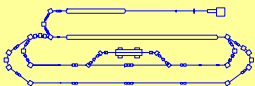




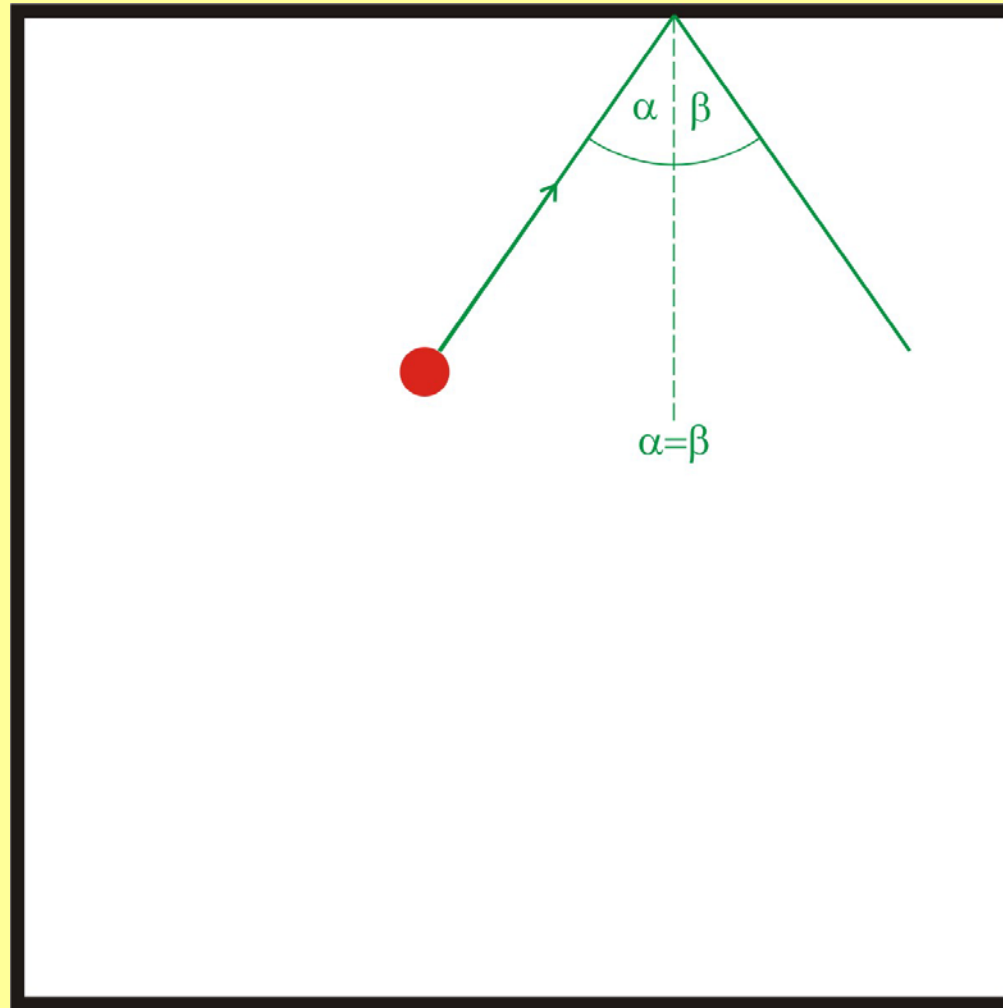
## Quantum Manifestations of Classical Chaos – Some Universal Features of Billiards and Nuclei

- Classical billiards and quantum billiards
- Random Matrix Theory (Wigner 1951 – Dyson 1962)
- Spectral properties of billiards and mesoscopic systems
- Microwave resonator as a model for the compound nucleus
  - S-Matrix fluctuations in the regime of overlapping resonances
  - Induced time-reversal symmetry breaking

Supported by DFG under SFB 634

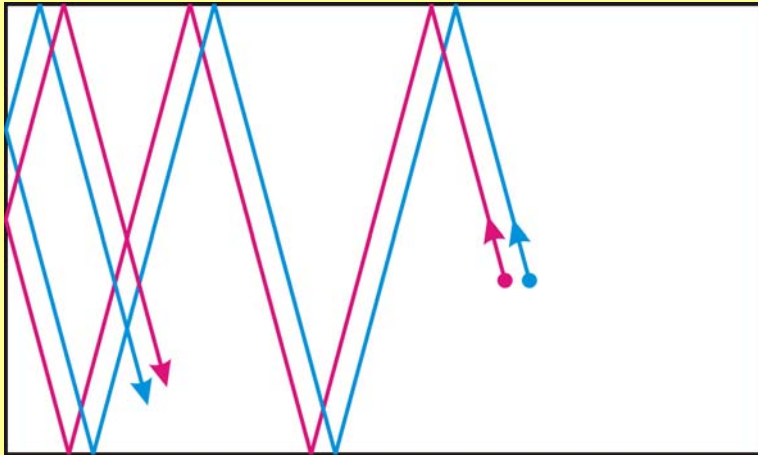


# Classical Billiard

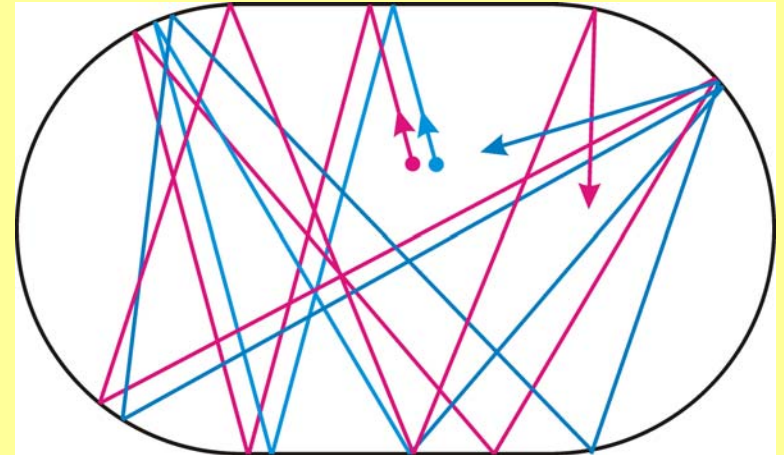


# Regular and Chaotic Dynamics

Regular



Bunimovich stadium (chaotic)



• Energy and  $p_x^2$  are conserved

• Equations of motion are integrable

• Predictable for infinite long times

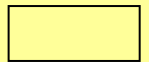
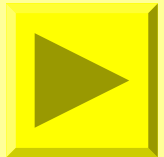
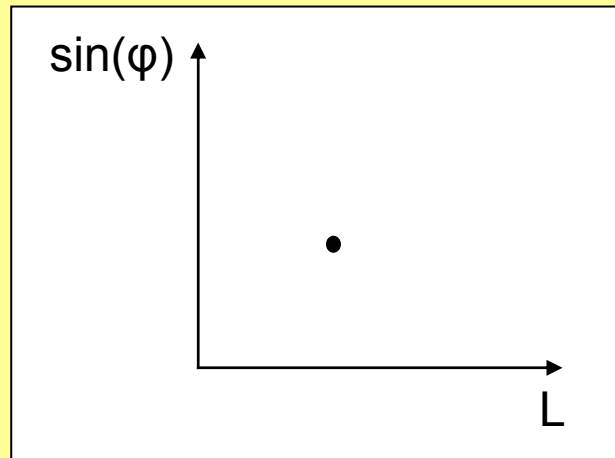
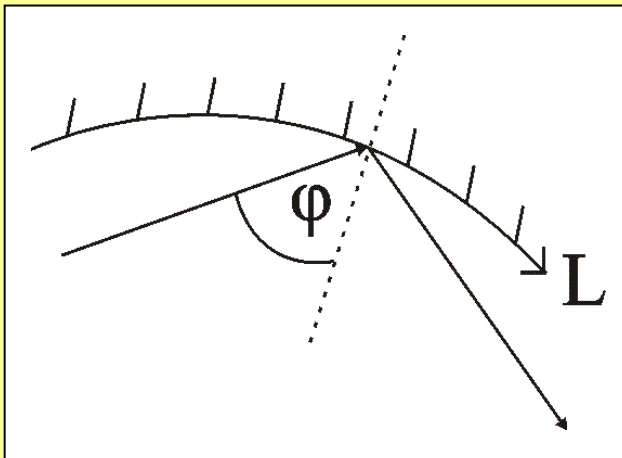
• Only energy is conserved

• Equations of motion are not integrable

• Predictable for a finite time only

# Tool: Poincaré Sections of Phase Space

- Parametrization of billiard boundary:  $L$
  - Momentum component along the boundary:  $\sin(\varphi)$
- } conjugate variables



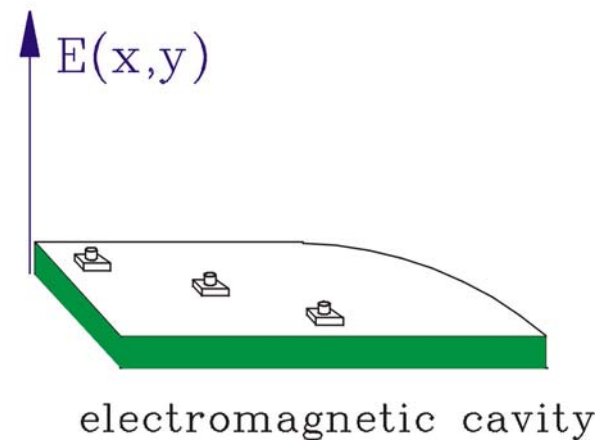
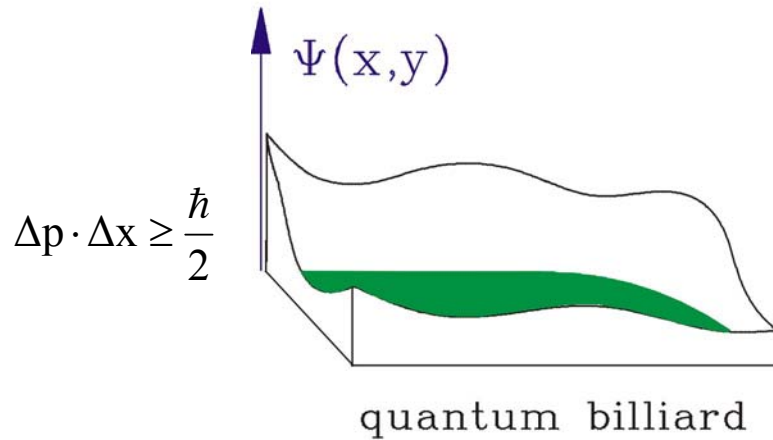
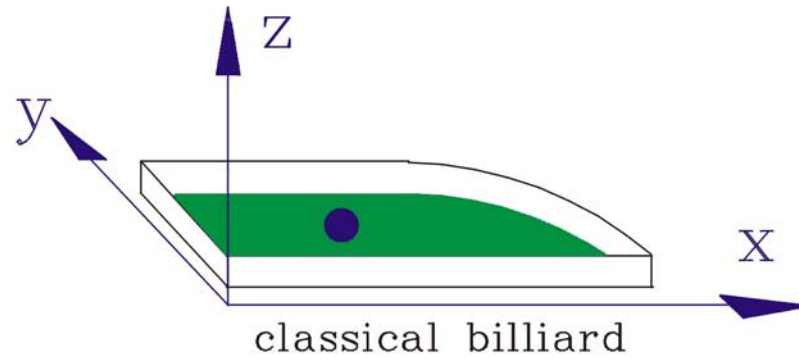
# Small Changes → Large Actions

- Sensitivity of the solutions of a deterministic problem with respect to small changes in the initial conditions is called **Deterministic Chaos**
- Beyond a fixed, for the system **characteristic time** becomes every prediction impossible. The system behaves in such a way as if not determined by physical laws but randomness

# Our Main Interest

- How are these properties of classical systems transformed into corresponding quantum-mechanical systems ?  
→ Quantum chaos ?
- What might we learn from generic features of billiards and mesoscopic systems (hadrons, nuclei, atoms, molecules, metal clusters, quantum dots) ?

# The Quantum Billiard and its Simulation



# Schrödinger ↔ Helmholtz

quantum billiard

2D microwave cavity:  $h_z < \lambda_{\min}/2$

$$(\Delta + k^2)\Psi = 0$$

$$(\Delta + k^2)E_z = 0$$

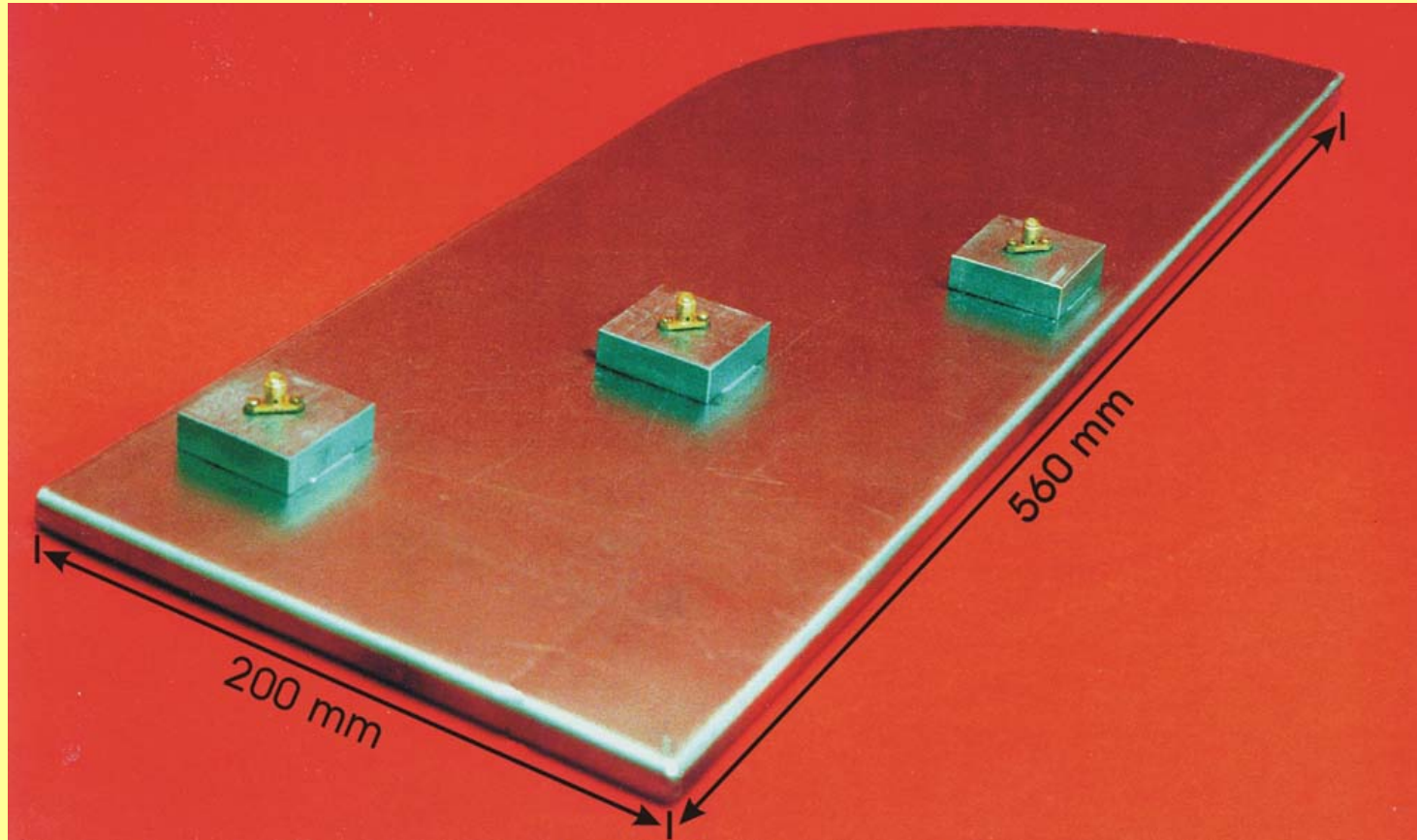
$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k = \frac{2\pi f}{c}$$

