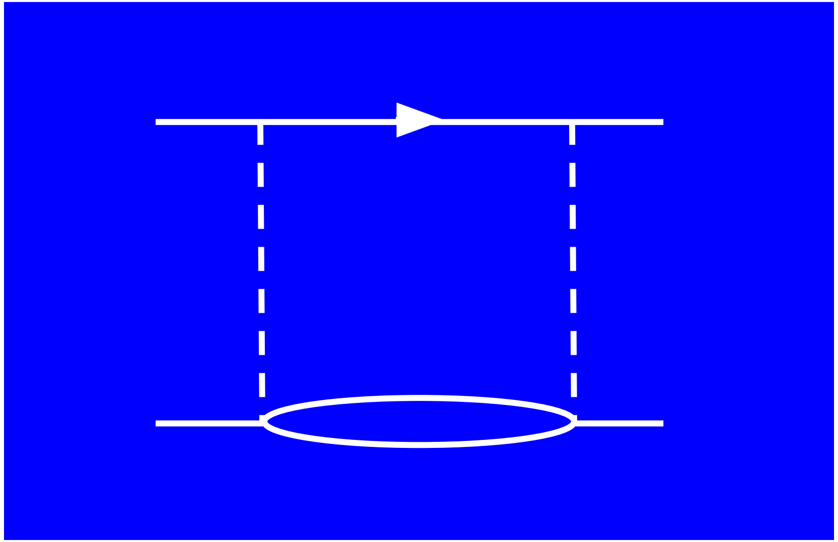
# Muonic deuterium results:
# Nuclear Structure Corrections from TPE




**In collaboration with:**
**Andreas Ekström**
**Nir Nevo Dinur**
**Chen Ji**
**Sonia Bacca**
**Nir Barnea**

THE UNIVERSITY OF BRITISH COLUMBIA

TRIUMF

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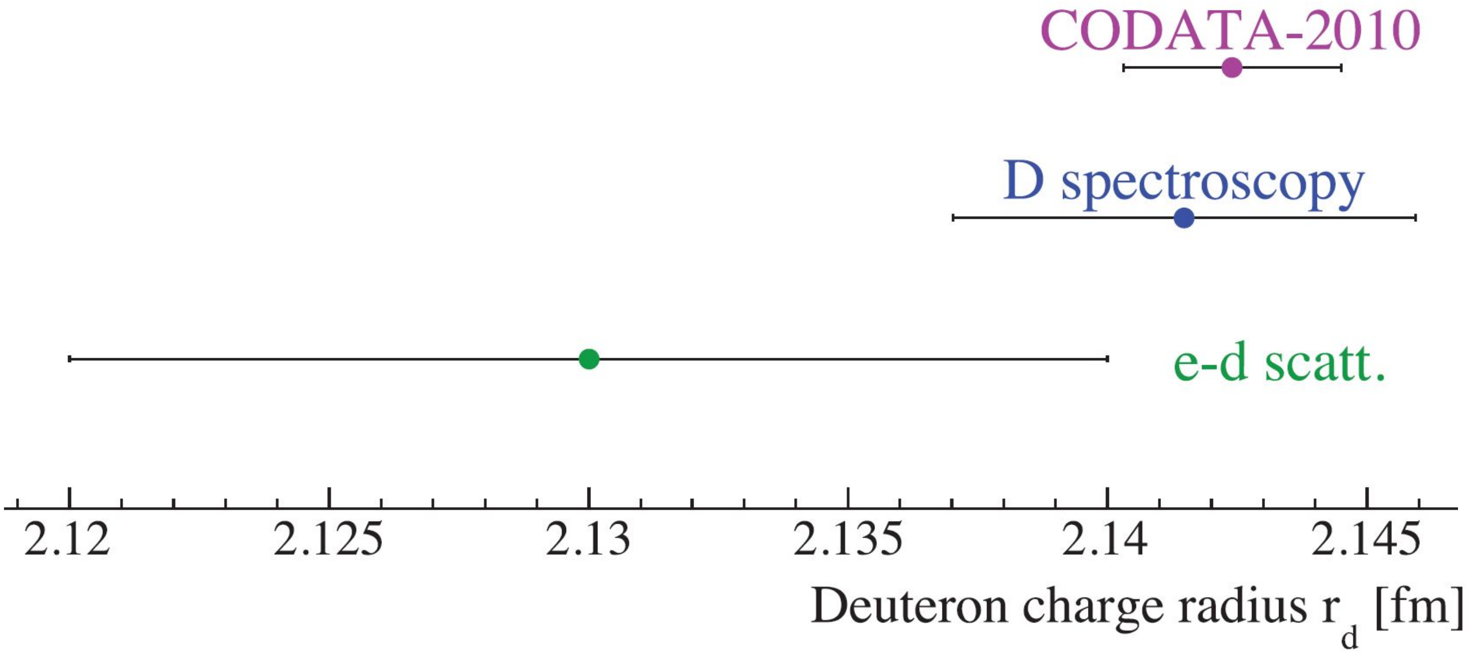


[Pohl *et. al.* Science, Vol 353, 6300, 2016]

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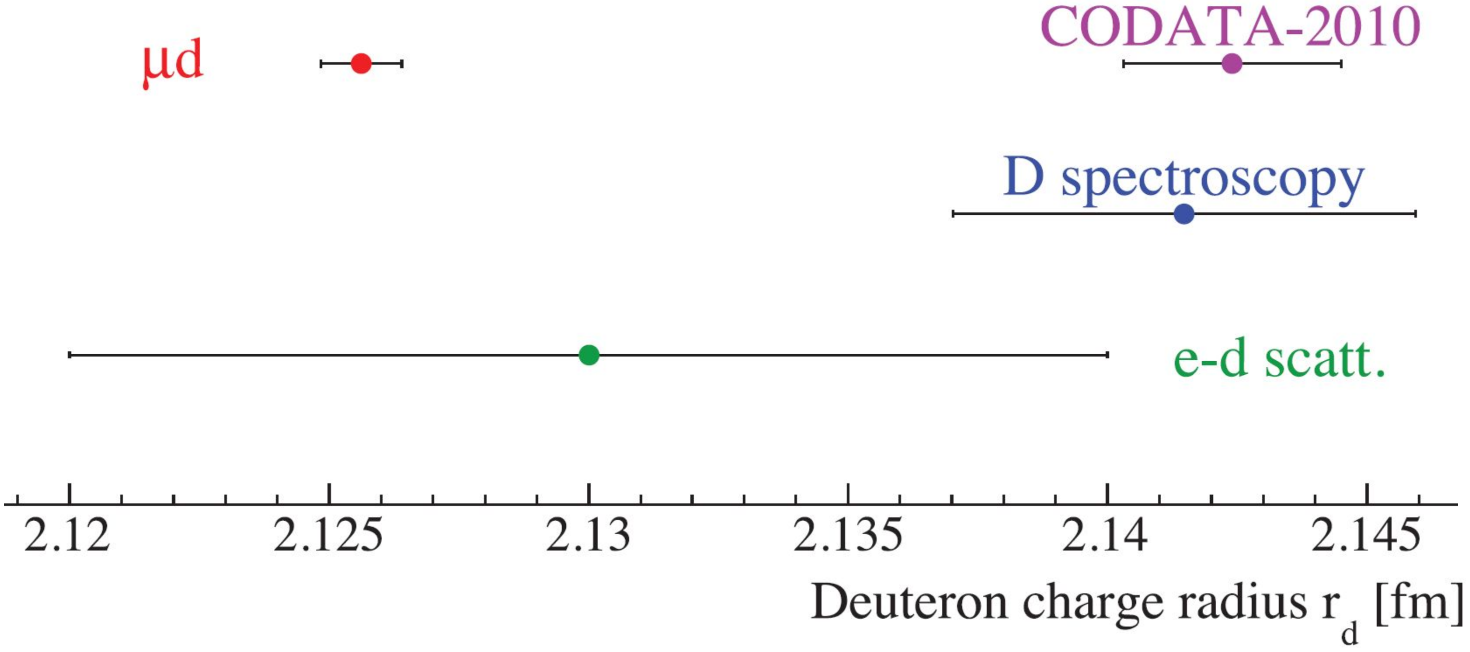


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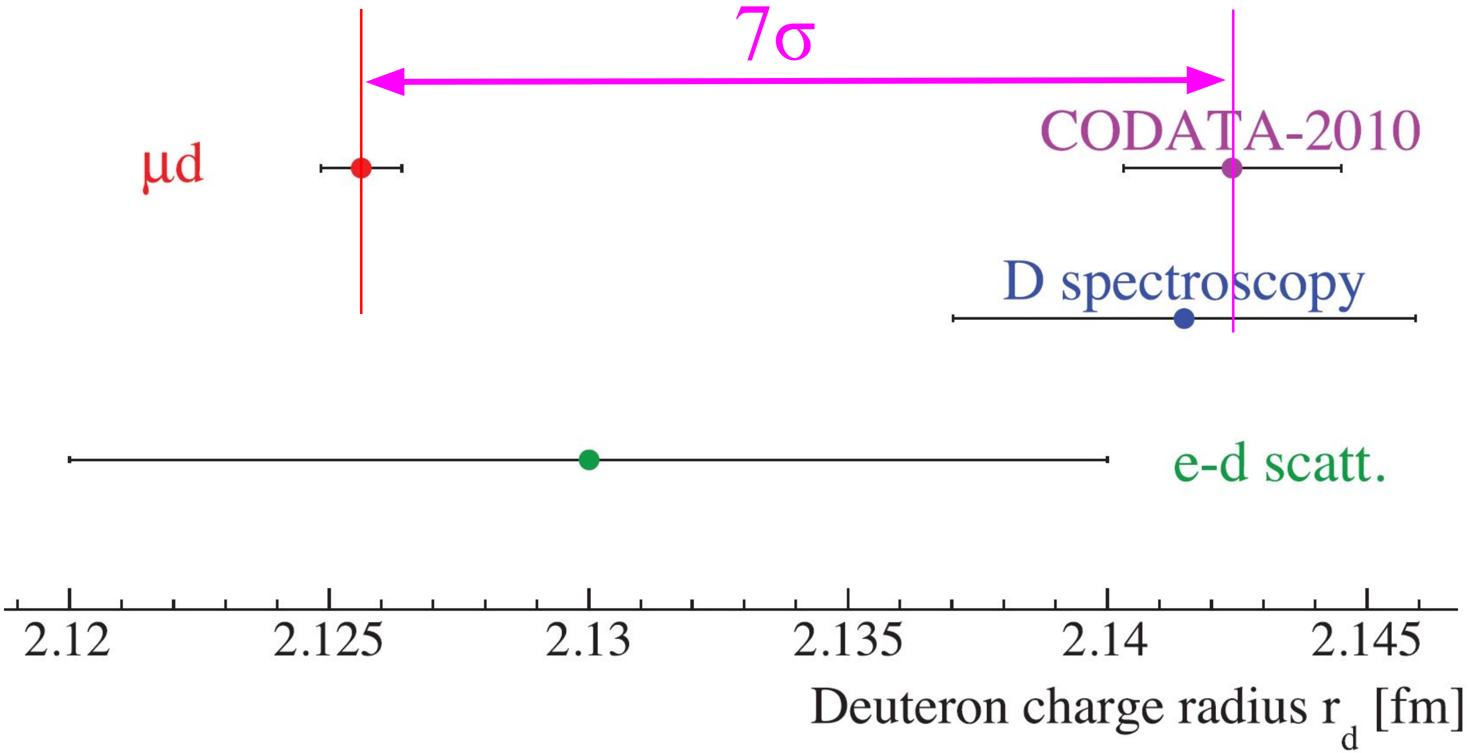


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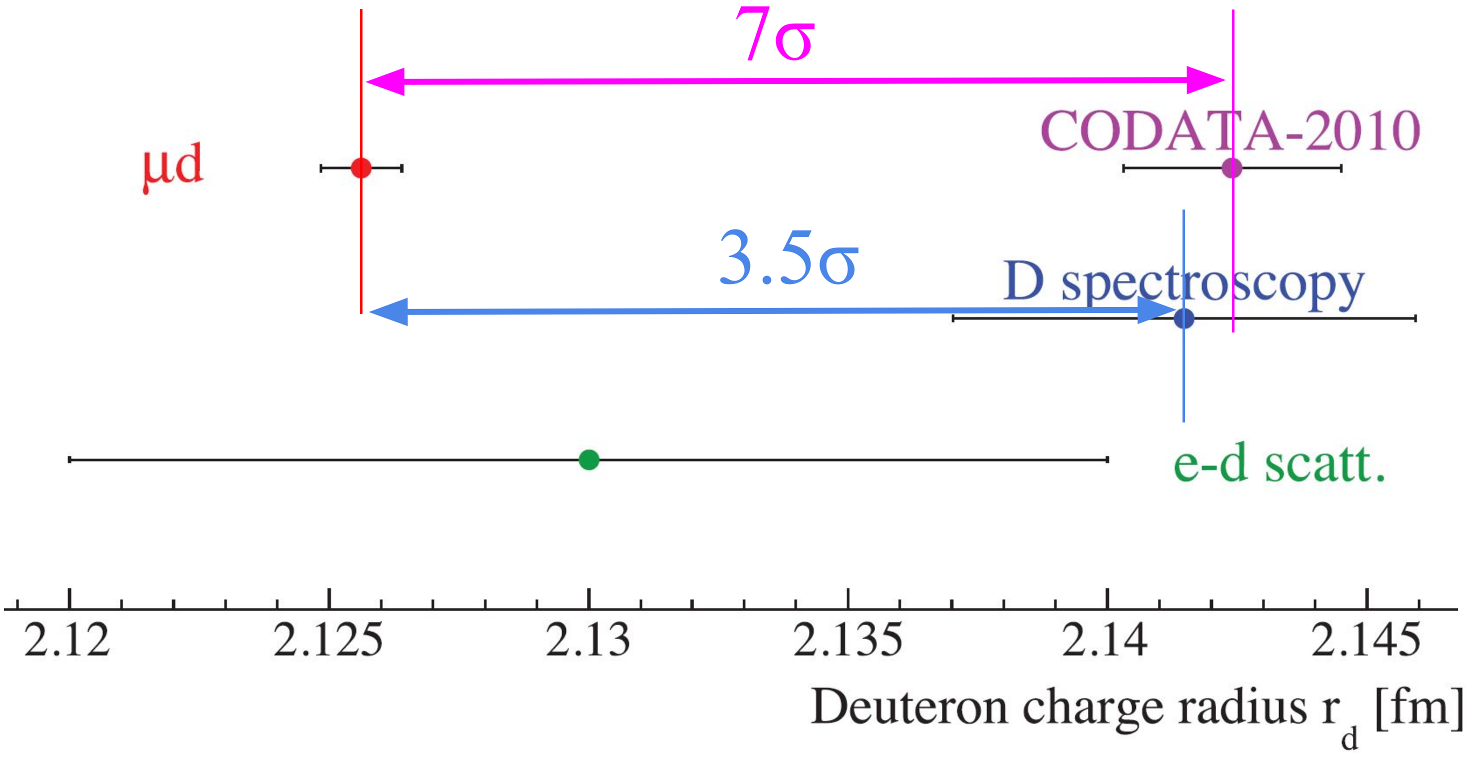


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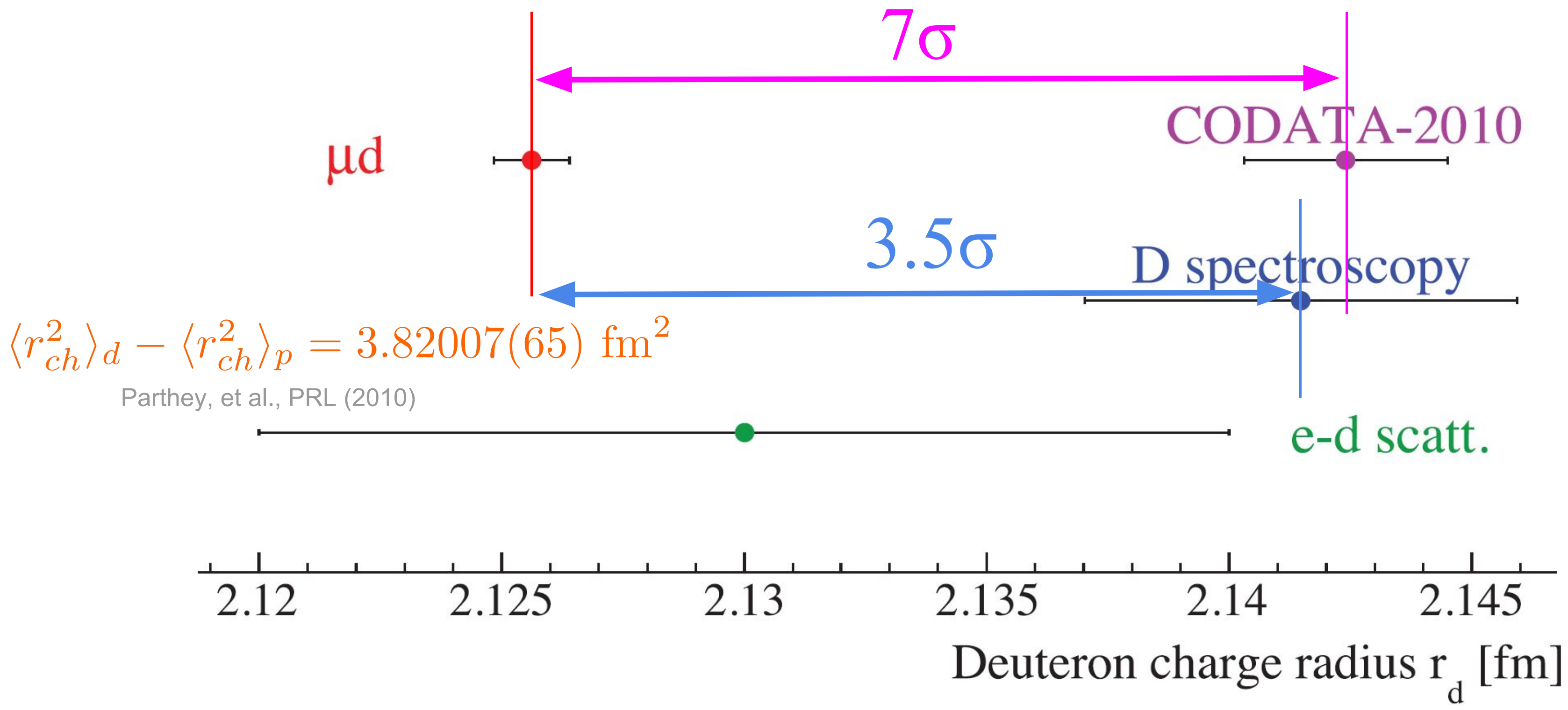


