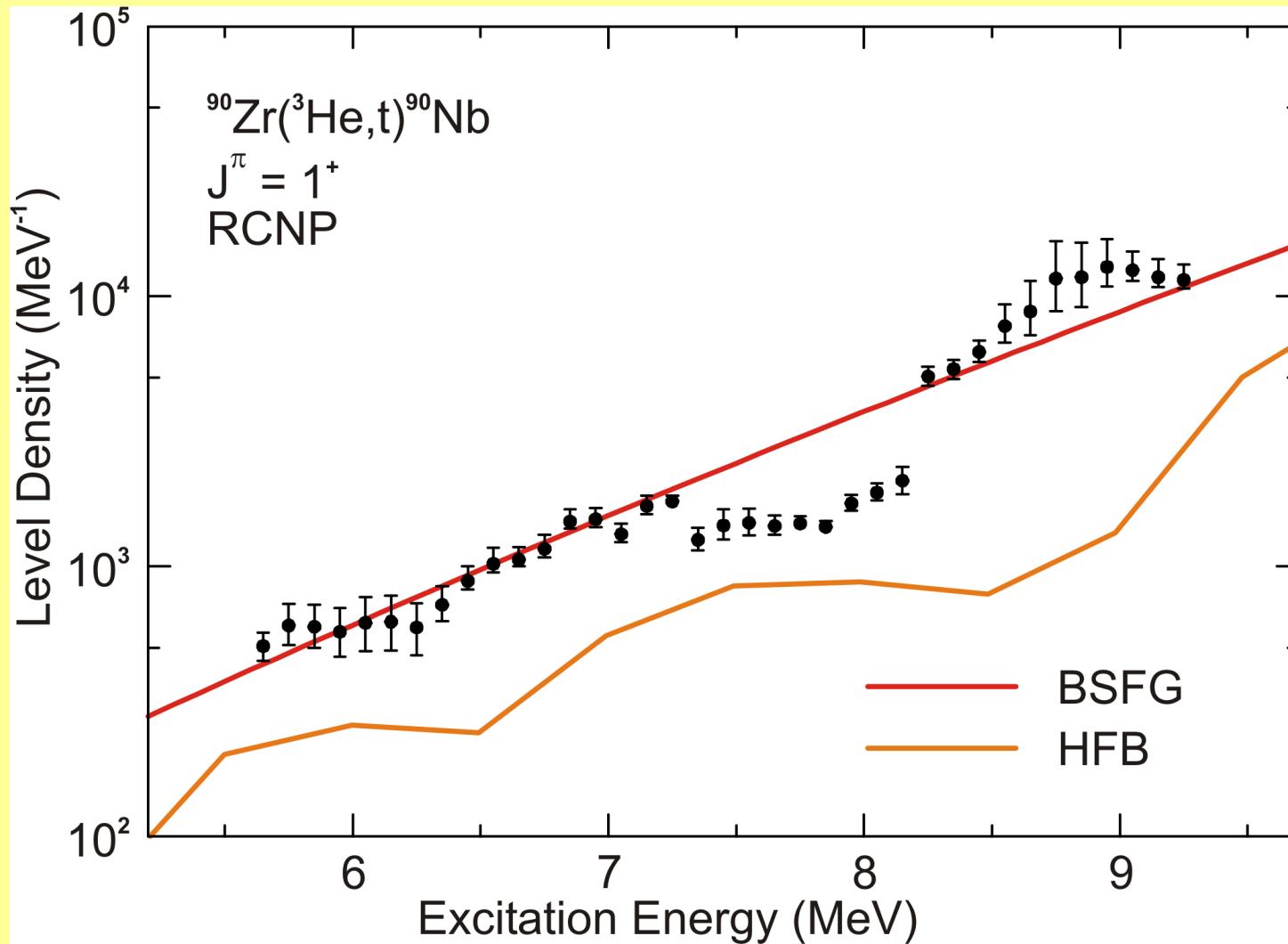
- Expectation values at inverse temperature  $\beta = 1/kT$

$$E_J^\pi(\beta) = \frac{\int dE' e^{-\beta E'} E' \rho_J^\pi(E')}{Z_J^\pi(\beta)}$$