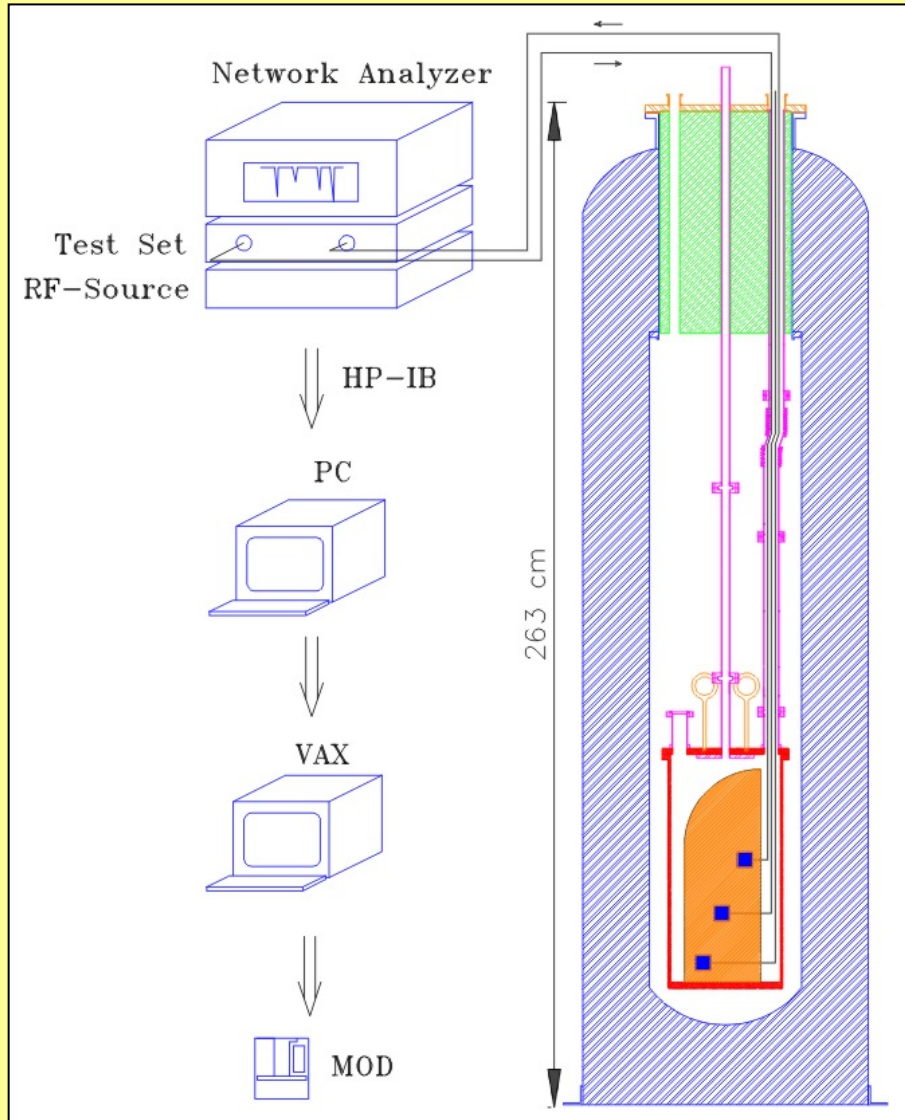
Helmholtz equation and Schrödinger equation are equivalent in 2D. The motion of the quantum particle in its potential can be simulated by electromagnetic waves inside a two-dimensional microwave resonator



# Superconducting Niobium Microwave Resonator



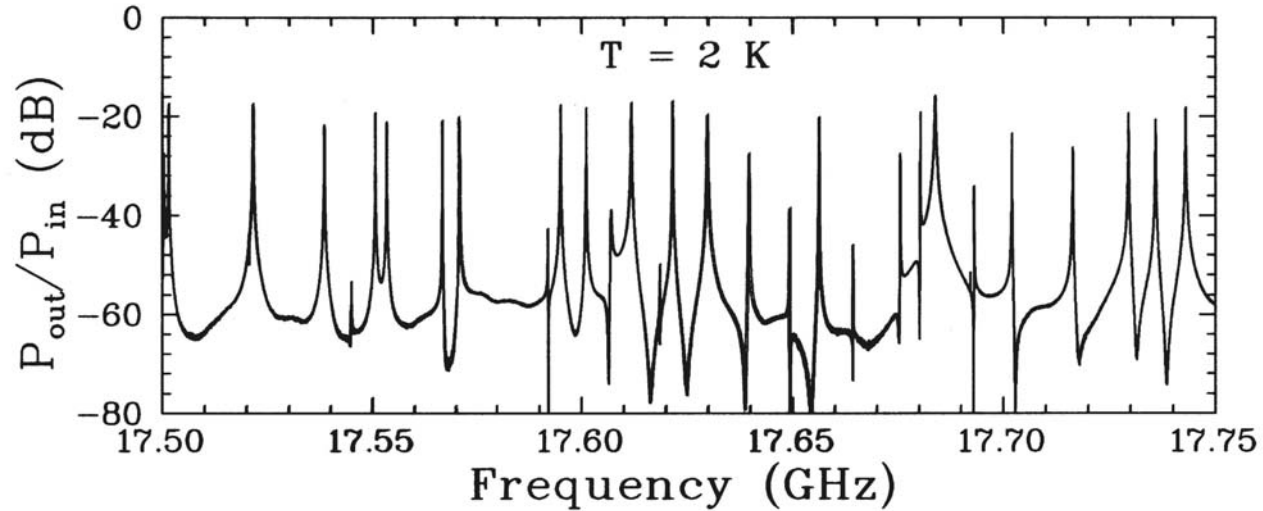
# Experimental Setup



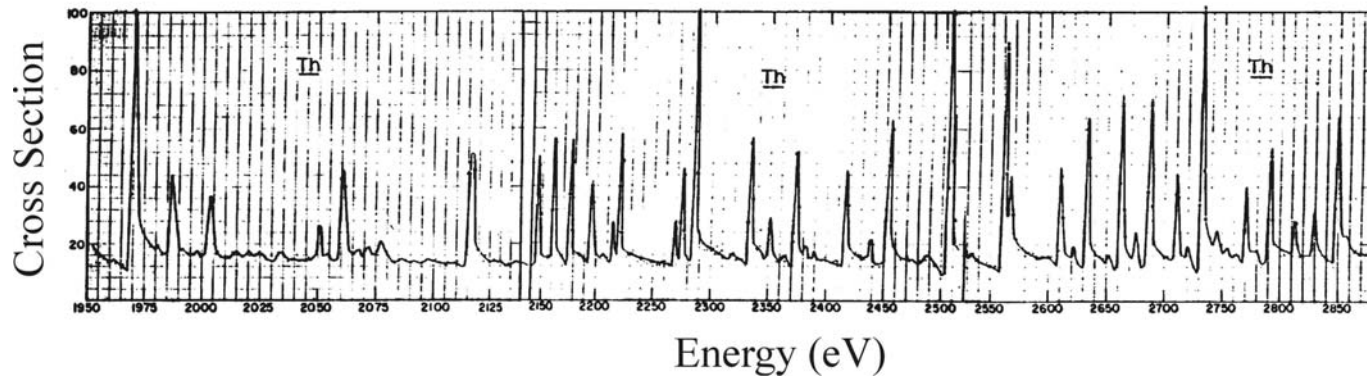
- Superconducting cavities
- LHe ( $T = 4.2 \text{ K}$ )
- $f = 45 \text{ MHz} \dots 50 \text{ GHz}$
- $10^3 \dots 10^4$  eigenfrequencies
- $Q = f/\Delta f \approx 10^6$

# Stadium Billiard $\leftrightarrow n + {}^{232}\text{Th}$

Transmission spectrum for the stadium billiard



Spectrum of neutron resonances in  ${}^{232}\text{Th} + n$



## News and Views

## Neutron Capture and Nuclear Constitution

THE new views of nuclear structure and the processes involved in neutron capture, presented by Prof. Niels Bohr in an address which appears elsewhere in this issue, were expounded by him in a lecture to the Chemical and Physical Society of University College, London, on February 11 and were illustrated by two pictures here reproduced. The first of these is intended to convey an idea of events arising out of a collision between a neutron and the nucleus. Imagine a shallow basin with a number of billiard balls in it as shown in the accompanying figure. If the basin were empty, then upon striking a ball from the outside, it would go down one slope and pass out on the opposite side with its original velocity. But with other balls in the basin, there would not be a free passage of this kind. The struck ball would divide its energy first with one of the balls in the basin, these two would similarly

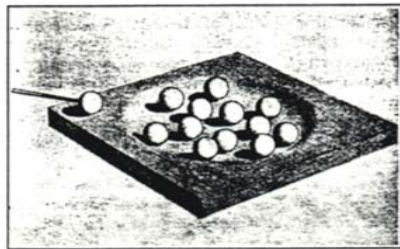


FIG. 1.

share their energies with others, and so on until the original kinetic energy was divided among all the balls. If the basin and the balls are regarded as perfectly smooth and elastic, the collisions would continue until the kinetic energy happens again to be concentrated upon a ball close to the edge. This ball would then escape from the basin and the remainder of the balls would be left with insufficient total energy for any of them to climb the slope. The picture illustrates, therefore, "that the excess energy of the incident neutron will be rapidly divided among all the nuclear particles with the result that for some time afterwards no single particle will possess sufficient kinetic energy to leave the nucleus".

## Nuclear Energy Levels

THE second figure illustrates the character of the distribution of energy levels for a nucleus of not too small atomic weight. The lowest lines represent the levels with an excitation of the same order of magnitude as ordinary excited  $\gamma$ -ray states. According

to the views developed in Prof. Bohr's address, the levels will for increasing excitation rapidly become closer to one another and will, for an excitation of about 15 million electron volts, corresponding to a collision between a nucleus and a high-speed neutron, be continuously distributed, whereas in the region of small excess energy of about 10 million volts excitation they will still be sharply separated. This is illustrated by the two lenses of high magnification placed over the level-diagram in the two above-mentioned regions. The dotted line in the middle of the field of the lower magnifying glass represents zero excess energy, and the fact that one of the levels

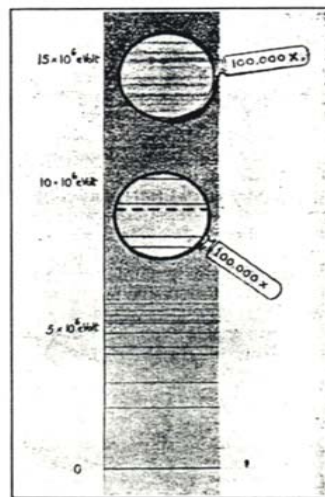


FIG. 2.

is very close to this line (about  $\frac{1}{2}$  volt distant) corresponds to the possibility of selective capture for very slow neutrons. The average distance between the neighbouring levels will in this energy region be about ten volts as estimated from the statistics for the occurrence of selective capture. The diagram shows no upper limit to the levels, and these actually extend to very high energy values. If it were possible to experiment with neutrons or protons of energies above a hundred million volts, several charged or uncharged particles would eventually leave the nucleus as a result of the encounter; and, adds Prof. Bohr, "with particles of energies of about a thousand million volts, we must even be prepared for the collision to lead to an explosion of the whole nucleus".

# Random Matrices $\leftrightarrow$ Level Schemes

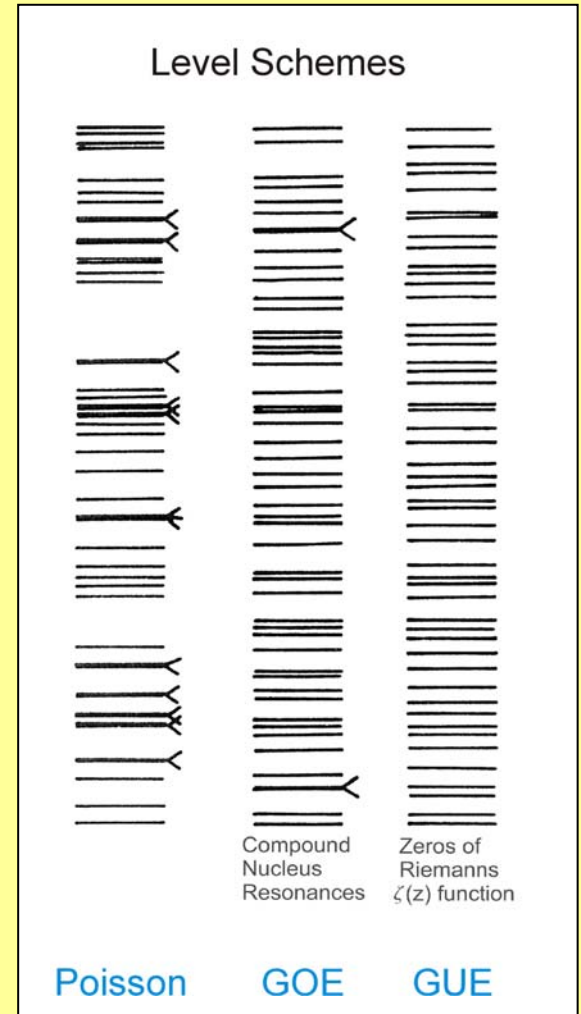
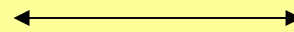
Random  
Matrix

$$H = \begin{pmatrix} H_{11} & \cdots & H_{1N} \\ \vdots & \ddots & \vdots \\ H_{N1} & \cdots & H_{NN} \end{pmatrix}$$

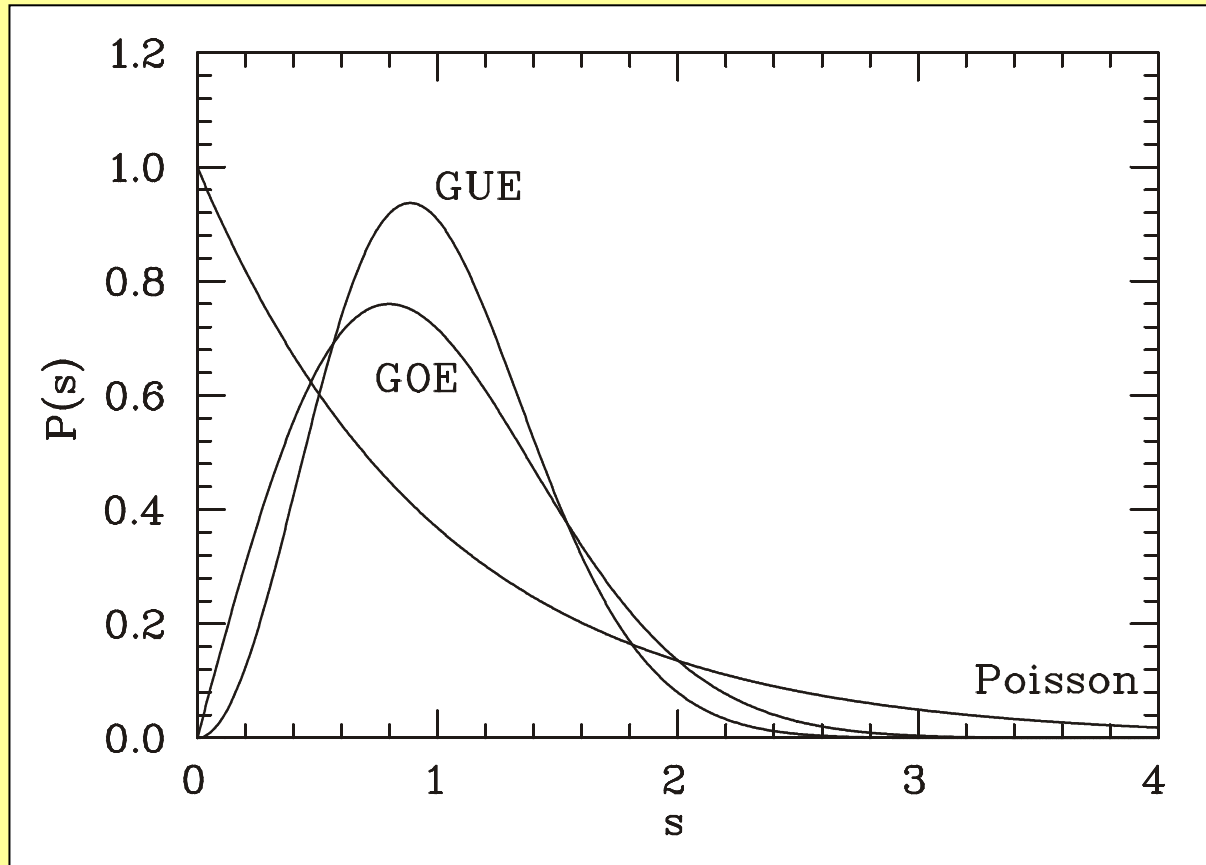


Eigenvalues

$$H\phi_n = E_n\phi_n$$



# Nearest Neighbor Spacings Distribution

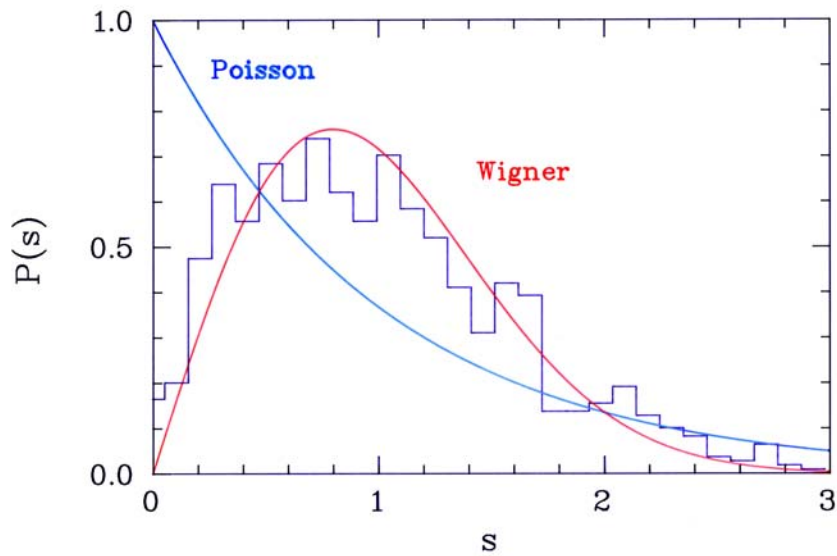


● GOE and GUE ↔ "Level Repulsion"

