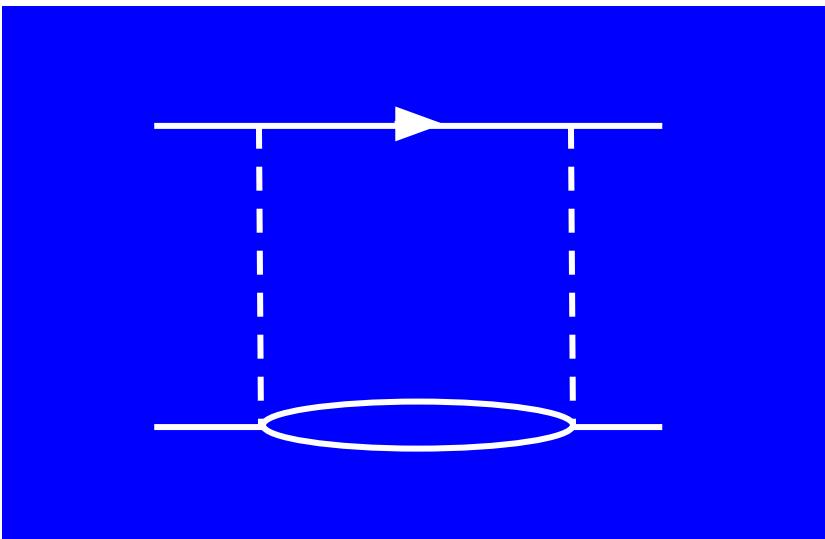


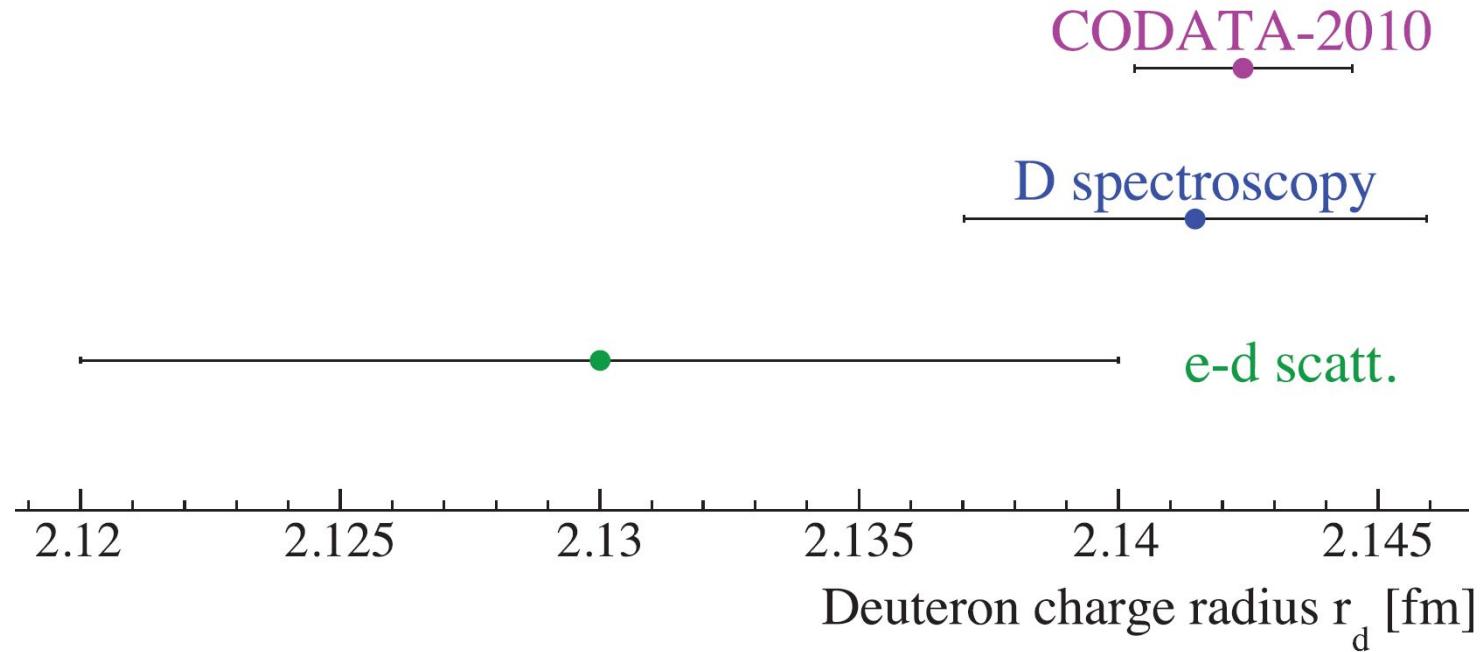
Muonic deuterium results: Nuclear Structure Corrections from TPE



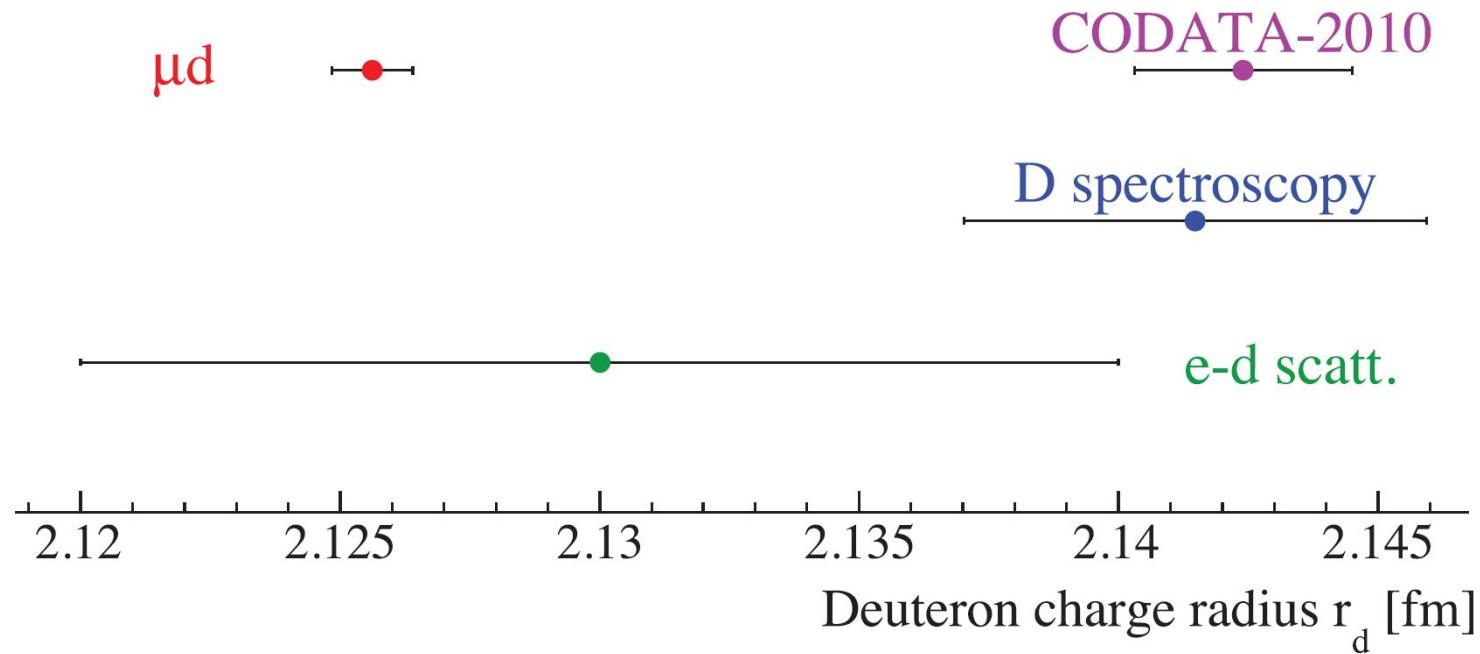
In collaboration with:
Andreas Ekström
Nir Nevo Dinur
Chen Ji
Sonia Bacca
Nir Barnea



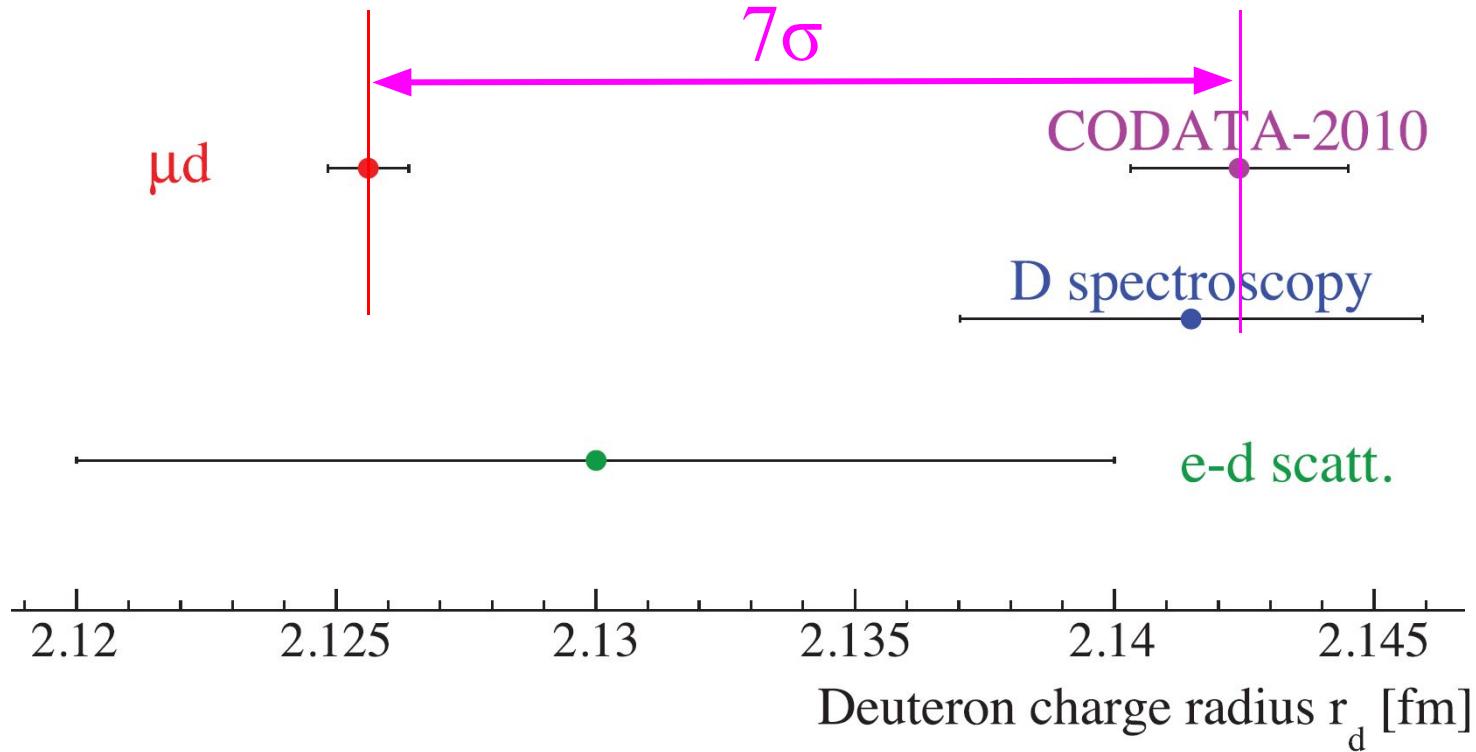
There is a discrepancy between eD and μ D data



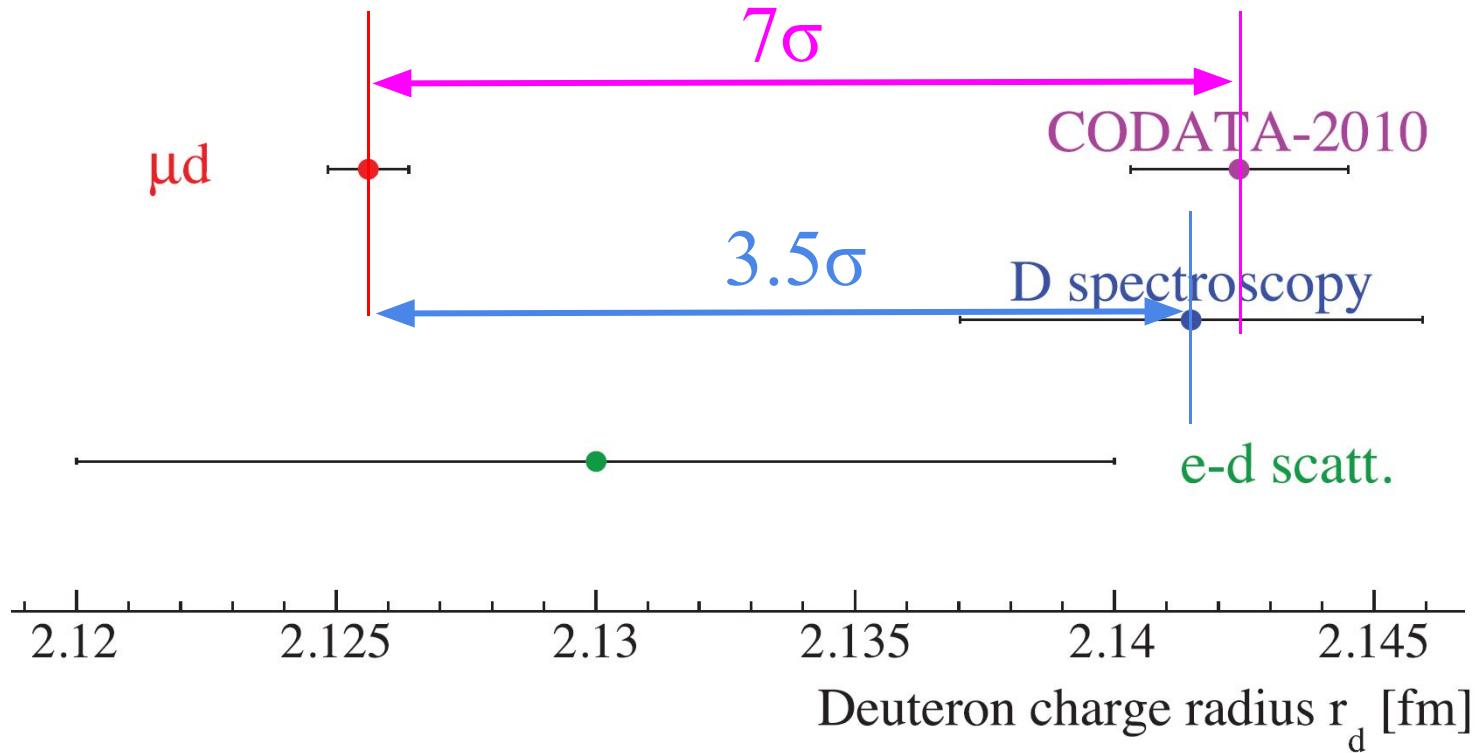
There is a discrepancy between eD and μ D data