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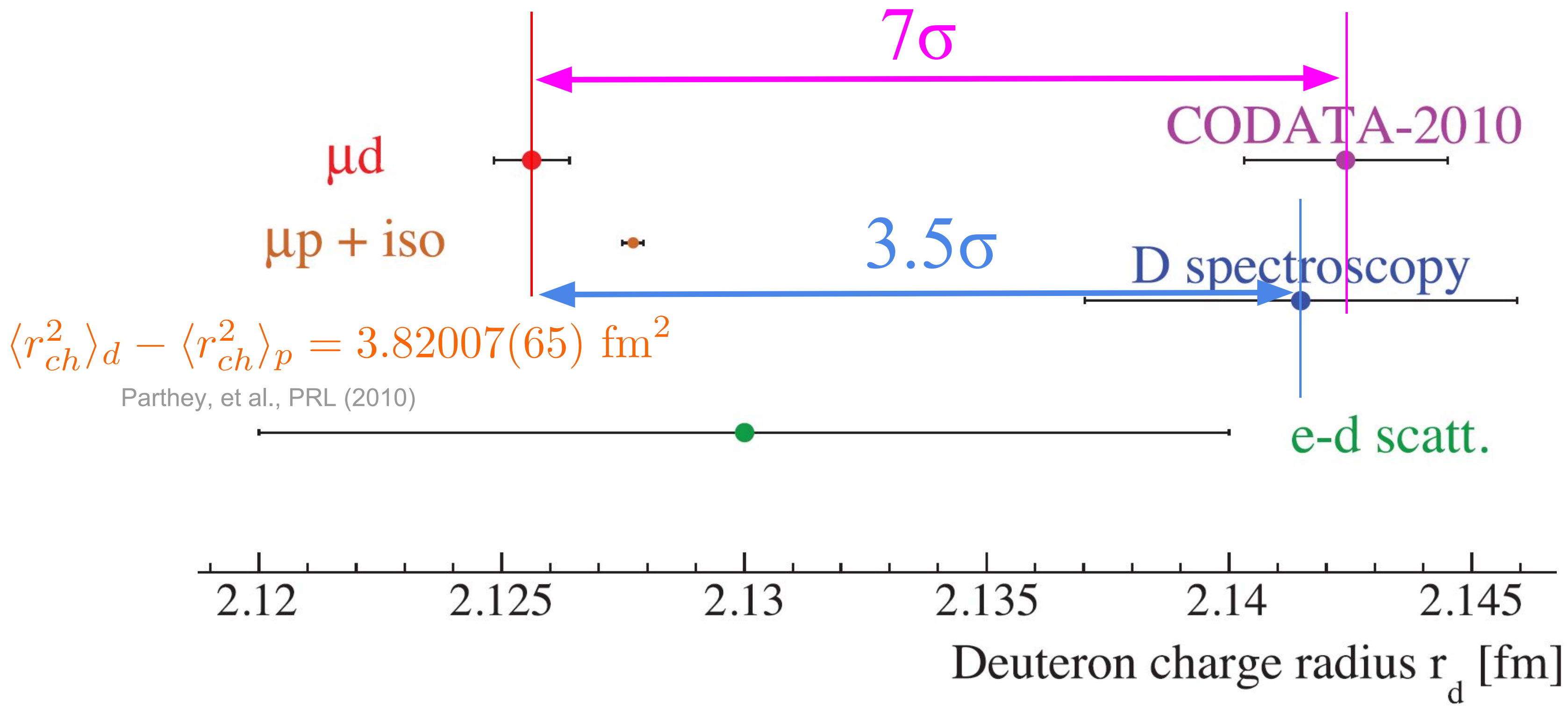


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$$\Delta E_{LS} = \delta_{QED} + \delta_{TPE} + \delta_{FS}(R_c).$$

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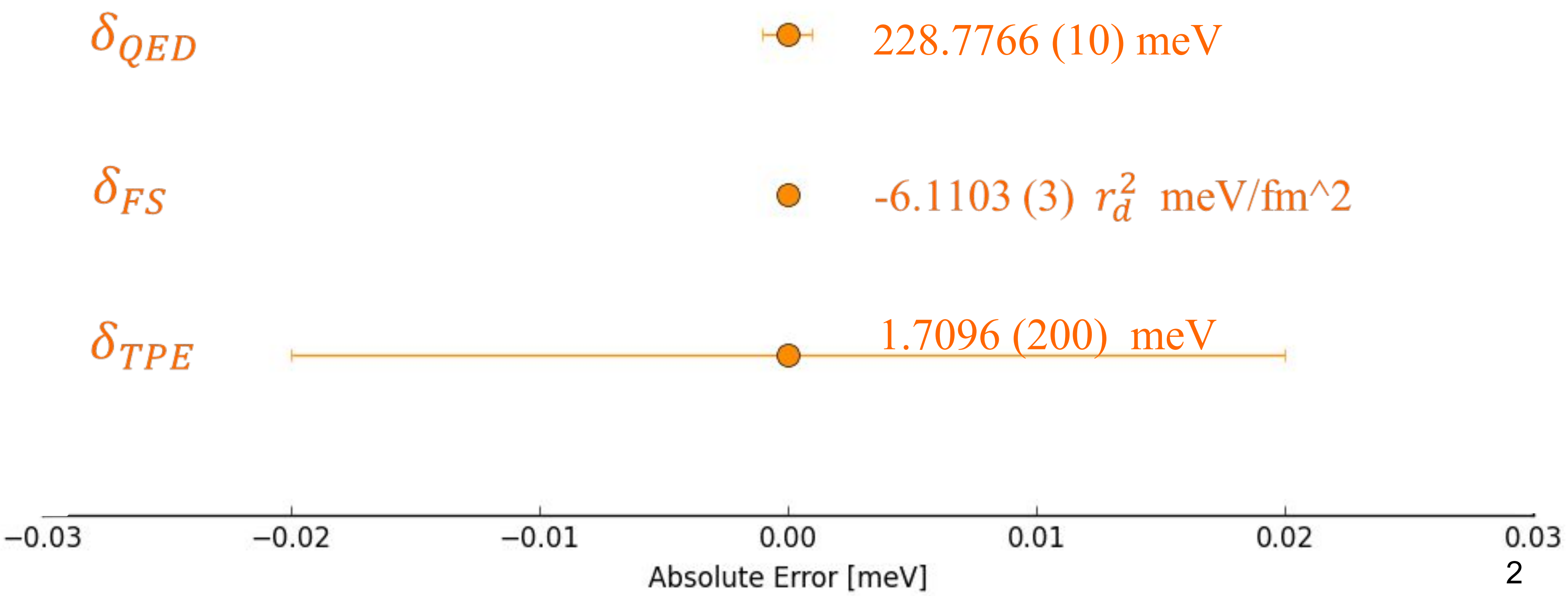

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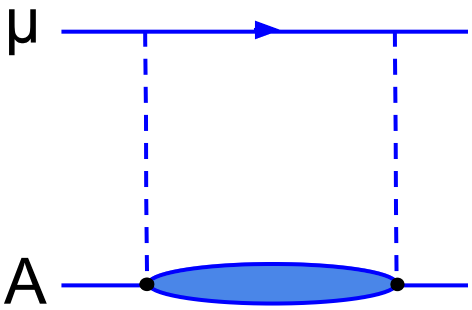
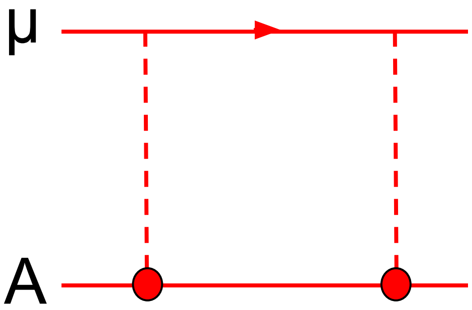
Nuclear Nucleonic




$\delta^{A}_{TPE}$




$\delta^{N}_{TPE}$

# Deuteron Calculations

- Expand the Schrödinger equation in the harmonic oscillator basis and diagonalize

$$\{N_{max}, \hbar\Omega\}$$

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- Expand the Schrödinger equation in the harmonic oscillator basis and diagonalize

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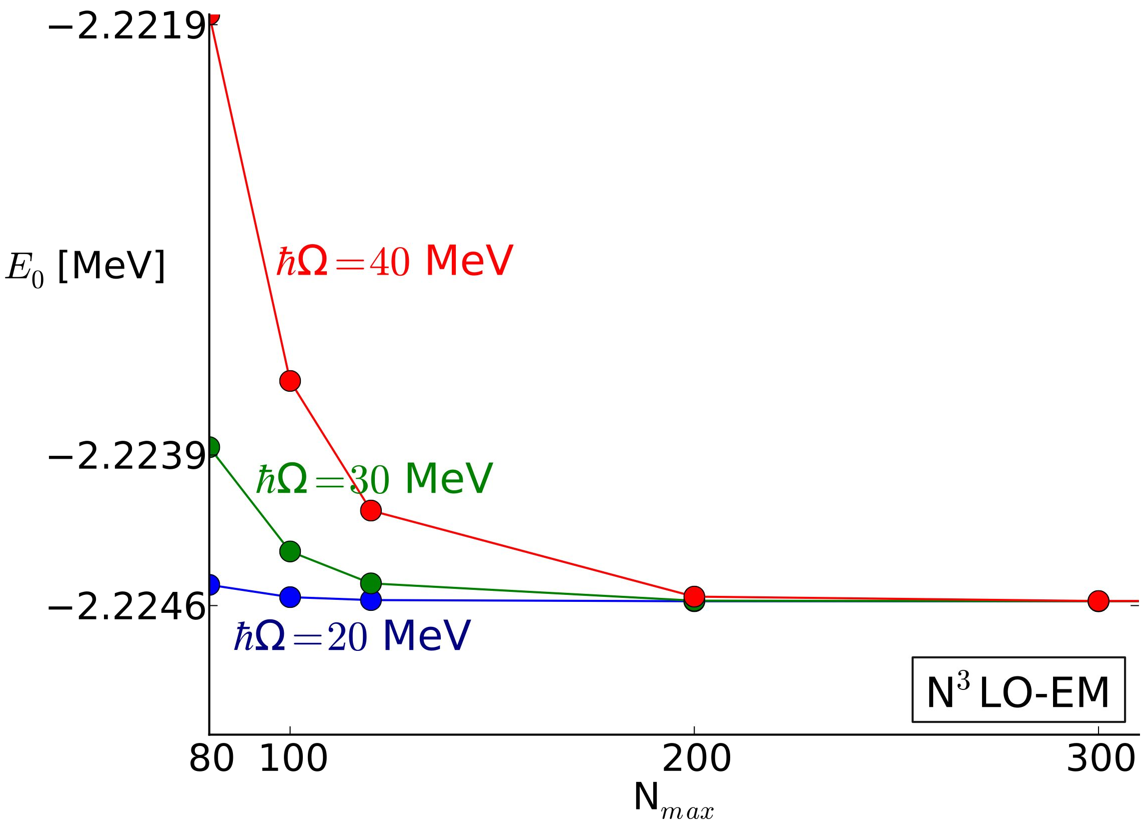
- Results independent on the model space size, and the HO frequency

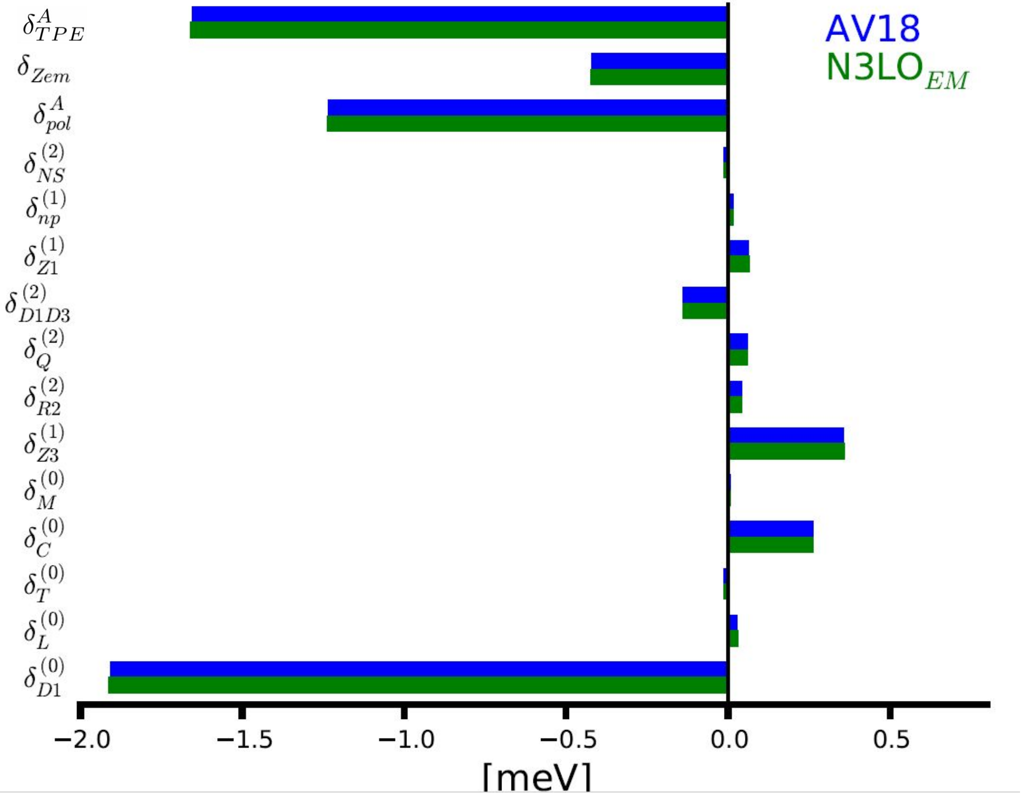


# Deuteron Calculations

- Benchmark with available literature

| | | $E_0$ [MeV] | $\langle r^2_{str}\rangle_d^{1/2}$ [fm] | $Q_d$ [fm$^2$] | $P_D$ [%] |
|---|---|---|---|---|---|
| **N3LO** | **This Work** | **2.2246** | **1.978** | **0.285** | **4.51** |
| | Entem *et al* [1] | 2.2246 | 1.978 | 0.285 | 4.51 |
| **AV18** | **This Work** | **2.2246** | **1.967** | **0.270** | **5.76** |
| | Wiringa *et al* [2] | 2.2246 | 1.967 | 0.270 | 5.76 |

# Calculation of TPE




- Each correction is an integral over the response

$$\delta \propto \int d\omega \, g(\omega) S_{\hat{O}}(\omega)$$

[Hernandez et. al, 2014,2016]

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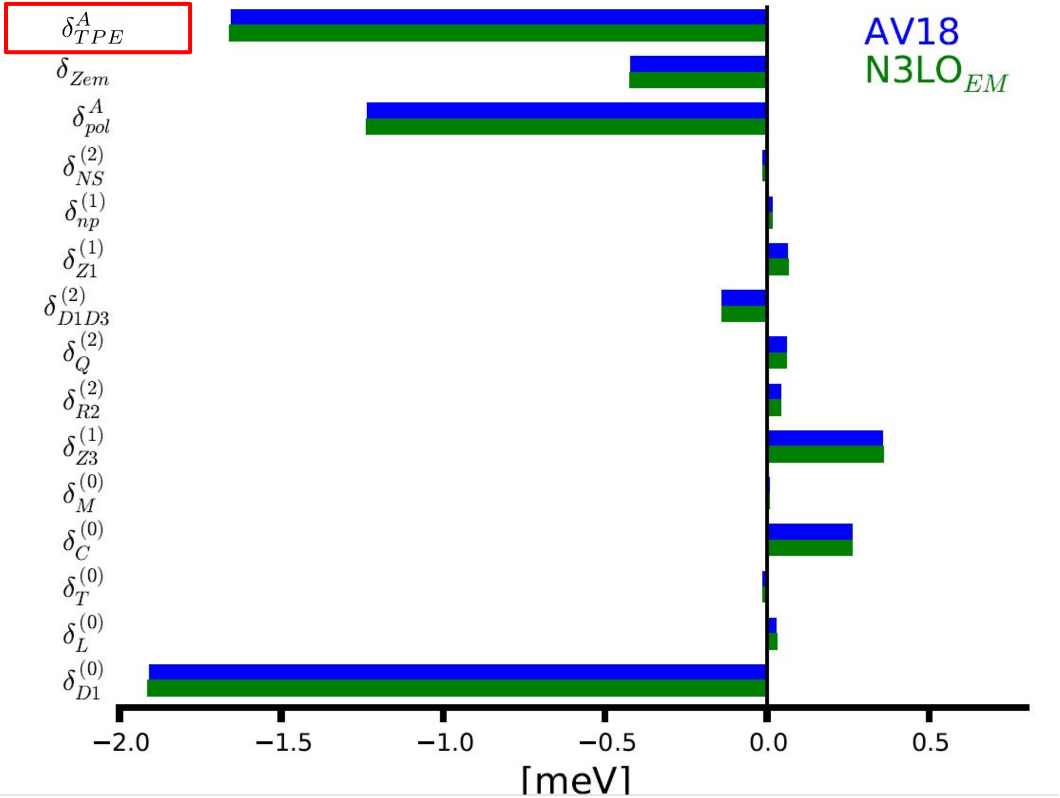