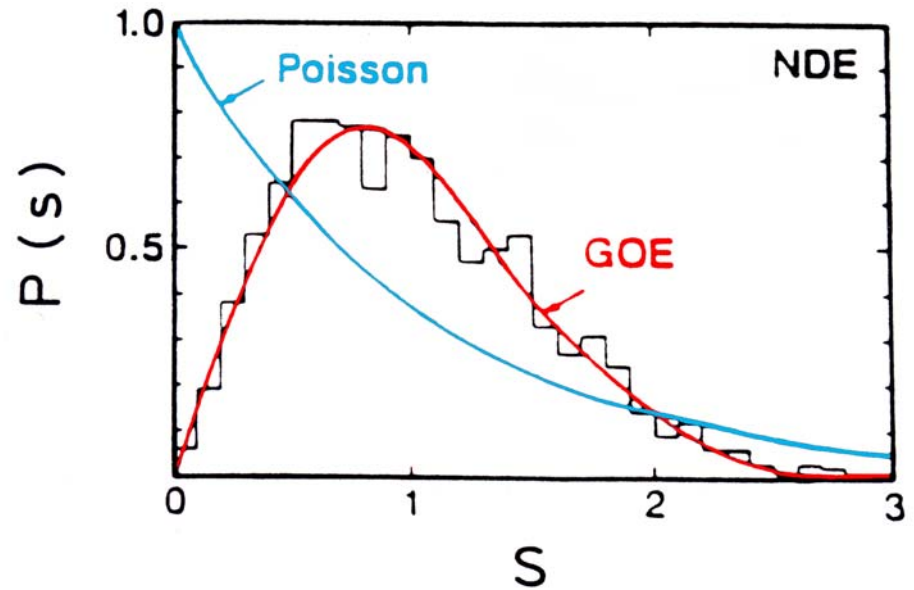
● Poissonian Random Numbers ↔ "Level Clustering"

# Nearest Neighbor Spacings Distribution

stadium billiard



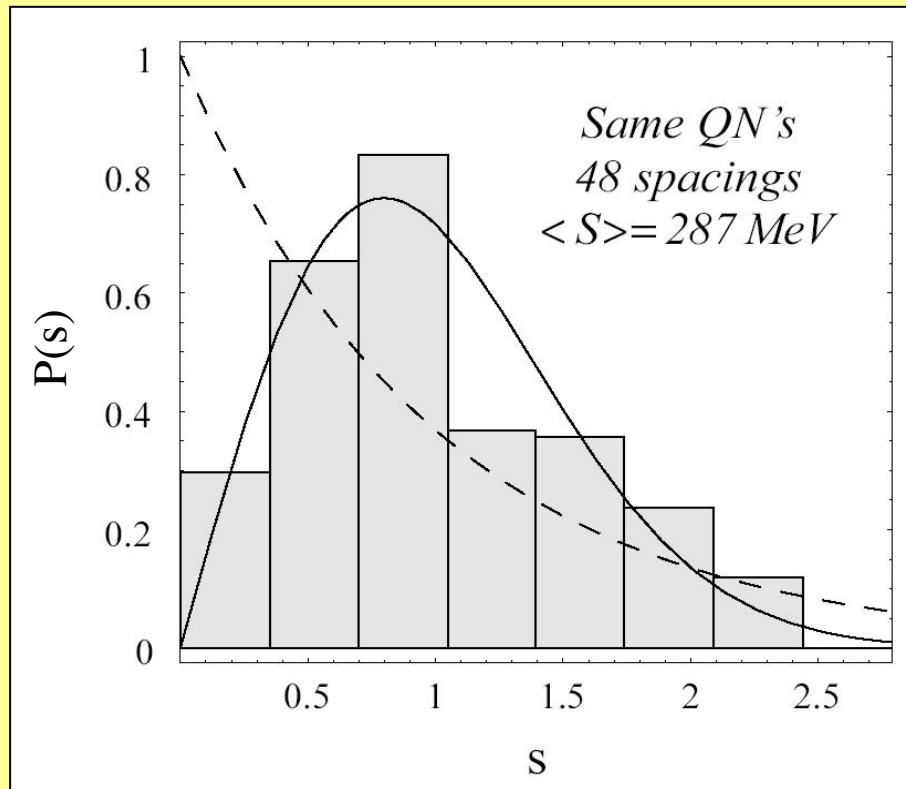
nuclear data ensemble



● Universal (generic) behaviour of the two systems

# Universality in Mesoscopic Systems: Quantum Chaos in Hadrons

- Combined data from measured baryon and meson mass spectra up to 2.5 GeV (from PDG)
- Spectra can be organized into multiplets characterized by a set of definite quantum numbers: isospin, spin, parity, strangeness, baryon number, ...



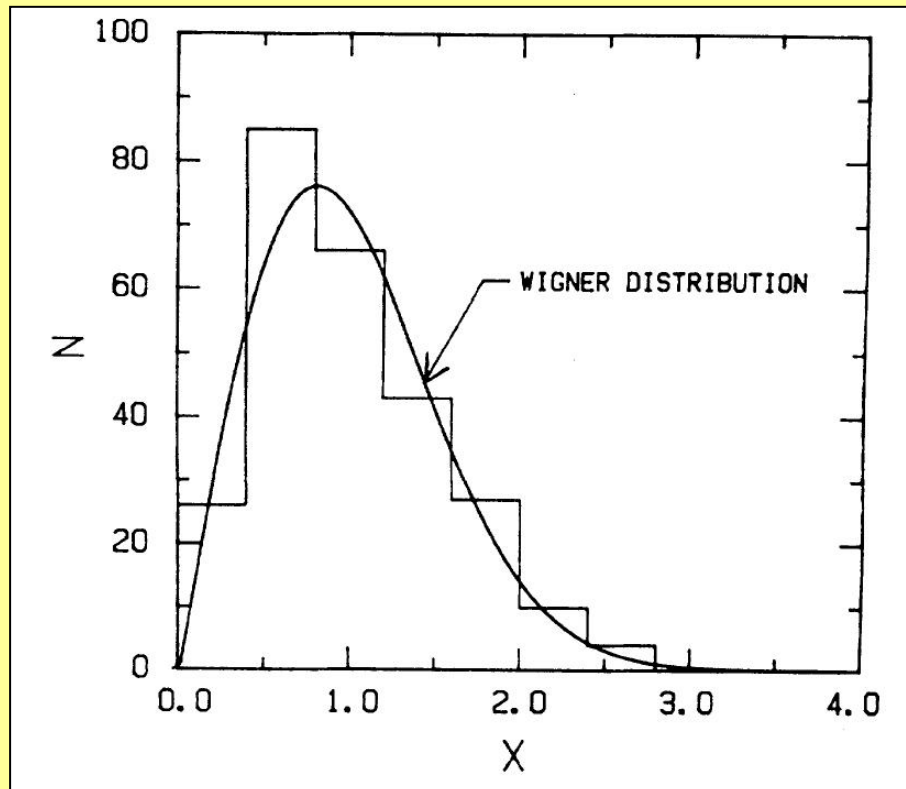
• Scale:  $10^{-16} \text{ m}$

Pascalutsa (2003)



# Universality in Mesoscopic Systems: Quantum Chaos in Atoms

- 8 sets of atomic spectra of highly excited neutral and ionized rare earth atoms combined into a data ensemble
- States of same total angular momentum and parity

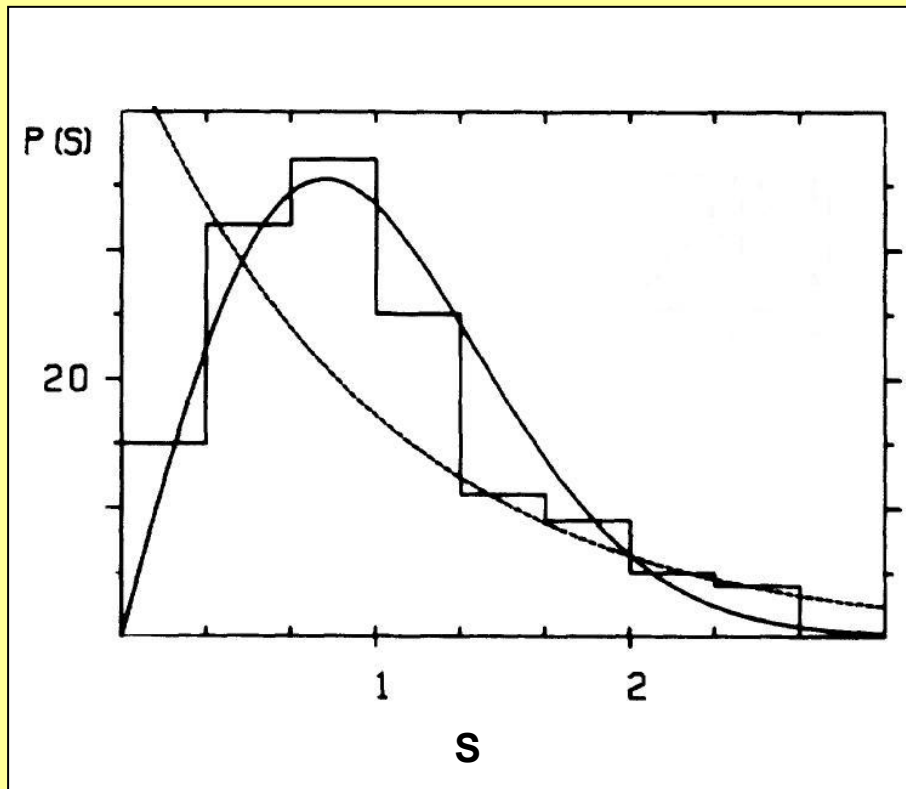


• Scale:  $10^{-10}$  m

Camarda + Georgopoulos (1983)

# Universality in Mesoscopic Systems: Quantum Chaos in Molecules

- Vibronic levels of  $\text{NO}_2$
- States of same quantum numbers



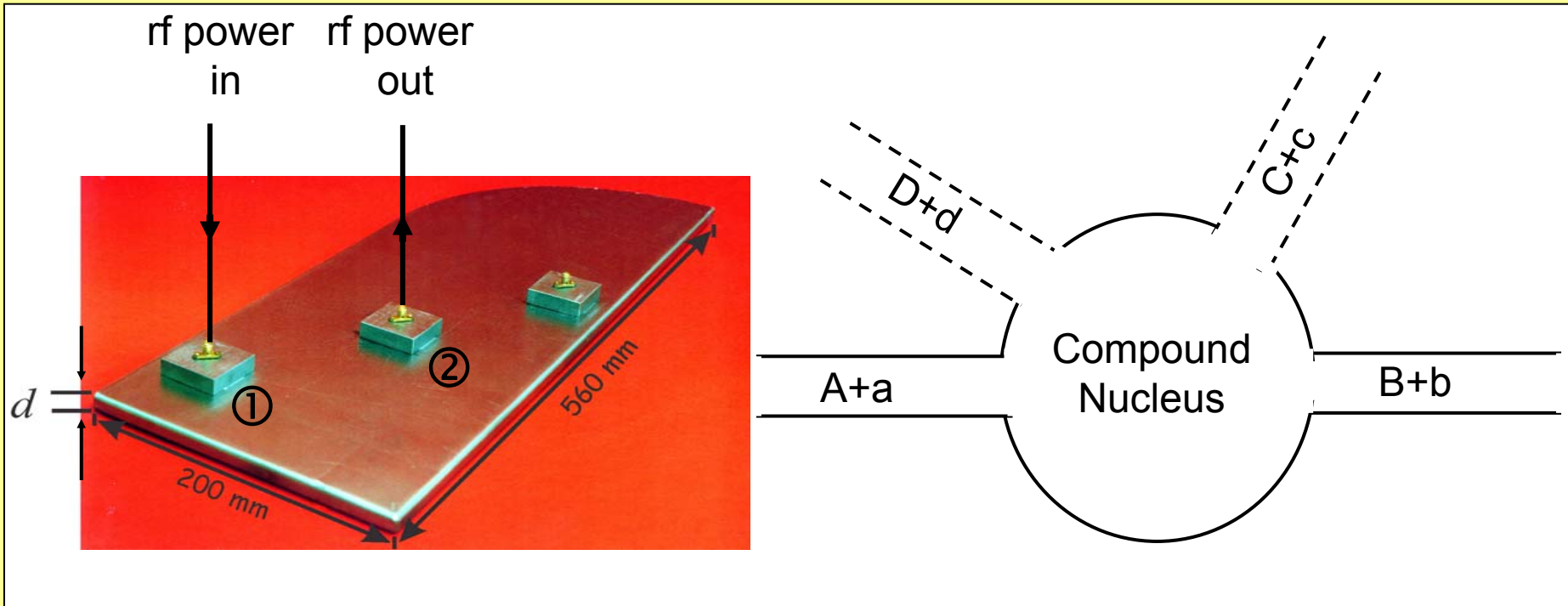
• Scale:  $10^{-9}$  m

Zimmermann et al. (1988)

# Conjecture of Bohigas, Giannoni + Schmit (1984)

- How is the behaviour of the classical system transferred to the quantum system ?
- Answer: There is a one-to-one correspondence between billiards and mesoscopic systems on all scales
- For chaotic systems, the spectral fluctuation properties of eigenvalues coincide with the predictions of random-matrix theory (RMT) for matrices of the same symmetry class
- Numerous tests of various spectral properties (NNSD,  $\Sigma^2$ ,  $\Delta_3$ , ...) and wave functions exist
- Our aim: to test this conjecture in scattering systems, i.e. in open chaotic microwave billiards particularly in the regime of weakly overlapping resonances

# Microwave Resonator as a Model for the Compound Nucleus



- Microwave power is **emitted** into the resonator by antenna ① and the output signal is **received** by antenna ②  
→ **Open scattering system**
- The antennas act as **single scattering channels**
- Absorption into the walls is modelled by **additive channels**

# Scattering Matrix Description

- Scattering matrix for both scattering processes

$$\hat{S}(E) = \mathbb{1} - 2\pi i \hat{W}^T (E\mathbb{1} - \hat{H} + i\pi \hat{W}\hat{W}^T)^{-1} \hat{W}$$

## Compound-nucleus reactions

nuclear Hamiltonian

←  $\hat{H}$  →

coupling of quasi-bound states to channel states

←  $\hat{W}$  →

## Microwave billiard

resonator Hamiltonian

coupling of resonator states to antenna states and to the walls

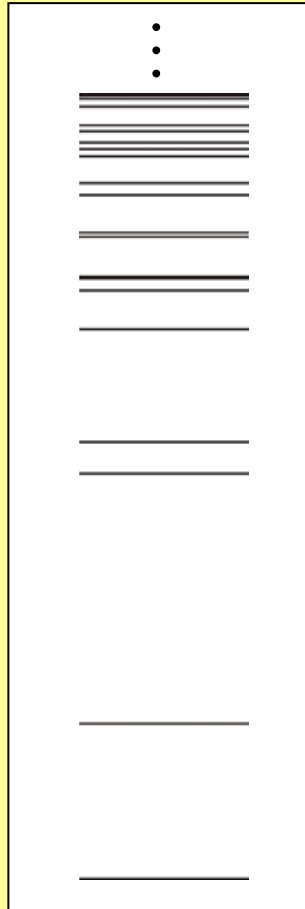
- Experiment:

complex S-matrix elements

- RMT description:** replace  $\hat{H}$  by a  $\begin{matrix} \text{GOE} \\ \text{GUE} \end{matrix}$  matrix for  $\begin{matrix} \text{T-inv} \\ \text{T-noninv} \end{matrix}$  systems