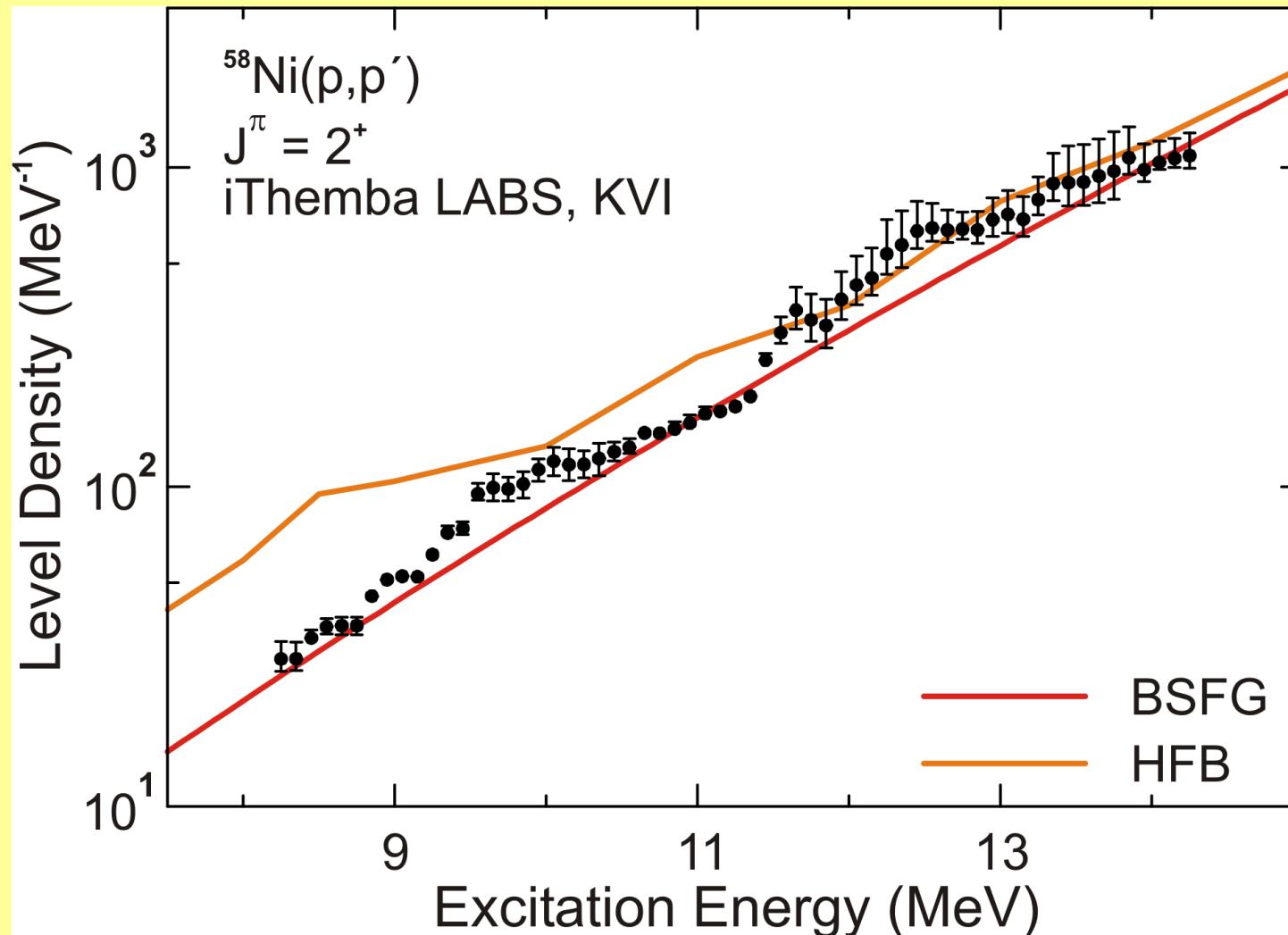
- Level density from inverse Laplace transform in the saddle-point approximation

$$\rho_J^\pi(E) = \frac{e^{\beta E_J^\pi + \ln Z_J^\pi(\beta)}}{\sqrt{-2\pi \frac{dE_J^\pi(\beta)}{d\beta}}}$$