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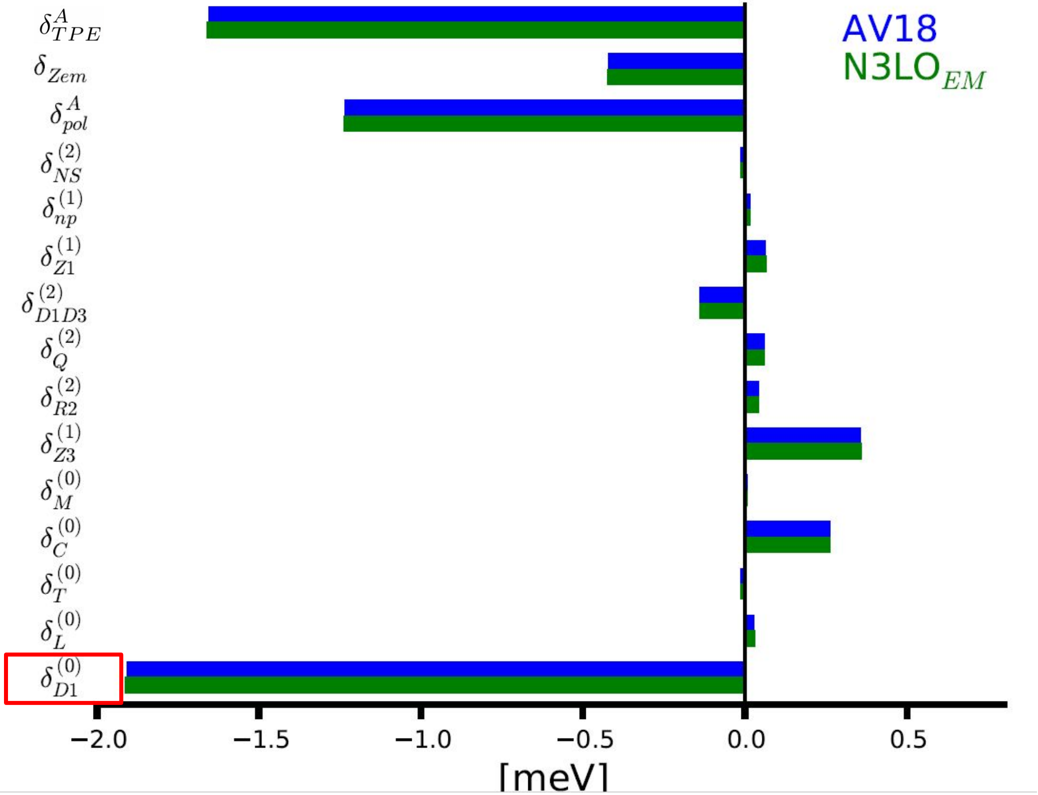


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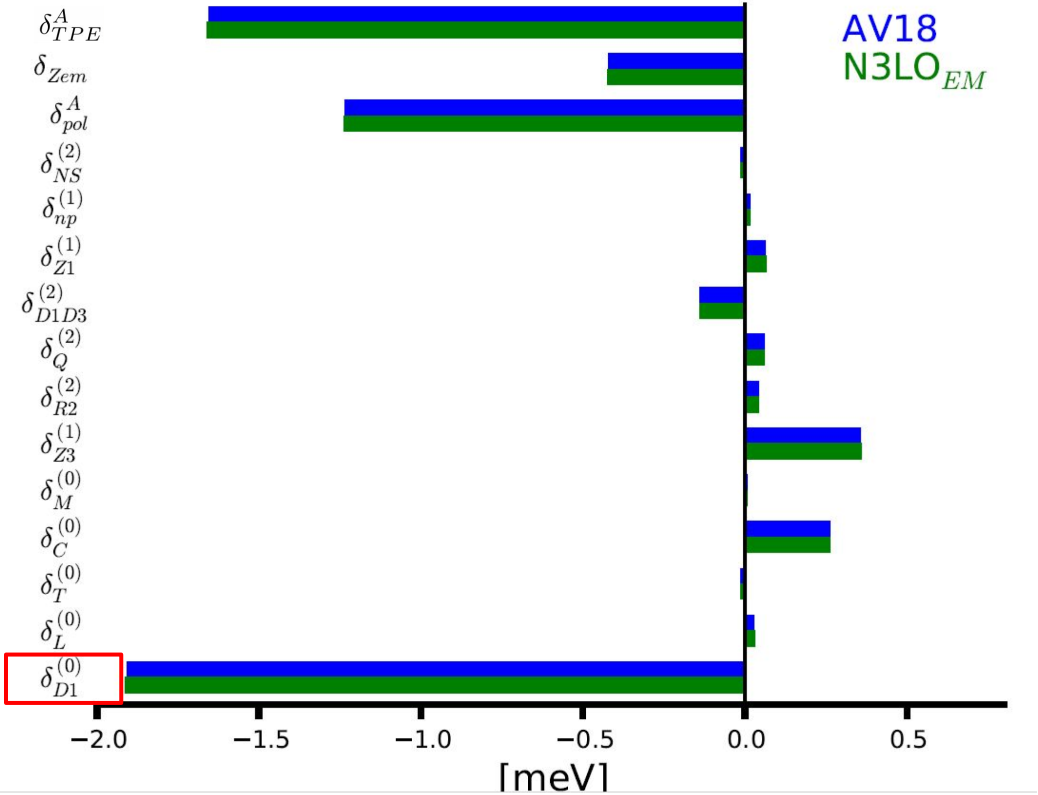


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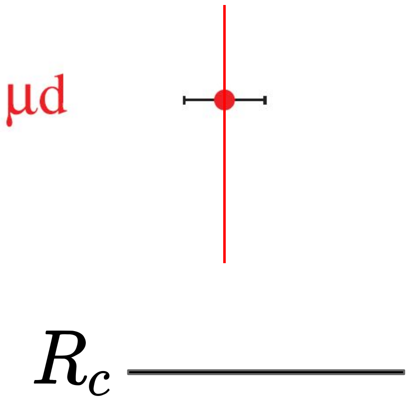
$$\delta_{TPE} = \delta^A_{TPE} + \delta^N_{TPE}$$

$$\delta_{TPE}(Our\ Work) = -1.718(22)\ meV$$

[Hernandez et. al, 2014,2016]

# TPE discrepancy

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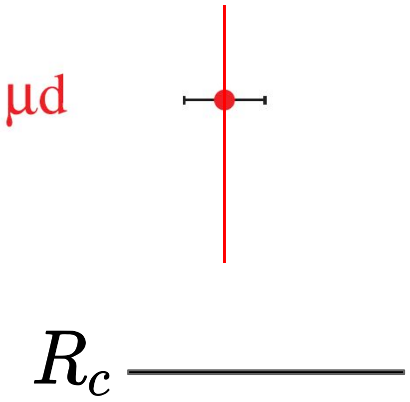


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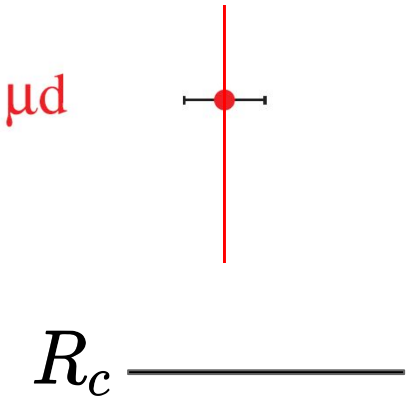


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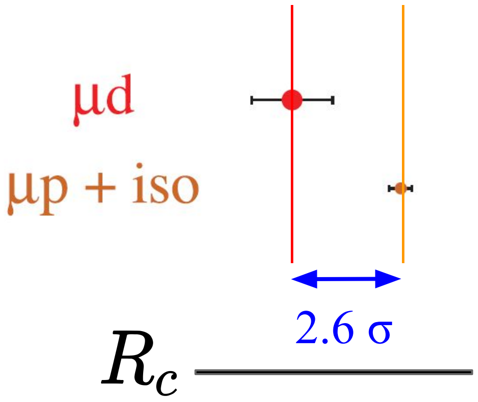


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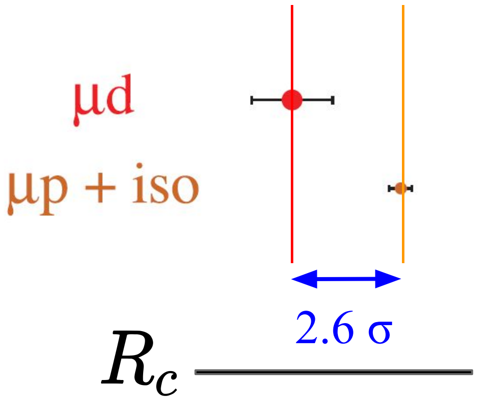


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[Pohl *et. al.* Science, Vol 353, 6300, 2016]

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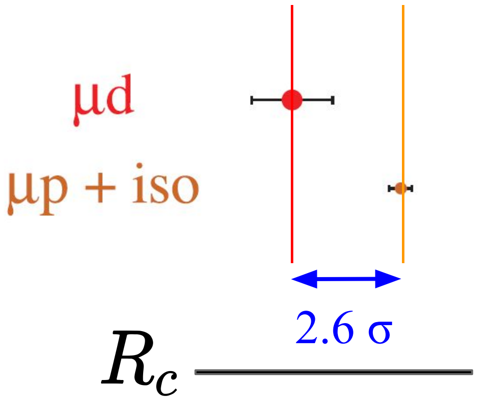


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- Theoretical TPE is 6 times larger than experimental uncertainty

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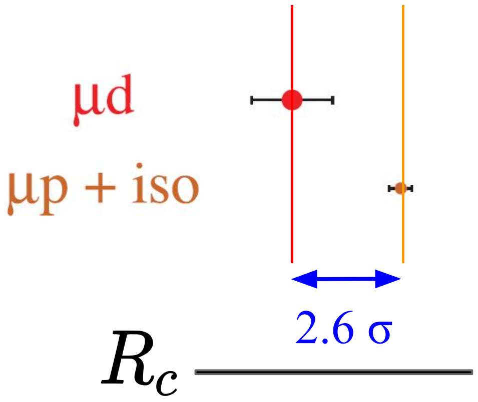


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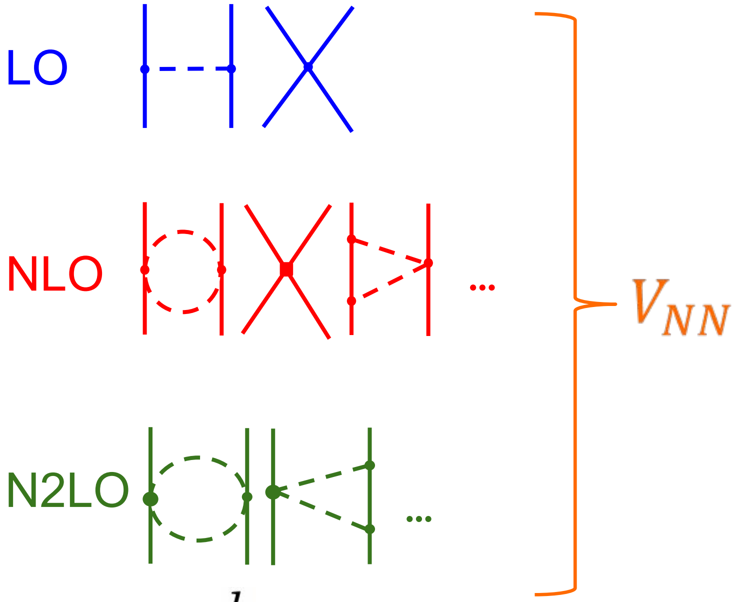
- Theoretical TPE is 6 times larger than experimental uncertainty
- A thorough analysis may change our ~1% uncertainty and shed light on disagreement in $\delta_{TPE}$

# Improving the uncertainty estimates

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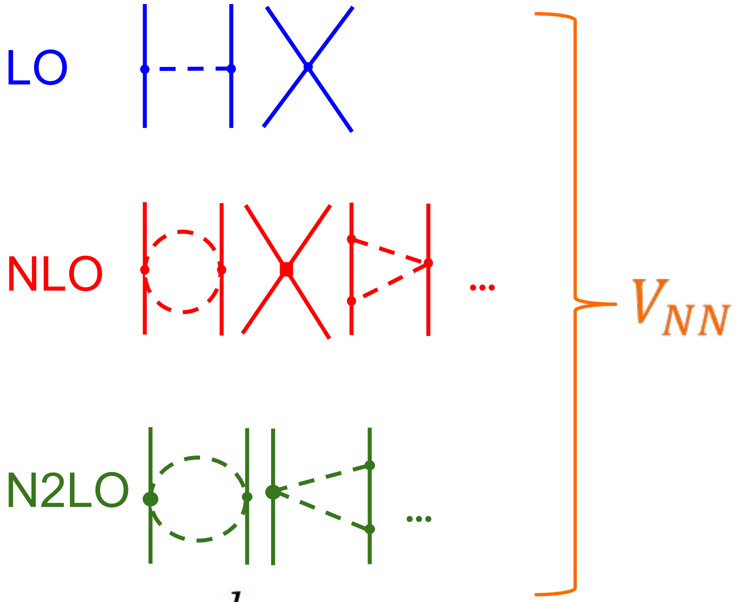
$$Obs(c_\mu, \Lambda, T_{Lab}^{Max}, k)$$

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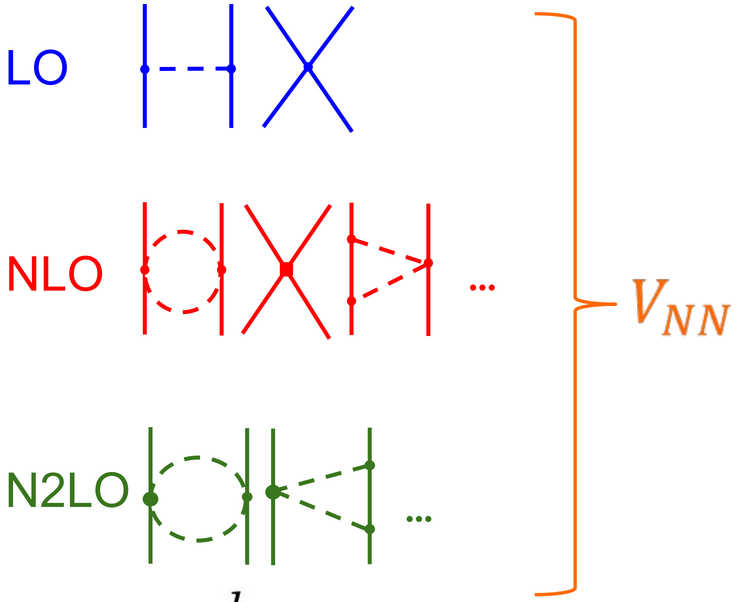


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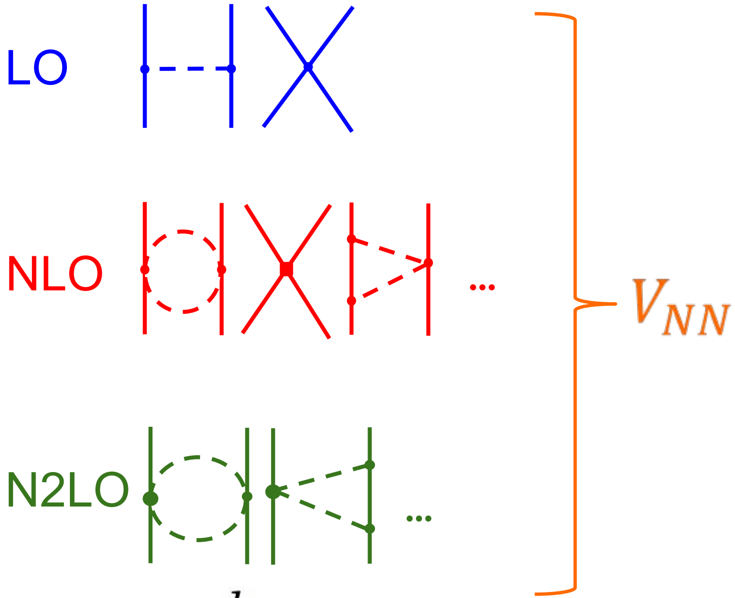


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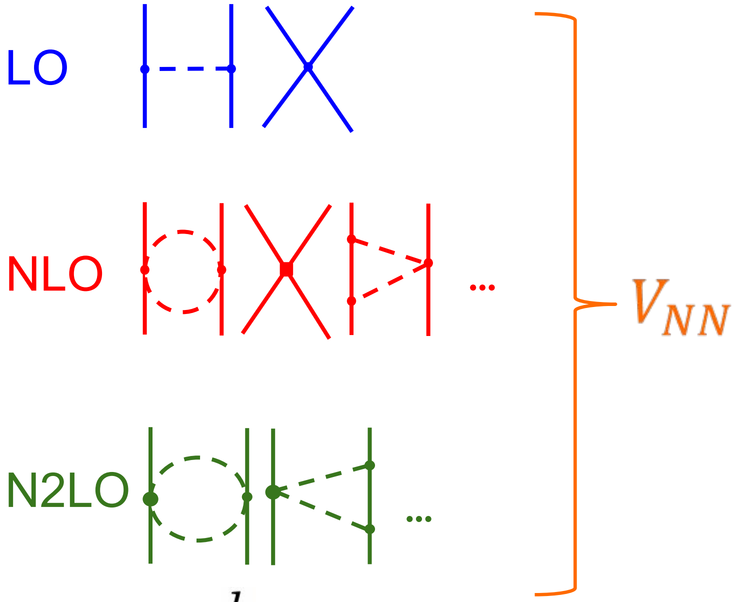