# Excitation Spectra

atomic nucleus

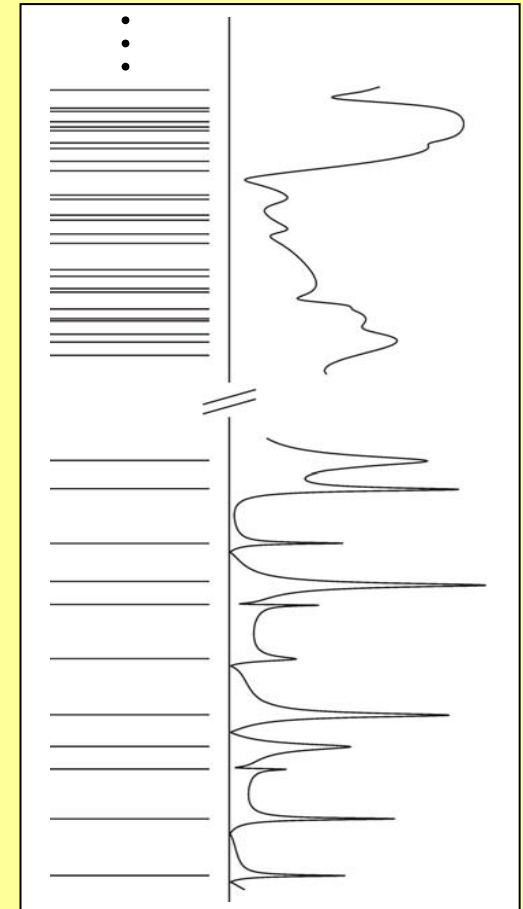


$$\rho \sim \exp(E^{1/2})$$

overlapping resonances  
for  $\Gamma/D > 1$   
**Ericson fluctuations**

isolated resonances  
for  $\Gamma/D \ll 1$

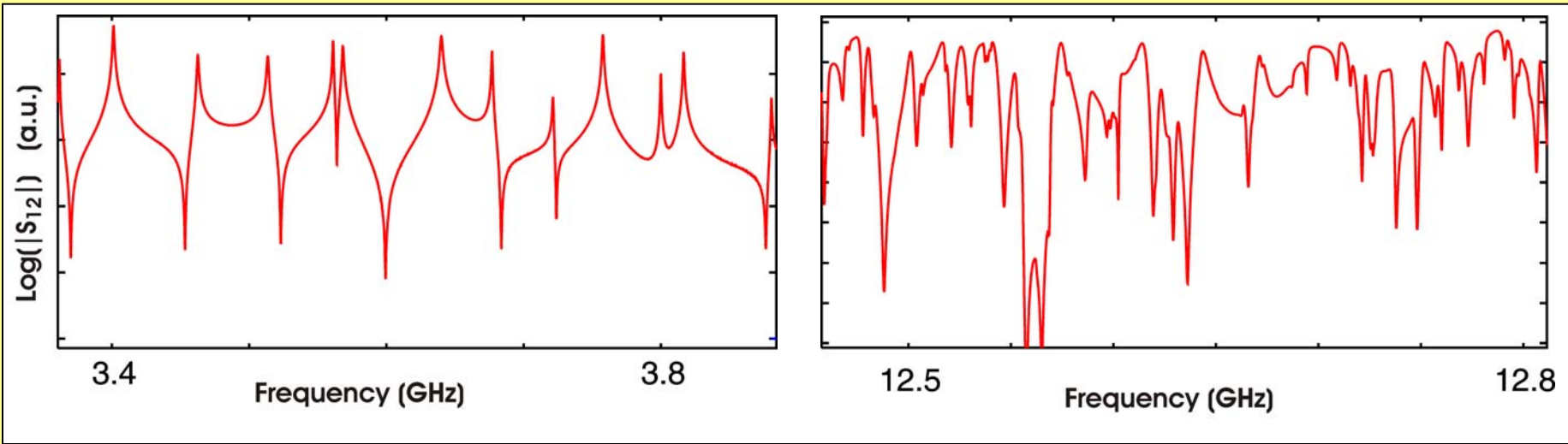
microwave cavity



$$\rho \sim f$$

- Universal description of spectra and fluctuations:  
Verbaarschot, Weidenmüller + Zirnbauer (1984)

# Spectra and Correlation of S-Matrix Elements



• Regime of isolated resonances

•  $\Gamma/D$  small

• Resonances: eigenvalues

• Overlapping resonances

•  $\Gamma/D \sim 1$

• Fluctuations:  $\Gamma_{\text{coh}}$

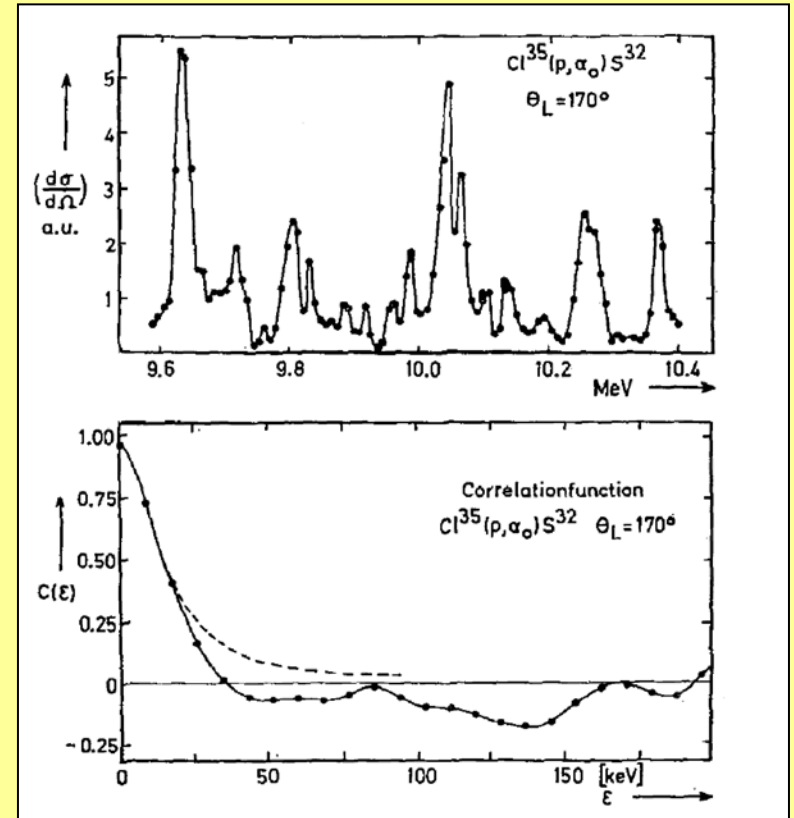
Correlation function: 
$$C(\varepsilon) = \langle S(f)S^*(f + \varepsilon) \rangle - \langle S(f) \rangle \langle S^*(f + \varepsilon) \rangle$$

# Ericson's Prediction for $\Gamma > D$

- Ericson fluctuations (1960):

$$|C(\varepsilon)|^2 \propto \frac{\Gamma_{coh}^2}{\Gamma_{coh}^2 + \varepsilon^2}$$

- Correlation function is Lorentzian
- Measured 1964 for overlapping compound nuclear resonances
- Now observed in lots of different systems: molecules, quantum dots, laser cavities, microwave cavities, ...



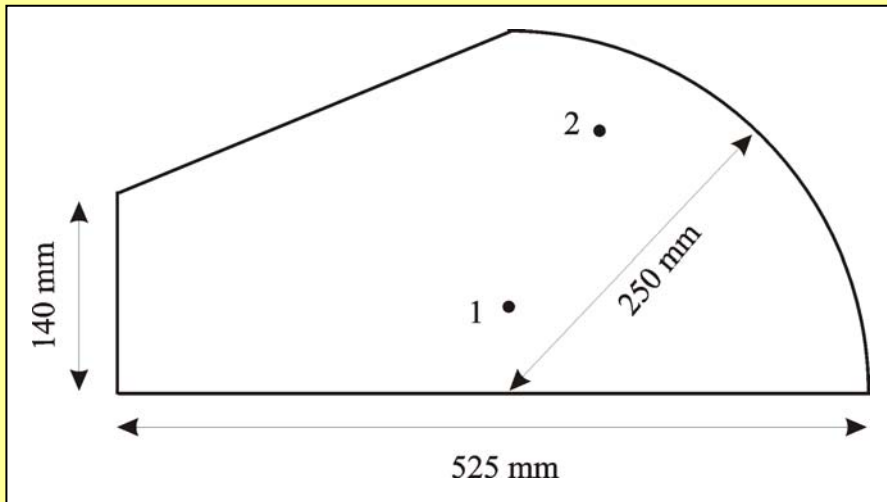
P. v. Brentano et al., PL 9 (1964) 48

- Different theoretical approaches: Ericson → energy and time domain  
 VWZ → RMT  
 Blümel and Smilansky → semiclassical approach
- Applicable for  $\Gamma/D \gg 1$  and for many open channels only



# Fluctuations in a Fully Chaotic Cavity with T-Invariance

- Tilted stadium (Primack + Smilansky, 1994)



- Height of cavity 15 mm
- Becomes 3D at 10.1 GHz
- GOE behaviour checked
- Measure full complex S-matrix for two antennas:  $S_{11}$ ,  $S_{22}$ ,  $S_{12}$

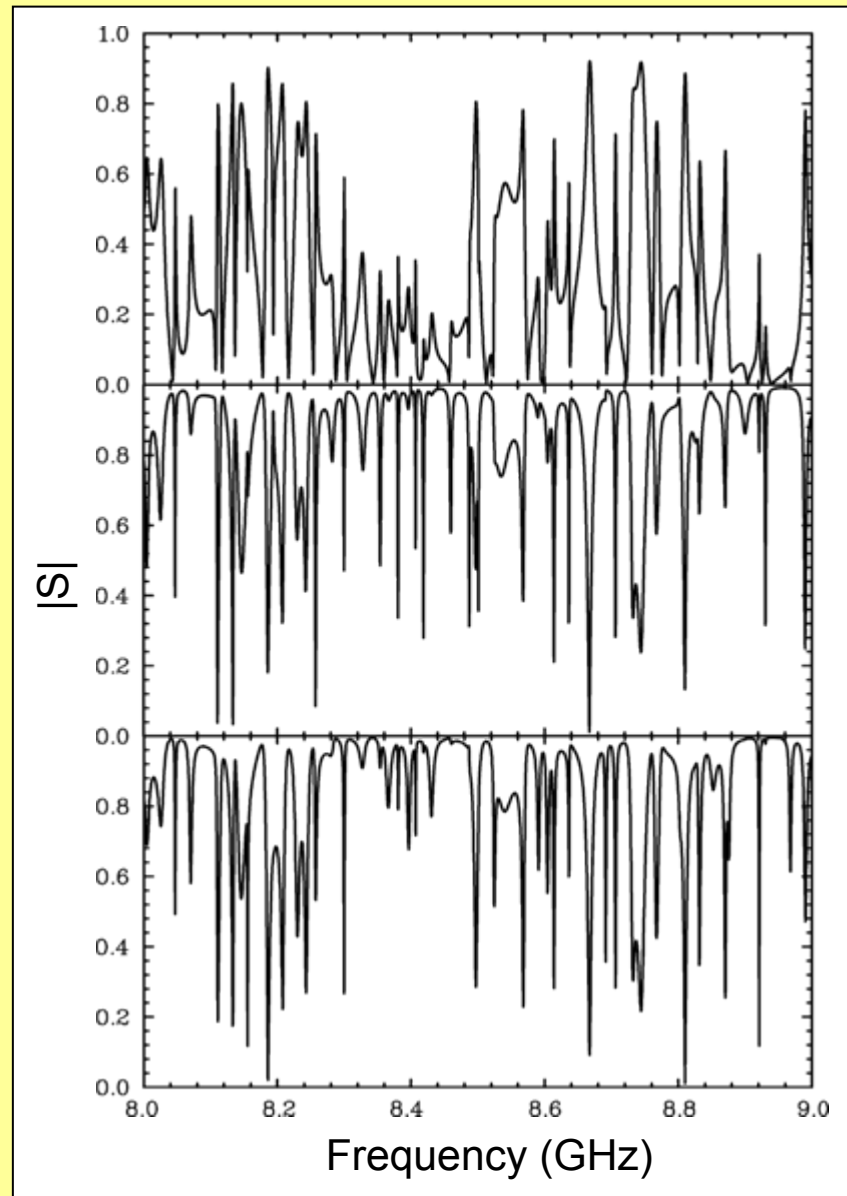
# Spectra of S-Matrix Elements

Example: 8-9 GHz

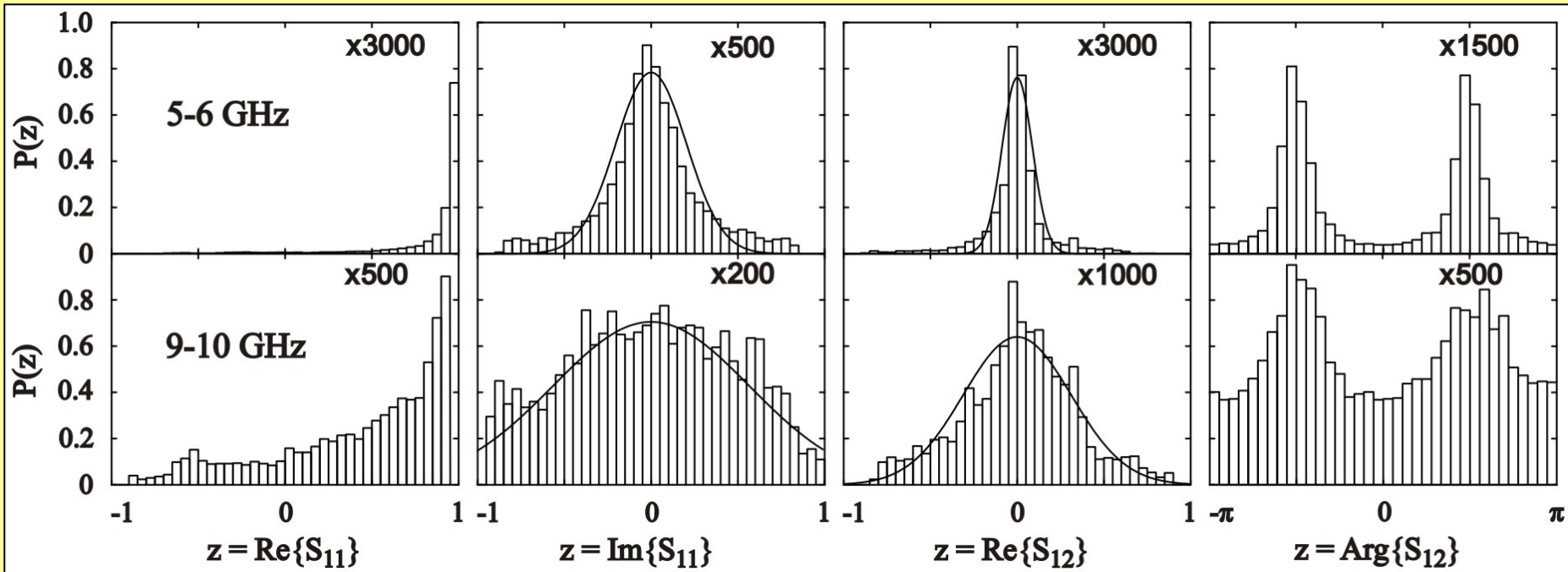
$S_{12} \rightarrow$

$S_{11} \rightarrow$

$S_{22} \rightarrow$



# Distributions of S-Matrix Elements



- Ericson regime:  $\text{Re}\{S\}$  and  $\text{Im}\{S\}$  should be Gaussian and phases uniformly distributed
- Clear deviations for  $\Gamma/D \approx 1$  which still need to be modeled theoretically

# Road to Analysis of the Measured Fluctuations

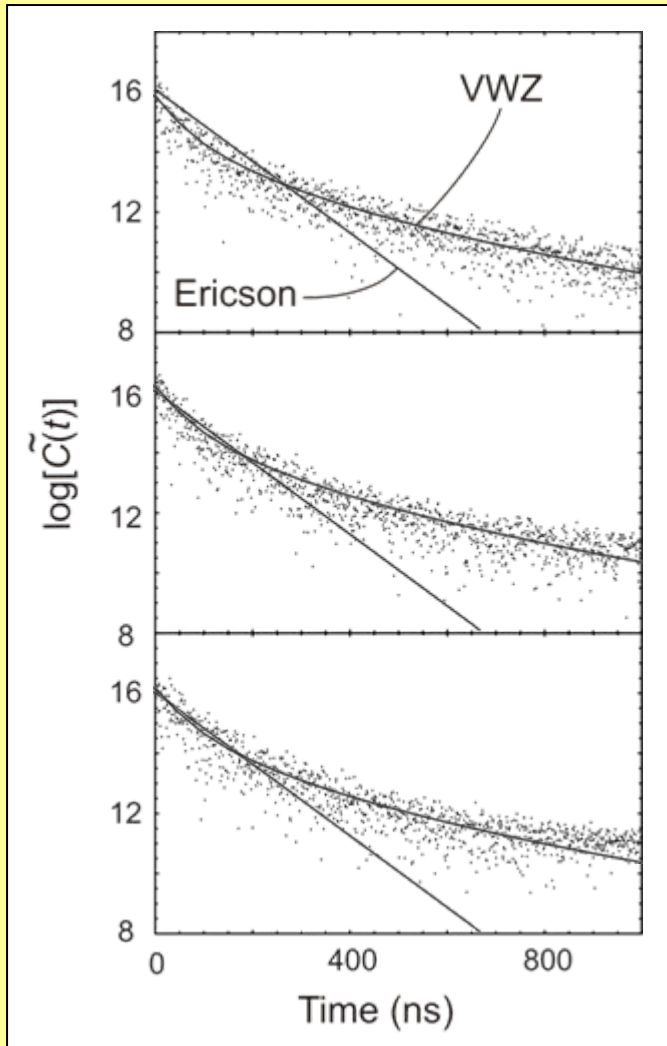
- Problem: adjacent points in  $C(\varepsilon)$  are correlated
- Solution: FT of  $C(\varepsilon) \rightarrow$  uncorrelated Fourier coefficients  $\tilde{C}(t)$   
Ericson (1965)
- Development: Non Gaussian fit and test procedure