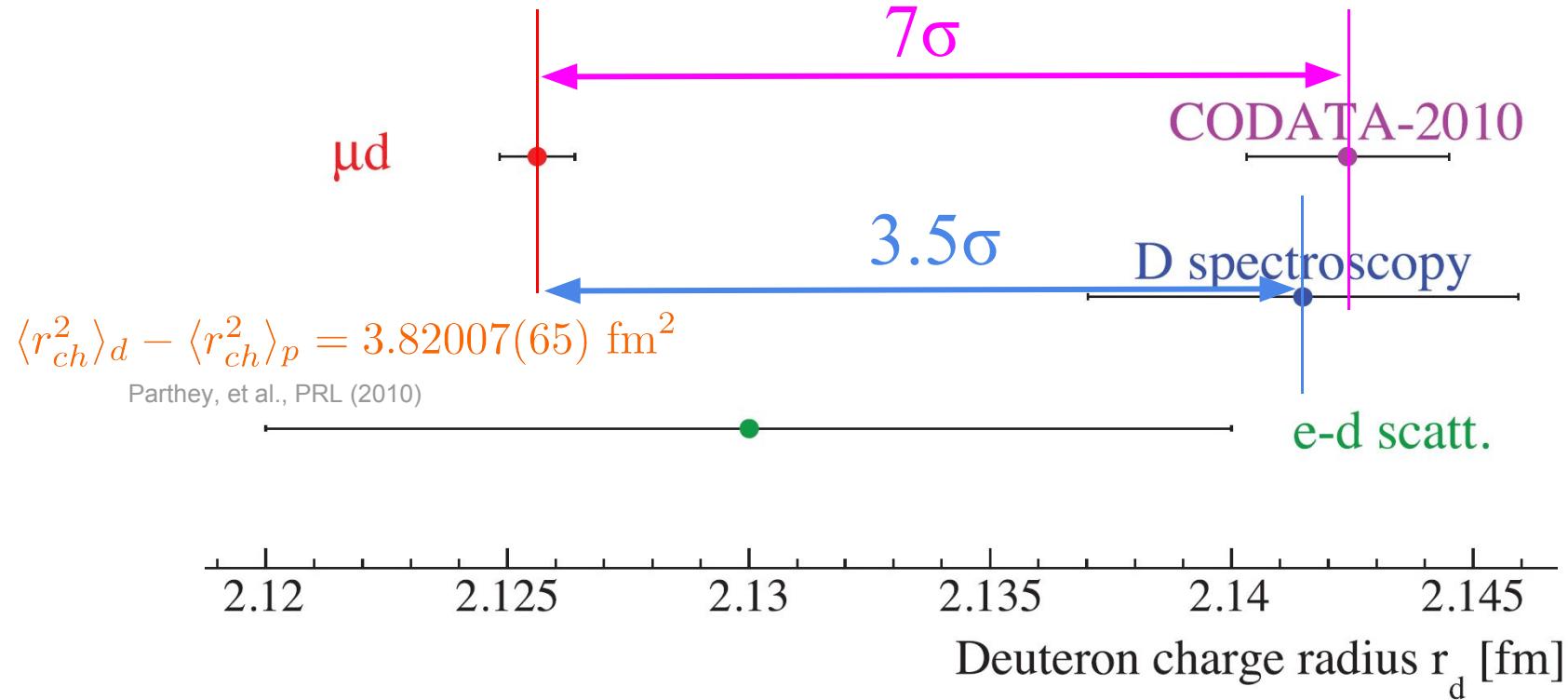
There is a discrepancy between eD and μ D data



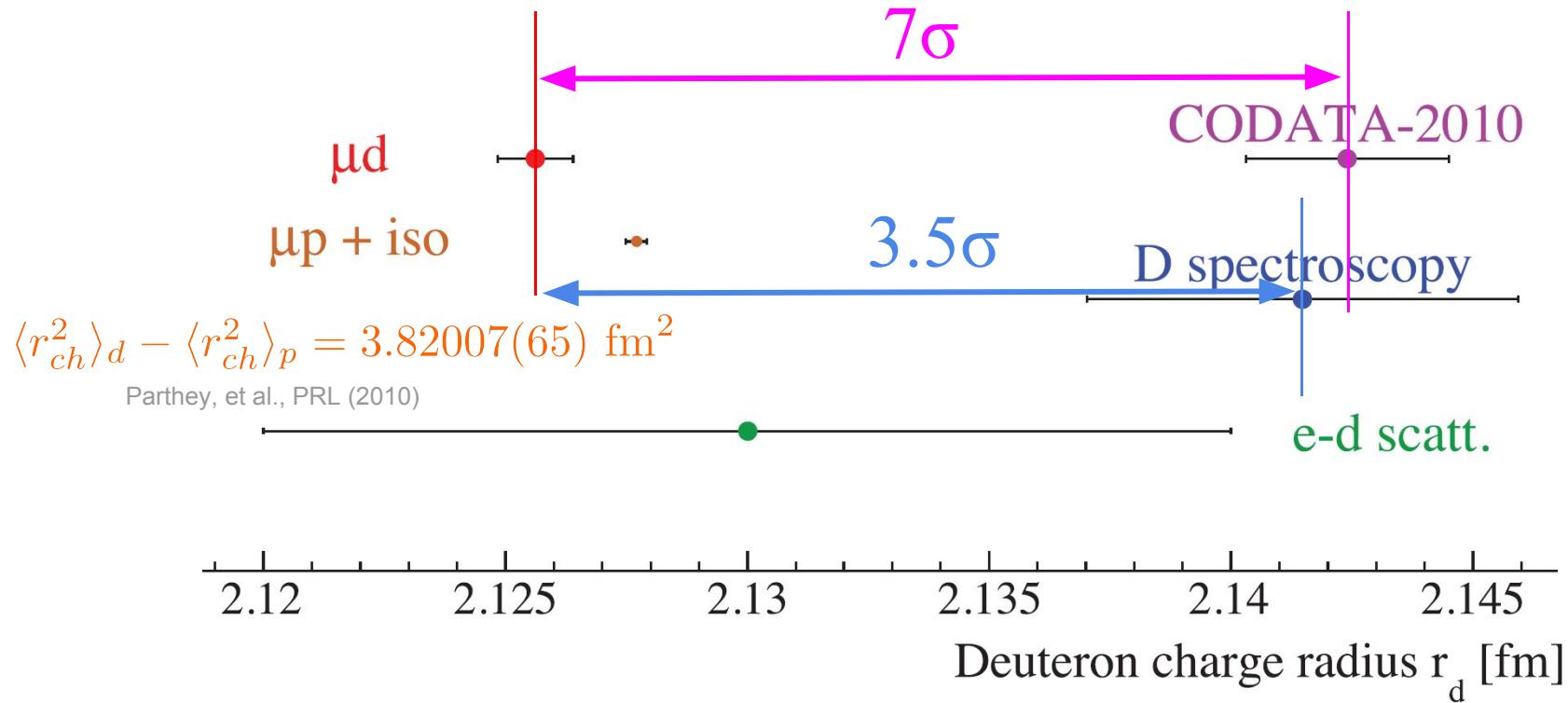
There is a discrepancy between eD and μ D data



There is a discrepancy between eD and μ D data



There is a discrepancy between eD and μ D data



The total Lamb shift error budget

$$\Delta E_{LS} = \delta_{QED} + \delta_{TPE} + \delta_{FS}(R_c).$$

The total Lamb shift error budget

$$\Delta E_{LS} = \delta_{QED} + \delta_{TPE} + \delta_{FS}(R_c).$$

δ_{QED}



228.7766 (10) meV

δ_{FS}

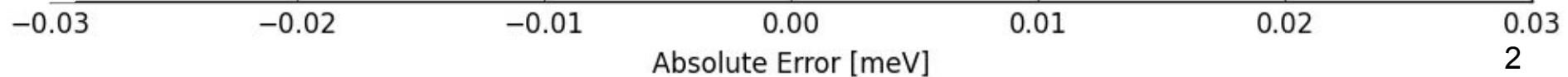


-6.1103 (3) r_d^2 meV/fm²

δ_{TPE}



1.7096 (200) meV



TPE decomposition

$$\Delta E_{LS} = \delta_{QED} + \delta_{TPE} + \delta_{FS}(R_c).$$

TPE decomposition

$$\Delta E_{LS} = \delta_{QED} + \delta_{TPE} + \delta_{FS}(R_c).$$

$$\delta_{TPE} = \delta_{TPE}^A + \delta_{TPE}^N$$

The diagram illustrates the decomposition of the total Pion Exchange (TPE) energy into two components: Nuclear and Nucleonic. It consists of two horizontal brackets, one blue and one red, positioned above the terms δ_{TPE}^A and δ_{TPE}^N respectively. Below each bracket is a label: "Nuclear" under the blue bracket and "Nucleonic" under the red bracket.

δ_{TPE}^A δ_{TPE}^N

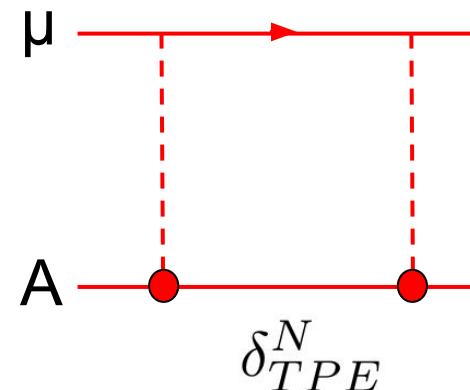
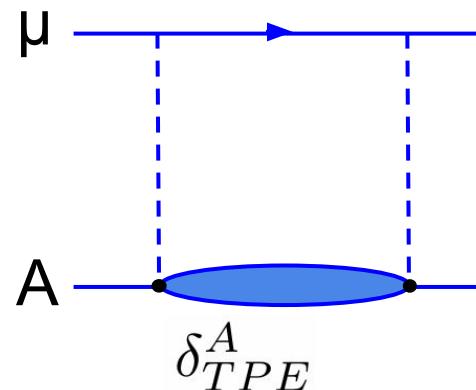
Nuclear Nucleonic

TPE decomposition

$$\Delta E_{LS} = \delta_{QED} + \delta_{TPE} + \delta_{FS}(R_c).$$

$$\delta_{TPE} = \delta_{TPE}^A + \delta_{TPE}^N$$

 
Nuclear Nucleonic



Deuteron Calculations

- Expand the Schrödinger equation in the harmonic oscillator basis and diagonalize

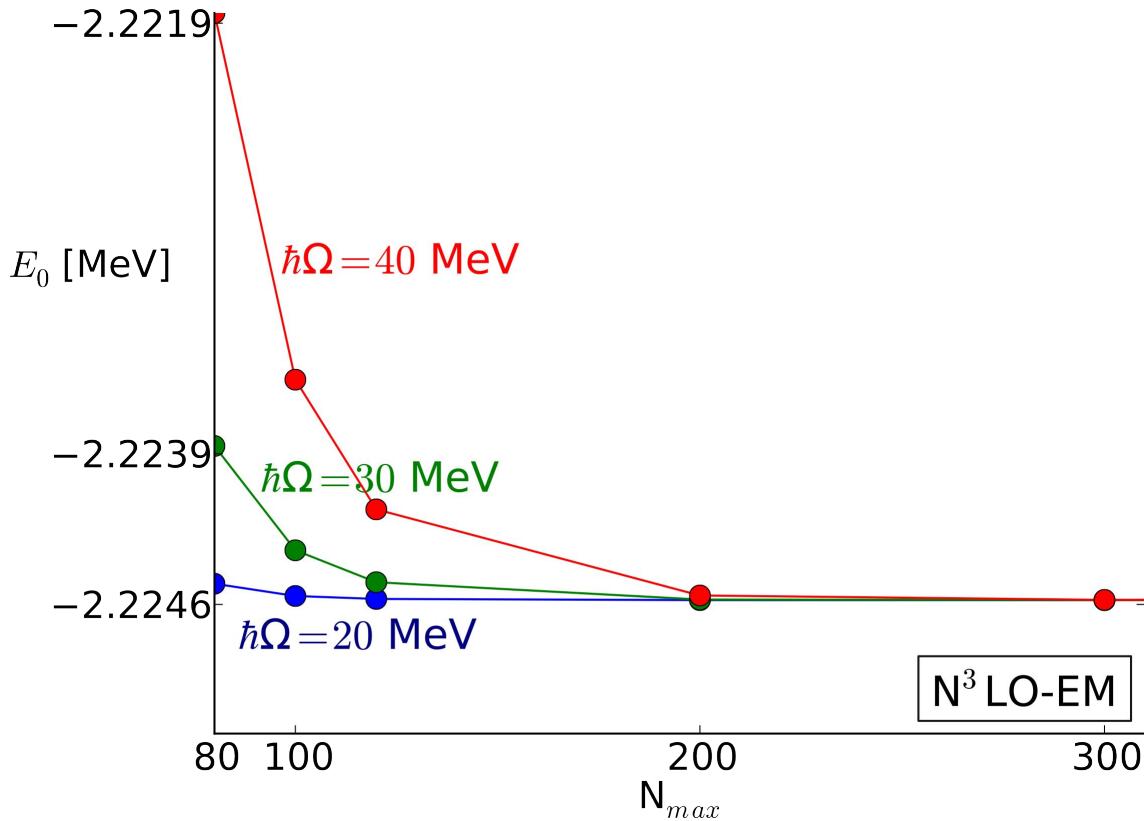
$$\{N_{max}, \hbar\Omega\}$$

Deuteron Calculations

- Expand the Schrödinger equation in the harmonic oscillator basis and diagonalize