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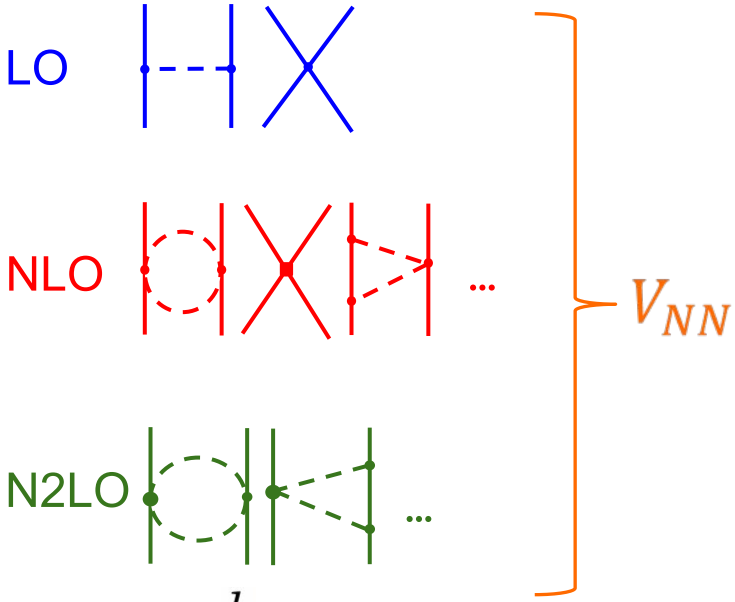


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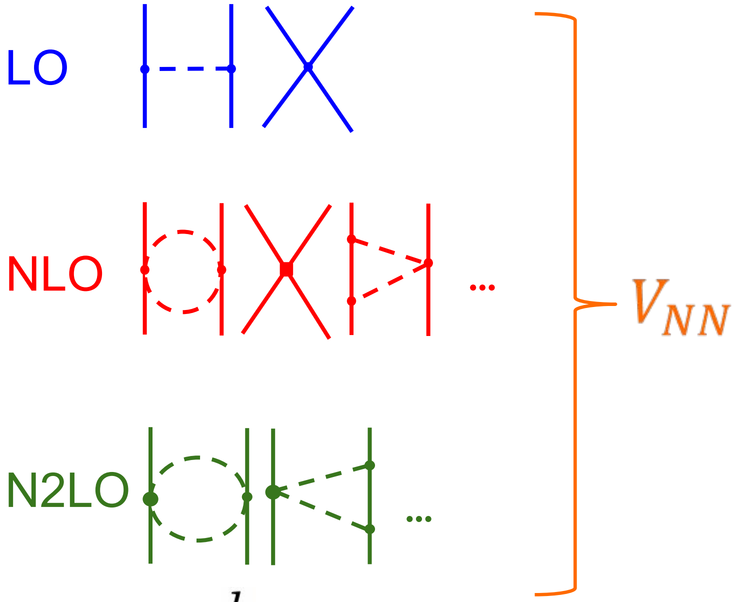


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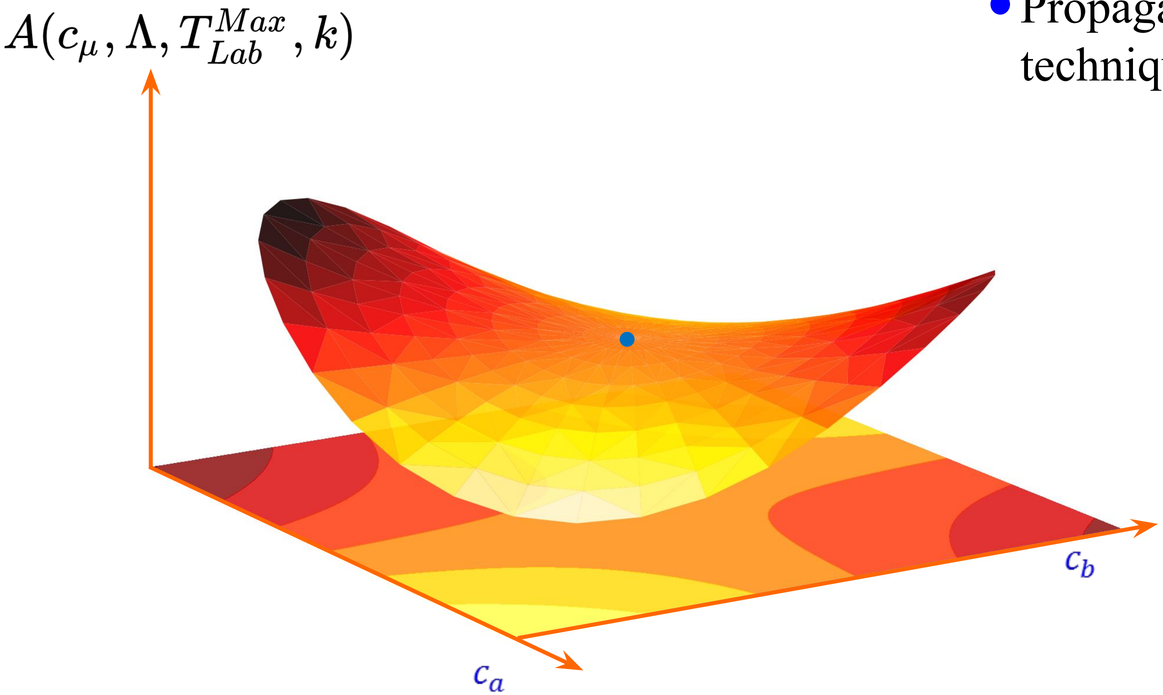
Single Nucleon: $\delta_{TPE}^{N}$

Higher Order Corrections: $O(\alpha^6)$

Ekström et al., PRL (2013), JPG (2015), Carlsson et al., PRX (2016)

# Statistical uncertainties




- Propagate uncertainty using standard techniques

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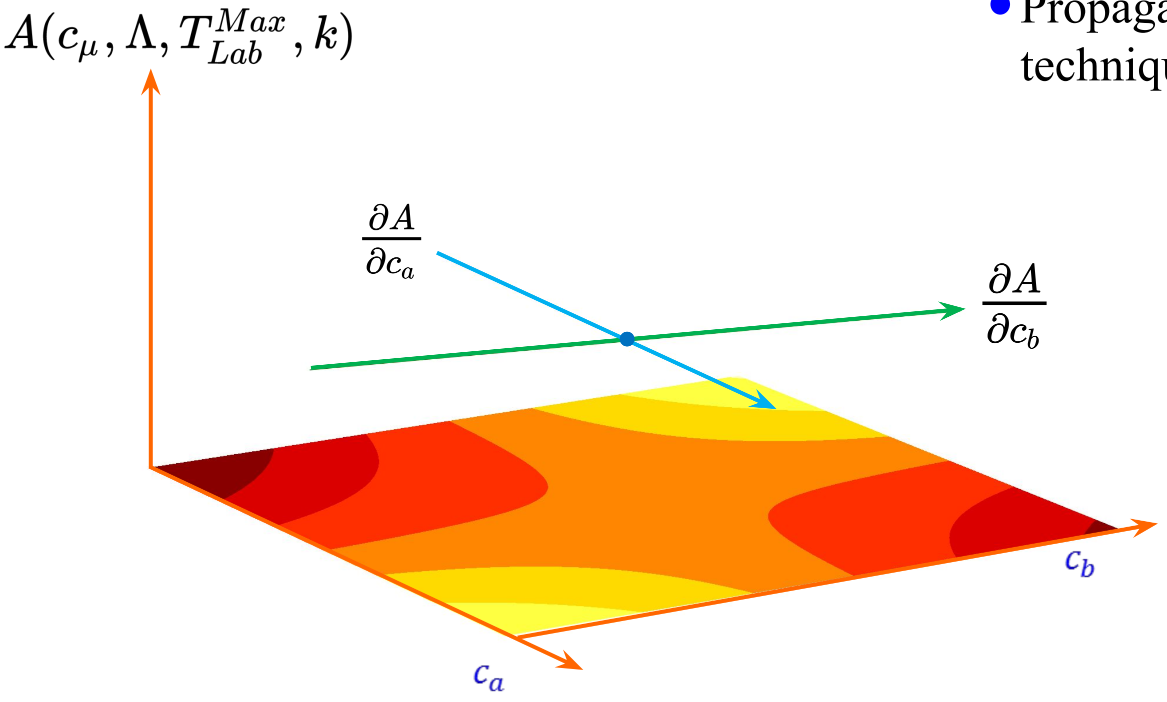


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$$J_{A,i} = \frac{\partial A}{\partial c_{\mu,i}}$$

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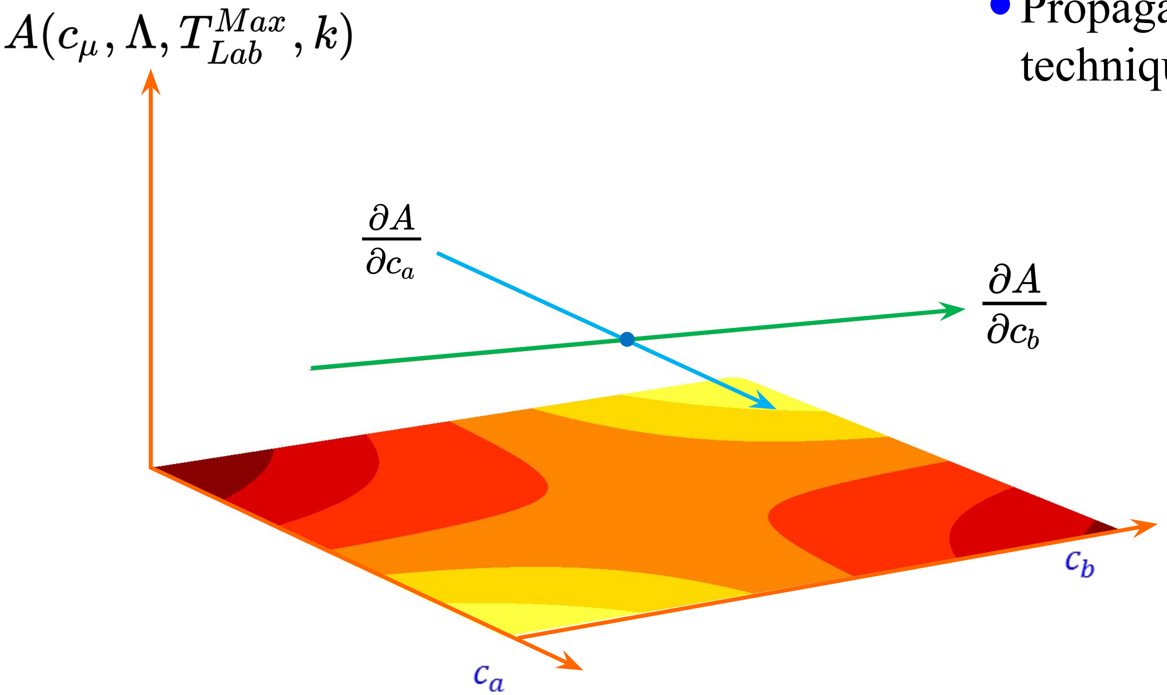


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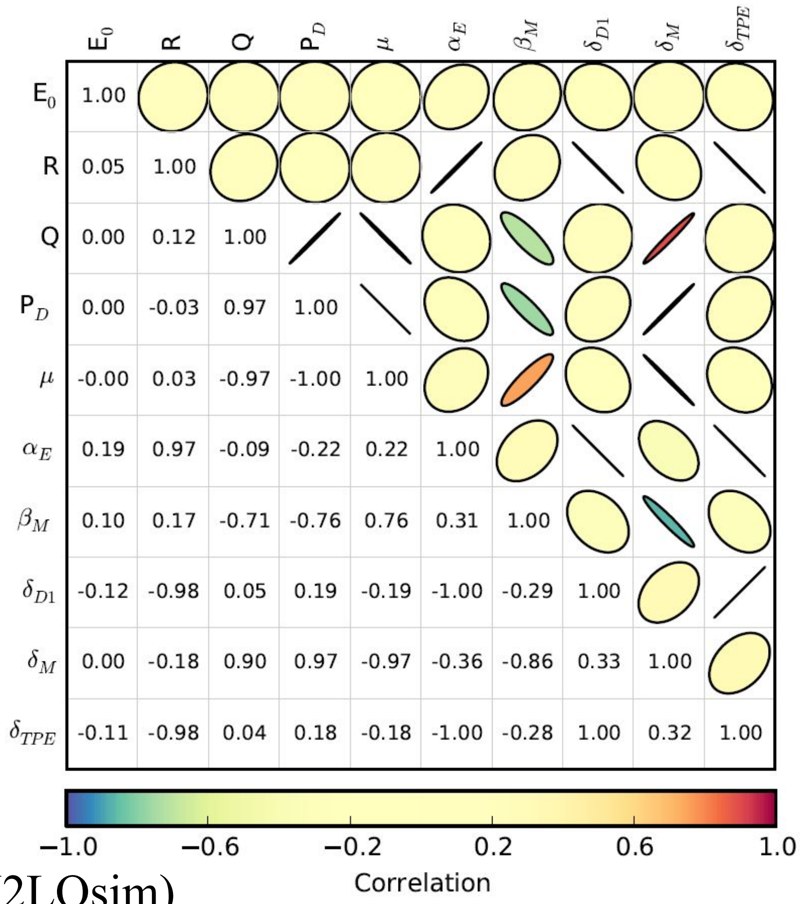
$$J_{A,i} = \frac{\partial A}{\partial c_{\mu,i}}$$

$$\mathrm{Cov}(A,B) = \boldsymbol{J}_A \mathrm{Cov}(c_\mu) \boldsymbol{J}_B^T$$

$$\sigma_{A,stat} = \sqrt{\mathrm{Cov}(A,A)}$$

# Correlation analysis




(N2LOsim)

- Serves as a check of the error propagation formalism

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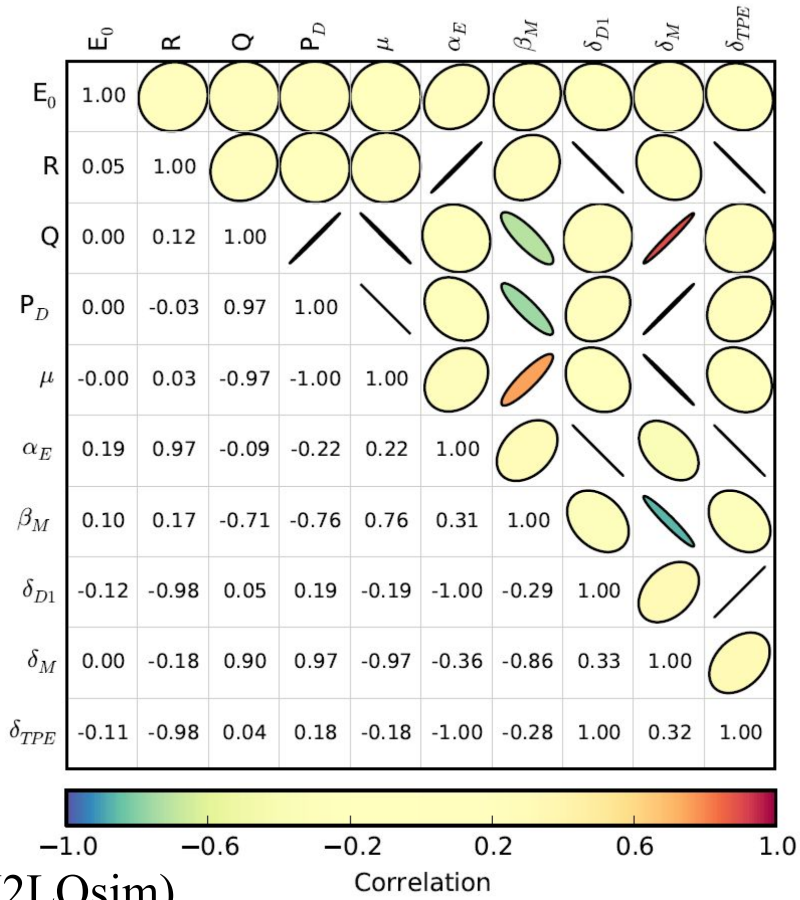