# Fourier Transform vs. Autocorrelation Function

Time domain

Example 8-9 GHz

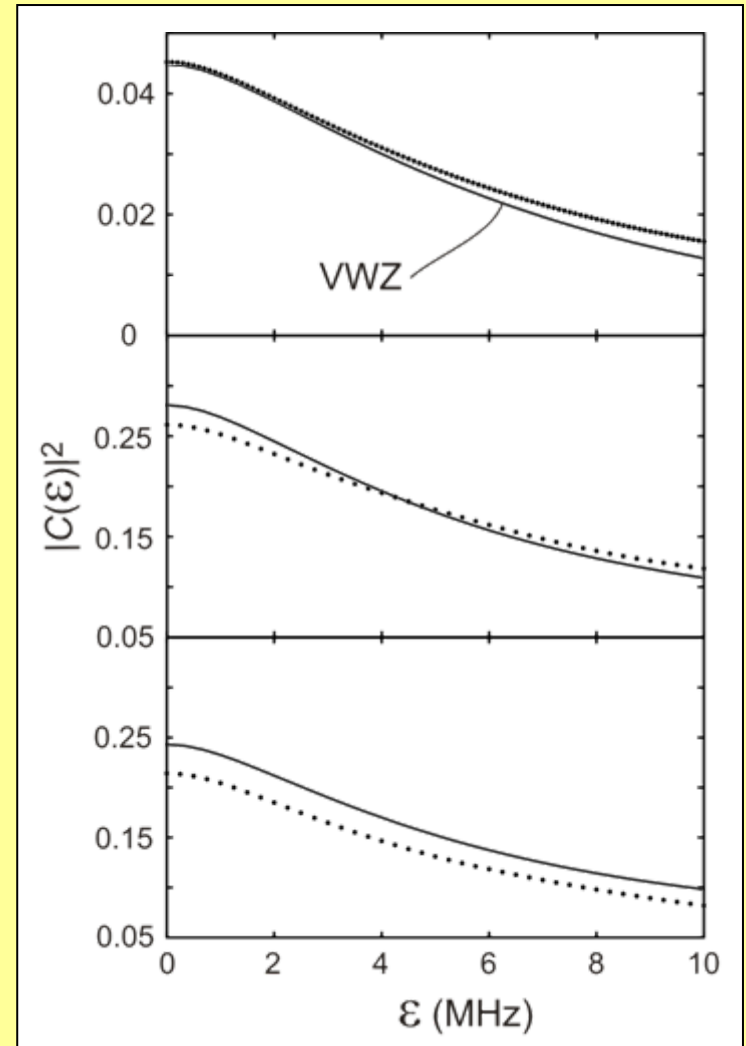
Frequency domain



$\leftarrow S_{12} \rightarrow$

$\leftarrow S_{11} \rightarrow$

$\leftarrow S_{22} \rightarrow$



# Exact RMT Result for GOE Systems

- Verbaarschot, Weidenmüller and Zirnbauer (VWZ) 1984 for arbitrary  $\Gamma/D$  :

- VWZ-integral:

$$C = C(T_i, D; \epsilon)$$

$$C_{ab}(\epsilon) = \frac{1}{8} \int_0^\infty d\lambda_1 \int_0^\infty d\lambda_2 \int_0^1 d\lambda \mu(\lambda, \lambda_1, \lambda_2) \\ \times \exp(-i\pi\epsilon(\lambda_1 + \lambda_2 + 2\lambda)/D) \\ \times J_{ab}(\lambda, \lambda_1, \lambda_2) \\ \times \prod_e \frac{(1 - T_e\lambda)}{((1 + T_e\lambda_1)(1 + T_e\lambda_2))^{1/2}}$$

$$\mu(\lambda, \lambda_1, \lambda_2) = \frac{\lambda(1-\lambda)|\lambda_1 - \lambda_2|}{(\lambda + \lambda_1)^2(\lambda + \lambda_2)^2(\lambda_1\lambda_2(1 + \lambda_1)(1 + \lambda_2))^{1/2}}$$

$$J_{ab}(\lambda, \lambda_1, \lambda_2) = \delta_{ab} T_a^2 (1 - T_a) \\ \times \left( \frac{\lambda_1}{1 + T_a\lambda_1} + \frac{\lambda_2}{1 + T_a\lambda_2} + \frac{2\lambda}{1 - T_a\lambda} \right) \\ + (1 + \delta_{ab}) T_a T_b \\ + \left( \frac{\lambda_1(1 + \lambda_1)}{(1 + T_a\lambda_1)(1 + T_b\lambda_1)} + \frac{\lambda_2(1 + \lambda_2)}{(1 + T_a\lambda_2)(1 + T_b\lambda_2)} \right. \\ \left. + \frac{2\lambda(1 - \lambda)}{(1 - T_a\lambda)(1 - T_b\lambda)} \right)$$

- Rigorous test of VWZ: isolated resonances, i.e.  $\Gamma \ll D$
- First test of VWZ in the intermediate regime, i.e.  $\Gamma/D \approx 1$ , with high statistical significance only achievable with microwave billiards
- Note: nuclear cross section fluctuation experiments yield only  $|S|^2$

## Corollary: Hauser-Feshbach Formula

• For  $\Gamma \gg D$ :  $\overline{S_{ab}(f)S_{ab}^*(f + \varepsilon)} \rightarrow (1 + \delta_{ab}) \frac{i\Gamma}{\varepsilon + i\Gamma} \frac{T_a T_b}{\sum_c T_c}$

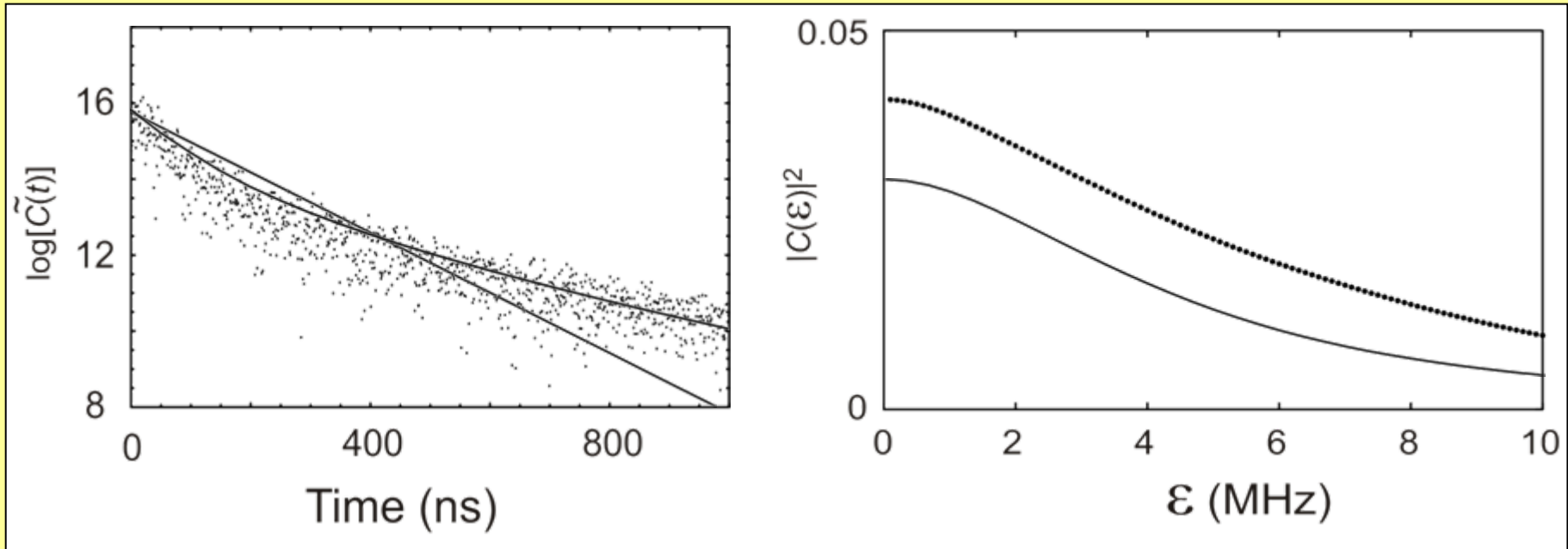
• Elastic enhancement factor  $W$

• Distribution of S-matrix elements yields

$$W = \left( \overline{|S_{11}^{fl}|^2} \cdot \overline{|S_{22}^{fl}|^2} \right)^{1/2} / \overline{|S_{12}^{fl}|^2} \approx 2$$

• Over the whole measured frequency range  $1 < f < 10$  GHz we find  $3.5 > W > 2$  in qualitative accordance with VWZ

# What Happens in the Region of 3D Modes ?



• VWZ curve in  $\tilde{C}(t)$  progresses through the cloud of points but it passes too high  $\rightarrow$  GOF test rejects VWZ

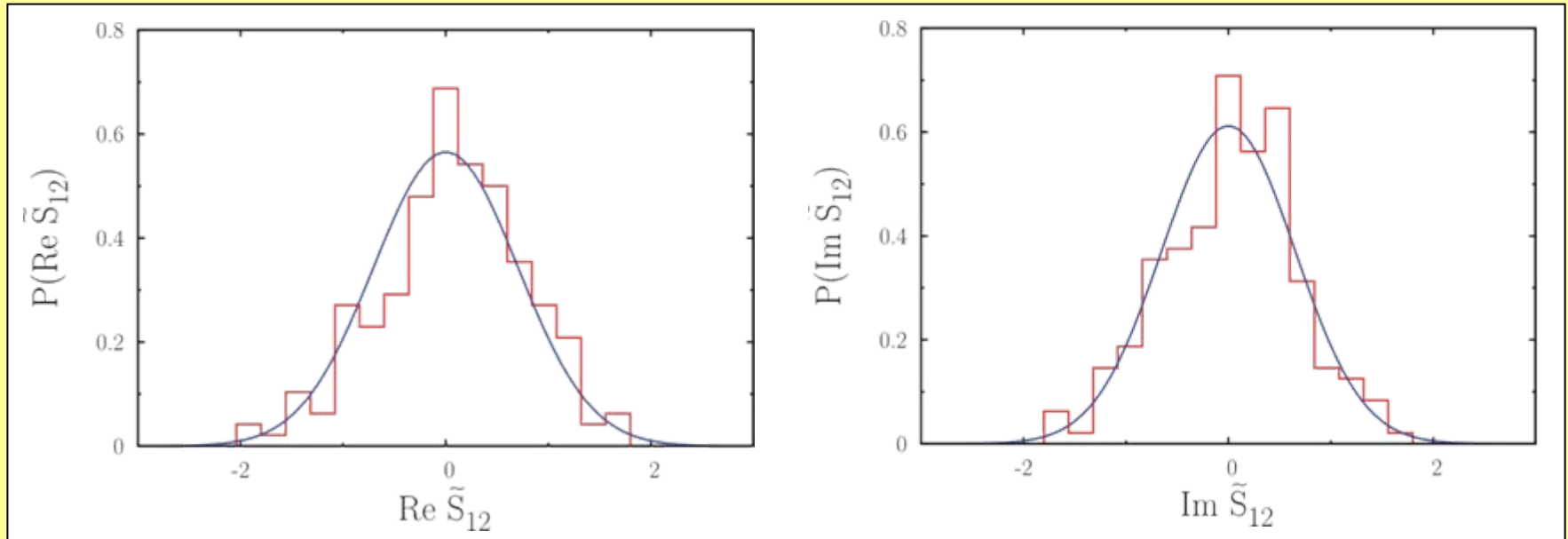
• This behaviour is clearly visible in  $C(\epsilon)$

• Behaviour can be modelled through

$$\hat{H} = \begin{pmatrix} H_1^{GOE} & 0 \\ 0 & H_2^{GOE} \end{pmatrix}$$



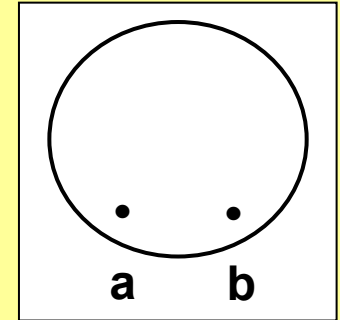
# Distribution of Fourier Coefficients



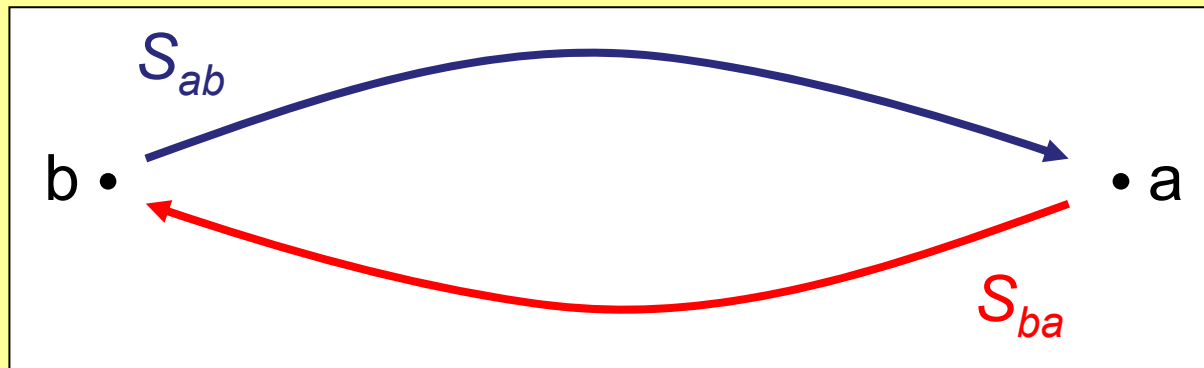
- Distributions are Gaussian with the same variances
- Remember: Measured S-matrix elements were non-Gaussian
- This still remains to be understood

# Induced Time-Reversal Symmetry Breaking (TRSB) in Billiards

- T-symmetry breaking caused by a magnetized ferrite



- Coupling of microwaves to the FMR depends on the direction  $a \leftrightarrow b$



- Principle of detailed balance:

$$|S_{ab}|^2 = |S_{ba}|^2$$

- Principle of reciprocity:

$$S_{ab} = S_{ba}$$

# Detailed Balance in Nuclear Reactions

Volume 56B, number 2

PHYSICS LETTERS

14 April 1975

## TIME-REVERSIBILITY VIOLATION AND ISOLATED NUCLEAR RESONANCES<sup>☆</sup>

J.M. PEARSON

*Laboratoire de Physique Nucléaire, Département de Physique, Université de Montréal, Montréal, Canada*

and

A. RICHTER \*

*Institut für Kernphysik, Technische Hochschule Darmstadt, Darmstadt, W Germany*

Received 21 January 1975

It is pointed out that measurements of differential cross-sections in nuclear reactions proceeding via an isolated resonance can provide in principle a test for time reversibility.