$$\{N_{max}, \hbar\Omega\}$$

- Results independent on the model space size, and the HO frequency

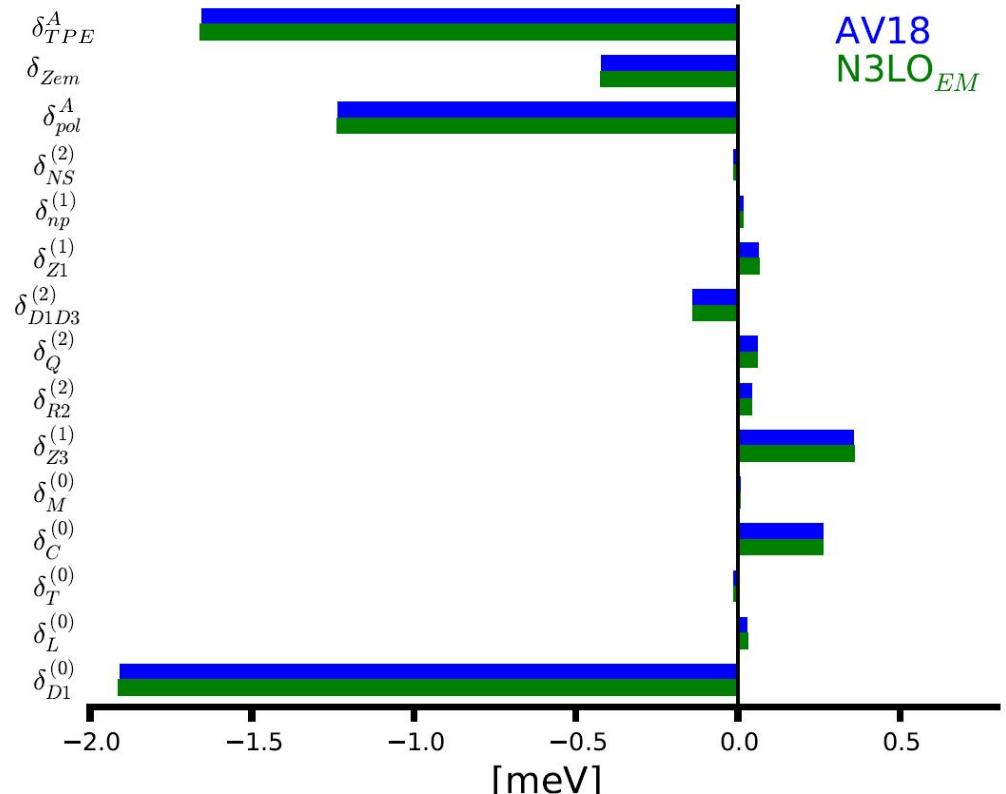


Deuteron Calculations

- Benchmark with available literature

		E_0 [MeV]	$\langle r_{str}^2 \rangle_d^{1/2}$ [fm]	Q_d [fm 2]	P_D [%]
N3LO	This Work	2.2246	1.978	0.285	4.51
	Entem <i>et al</i> [1]	2.2246	1.978	0.285	4.51
AV18	This Work	2.2246	1.967	0.270	5.76
	Wiringa <i>et al</i> [2]	2.2246	1.967	0.270	5.76

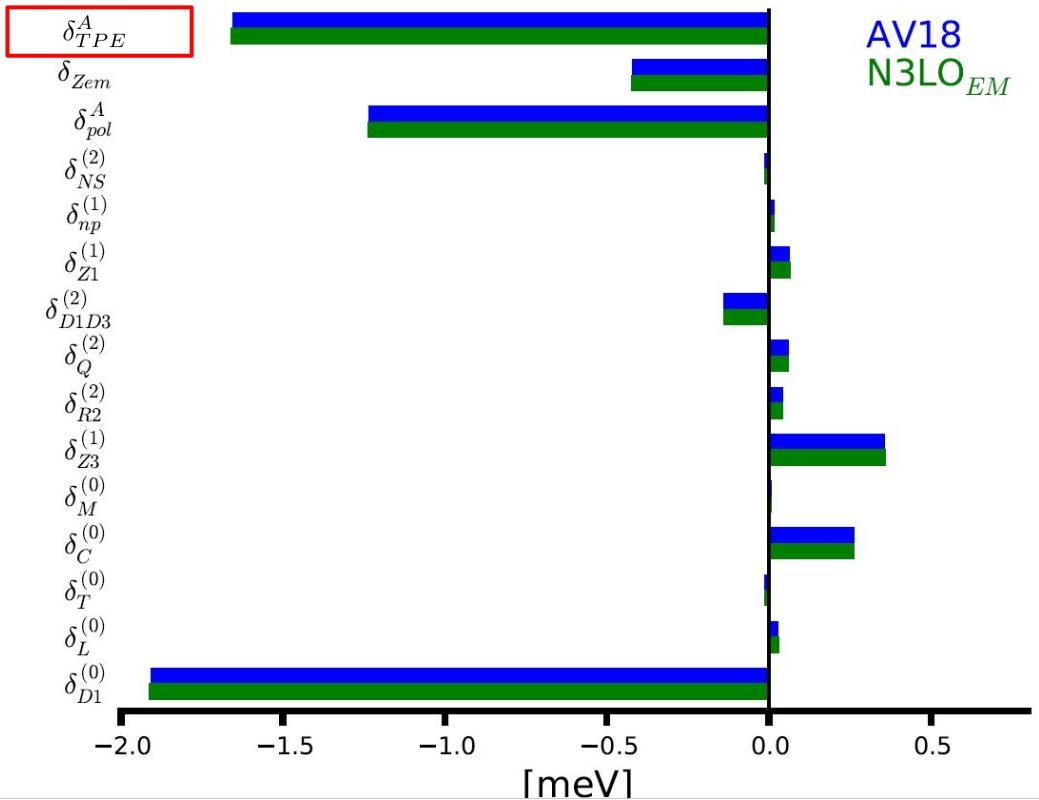
Calculation of TPE



- Each correction is an integral over the response

$$\delta \propto \int d\omega g(\omega) S_{\hat{O}}(\omega)$$

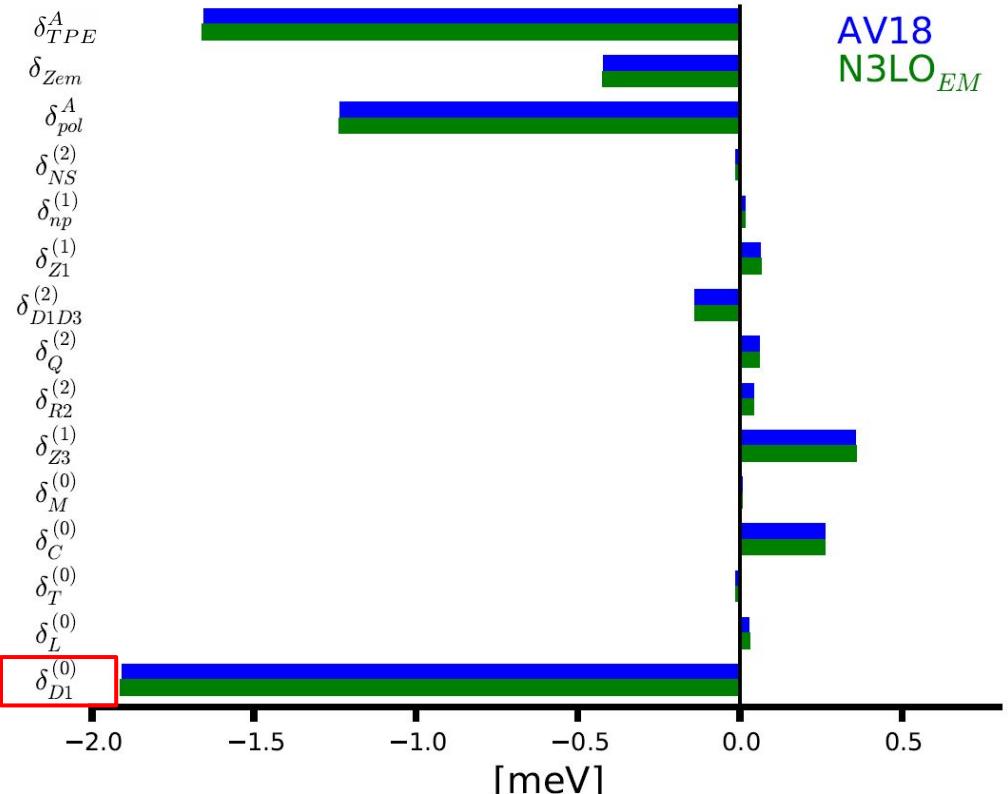
Calculation of TPE



- Each correction is an integral over the response

$$\delta \propto \int d\omega g(\omega) S_{\hat{O}}(\omega)$$

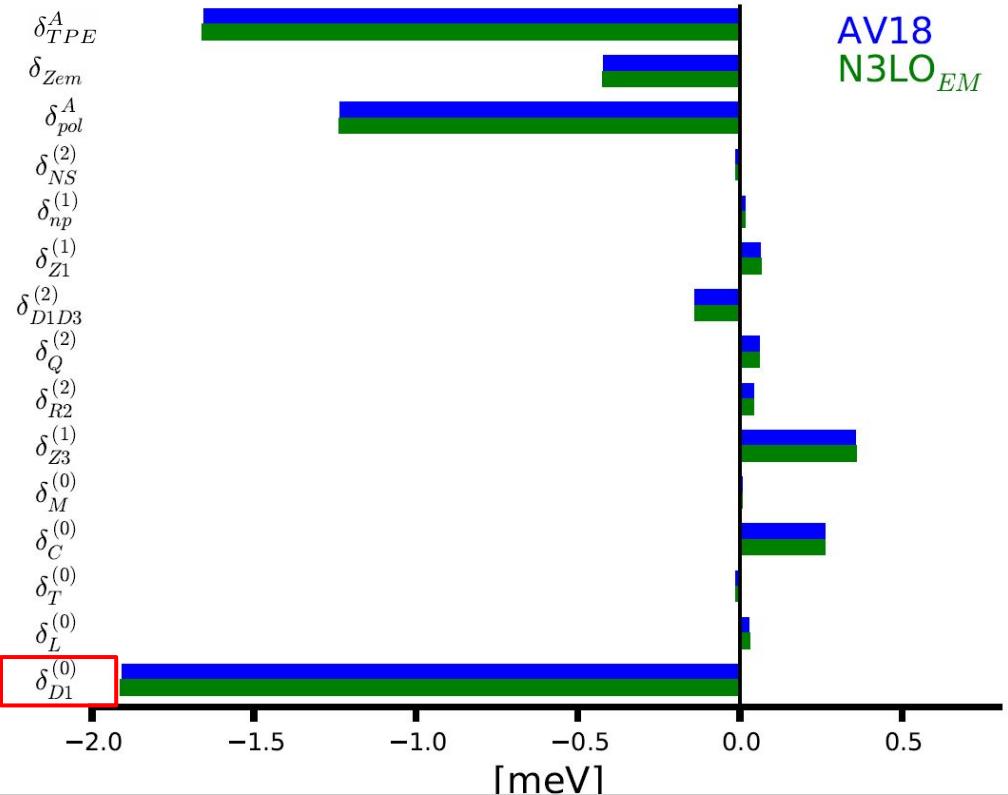
Calculation of TPE



- Each correction is an integral over the response

$$\delta \propto \int d\omega g(\omega) S_{\hat{O}}(\omega)$$

Calculation of TPE



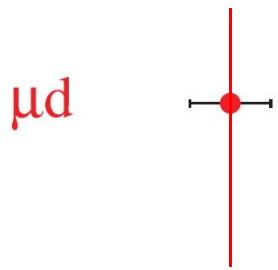
- Each correction is an integral over the response