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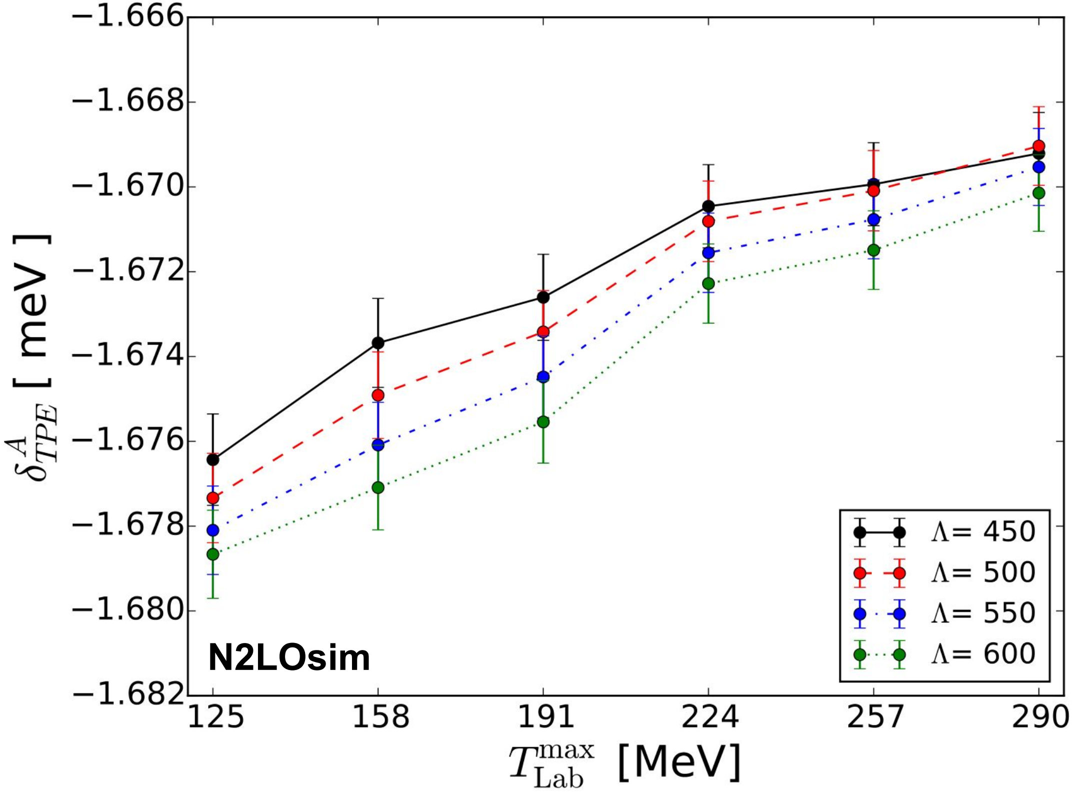
- We observe strong correlations between
  - $\{P_d, \mu_d\}$
  - $\{R(^2\mathrm{H}), \alpha_E\}$
  - $\{R(^2\mathrm{H}), \delta_{TPE}\}$

# Statistical uncertainties



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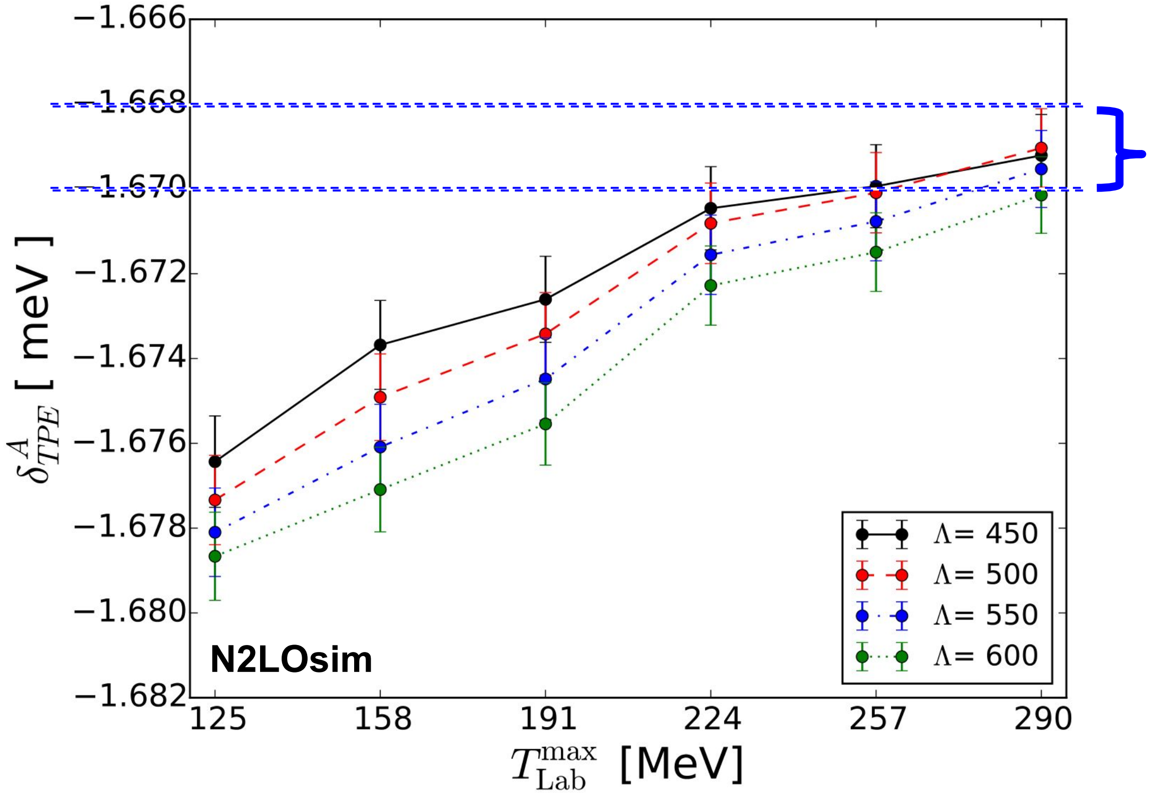
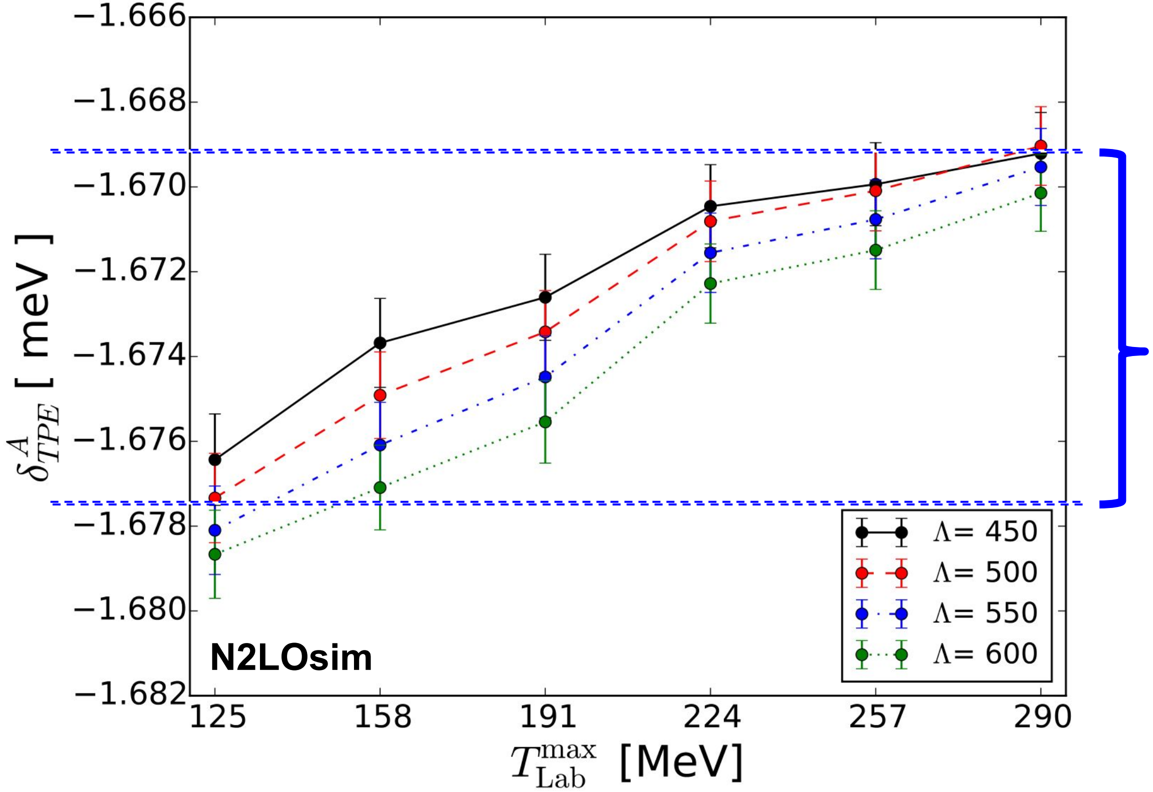



Statistical uncert. $c_\mu$

| Correction | % Uncert. |
| --- | --- |
| **Statistical** | 0.06 |

# Sytematic Tlab Uncertainties

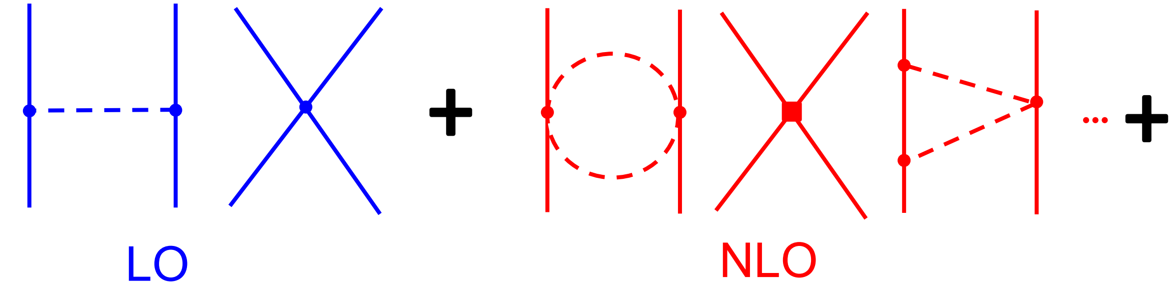



Systematic Tlab uncert.

| Correction | % Uncert. |
|---|---|
| **Statistical** | 0.06 |
| **Tlab Sys.** | 0.2 |

# Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^{k} c_{\nu} \left( \frac{Q}{\Lambda_{\chi}} \right)^{\nu} =$$



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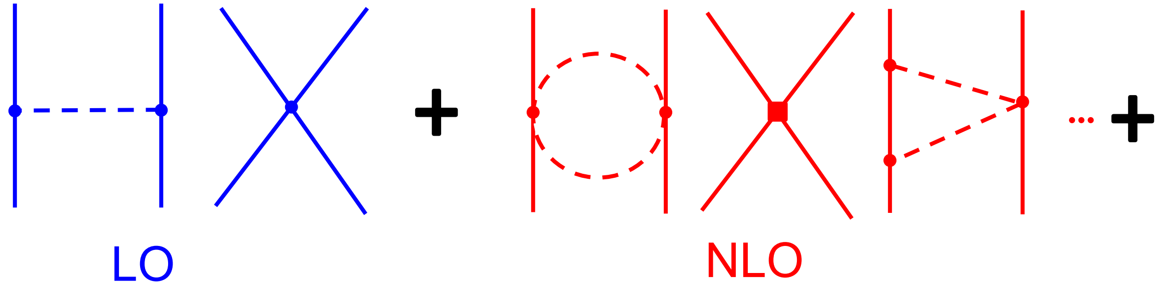


- Expand observable in the same Chiral EFT pattern,

$$A^{N^k LO}(p) = A_0 \sum_{\nu=0}^{k+1} \beta_\nu(p) Q^\nu \qquad Q = max\left\{\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right\}$$

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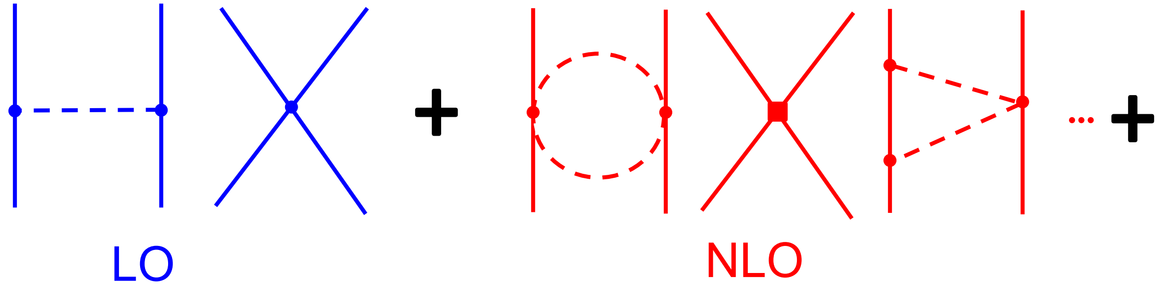


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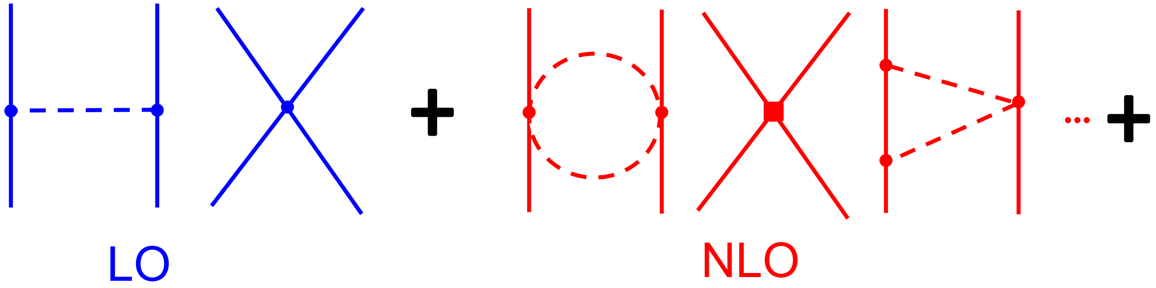


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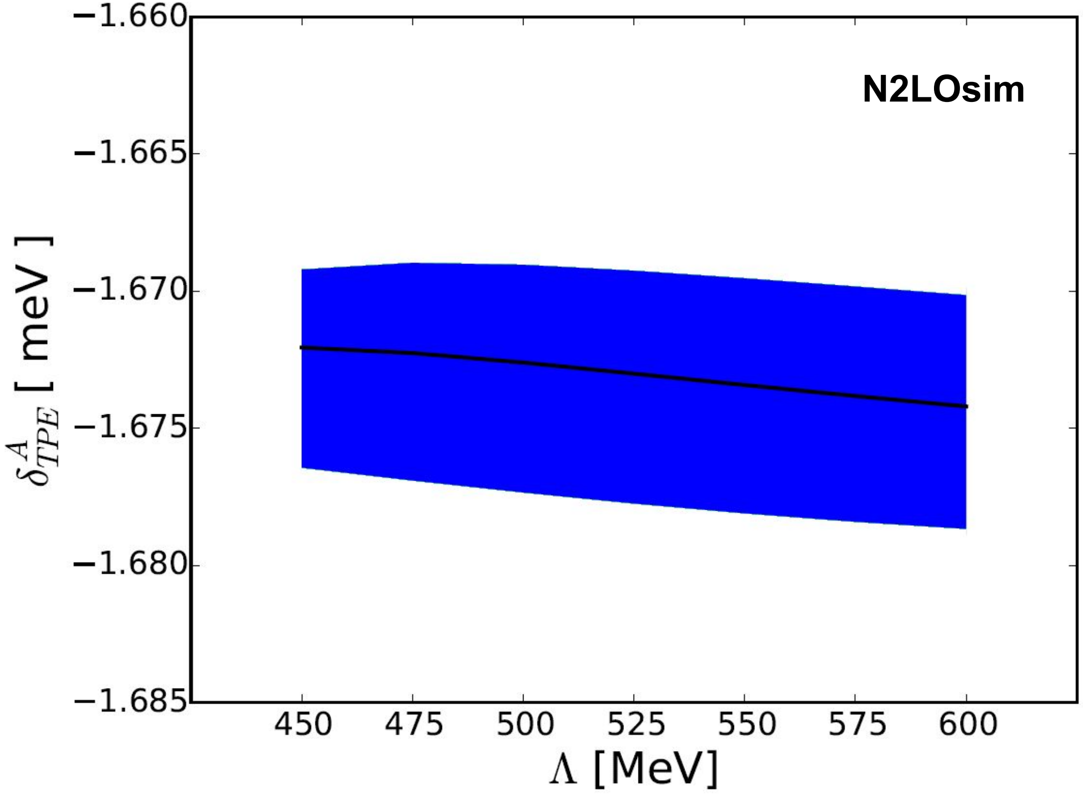
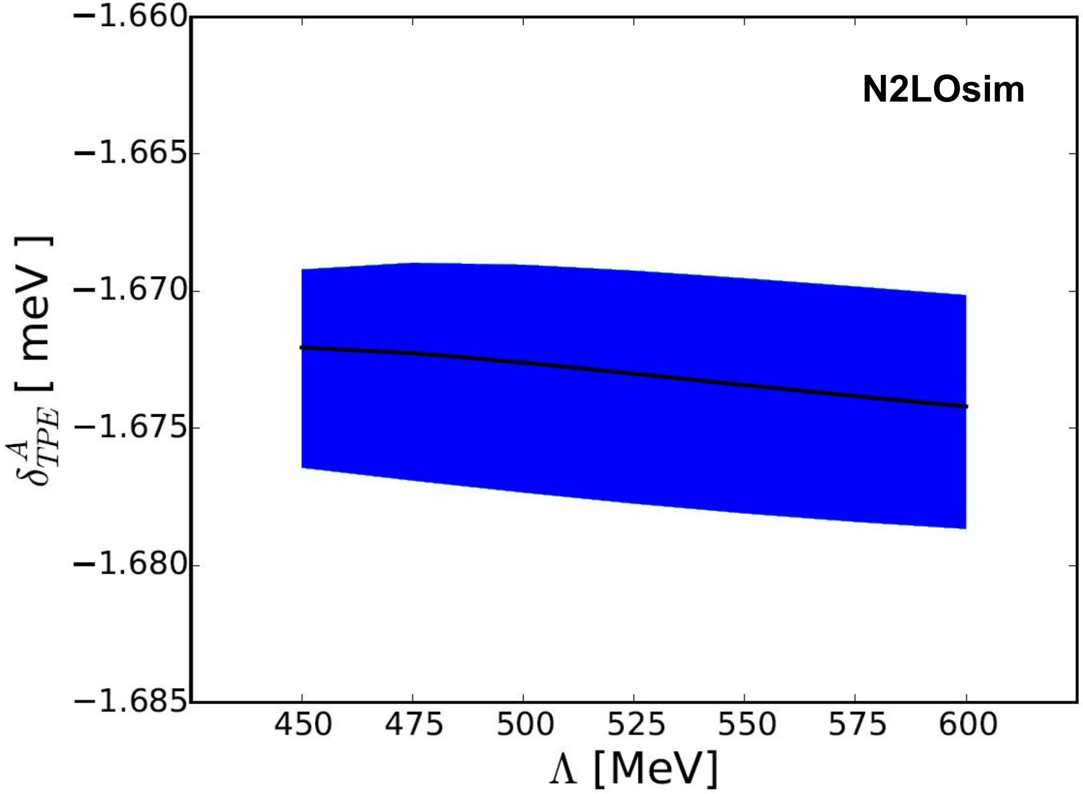
$$\sigma_{A,sys}^{N^kLO}(p) = A_0 Q^{k+2}\ max\{|\beta_0|, ..., |\beta_{k+1}|\}$$