- Search for Time-Reversal Symmetry Breaking (TRSB) in nuclear reactions

# Detailed Balance in Nuclear Reactions

2.A.1:  
2.C

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## TEST OF DETAILED BALANCE AT ISOLATED RESONANCES IN THE REACTIONS $^{27}\text{Al} + \text{p} \rightleftharpoons ^{24}\text{Mg} + \alpha$ AND TIME REVERSIBILITY †

H. DRILLER †† and E. BLANKE †††

*Institut für Experimentalphysik, Ruhr Universität Bochum, 4630 Bochum, Germany*

H. GENZ, A. RICHTER and G. SCHRIEDER

*Institut für Kernphysik, Technische Hochschule Darmstadt, 6100 Darmstadt, Germany*

and

J. M. PEARSON ‡

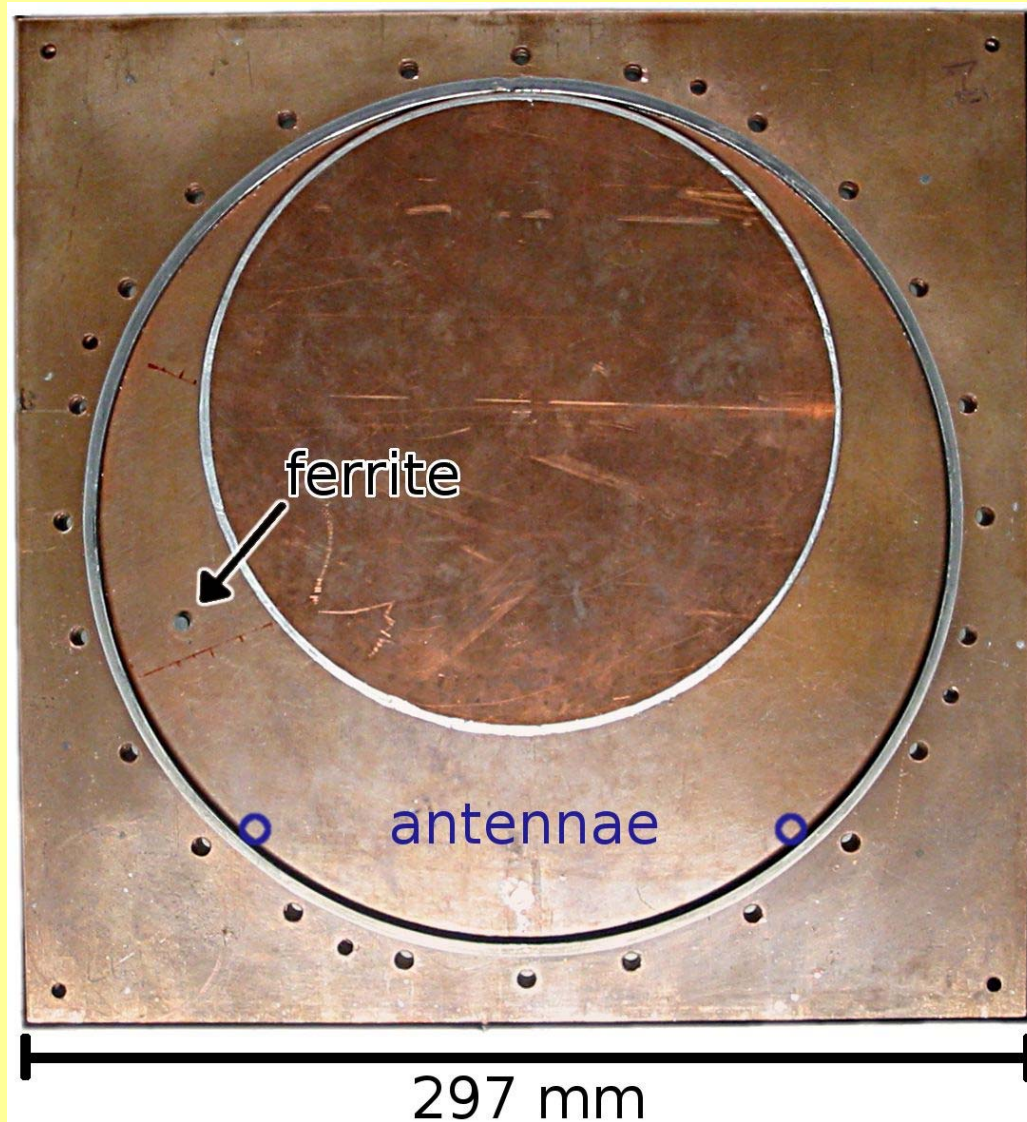
*Laboratoire de Physique Nucléaire, Département de Physique, Université de Montréal, Montréal, Québec, Canada*

Received 15 September 1978

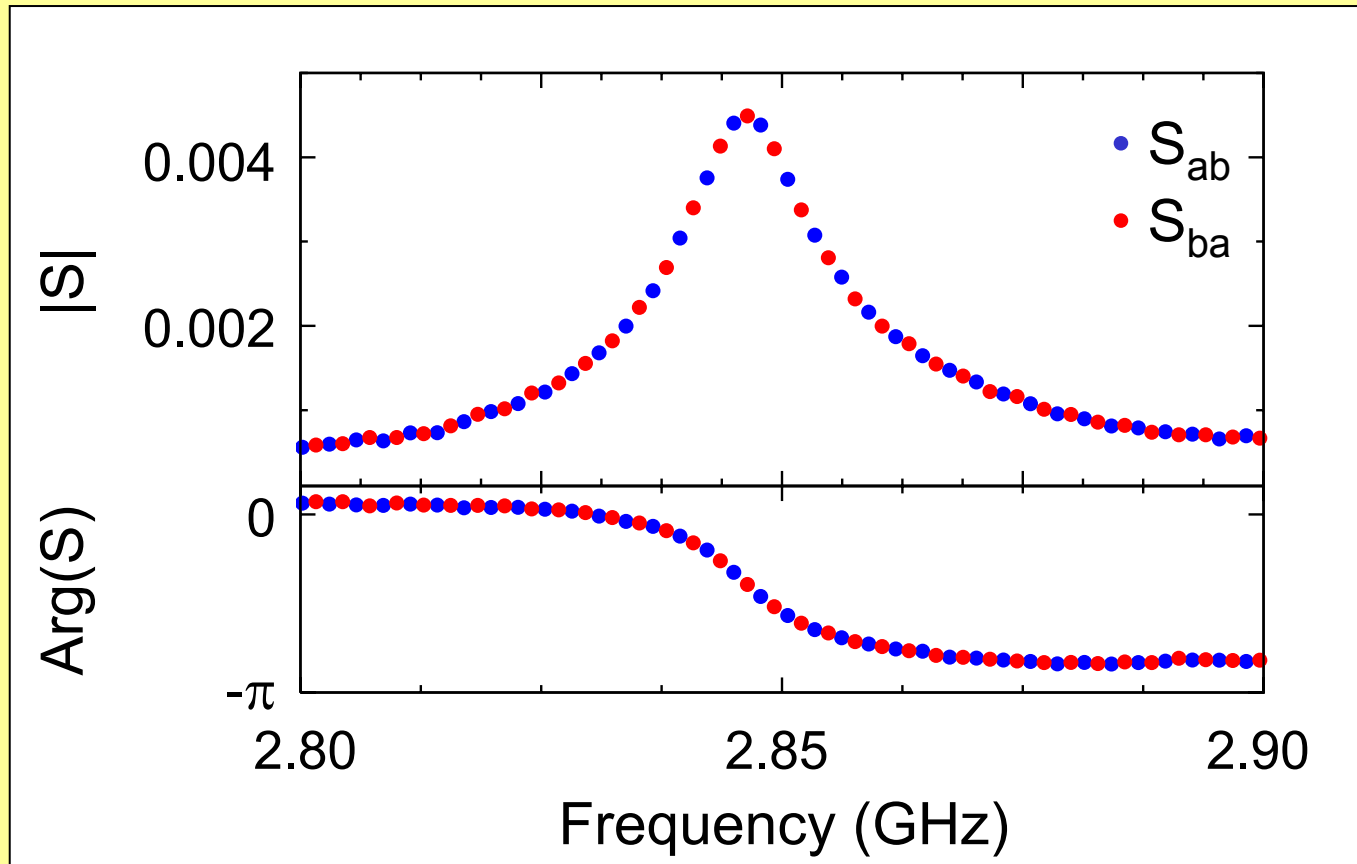
**Abstract:** The principle of detailed balance has been tested in the reactions  $^{27}\text{Al}(\text{p}, \alpha_0)^{24}\text{Mg}$  ( $Q = 1.600$  MeV) and  $^{24}\text{Mg}(\alpha, \text{p}_0)^{27}\text{Al}$  ( $Q = -1.600$  MeV) at bombarding energies  $E_p^{\text{lab}} = 1.35\text{--}1.46$  MeV and  $E_\alpha^{\text{lab}} = 3.38\text{--}3.52$  MeV, respectively. Protons and  $\alpha$ -particles were detected at  $\theta_{\text{c.m.}} = 177.7^\circ$ . The relative strengths of two resonances at  $E_x = 12.901$  MeV ( $J^\pi = 2^+$ ) and  $E_x = 12.974$  MeV ( $J^\pi = 1^-$ ) in  $^{28}\text{Si}$  excited in the forward and backward reaction agree within the experimental uncertainty  $\delta = 0.0025 \pm 0.0192$ . This experimental result is converted into a difference of phase angles for reduced widths amplitudes,  $\Delta\xi = (0.3 \pm 3)^\circ$ , which is consistent with time reversibility.

● Search for TRSB in nuclear reactions → upper limits

# Isolated Resonances - Setup

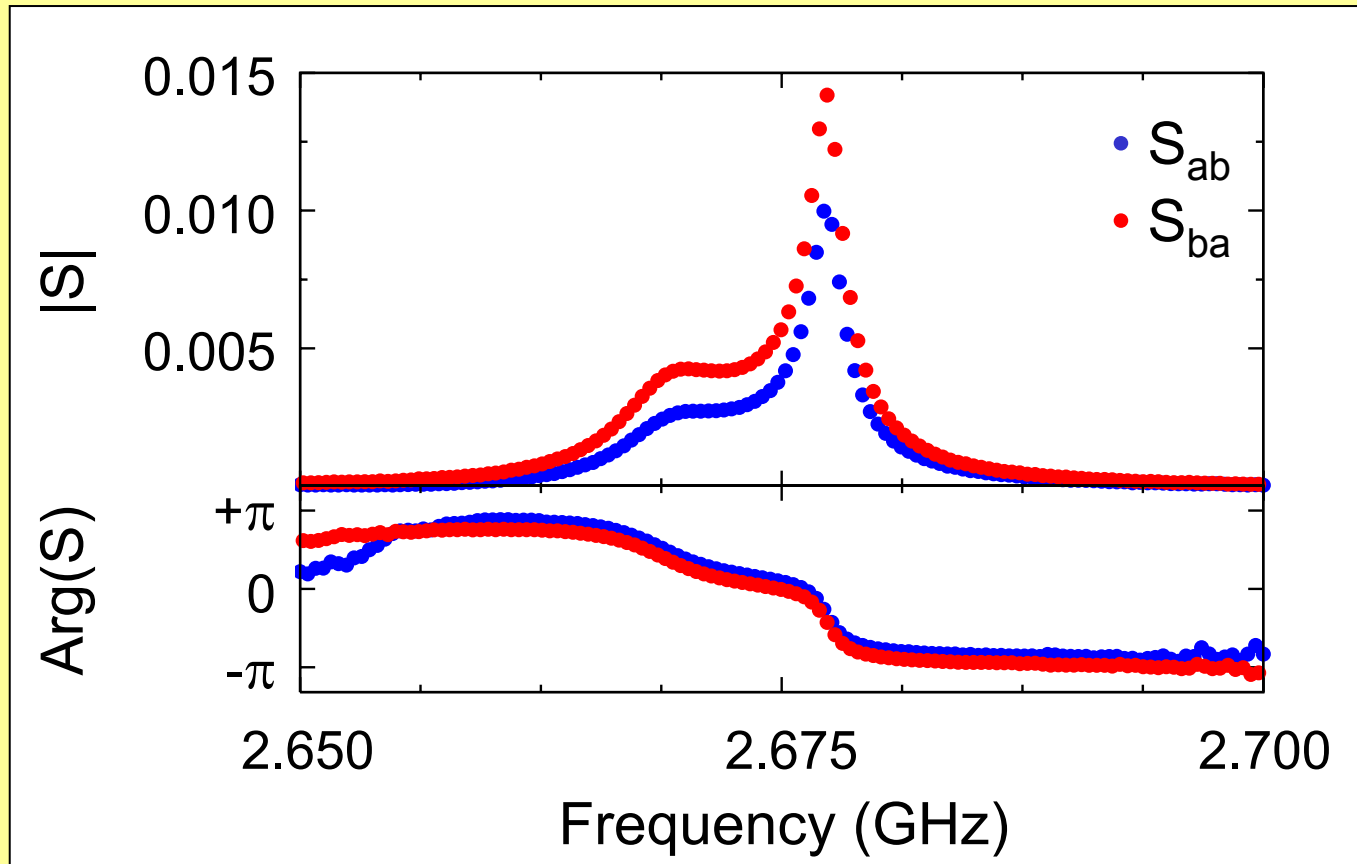


# Isolated Resonances - Singlets



● Reciprocity holds  $\rightarrow$  TRSB cannot be detected this way

# Isolated Doublets of Resonances



● Violation of reciprocity due to interference of two resonances

# Scattering Matrix and TRSB

- Scattering matrix element

$$S_{ab}(\omega) = \delta_{ab} - 2\pi i \langle a | \hat{W}^+ (\omega - \hat{H}^{eff})^{-1} \hat{W} | b \rangle$$

- Decomposition of effective Hamiltonian

$$\hat{H}^{eff} = \hat{H}^a + \hat{H}^s$$



$$\begin{pmatrix} 0 & H_{12}^a \\ -H_{12}^a & 0 \end{pmatrix}$$

- Ansatz for TRSB incorporating the FMR and its selective coupling to the microwaves



# TRSB Matrix Element

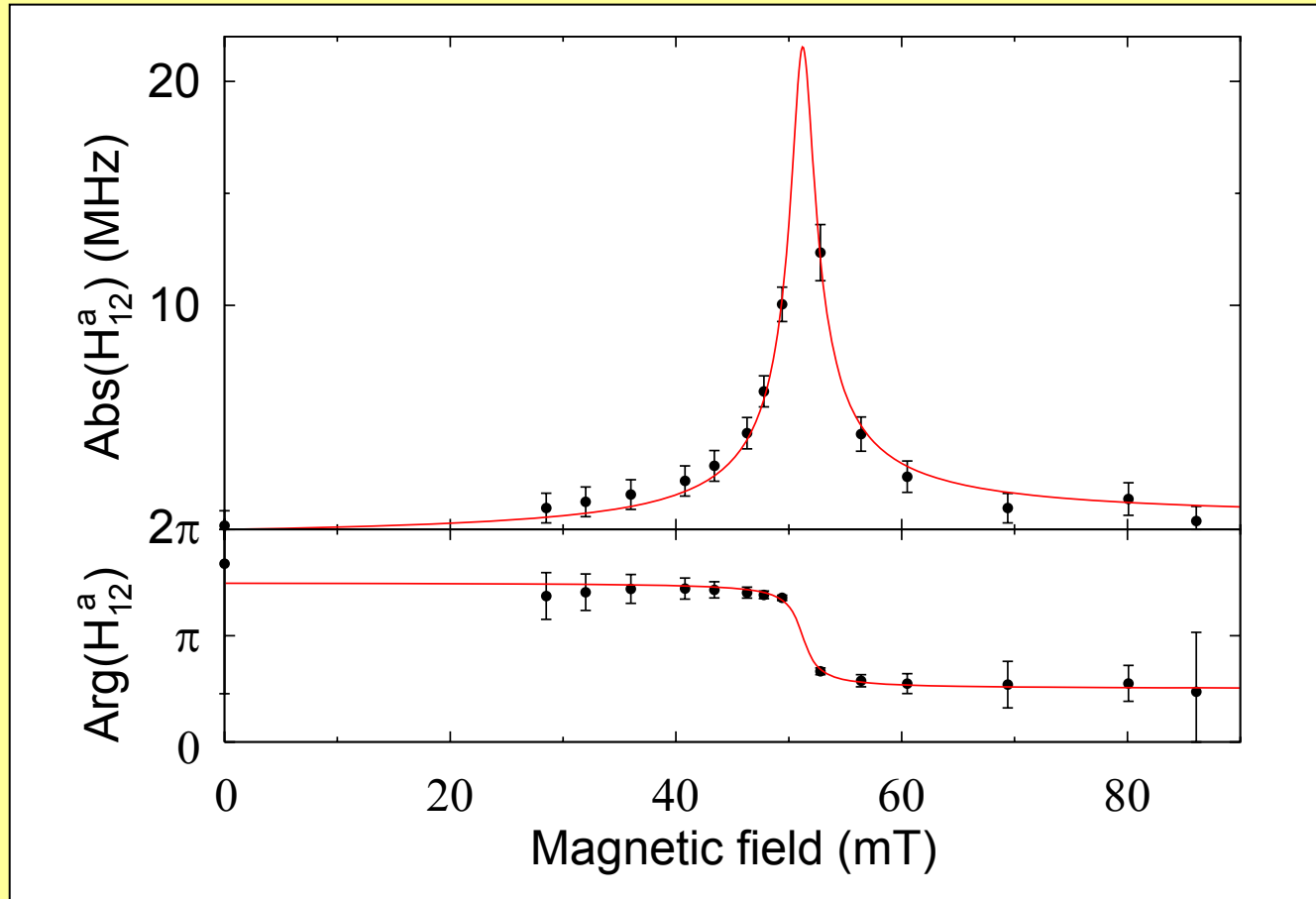
$$\bullet H_{12}^a(B) = \frac{i\pi}{2} \cdot \lambda \cdot B \cdot T \cdot \frac{\omega_M^2}{\omega_0(B) - \bar{\omega} - i/T}$$