$$\delta \propto \int d\omega g(\omega) S_{\hat{O}}(\omega)$$

$$\delta_{TPE} = \delta_{TPE}^A + \delta_{TPE}^N$$

$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

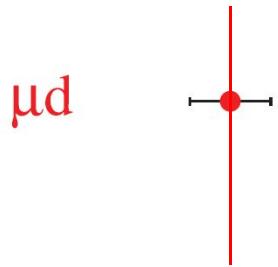
TPE discrepancy



R_c —————

$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

TPE discrepancy

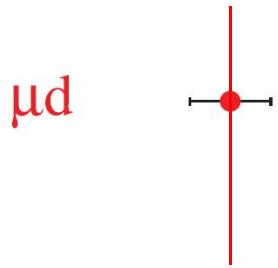


R_c —————

$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

$$\delta_{TPE}(\text{Pachucki}) = -1.717(20) \text{ meV}$$

TPE discrepancy



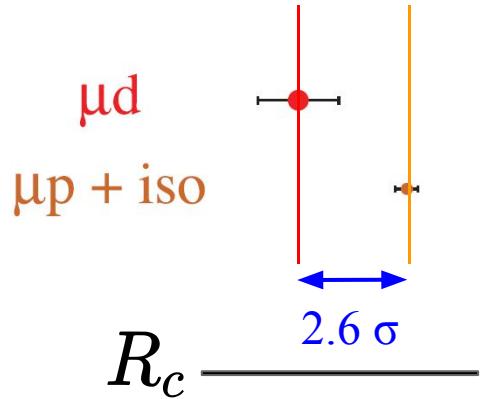
R_c —————

$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

$$\delta_{TPE}(\text{Pachucki}) = -1.717(20) \text{ meV}$$

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

TPE discrepancy

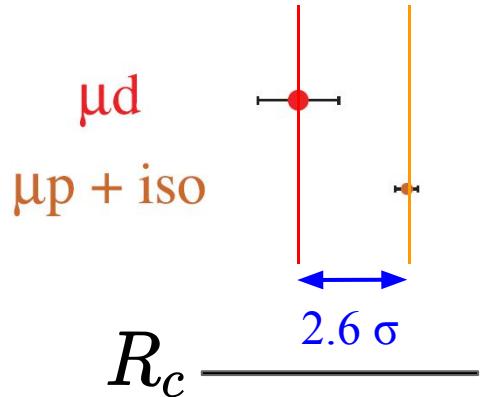


$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

$$\delta_{TPE}(\text{Pachucki}) = -1.717(20) \text{ meV}$$

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

TPE discrepancy



$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

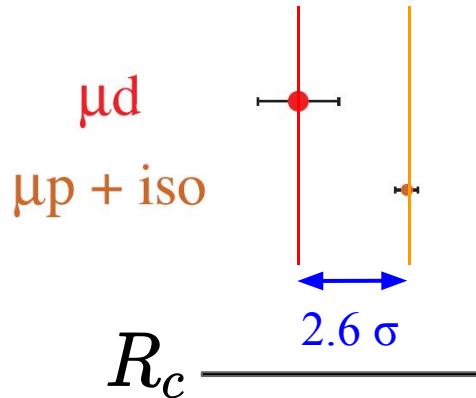
$$\delta_{TPE}(\text{Pachucki}) = -1.717(20) \text{ meV}$$

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

$$\delta_{TPE} = \delta_{QED} + \delta_{FS}(R_c) - \Delta E_{LS} \rightarrow \delta_{TPE}(\text{Exp.}) = -1.7638(68) \text{ meV}$$

[Pohl et. al. Science, Vol 353, 6300, 2016]

TPE discrepancy



$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

$$\delta_{TPE}(\text{Pachucki}) = -1.717(20) \text{ meV}$$

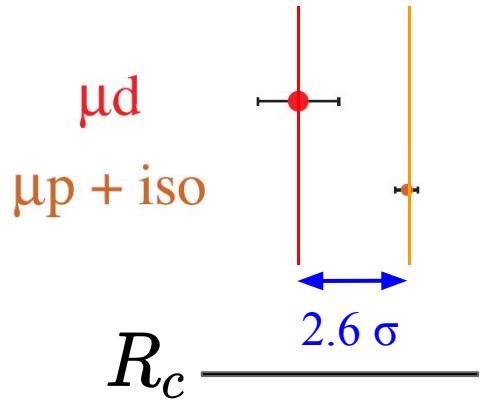
$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

$$\delta_{TPE} = \delta_{QED} + \delta_{FS}(R_c) - \Delta E_{LS} \rightarrow \delta_{TPE}(\text{Exp.}) = -1.7638(68) \text{ meV}$$

[Pohl et. al. Science, Vol 353, 6300, 2016]

- Theoretical TPE is 6 times larger than experimental uncertainty

TPE discrepancy



$$\delta_{TPE}(\text{Our Work}) = -1.718(22) \text{ meV}$$

$$\delta_{TPE}(\text{Pachucki}) = -1.717(20) \text{ meV}$$

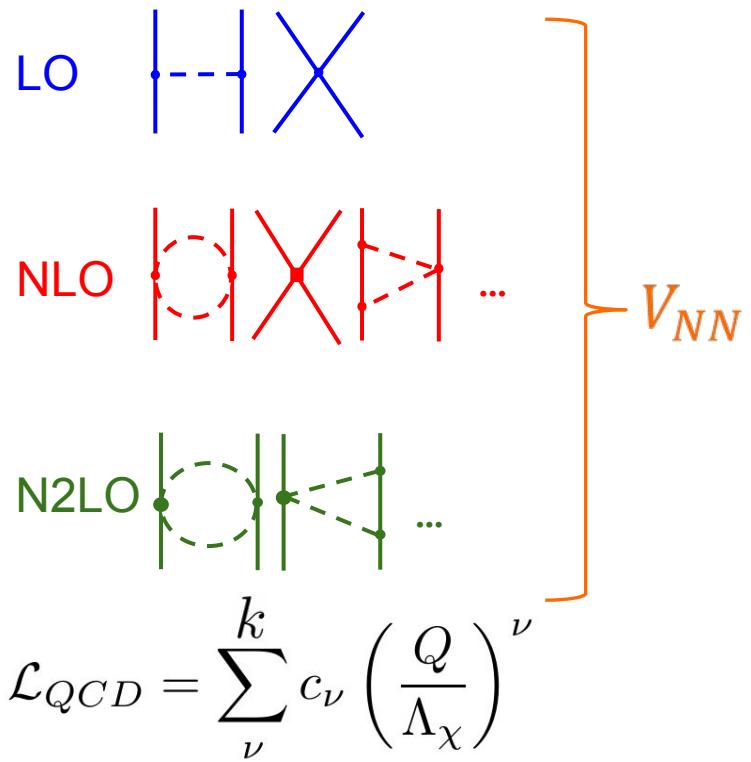
$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

$$\delta_{TPE} = \delta_{QED} + \delta_{FS}(R_c) - \Delta E_{LS} \rightarrow \delta_{TPE}(\text{Exp.}) = -1.7638(68) \text{ meV}$$

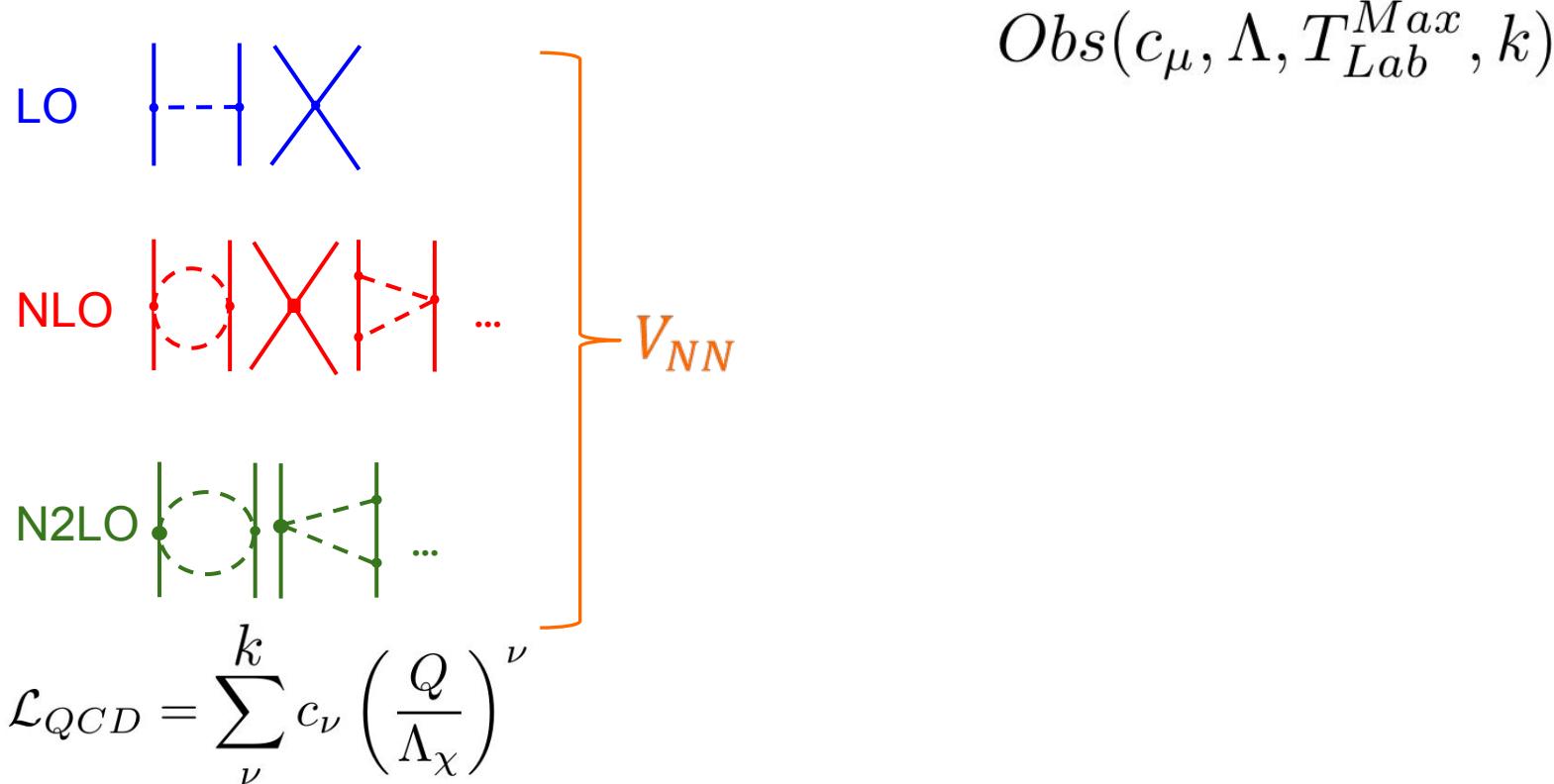
[Pohl et. al. Science, Vol 353, 6300, 2016]

- Theoretical TPE is 6 times larger than experimental uncertainty
- A thorough analysis may change our $\sim 1\%$ uncertainty and shed light on disagreement in δ_{TPE}