# Chiral truncation uncertainties




- Tlab variation

# Chiral truncation uncertainties

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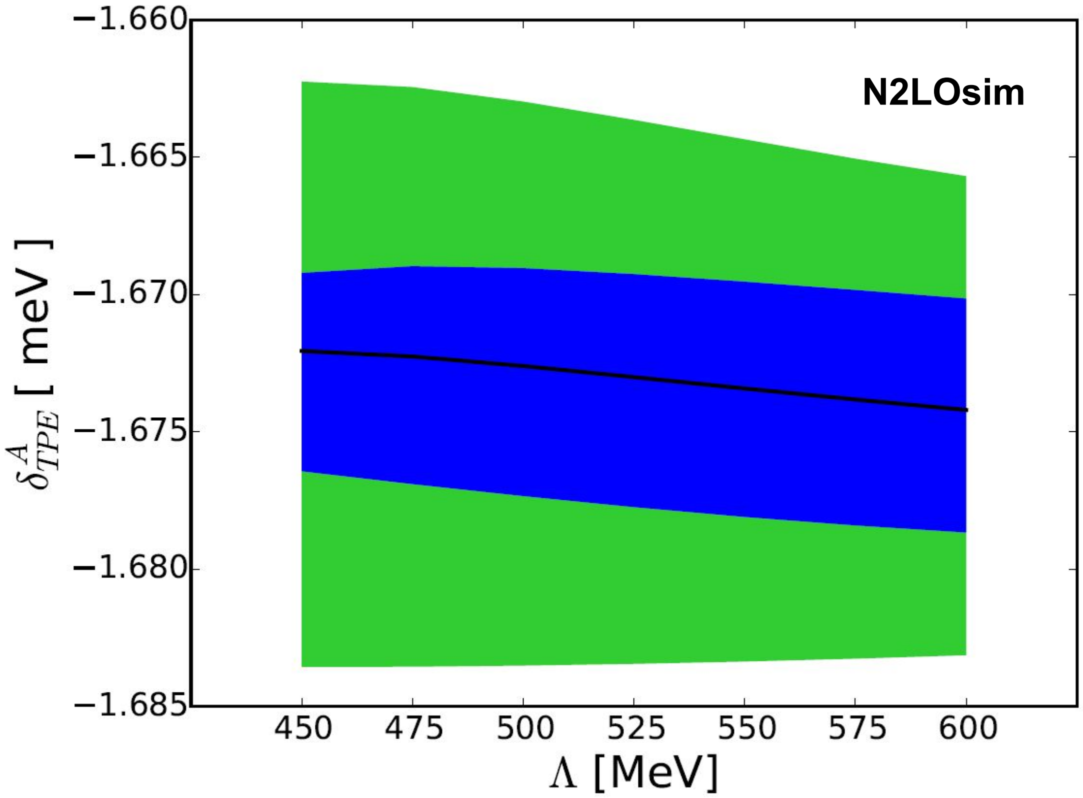


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- Estimate momentum scale of TPE

$$\langle\omega\rangle_{D1} = \frac{\int d\omega\, \omega \sqrt{\frac{2m_r}{\omega_N}}\ S_{D1}(\omega)}{\int d\omega\, \sqrt{\frac{2m_r}{\omega_N}}\ S_{D1}(\omega)}.$$

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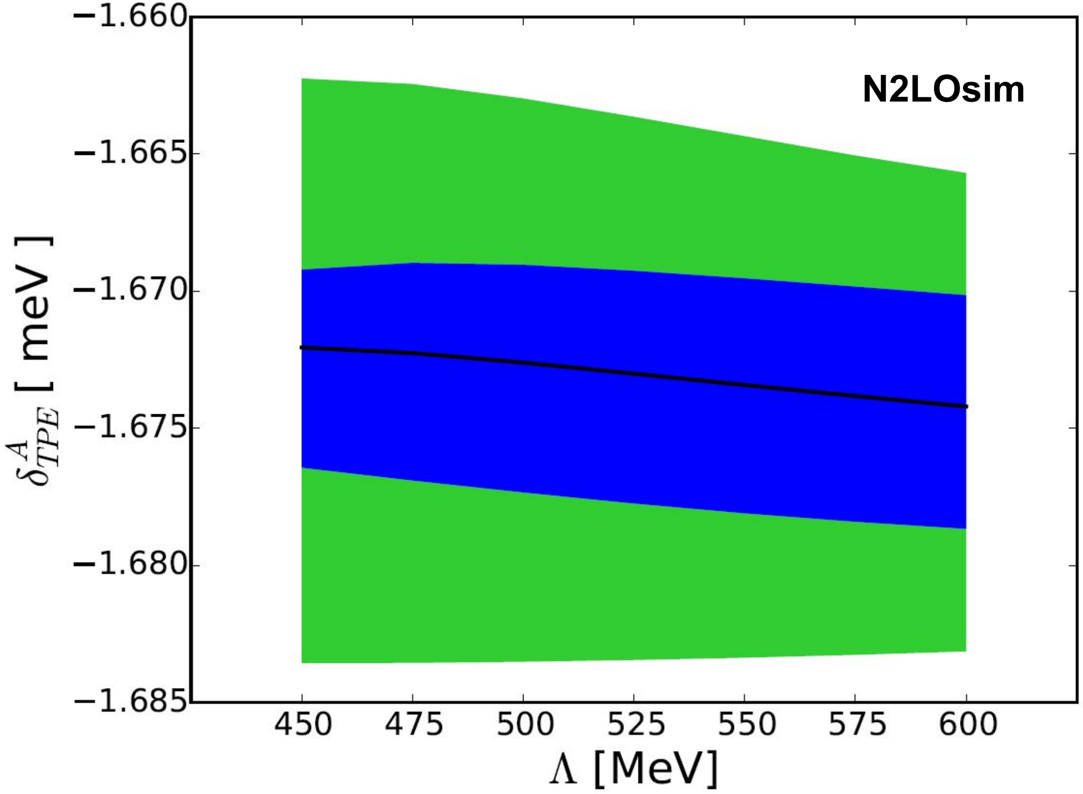


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| Correction | % Uncert. |
|---|---|
| Chiral Trunc. | 0.4 |

# Additional uncertainties

Two body currents + relativistic corr.

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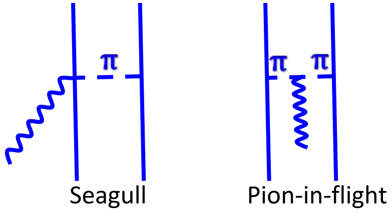


$$\rho, \vec{j}$$

| Correction | % Uncert. |
|---|---|
| **NLO MEC** | 0.05 |
| **Rel. Corr.** | 0.05 |

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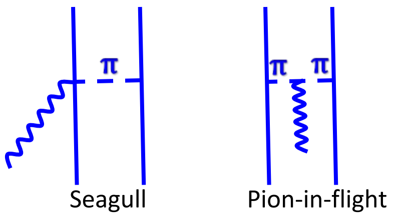
Two body currents + relativistic corr.




$\rho, \vec{j}$

Eta Expansion $\eta$

$$\delta^{A}_{TPE} = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + O(\eta^3)$$

| Correction | % Uncert. |
|---|---|
| **NLO MEC** | 0.05 |
| **Rel. Corr.** | 0.05 |
| **Eta Exp.** | 0.3 |

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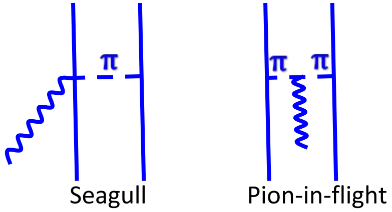
Two body currents + relativistic corr.

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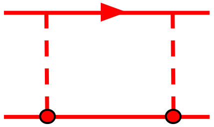
Single Nucleon Physics

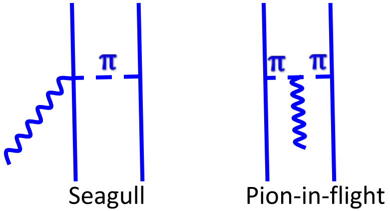


$\delta^{N}_{TPE}$

| Correction | % Uncert. |
|---|---|
| **NLO MEC** | 0.05 |
| **Rel. Corr.** | 0.05 |
| **Eta Exp.** | 0.3 |
| **Nucleon** | 0.6 |

# Additional uncertainties

Two body currents + relativistic corr.

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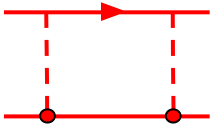
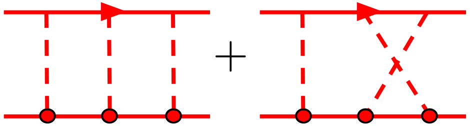


$\delta^N_{TPE}$

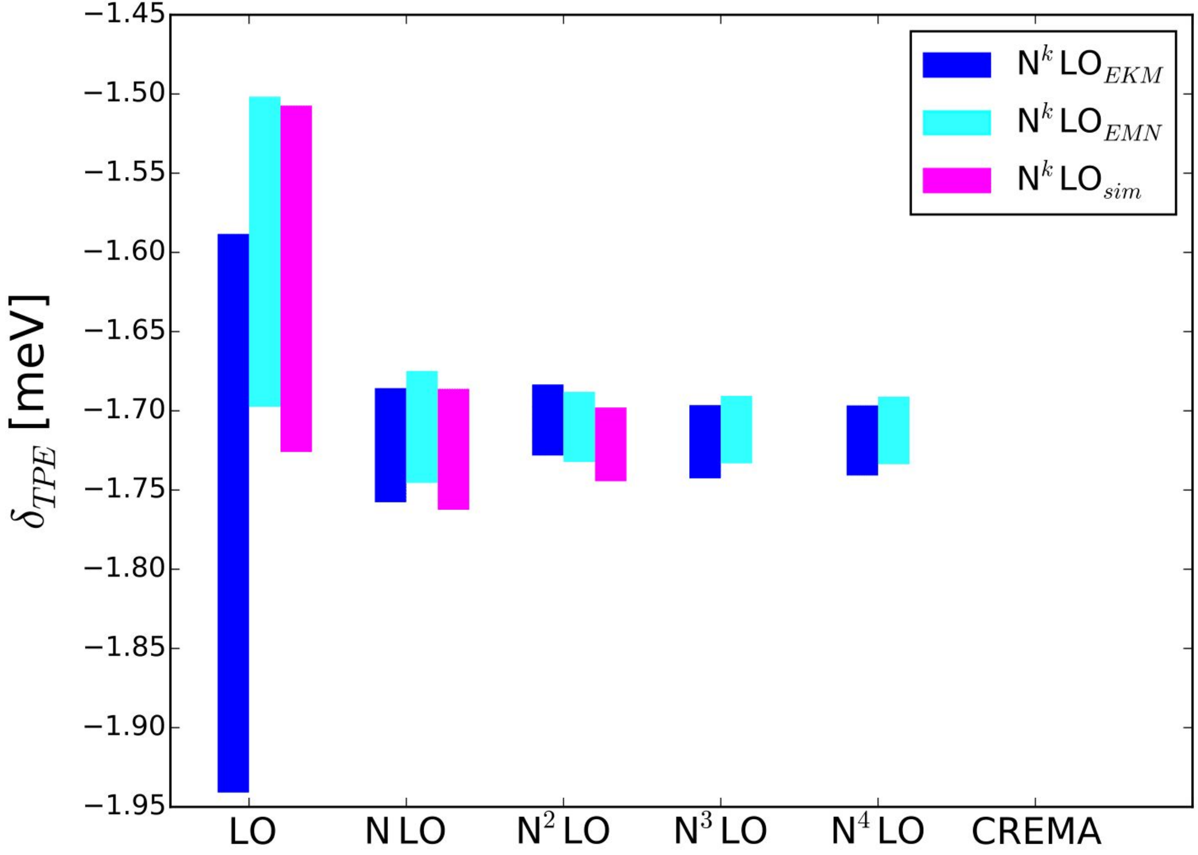
Atomic Physics uncert.



$$+ \cdots \Big\} \; O(\alpha^6)$$

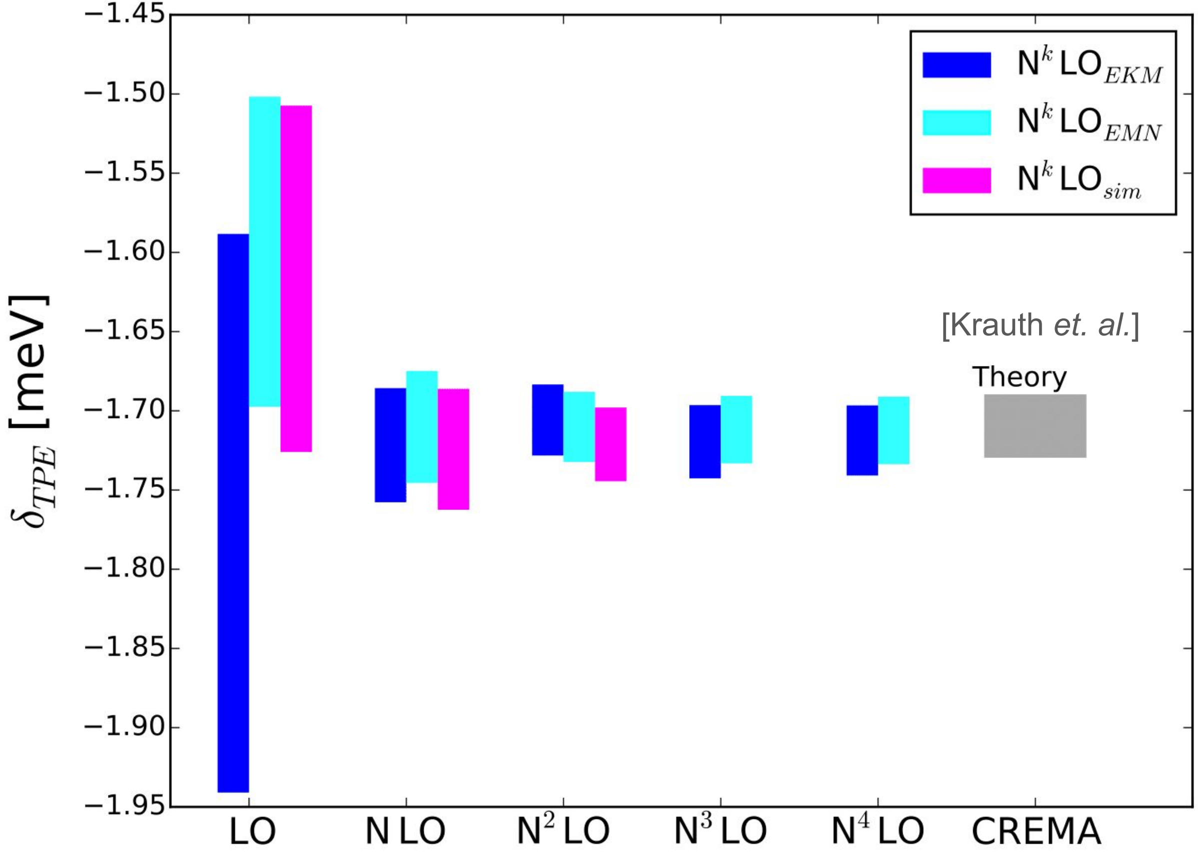
| Correction | % Uncert. |
|---|---|
| **NLO MEC** | 0.05 |
| **Rel. Corr.** | 0.05 |
| **Eta Exp.** | 0.3 |
| **Nucleon** | 0.6 |
| **Atomic Phys.** | 1.0 |

# The total uncertainty




[arXiv: 1711.01199]