↑                    ↑                    ↑                    ↑

coupling    external    spin    magnetic  
strength    field    relaxation    susceptibility  
time

• Fit parameters:  $\lambda$  and  $\bar{\omega}$

# T-Violating Matrix Element

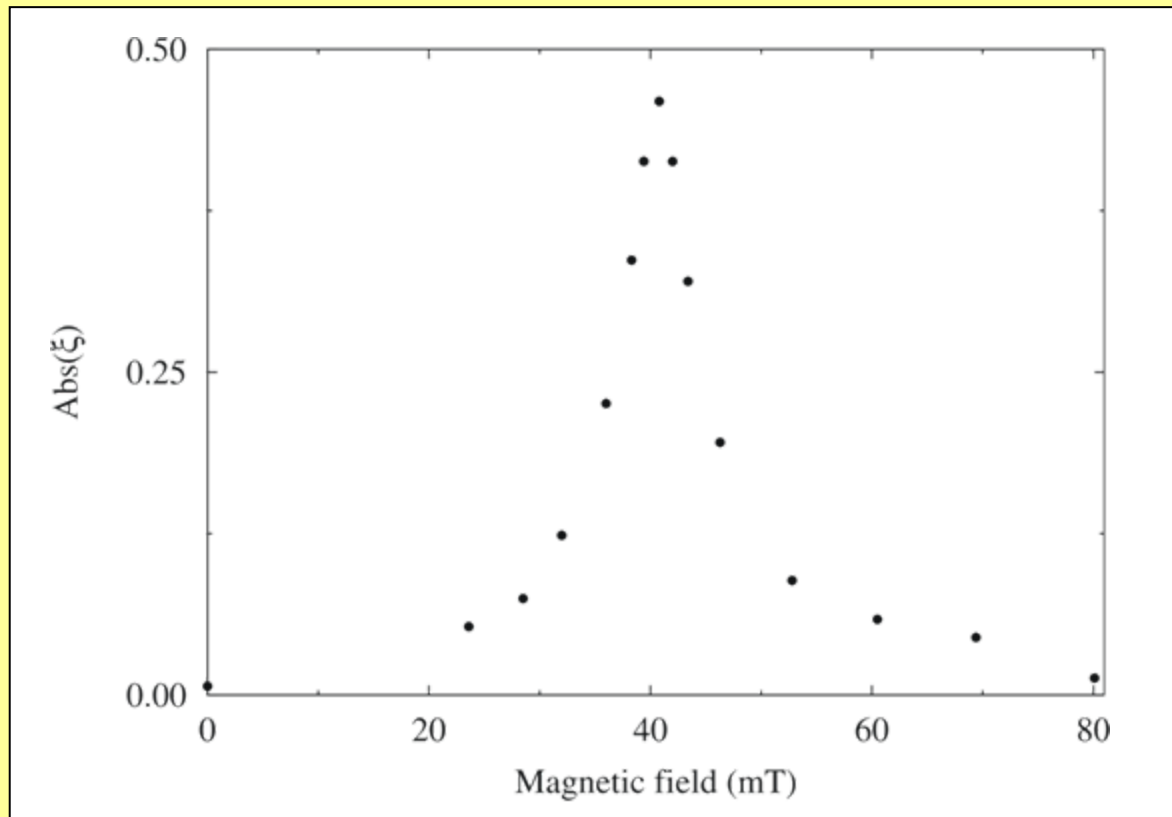


- T-violating matrix element shows resonance like structure
- Successful description of dependence on magnetic field

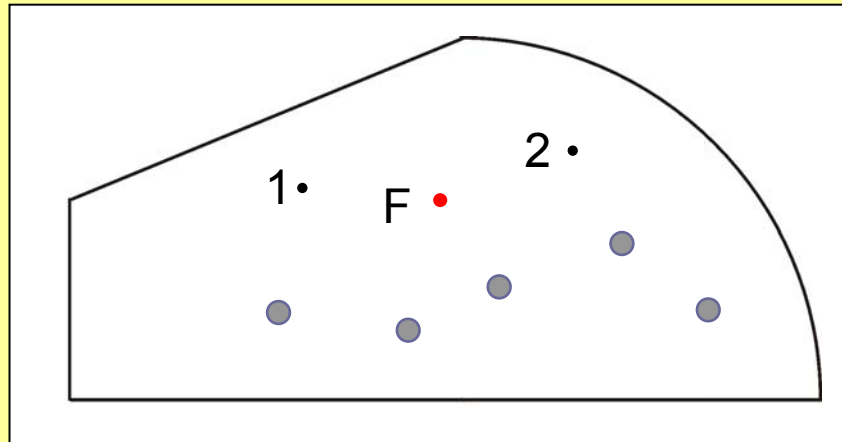
# Relative Strength of T-Violation

- Compare: TRSB matrix element  $H_{12}^a$  to the energy difference of two eigenvalues of the T-invariant system

$$\xi = \left| \frac{2H_{12}^a}{E_1^s - E_2^s} \right|$$



# TRSB in the Region of Overlapping Resonances ( $\Gamma \gtrsim D$ )



- Antenna 1 and 2 in a 2D tilted stadium billiard
  - Magnetized ferrite F in the stadium
  - Place an additional Fe - scatterer into the stadium and move it up to 12 different positions in order to improve the statistical significance of the data sample
- distinction between GOE and GUE behaviour becomes possible

# Search for Time-Reversal Symmetry Breaking in Nuclei

VOLUME 19, NUMBER 9

PHYSICAL REVIEW LETTERS

28 AUGUST 1967

## UPPER LIMIT OF $T$ NONCONSERVATION IN THE REACTIONS $^{24}\text{Mg} + \alpha \rightleftharpoons ^{27}\text{Al} + p$

W. von Witsch, A. Richter, and P. von Brentano\*  
Max Planck Institut für Kernphysik, Heidelberg, Germany  
(Received 28 June 1967)

Time-reversal invariance has been tested via detailed balance in the compound-nuclear reactions  $^{24}\text{Mg} + \alpha \rightleftharpoons ^{27}\text{Al} + p$ . The relative differential cross sections agree within the experimental uncertainties, leading to an estimated upper limit for the ratio of the  $T$ -nonconserving to the  $T$ -conserving reaction amplitudes of  $(2-4) \times 10^{-3}$ . The same upper limit is found for the nuclear matrix elements which are odd with respect to time reversal.

VOLUME 51, NUMBER 5

PHYSICAL REVIEW LETTERS

1 AUGUST 1983

## Improved Experimental Test of Detailed Balance and Time Reversibility in the Reactions $^{27}\text{Al} + p \rightleftharpoons ^{24}\text{Mg} + \alpha$

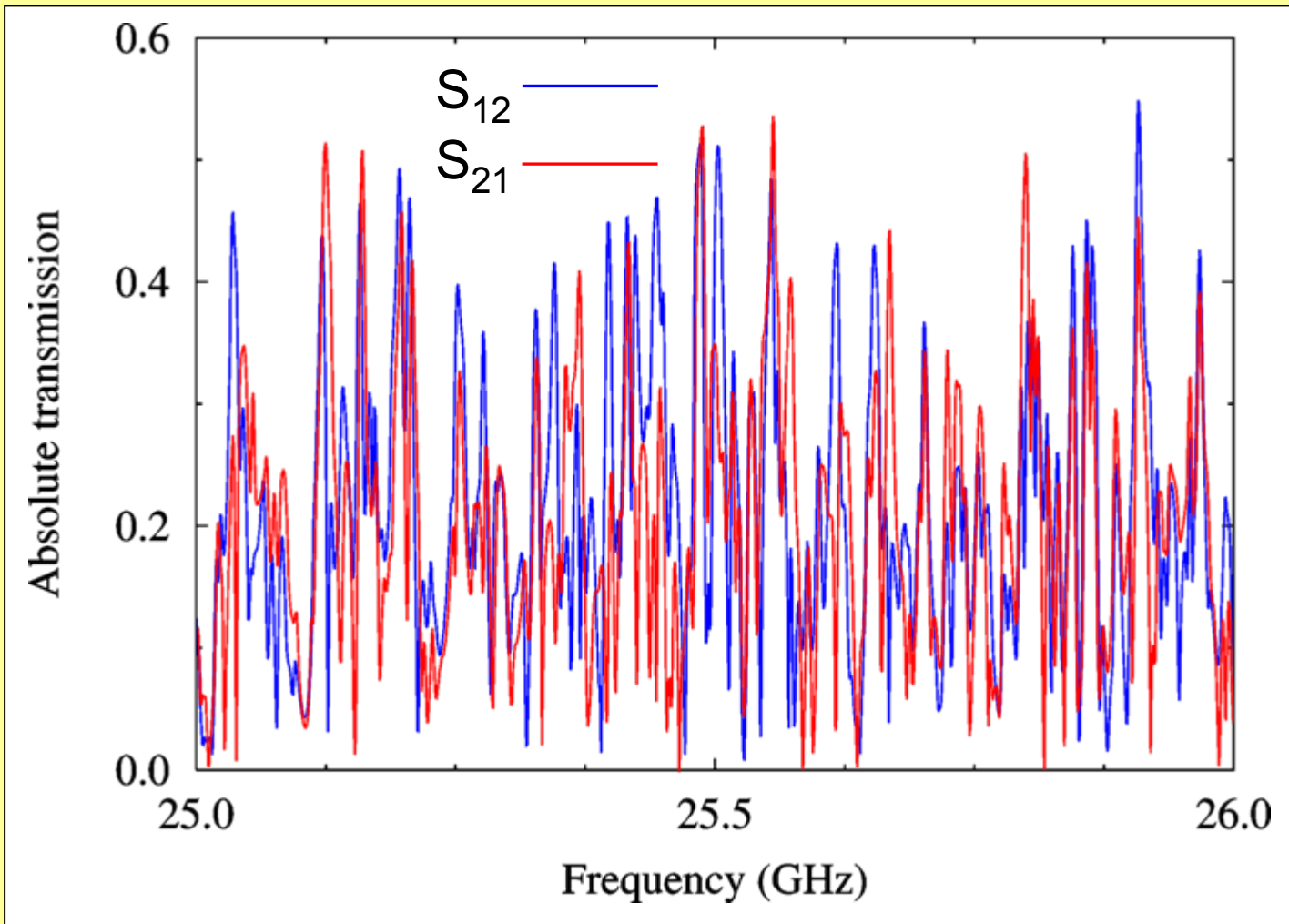
E. Blanke,<sup>(a)</sup> H. Driller,<sup>(b)</sup> and W. Glöckle  
*Abteilung für Physik und Astronomie, Ruhr Universität Bochum, D-4630 Bochum, Germany*

and

H. Genz, A. Richter, and G. Schrieder  
*Institut für Kernphysik, Technische Hochschule Darmstadt, D-6100 Darmstadt, Germany*  
(Received 25 April 1983)

A new test of the principle of detailed balance in the nuclear reactions  $^{27}\text{Al}(p, \alpha_0) ^{24}\text{Mg}$  and  $^{24}\text{Mg}(\alpha, p_0) ^{27}\text{Al}$  at bombarding energies  $7.3 \text{ MeV} \leq E_p \leq 7.7 \text{ MeV}$  and  $10.1 \text{ MeV} \leq E_\alpha \leq 10.5 \text{ MeV}$ , respectively, is reported. Measured relative differential cross sections agree within the experimental uncertainty  $\Delta = \pm 0.51\%$  and hence are consistent with time-reversal invariance. From this result an upper limit  $\xi \leq 5 \times 10^{-4}$  (80% confidence) is derived for a possible time-reversal-noninvariant amplitude in the reaction.

# Violation of Reciprocity



- Clear violation of reciprocity in the regime of  $\Gamma/D \gtrsim 1$

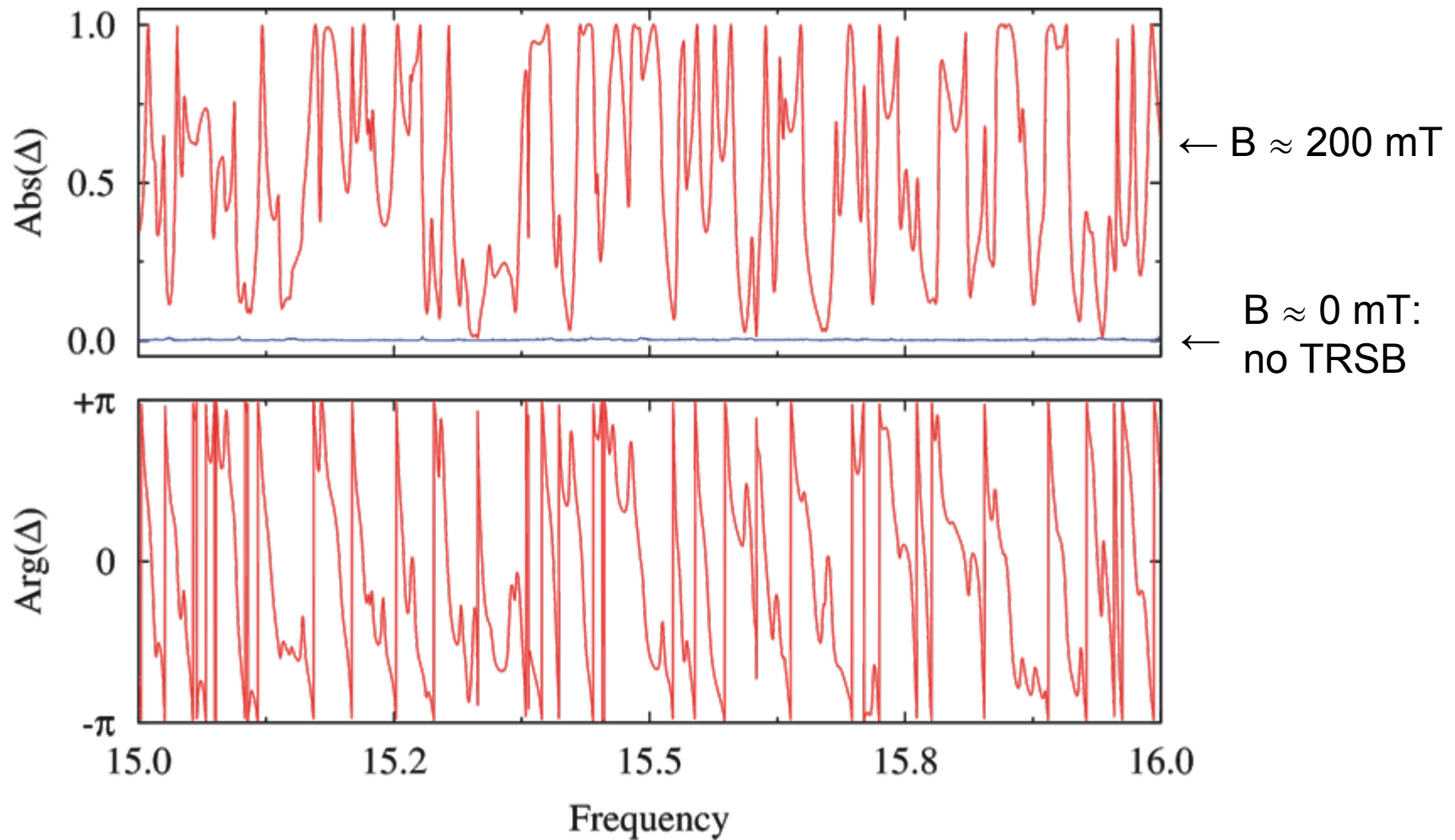
# Quantification of Reciprocity Violation

- The violation of reciprocity reflects degree of TRSB
- Definition of a contrast function

$$\Delta = \frac{S_{ab} - S_{ba}}{|S_{ab}| + |S_{ba}|}$$

- Quantification of reciprocity violation via  $\Delta$

# Magnitude and Phase of $\Delta$ Fluctuate





# S-Matrix Fluctuations and RMT

- Pure GOE → VWZ 1984
- Pure GUE → FSS (Fyodorov, Savin + Sommers) 2005  
V (Verbaarschot) 2007