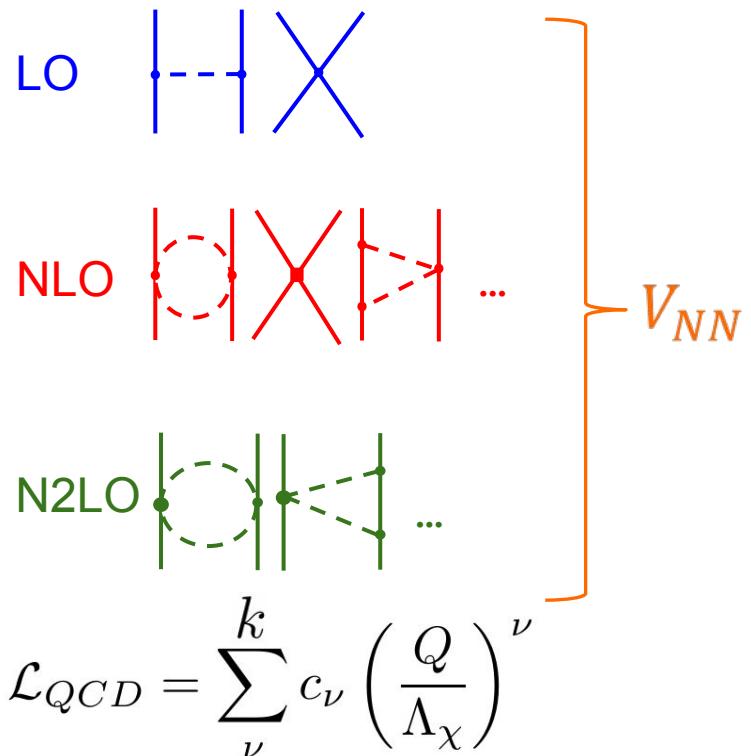
Improving the uncertainty estimates



Improving the uncertainty estimates



Improving the uncertainty estimates

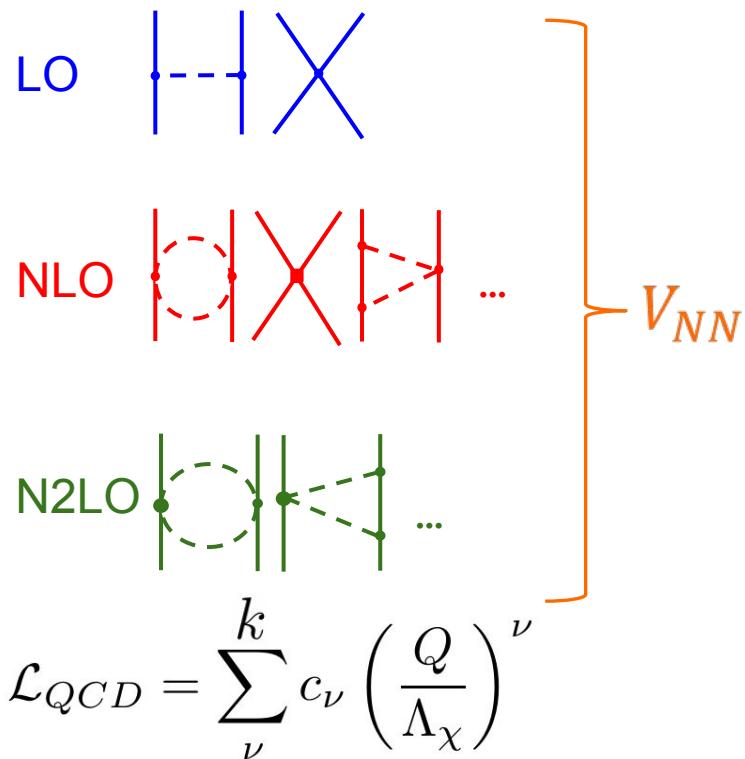


$$Obs(c_{\mu}, \Lambda, T_{Lab}^{Max}, k)$$

- Use N2LO potentials fit simultaneously to NN and πN data

Statistical uncertainties: c_{μ}

Improving the uncertainty estimates



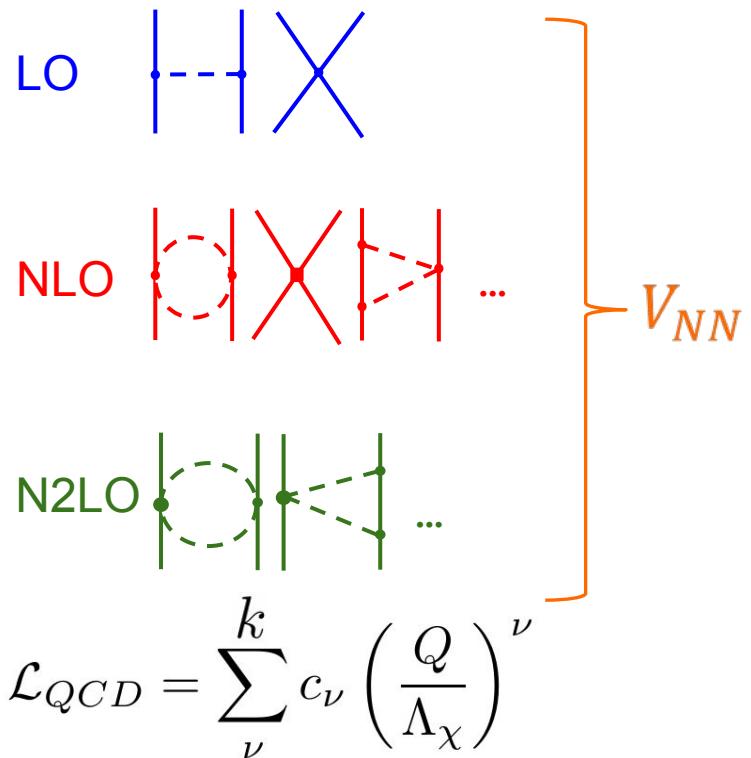
$$Obs(c_{\mu}, \Lambda, T_{Lab}^{Max}, k)$$

- Use N2LO potentials fit simultaneously to NN and πN data

Statistical uncertainties: c_{μ}

Systematic uncertainties: $\Lambda, T_{Lab}^{Max}, k$

Improving the uncertainty estimates



$$Obs(c_{\mu}, \Lambda, T_{Lab}^{Max}, k)$$

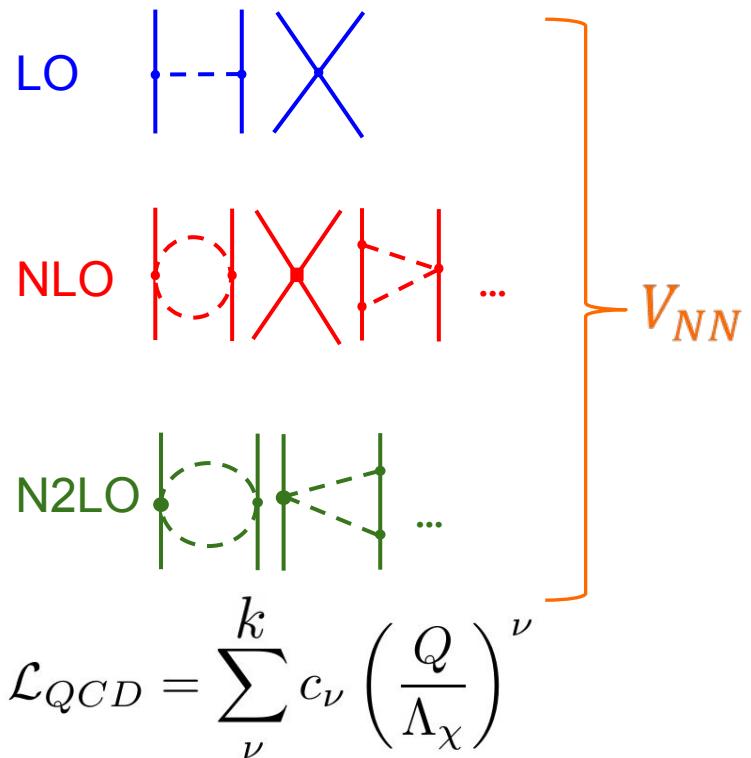
- Use N2LO potentials fit simultaneously to NN and πN data