# The total uncertainty

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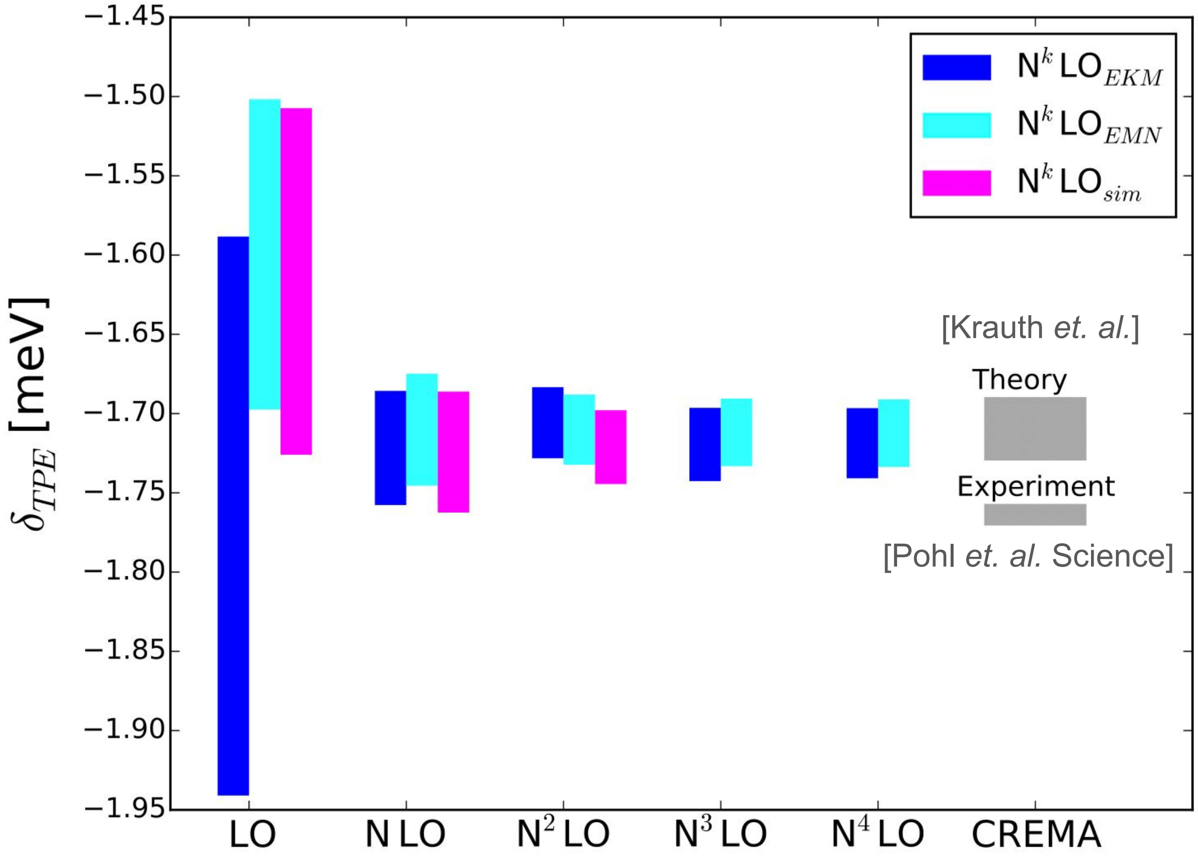


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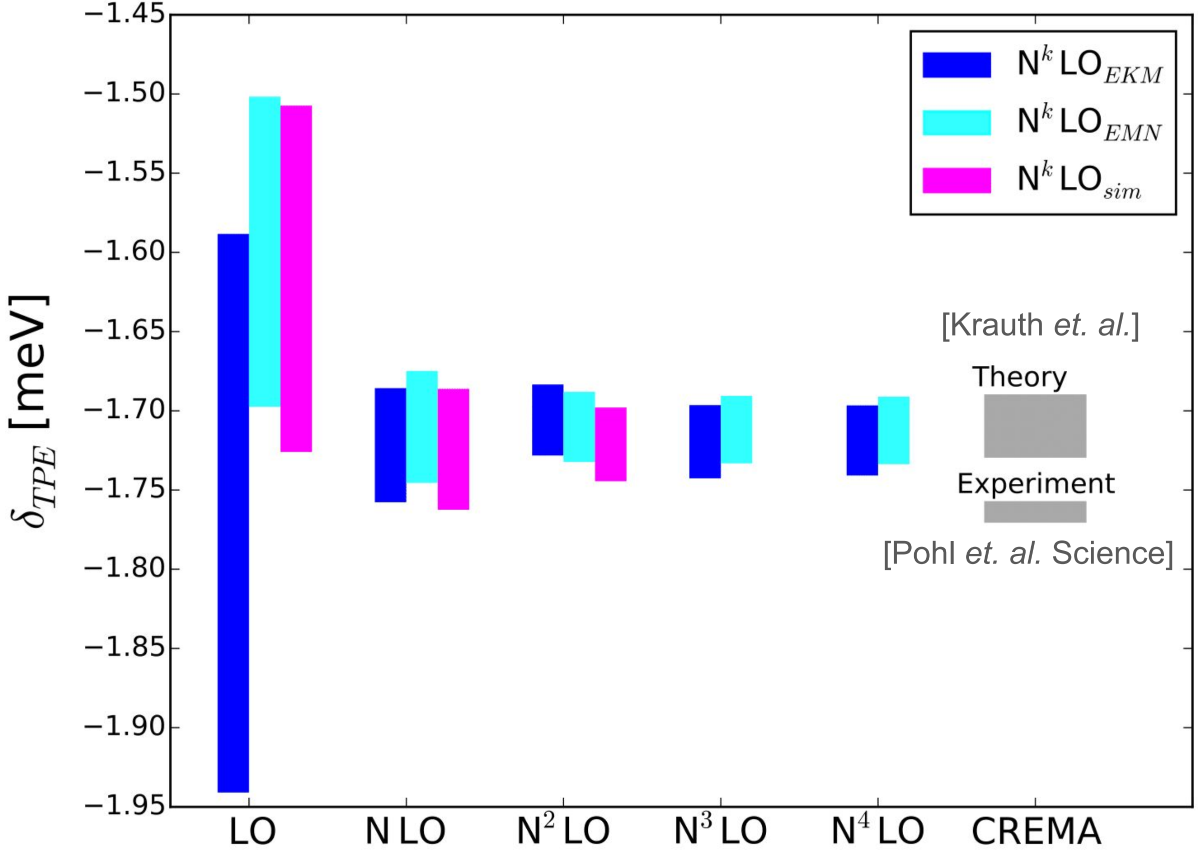



| Contribution | Uncertainty in meV |
|---|---|
| Nuclear physics (syst) | +0.008 −0.011 |
| Nuclear physics (stat) | ±0.001 |
| $\eta$-expansion | ±0.005 |
| Single-nucleon | ±0.0102 |
| Atomic physics | ±0.0172 |
| Total | +0.022 −0.024 |

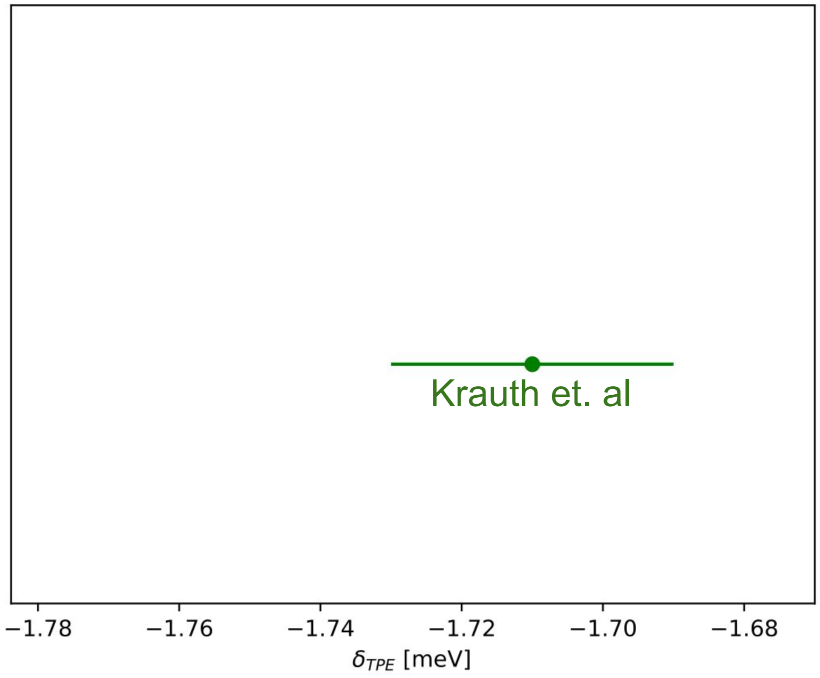
$$\delta_{TPE} = -1.715\ meV$$

[arXiv: 1711.01199]

# Summary

Krauth et. al. [2016]

$$\delta_{TPE}(Krauth\ et\ al.) = -1.710(20)\ meV$$



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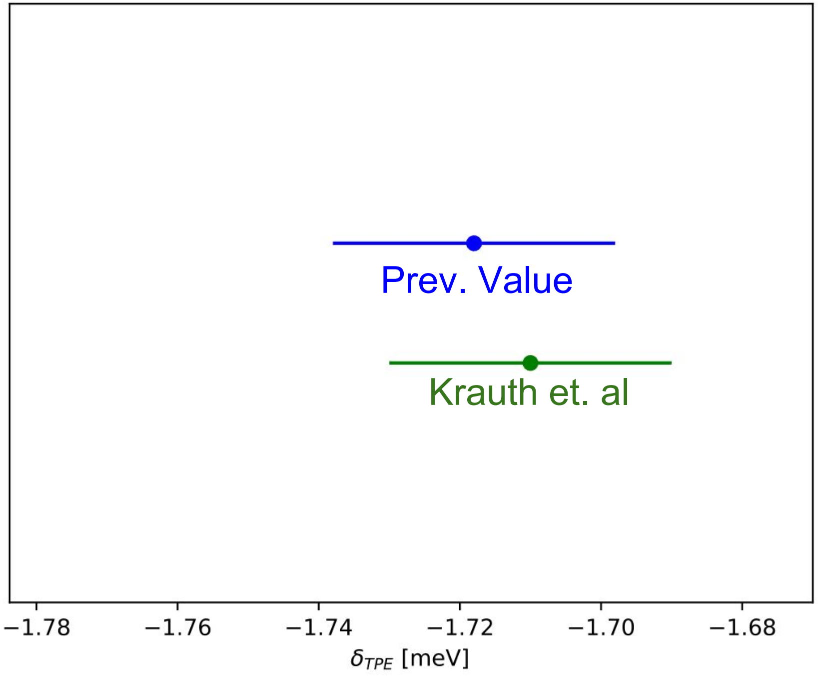
$$\delta_{TPE}(Krauth\ et\ al.) = -1.710(20)\ meV$$

Previous Value [2014,2016]

$$\delta_{TPE}(Prev\ Work) = -1.718(22)\ meV$$



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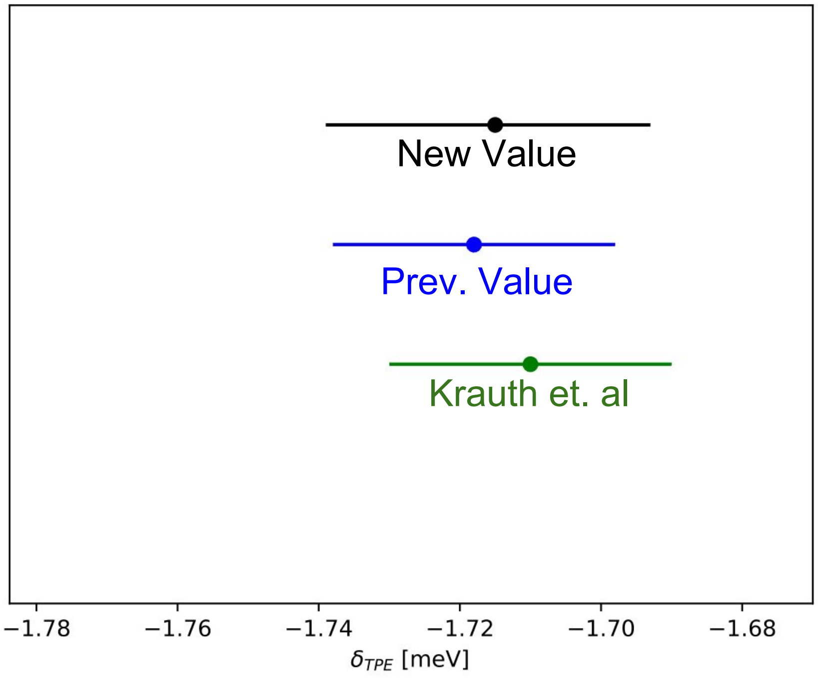

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Experimental

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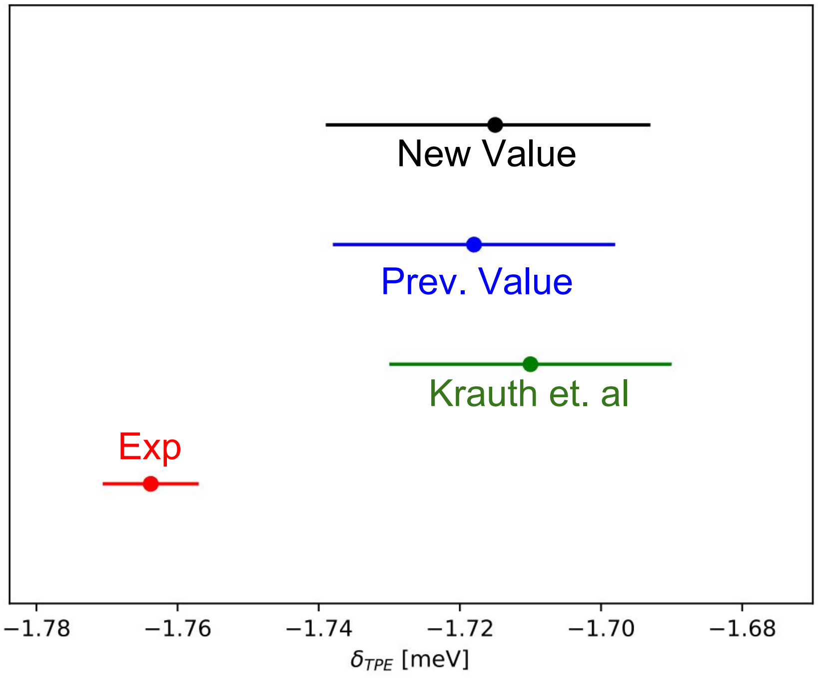

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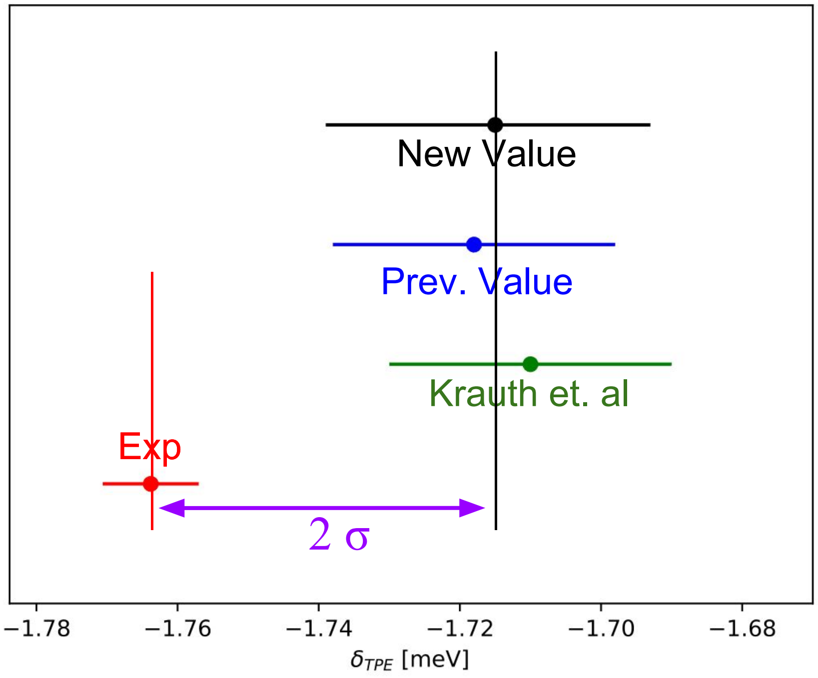

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|---|---|---|---|---|
| **Percent Uncert.** | $O(\eta^3)$ | 0.4 | 2.0 | 2.0 |
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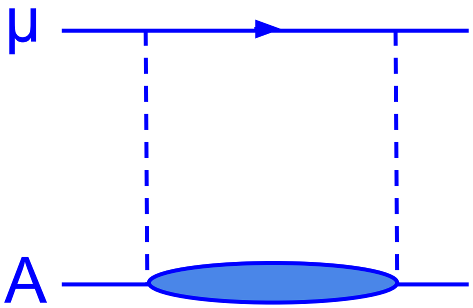
- Can we avoid the eta-expansion?