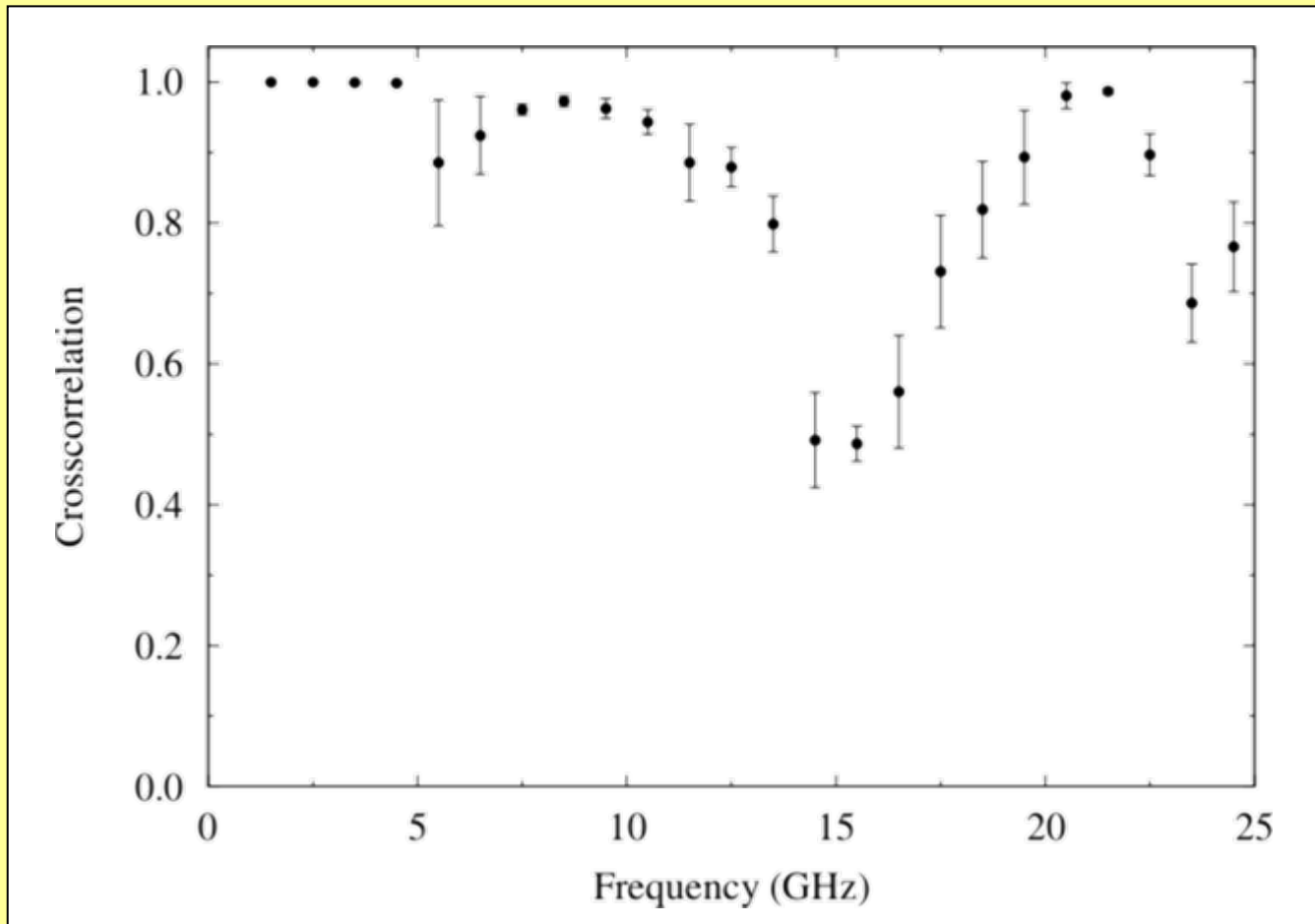
- Partial TRSB → analytical model under development  
(based on Pluhař, Weidenmüller, Zuk + Wegner, 1995)

- RMT → 
$$\hat{H} = \hat{H}^s + i\alpha \hat{H}^a$$

↑  
 $\alpha = 0$  GOE  
 $\alpha = 1$  GUE

- Full T symmetry breaking sets in experimentally already  
for  $\lambda \approx \alpha/D \approx 1$

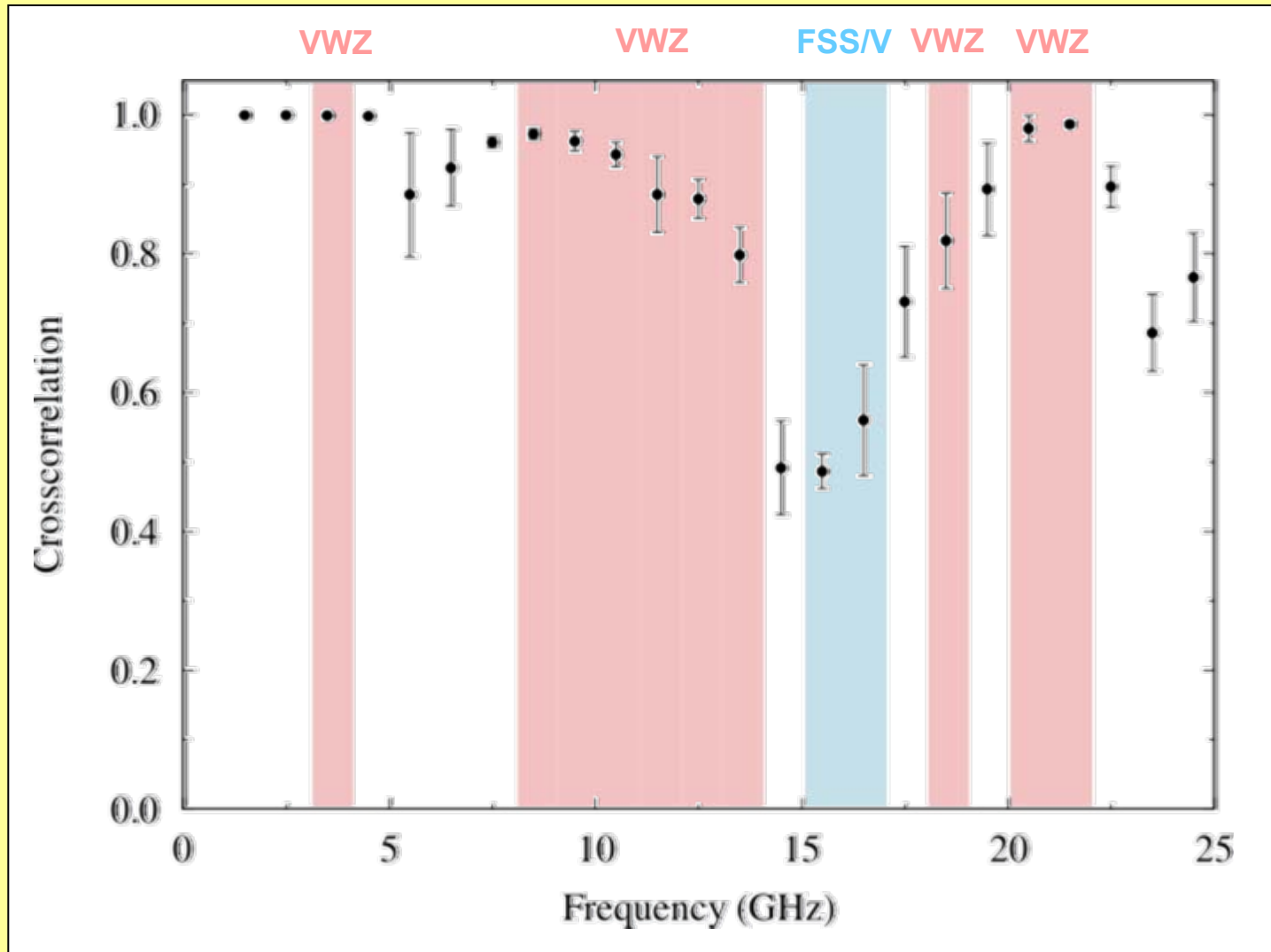
# Crosscorrelation between $S_{12}$ and $S_{21}$ at $\varepsilon = 0$



●  $C(S_{12}, S_{21}^*) = \begin{cases} 1 \text{ for GOE} \\ 0 \text{ for GUE} \end{cases}$

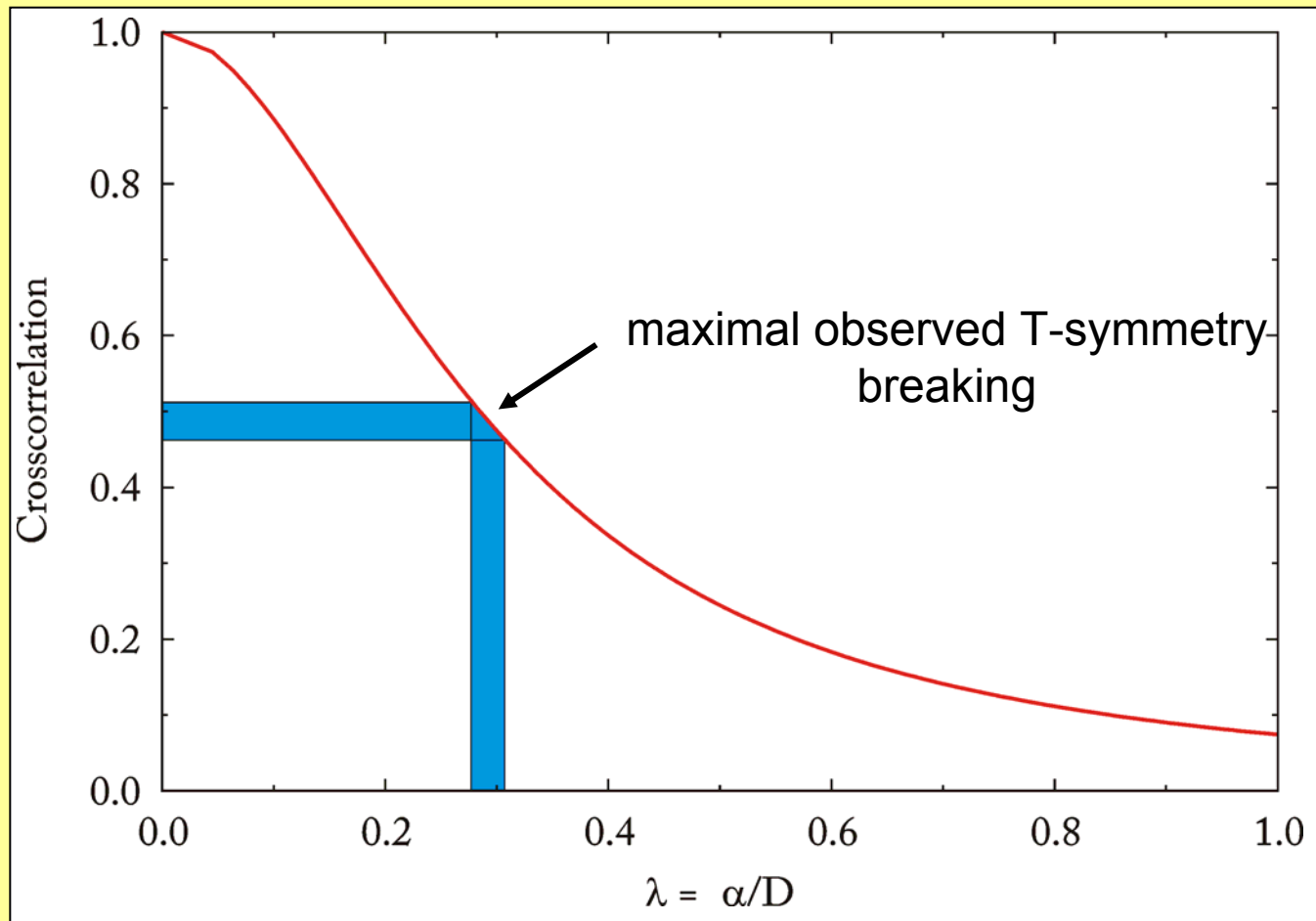
● Data: TRSB is incomplete  $\rightarrow$  mixed GOE / GUE system

# Test of VWZ and FSS / V Models



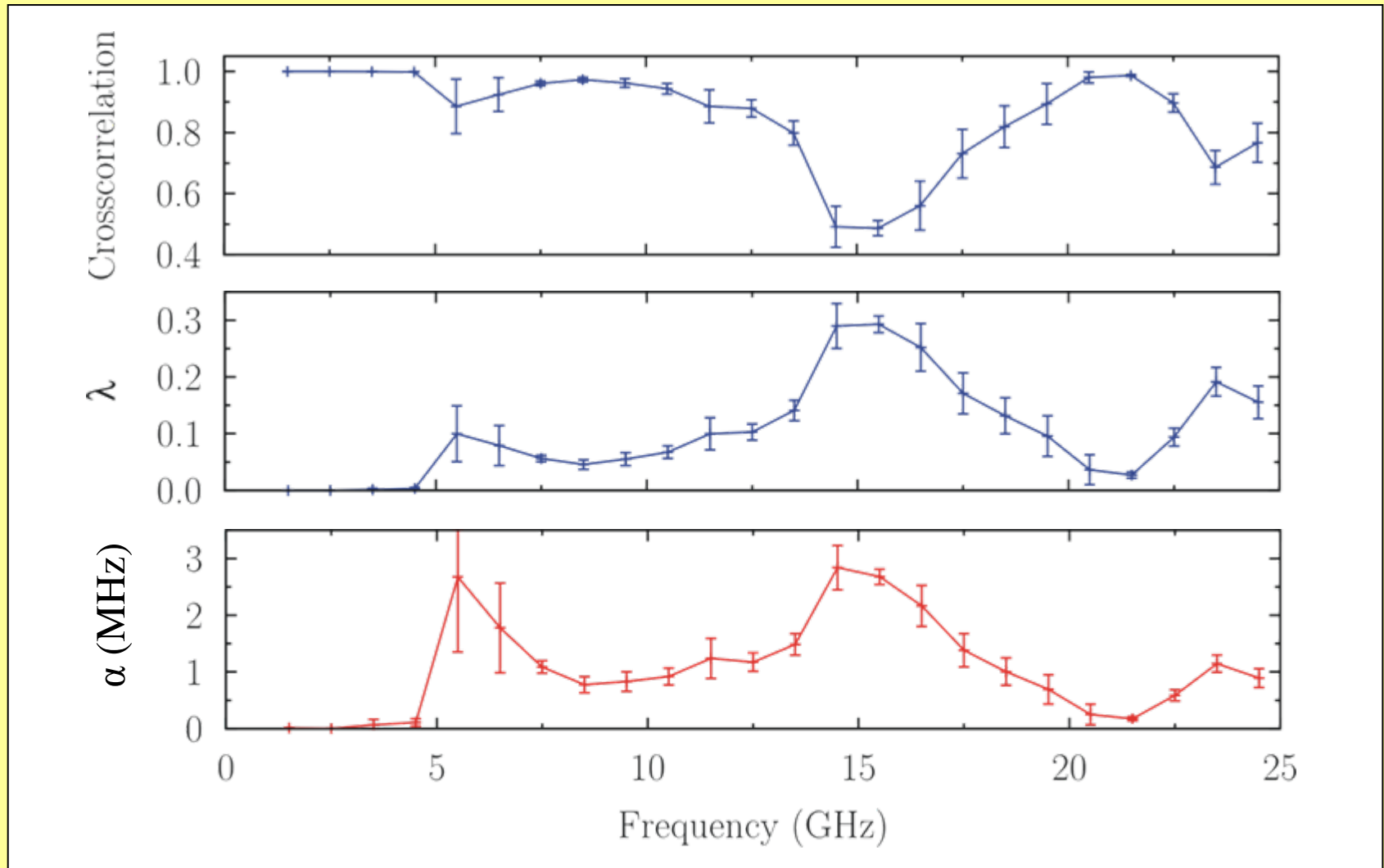
- Autocorrelation functions of S-matrix fluctuations can be described by VWZ for weak TRSB and by FSS / V for strong TRSB

# First Approach towards the TRSB Matrix Element based on RMT



- RMT  $\rightarrow \hat{H} = \hat{H}^s + i\alpha\hat{H}^a$ 
  - $\alpha = 0$  GOE
  - $\alpha = 1$  GUE
- Full T-breaking already sets in for  $\alpha \approx D$

# Determination of the rms Value of T-breaking Matrix Element



# Summary

- Investigated a chaotic T-invariant microwave resonator (i.e. a GOE system) in the regime of weakly overlapping resonances ( $\Gamma \approx D$ )
- Distributions of S-matrix elements are not Gaussian
- However, distribution of the 2400 uncorrelated Fourier coefficients of the scattering matrix is Gaussian
- Data are limited by rather small FRD errors, not by noise
- Data were used to test VWZ theory of chaotic scattering and the predicted non-exponential decay in time of resonator modes and the frequency dependence of the elastic enhancement factor are confirmed
- The most stringent test of the theory yet uses this large number of data points and a goodness-of-fit test

## Summary ctd.

- Investigated furthermore a chaotic T-noninvariant microwave resonator (i.e. a GUE system) in the regime of weakly overlapping resonances
- Principle of reciprocity is strongly violated ( $S_{ab} \neq S_{ba}$ )
- Data show, however, that TRSB is incomplete  
→ mixed GOE / GUE system
- Data were subjected to tests of VWZ theory (GOE) and FFS / V theory (GUE) of chaotic scattering
- S-matrix fluctuations are described in spectral regions of weak TRSB by VWZ and for strong TRSB by FSS / V
- Analytical model for partial TRSB is under development
- First approach using RMT shows that full TRSB sets already in when the symmetry breaking matrix element is of the order of the mean level spacing of the overlapping resonances