Statistical uncertainties: c_{μ}

Systematic uncertainties: $\Lambda, T_{Lab}^{Max}, k$

η, ρ, \vec{j}

Improving the uncertainty estimates



$$Obs(c_\mu, \Lambda, T_{Lab}^{Max}, k)$$

- Use N2LO potentials fit simultaneously to NN and πN data

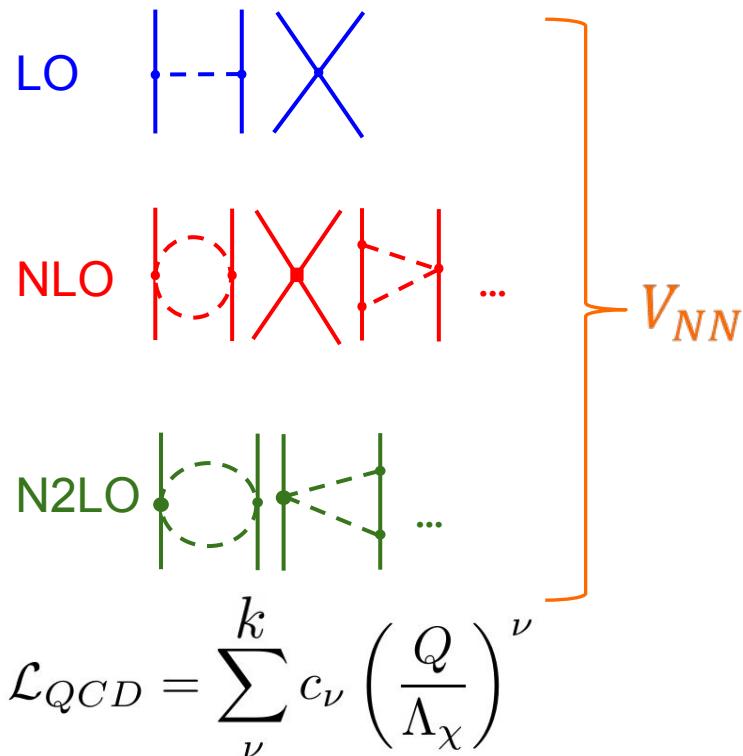
Statistical uncertainties: c_μ

Systematic uncertainties: $\Lambda, T_{Lab}^{Max}, k$

η, ρ, \vec{j}

Single Nucleon: δ_{TPE}^N

Improving the uncertainty estimates



$$Obs(c_{\mu}, \Lambda, T_{Lab}^{Max}, k)$$

- Use N2LO potentials fit simultaneously to NN and πN data

Statistical uncertainties: c_{μ}

Systematic uncertainties: $\Lambda, T_{Lab}^{Max}, k$

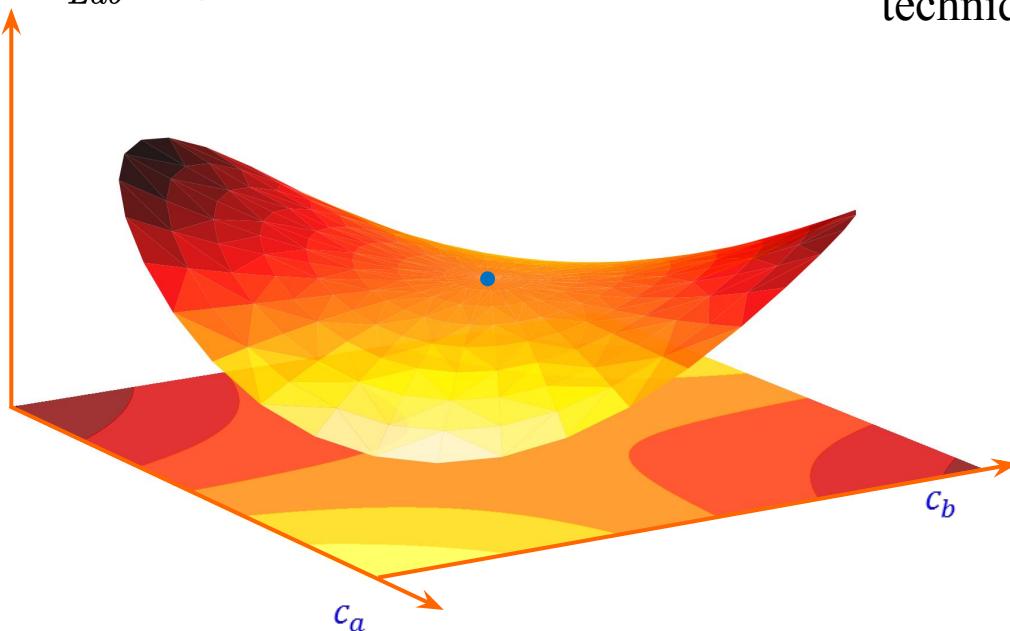
η, ρ, \vec{j}

Single Nucleon: δ_{TPE}^N

Higher Order Corrections: $O(\alpha^6)$

Statistical uncertainties

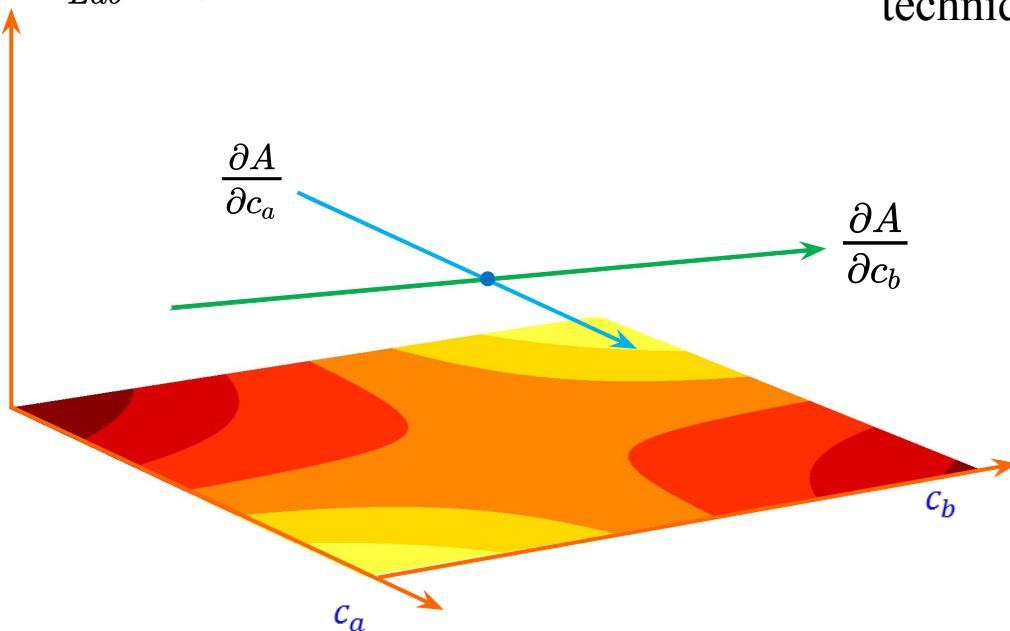
$$A(c_\mu, \Lambda, T_{Lab}^{Max}, k)$$



- Propagate uncertainty using standard techniques

Statistical uncertainties

$$A(c_\mu, \Lambda, T_{Lab}^{Max}, k)$$

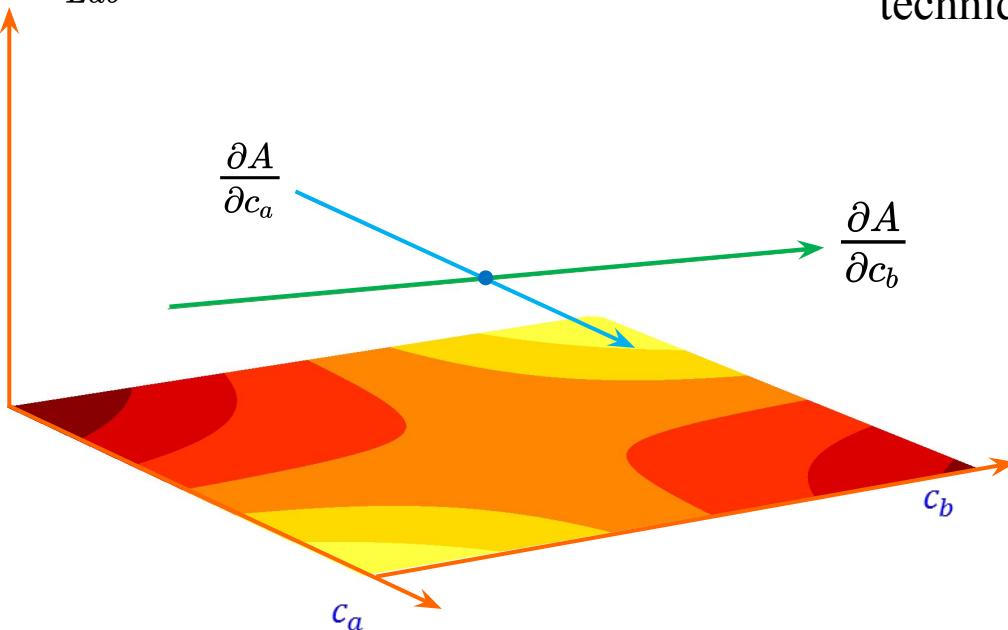


- Propagate uncertainty using standard techniques

$$J_{A,i} = \frac{\partial A}{\partial c_{\mu,i}}$$

Statistical uncertainties

$$A(c_\mu, \Lambda, T_{Lab}^{Max}, k)$$



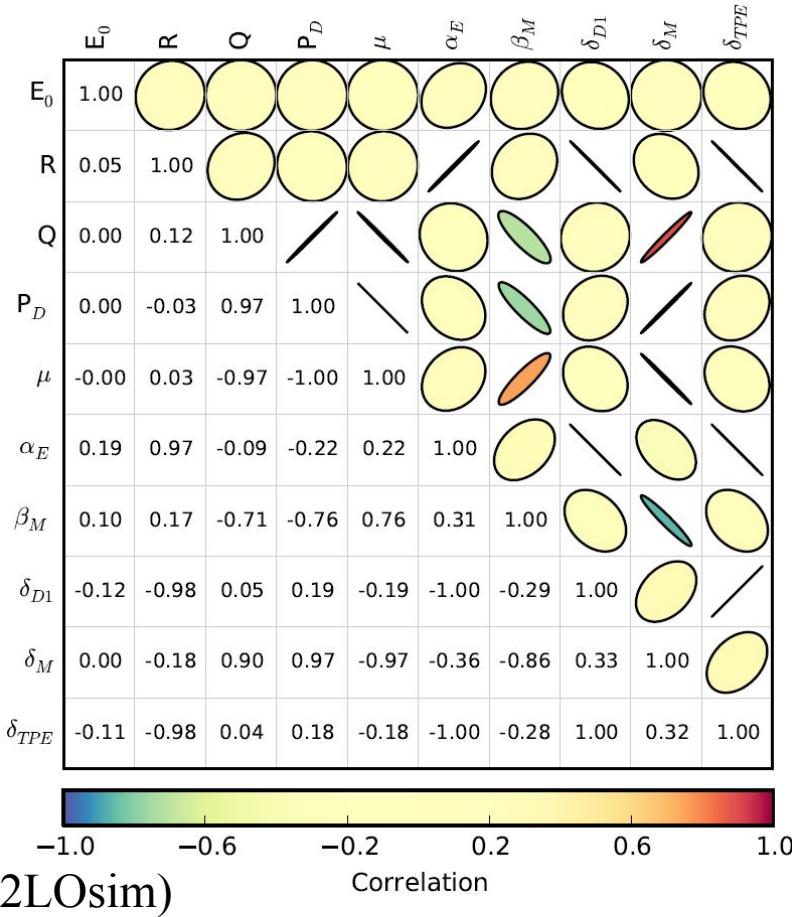
- Propagate uncertainty using standard techniques

$$J_{A,i} = \frac{\partial A}{\partial c_{\mu,i}}$$

$$\text{Cov}(A, B) = \mathbf{J}_A \text{Cov}(c_\mu) \mathbf{J}_B^T$$

$$\sigma_{A,stat} = \sqrt{\text{Cov}(A, A)}$$

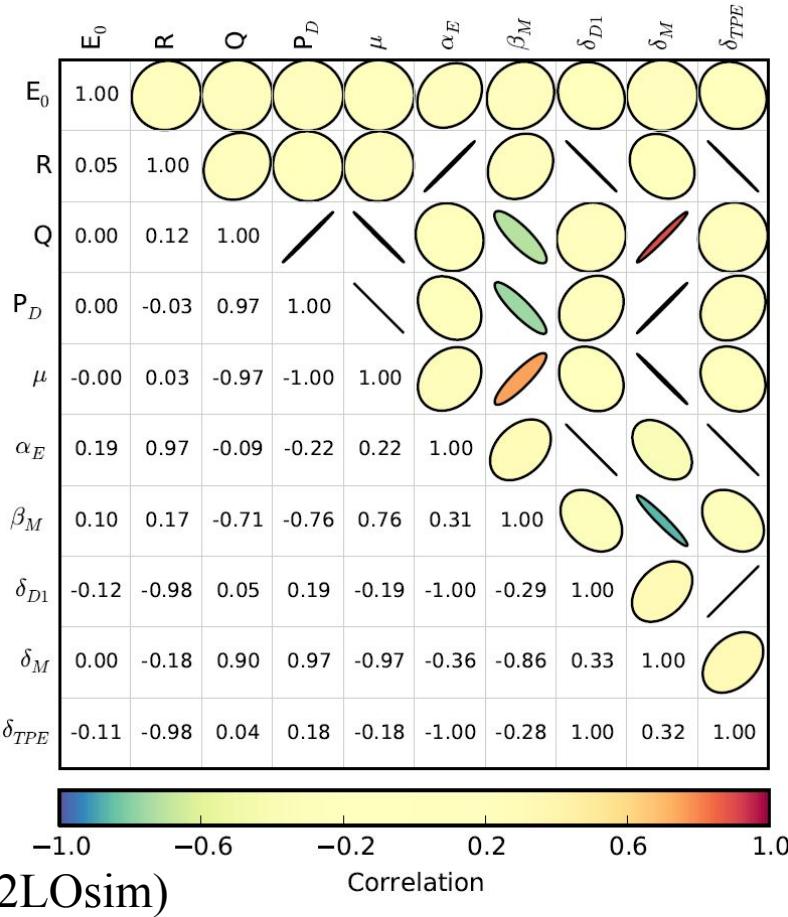
Correlation analysis



- Serves as a check of the error propagation formalism

$$\rho(A, B) = \frac{Cov(A, B)}{\sigma_A \sigma_B}$$

Correlation analysis

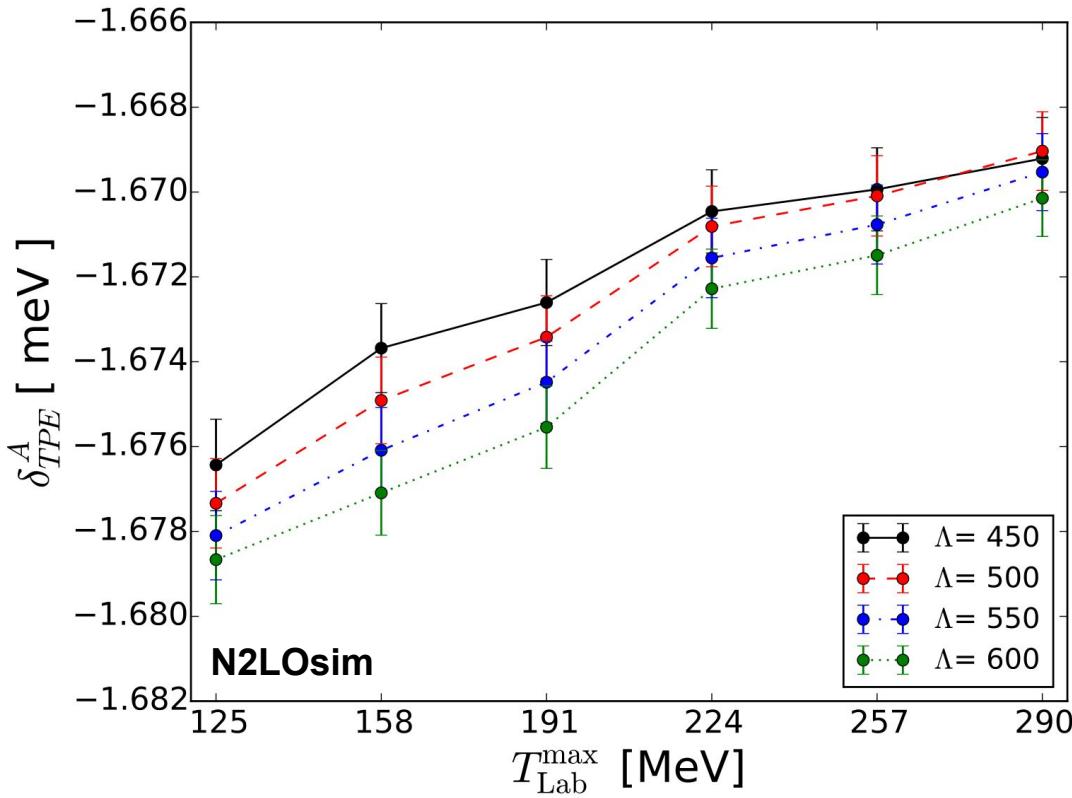


- Serves as a check of the error propagation formalism

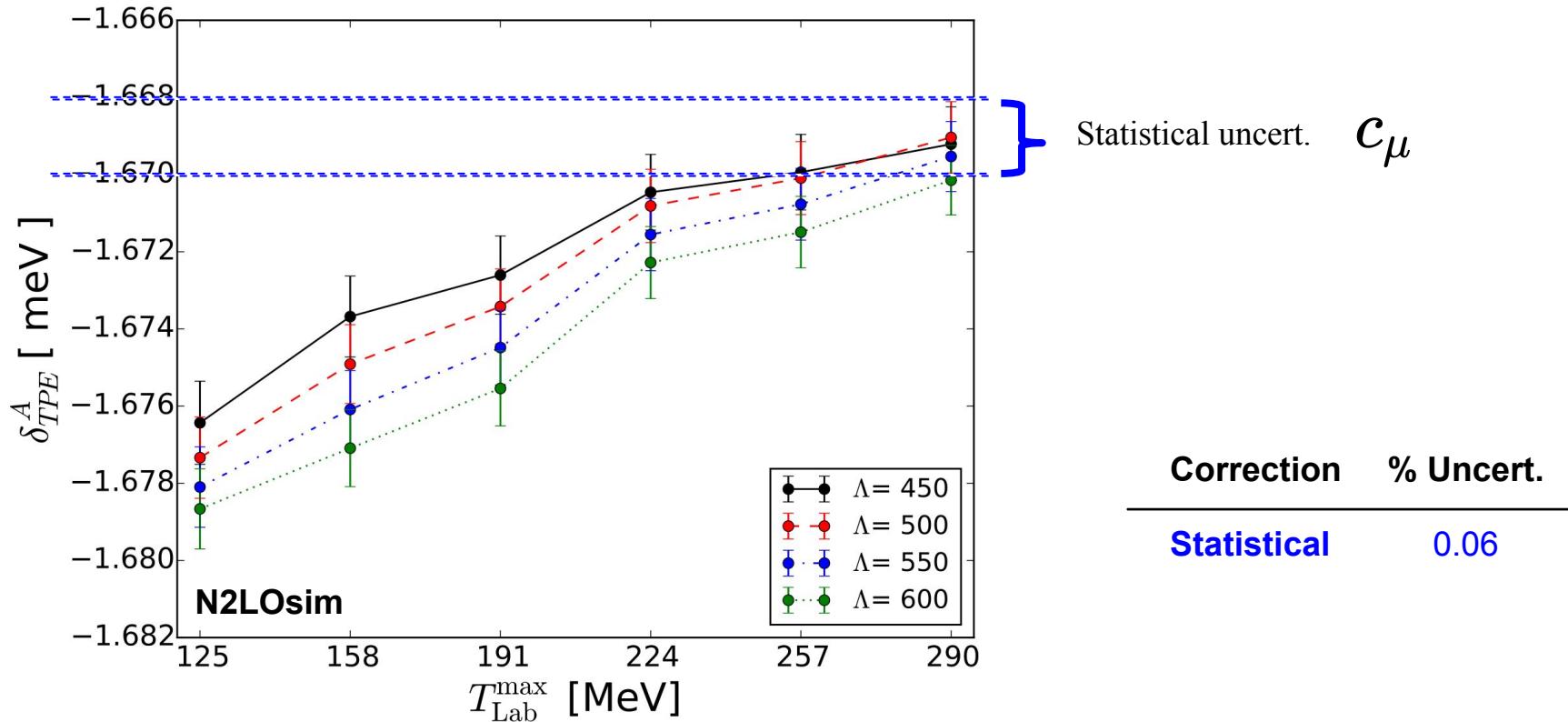
$$\rho(A, B) = \frac{Cov(A, B)}{\sigma_A \sigma_B}$$

- We observe strong correlations between
 - $\{P_d, \mu_d\}$
 - $\{R(^2\text{H}), \alpha_E\}$
 - $\{R(^2\text{H}), \delta_{TPE}\}$

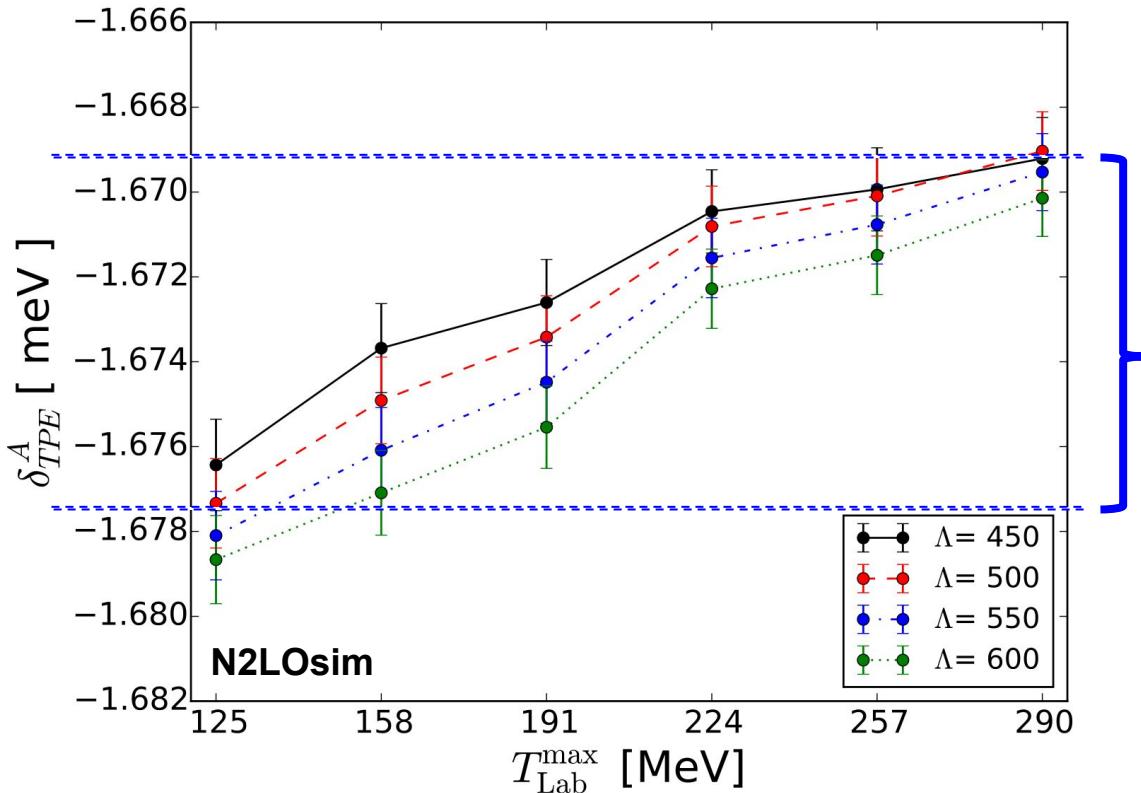
Statistical uncertainties



Statistical uncertainties



Sytematic Tlab Uncertainties



Correction	% Uncert.
Statistical	0.06
Tlab Sys.	0.2

Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^k c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu} = \text{LO diagram} + \text{NLO diagram} + \dots$$

The equation shows the chiral Lagrangian \mathcal{L}_{QCD} as a sum of terms. The first term is the LO (Leading Order) contribution, shown as a blue vertical line with a dot at the center, connected by a dashed horizontal line to another blue vertical line with a dot at its center. These two lines are crossed by a blue X-shaped internal line. The second term is the NLO (Next-to-Leading Order) contribution, shown as a red vertical line with a dot at the center, connected by a dashed horizontal line to another red vertical line with a dot at its center. These two lines are crossed by a red X-shaped internal line. A dashed red circle surrounds the leftmost vertical line. The third term is indicated by a red ellipsis followed by a plus sign.

Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^k c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu} = \text{LO} + \text{NLO} + \dots$$

The diagram illustrates the expansion of the QCD Lagrangian. It starts with a term at LO (Low Order) shown in blue, which consists of two vertical lines connected by a dashed horizontal line, with a crossed line vertex. This is followed by a plus sign. The next term is at NLO (Next-to-Low Order) shown in red, which includes the LO term plus a loop correction (a dashed circle around the LO vertex) and a crossed line vertex. This is followed by another plus sign.

- Expand observable in the same Chiral EFT pattern,

$$A^{N^k LO}(p) = A_0 \sum_{\nu=0}^{k+1} \beta_{\nu}(p) Q^{\nu}$$

$$Q = \max \left\{ \frac{p}{\Lambda_b}, \frac{m_{\pi}}{\Lambda_b} \right\}$$

Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^k c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu} = \text{LO} + \text{NLO} + \dots$$

- Expand observable in the same Chiral EFT pattern,

$$A^{N^k LO}(p) = A_0 \sum_{\nu=0}^{k+1} \beta_{\nu}(p) Q^{\nu} \quad Q = \max \left\{ \frac{p}{\Lambda_b}, \frac{m_{\pi}}{\Lambda_b} \right\}$$

- Truncation uncertainty can then be calculated according to

$$\sigma_{A,sys}^{N^k LO}(p) \approx Q \cdot |A_0 Q^{k+1} \beta_{k+1}|$$

Chiral truncation uncertainties

$$\mathcal{L}_{QCD} = \sum_{\nu}^k c_{\nu} \left(\frac{Q}{\Lambda_{\chi}} \right)^{\nu} = \text{LO} + \text{NLO} + \dots$$

The diagram illustrates the chiral expansion of the QCD Lagrangian. It shows the LO term (blue) and NLO term (red). The LO term consists of two vertical lines connected by a dashed horizontal line, with a blue cross connecting them. The NLO term adds a loop around the left vertical line and a red cross connecting the two lines.

- Expand observable in the same Chiral EFT pattern,

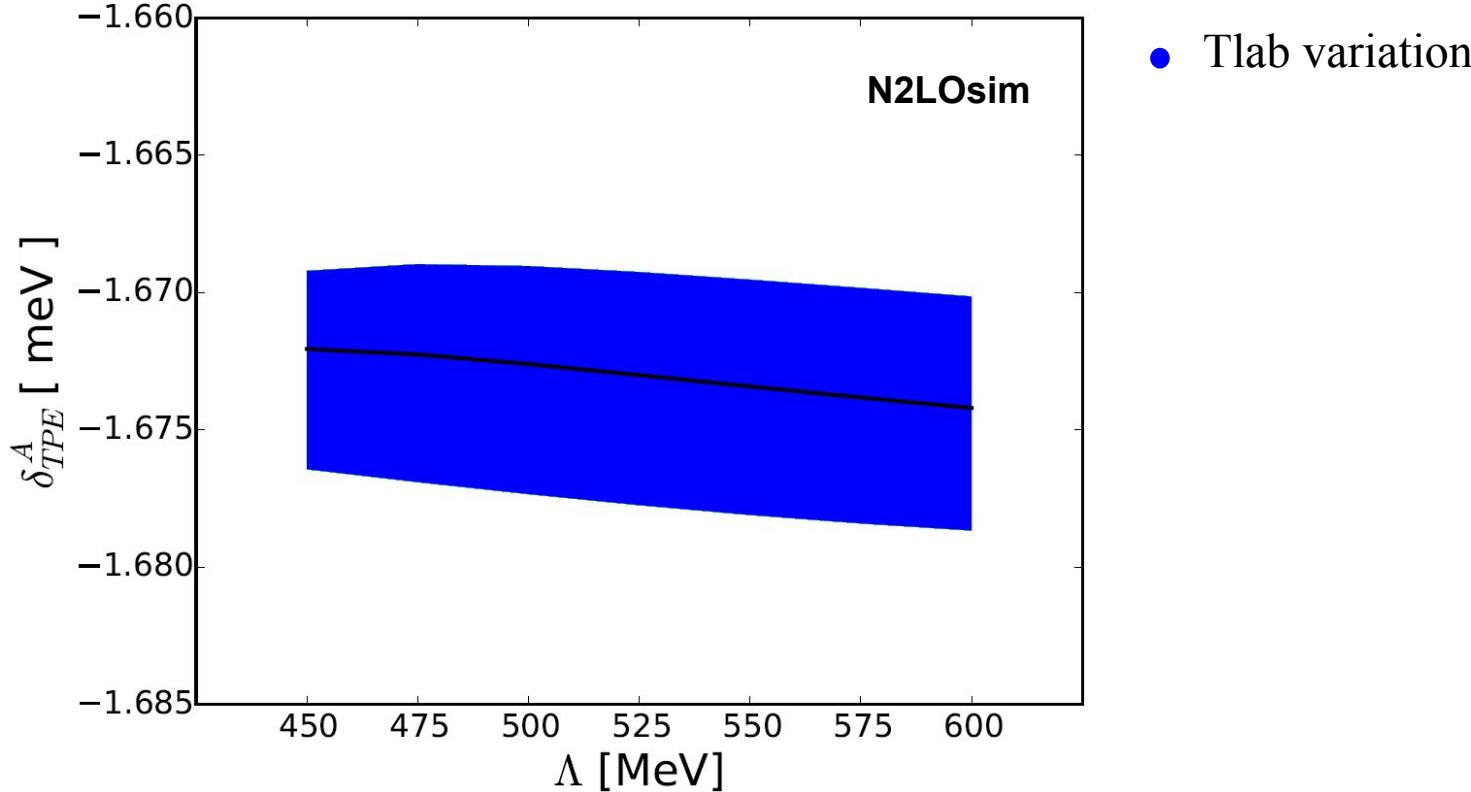
$$A^{N^k LO}(p) = A_0 \sum_{\nu=0}^{k+1} \beta_{\nu}(p) Q^{\nu} \quad Q = \max \left\{ \frac{p}{\Lambda_b}, \frac{m_{\pi}}{\Lambda_b} \right\}$$

- Truncation uncertainty can then be calculated according to

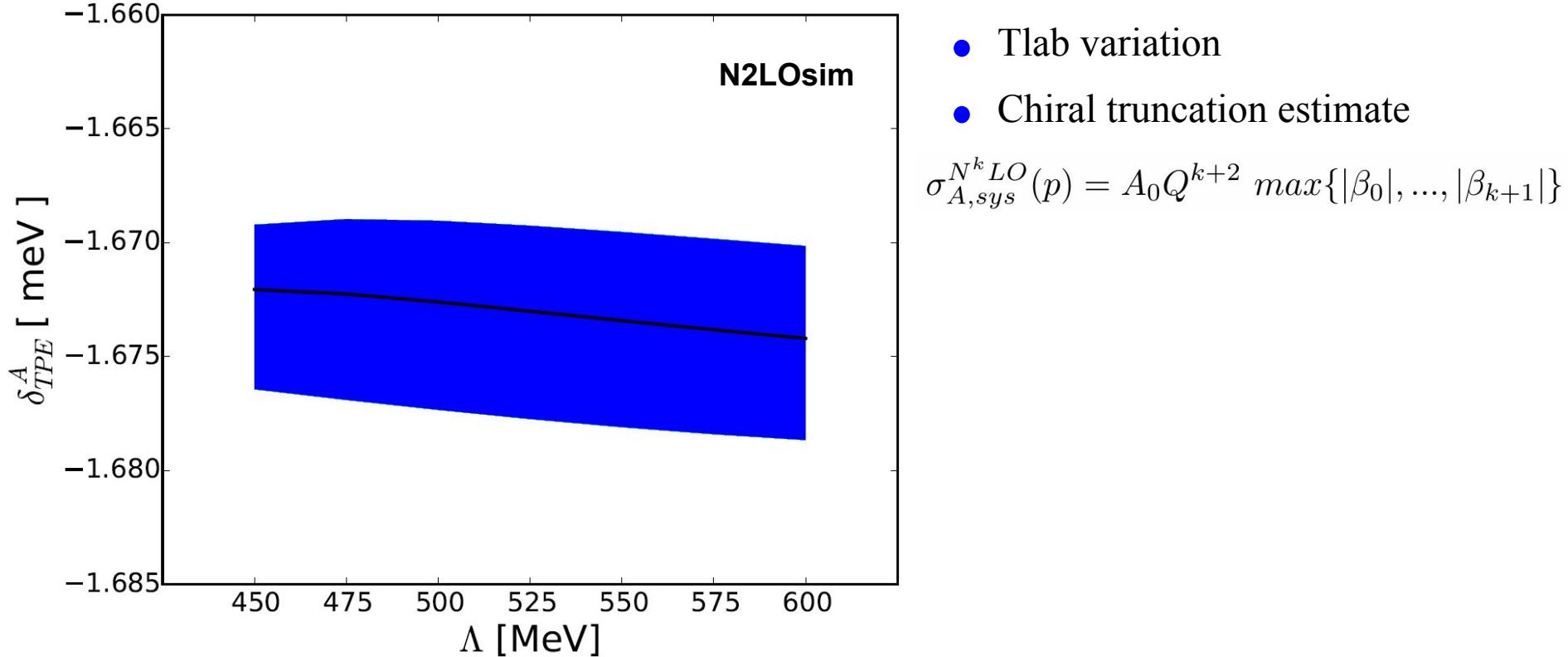
$$\sigma_{A,sys}^{N^k LO}(p) \approx Q \cdot |A_0 Q^{k+1} \beta_{k+1}|$$

$$\sigma_{A,sys}^{N^k LO}(p) = A_0 Q^{k+2} \max\{|\beta_0|, \dots, |\beta_{k+1}|\}$$