## Fine structure of level density: $A = 90$ , $J^\pi = 1^+$

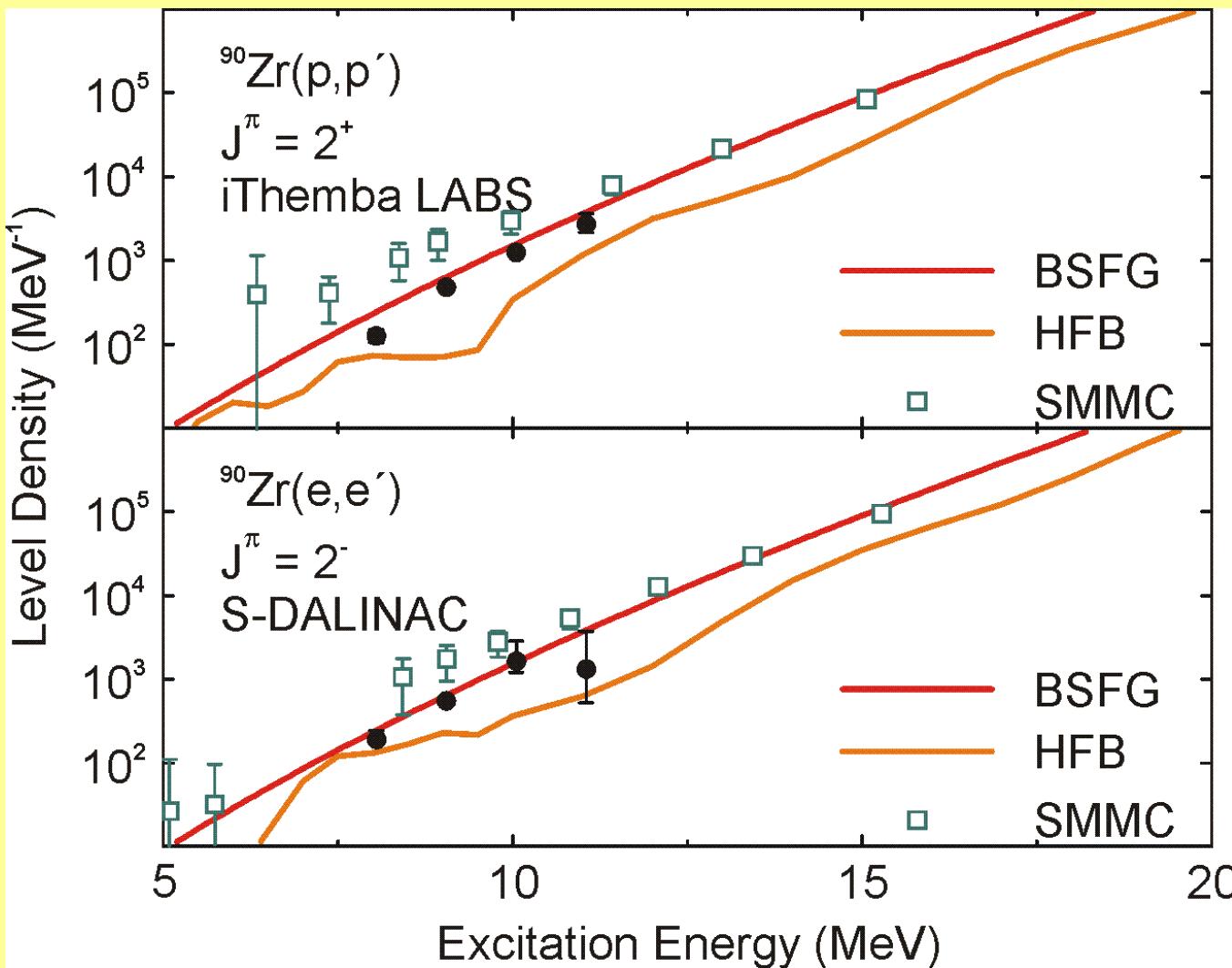


## Fine structure of level density: $A = 58$ , $J^\pi = 2^+$

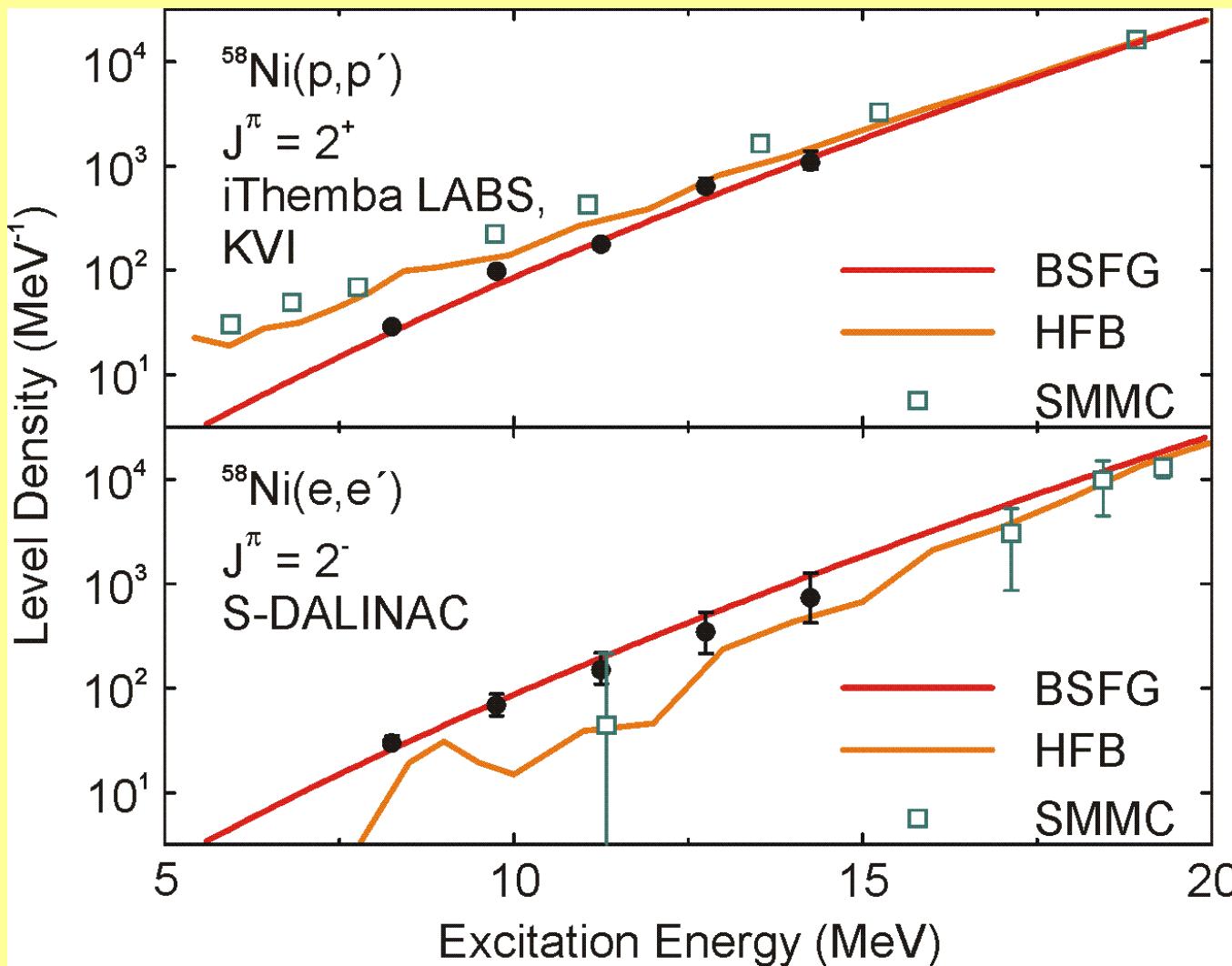


- Both for  $A = 90$  and  $A = 58$  level densities at this  $E_x$  seem not to be a smooth function