# η-less expansion



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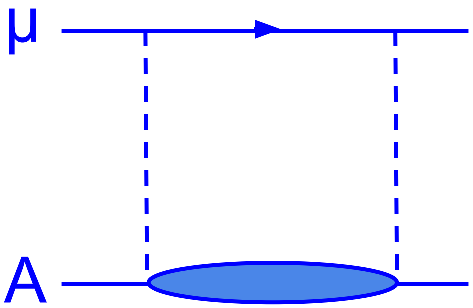


$$P_{NR} = -2m_r\phi^2(0)Z^2 \int \frac{d^3q}{(2\pi)^3}\left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{q^2+2m_r\omega_N}(1-e^{i\boldsymbol{q}\cdot\boldsymbol{R}})(1-e^{-i\boldsymbol{q}\cdot\boldsymbol{R}'})$$

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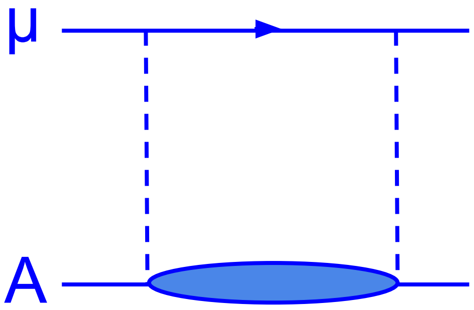


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- Insert intermediate states

$$\sum_{N\neq N_0} |N\rangle\langle N|$$

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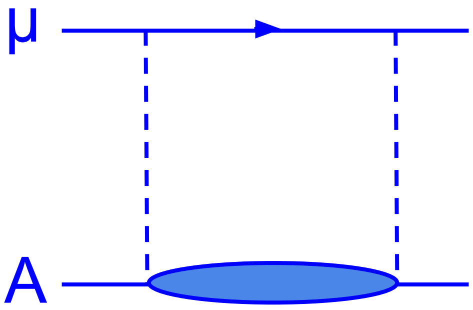


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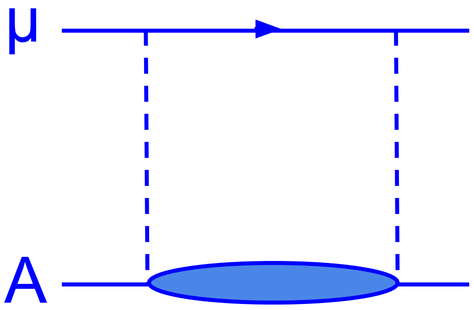


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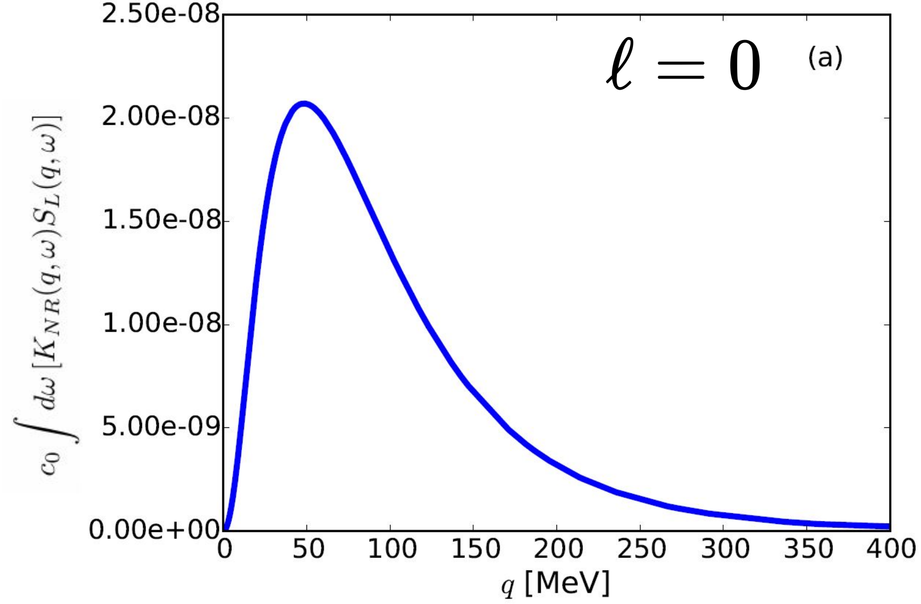
$$\delta^A_{NR} = -8(Z\alpha)^2|\phi(0)|^2 \int dq \int d\omega\; K_{NR}(q,\omega)S_L(q,\omega)$$

- Full treatment

$$\delta^A_{TPE} = -8(Z\alpha)^2|\phi(0)|^2 \int dq \int d\omega\, [K_L(q,\omega)S_L(q,\omega) + K_T(q,\omega)S_T(q,\omega) + K_S(q,\omega)S_T(0,\omega)]$$

# η-less expansion



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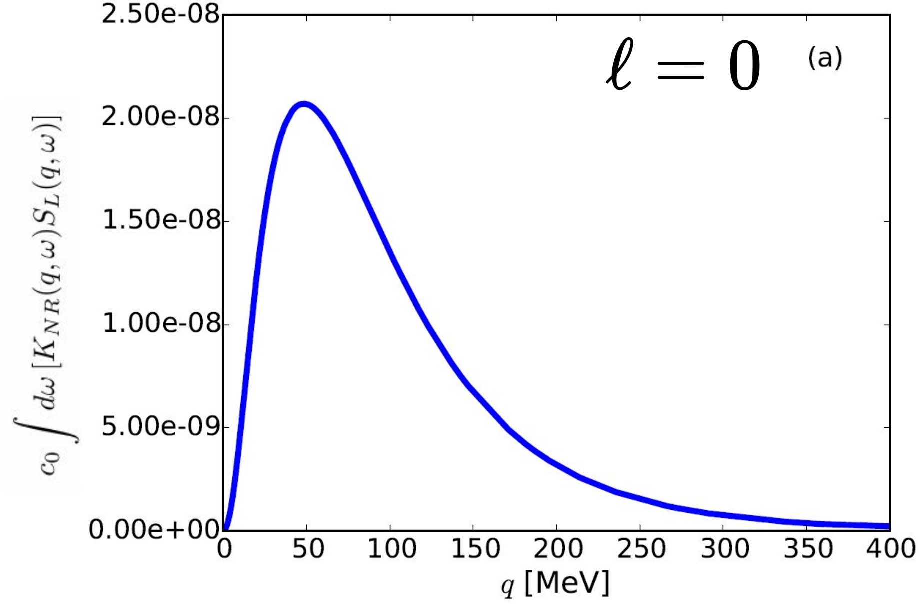


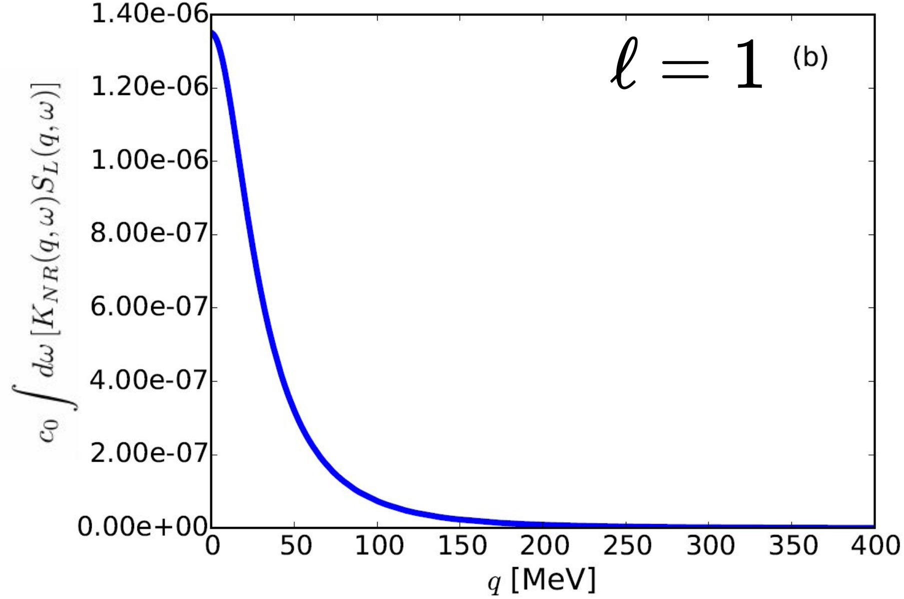

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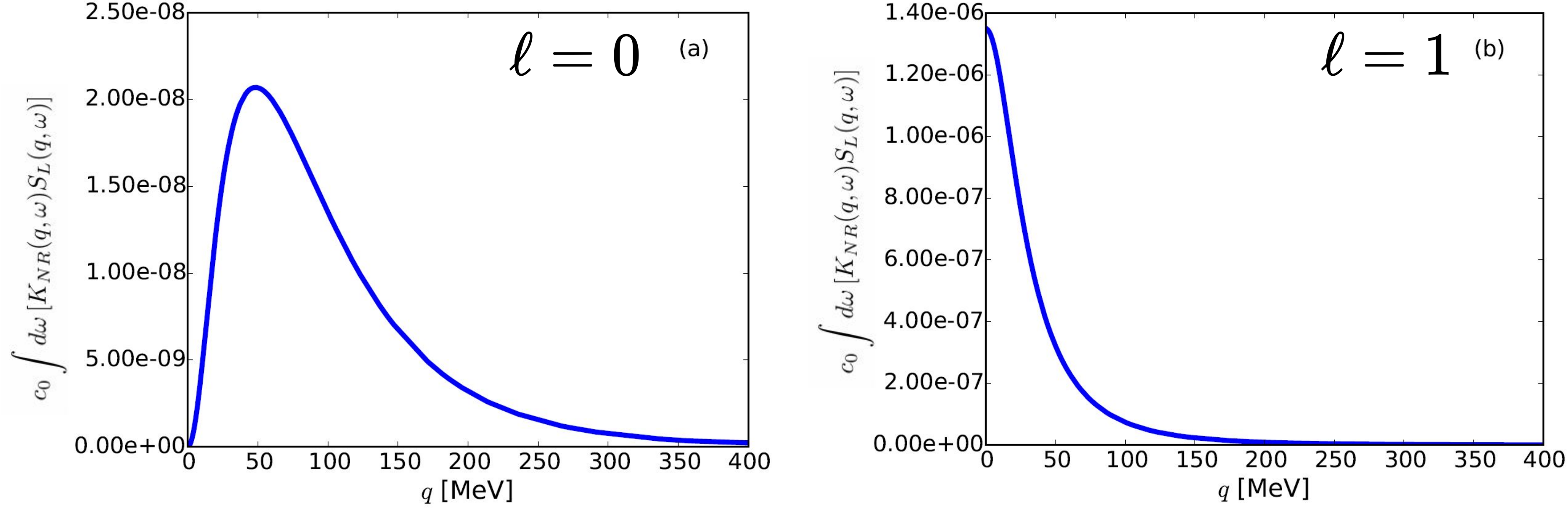
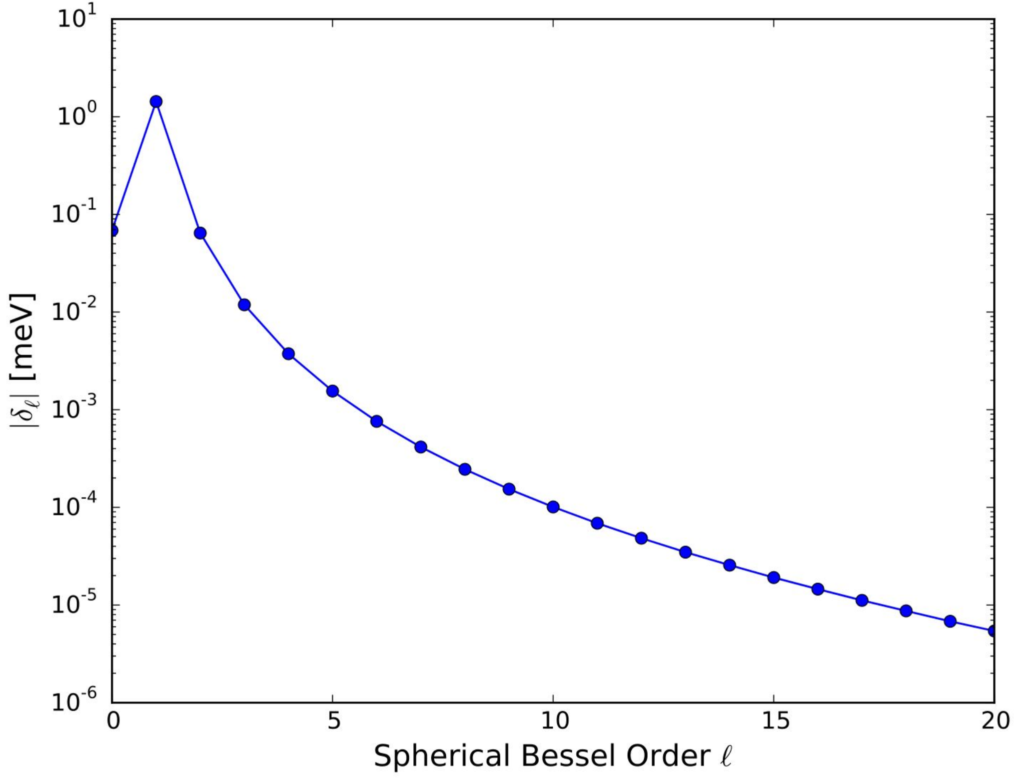



| $N^3$LO-EM | $\delta^{(0)}_{\ell=0}$ | $\delta^{(0)}_{\ell=1}$ | $\delta^{(0)}_{\ell=2}$ | $\delta^{(0)}_{\ell=3}$ | $\delta^{(0)}_{\ell=4}$ | $\sum \delta^{(0)}_{\ell}$ |
|---|---|---|---|---|---|---|
| [meV] | -0.069 | -1.436 | -0.064 | -0.012 | -0.004 | -1.585 |
| [meV] | [PLB 2014,. Proc. H.H.I 2016] | | | | | -1.590 |

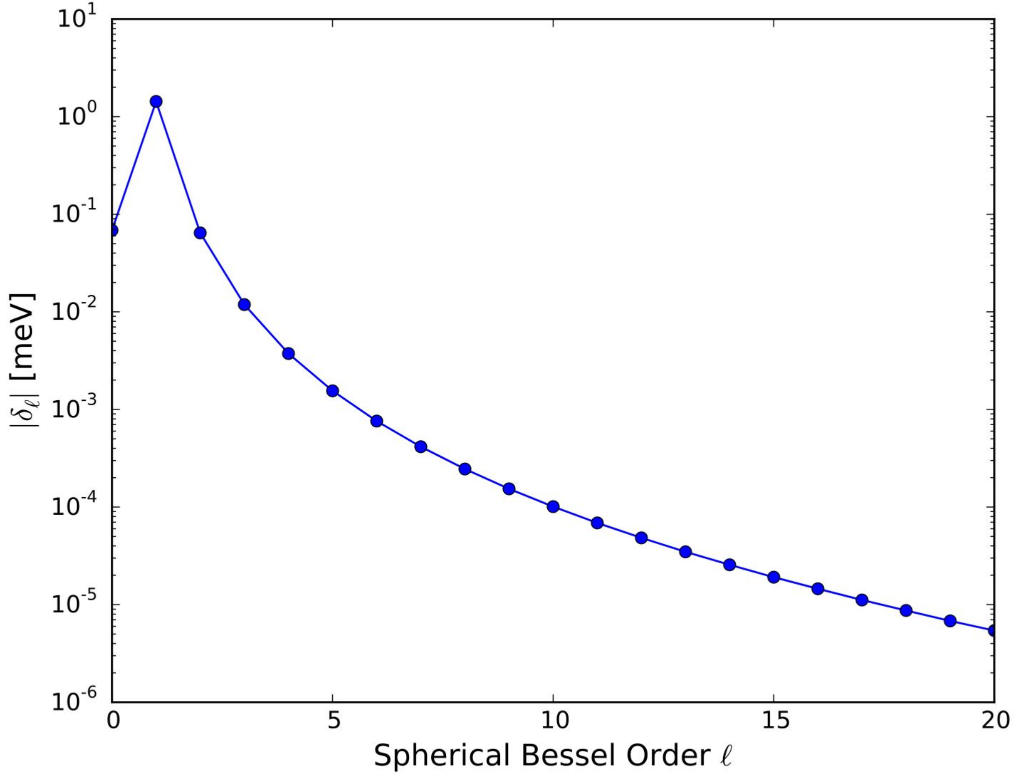
O. J. Hernandez, C. Ji, S. Bacca, N. Barnea in preparation.

# η-less expansion




| $\ell$ | $\delta_\ell$[meV] |
|---|---|
| 0 | -6.856010106985817E-002 |
| 1 | -1.43618685244822 |
| 2 | -6.442225521652188E-002 |
| 3 | -1.186645711405751E-002 |
| 4 | -3.741888013064462E-003 |
| 5 | -1.552676561183561E-003 |
| 6 | -7.602058875707891E-004 |
| 7 | -4.151874435608740E-004 |
| 8 | -2.454274817754850E-004 |
| 9 | -1.537989305734875E-004 |
| 10 | -1.010484483922072E-004 |
| 11 | -6.881334910974345E-005 |
| 12 | -4.837697294962907E-005 |
| 13 | -3.480144716570742E-005 |
| 14 | -2.561972635803522E-005 |
| 15 | -1.913783414930239E-005 |
| 16 | -1.455885571724146E-005 |
| 17 | -1.117441231993074E-005 |
| 18 | -8.713757524696711E-006 |
| 19 | -6.827246811968316E-006 |
| 20 | -5.428475472307609E-006 |
| Sum | -1.5882493506924E+00 |

# η-less expansion

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| 4 | -3.741888013064462E-003 |
| 5 | -1.552676561183561E-003 |
| 6 | -7.602058875707891E-004 |
| 7 | -4.151874435608740E-004 |
| 8 | -2.454274817754850E-004 |
| 9 | -1.537989305734875E-004 |
| 10 | -1.010484483922072E-004 |
| 11 | -6.881334910974345E-005 |
| 12 | -4.837697294962907E-005 |
| 13 | -3.480144716570742E-005 |
| 14 | -2.561972635803522E-005 |
| 15 | -1.913783414930239E-005 |
| 16 | -1.455885571724146E-005 |
| 17 | -1.117441231993074E-005 |
| 18 | -8.713757524696711E-006 |
| 19 | -6.827246811968316E-006 |
| 20 | -5.428475472307609E-006 |
| Sum | -1.5882493506924E+00 |

η-expansion Result:

$N^3LO_{EM}$ $-1.590$ [meV]

# Outlook

Uncertainty Analysis:

- Reduce atomic physics uncert. $O(\alpha^6)$

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Etaless Expansion

- Implement transverse corrections in A=2
- Apply formalism to A=3 systems
- Extend formalism for HFS