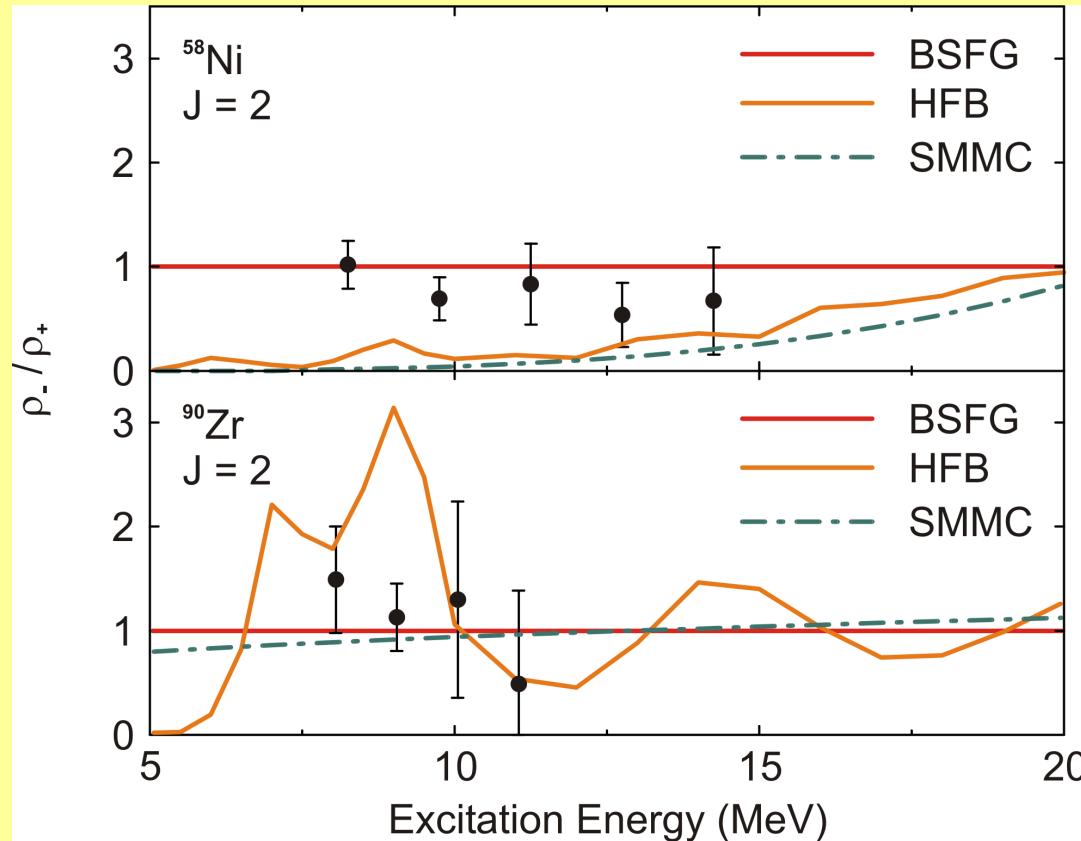
## Level density of $2^+$ and $2^-$ states: $^{90}\text{Zr}$



## Level density of $2^+$ and $2^-$ states: $^{58}\text{Ni}$



## Test of parity dependence of level densities



- Experiments: no parity dependence
- HFB and SMMC
  - $^{58}\text{Ni}$ : strong parity dependence:  $\rho_- \ll \rho_+$
  - $^{90}\text{Zr}$ : weak parity dependence:  $\rho_- \approx \rho_+$
- Problem

## Equilibration of parity-projected level densities

- $^{58}\text{Ni}$

$$\rho_- \approx \rho_+ \text{ at } E_x \approx 20 \text{ MeV}$$

- $^{90}\text{Zr}$

$$\rho_- \approx \rho_+ \text{ at } E_x \approx 5 - 10 \text{ MeV}$$

- Two energy scales which determine  $\rho_-/\rho_+$   
pair-breaking

- 5 – 6 MeV for intermediate mass nuclei  
shell gap between opposite-parity states near the Fermi level
  - depends strongly on the shell structure, e.g.  $^{68}\text{Zn}$   $\Delta_{pf-g9/2}$  is small

- Core breaking

- e.g. near shell closure  $^{58}\text{Ni}$   $\Delta_{sd-pf}$  transitions are important

- $\rho_-$  would be enlarged

## Summary and outlook

- Fine structure of giant resonances
- Wavelet analysis for a nearly model-independent background determination
- Fluctuation analysis
- Spin- and parity-resolved level densities in  $^{58}\text{Ni}$ ,  $^{90}\text{Zr}$ ,  $^{90}\text{Nb}$
- Comparison with current nuclear structure model predictions
- Indication for fine structure of level densities at high excitation energies
- No parity dependence for  $J = 2$  in  $^{58}\text{Ni}$  and  $^{90}\text{Zr}$
- Further applications to GTR, IVGDR, ISGQR ... in a wide range of nuclei