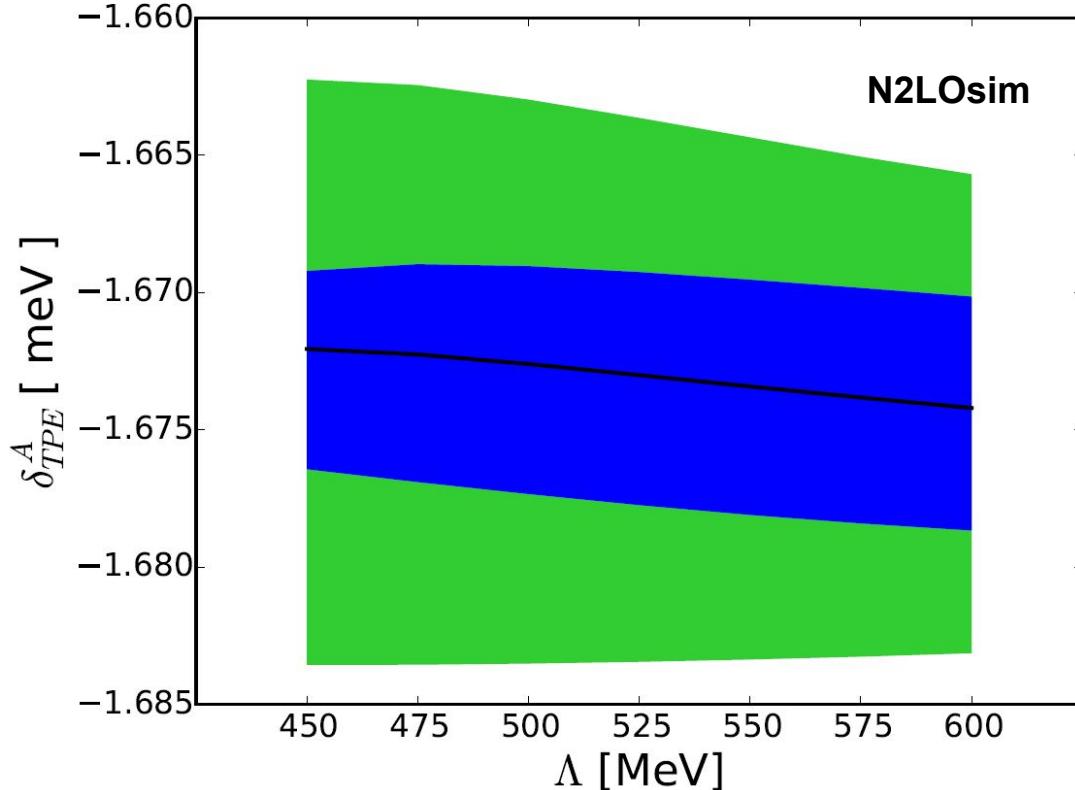
Chiral truncation uncertainties



Chiral truncation uncertainties



Chiral truncation uncertainties



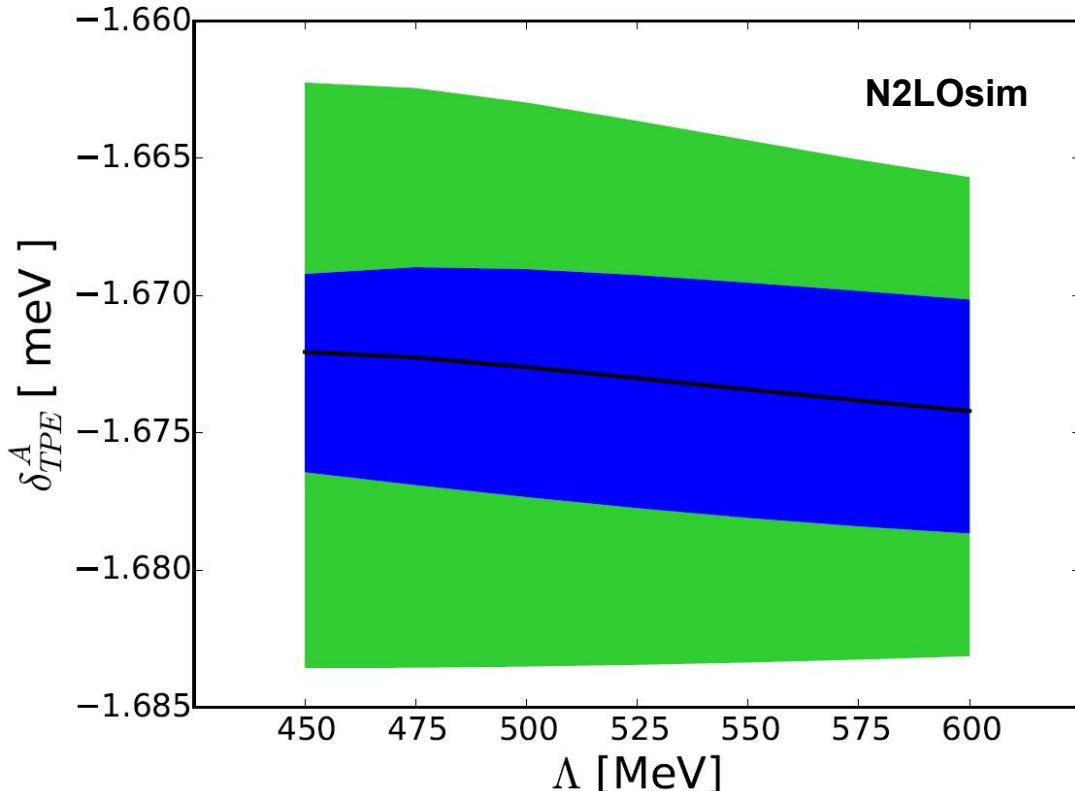
- Tlab variation
- Chiral truncation estimate

$$\sigma_{A,sys}^{N^k LO}(p) = A_0 Q^{k+2} \max\{|\beta_0|, \dots, |\beta_{k+1}|\}$$

- Estimate momentum scale of TPE

$$\langle \omega \rangle_{D1} = \frac{\int d\omega \omega \sqrt{\frac{2m_r}{\omega_N}} S_{D1}(\omega)}{\int d\omega \sqrt{\frac{2m_r}{\omega_N}} S_{D1}(\omega)}.$$

Chiral truncation uncertainties



- Tlab variation
- Chiral truncation estimate
- Estimate momentum scale of TPE

$$\sigma_{A,sys}^{N^k LO}(p) = A_0 Q^{k+2} \max\{|\beta_0|, \dots, |\beta_{k+1}|\}$$

$$\langle \omega \rangle_{D1} = \frac{\int d\omega \omega \sqrt{\frac{2m_r}{\omega_N}} S_{D1}(\omega)}{\int d\omega \sqrt{\frac{2m_r}{\omega_N}} S_{D1}(\omega)}.$$

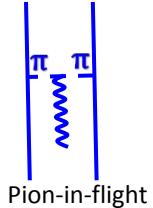
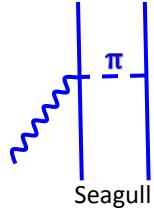
Correction	% Uncert.
Chiral Trunc.	0.4

Additional uncertainties

Two body currents + relativistic corr.

Additional uncertainties

Two body currents + relativistic corr.

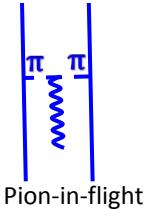
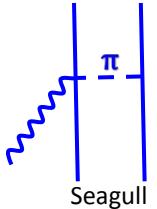


ρ, \vec{j}

Correction	% Uncert.
NLO MEC	0.05
Rel. Corr.	0.05

Additional uncertainties

Two body currents + relativistic corr.



ρ, \vec{j}

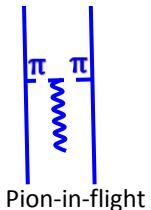
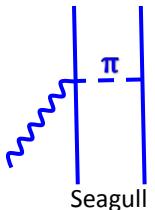
Eta Expansion η

$$\delta_{TPE}^A = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + O(\eta^3)$$

Correction	% Uncert.
NLO MEC	0.05
Rel. Corr.	0.05
Eta Exp.	0.3

Additional uncertainties

Two body currents + relativistic corr.



ρ, \vec{j}

Eta Expansion η

$$\delta_{TPE}^A = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + O(\eta^3)$$

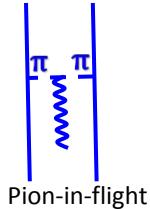
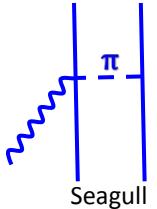
Single Nucleon Physics



Correction	% Uncert.
NLO MEC	0.05
Rel. Corr.	0.05
Eta Exp.	0.3
Nucleon	0.6

Additional uncertainties

Two body currents + relativistic corr.



ρ, \vec{j}

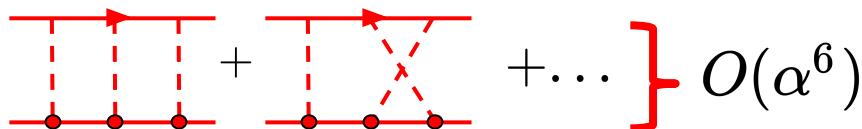
Eta Expansion η

$$\delta_{TPE}^A = \delta^{(0)} + \delta^{(1)} + \delta^{(2)} + O(\eta^3)$$

Single Nucleon Physics

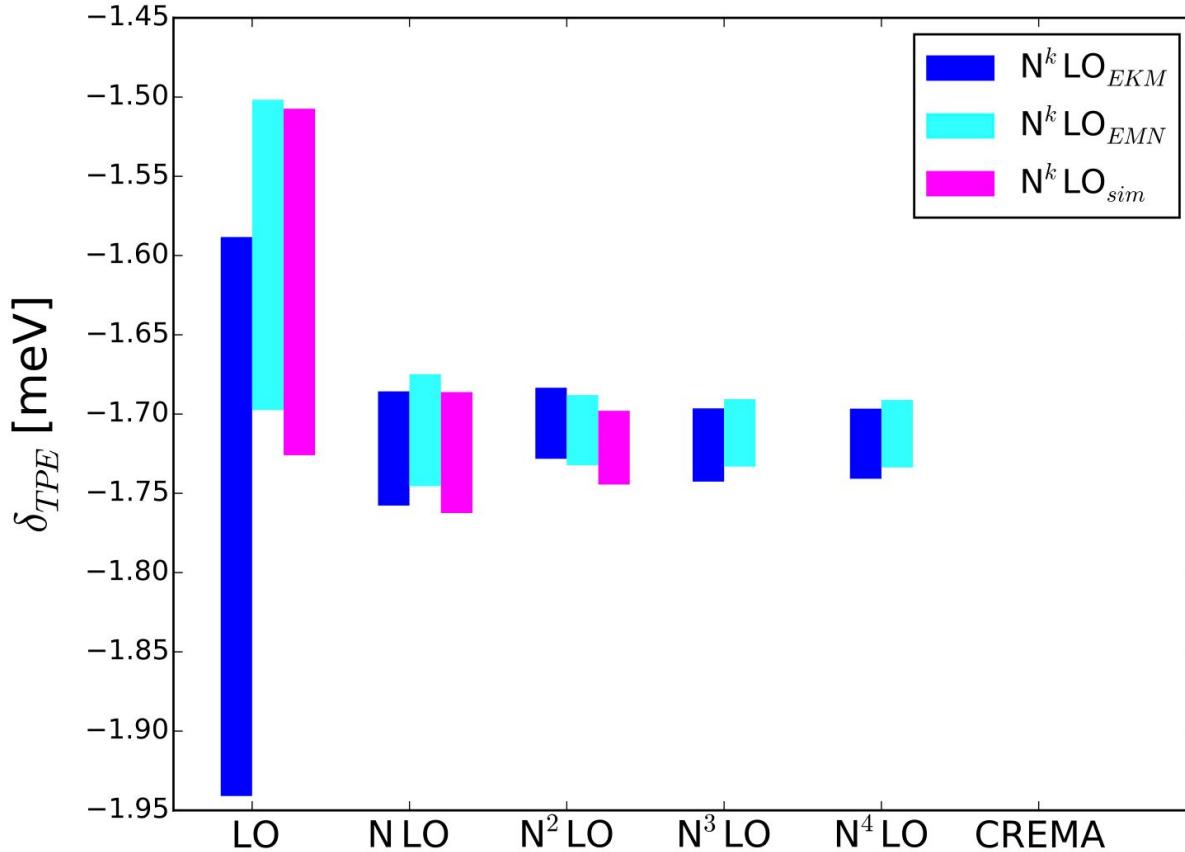


Atomic Physics uncert.

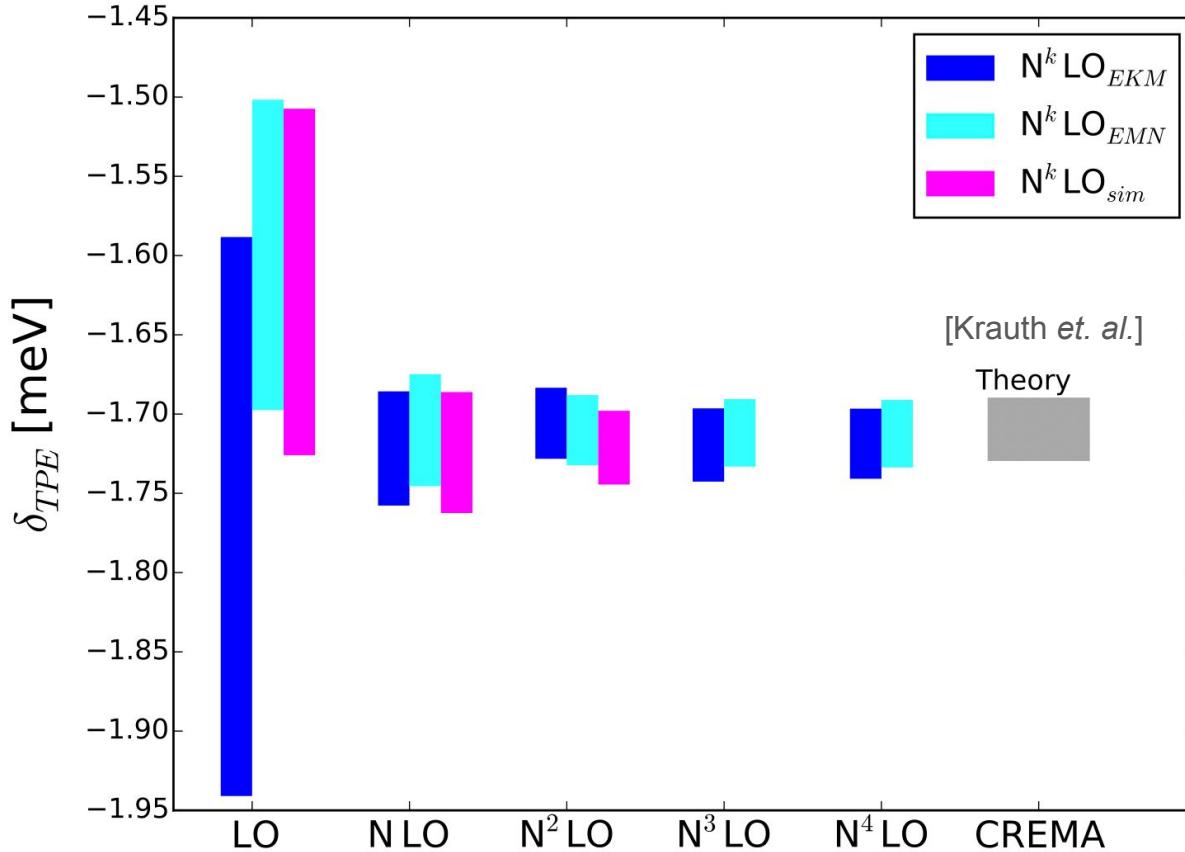


Correction	% Uncert.
NLO MEC	0.05
Rel. Corr.	0.05
Eta Exp.	0.3
Nucleon	0.6
Atomic Phys.	1.0

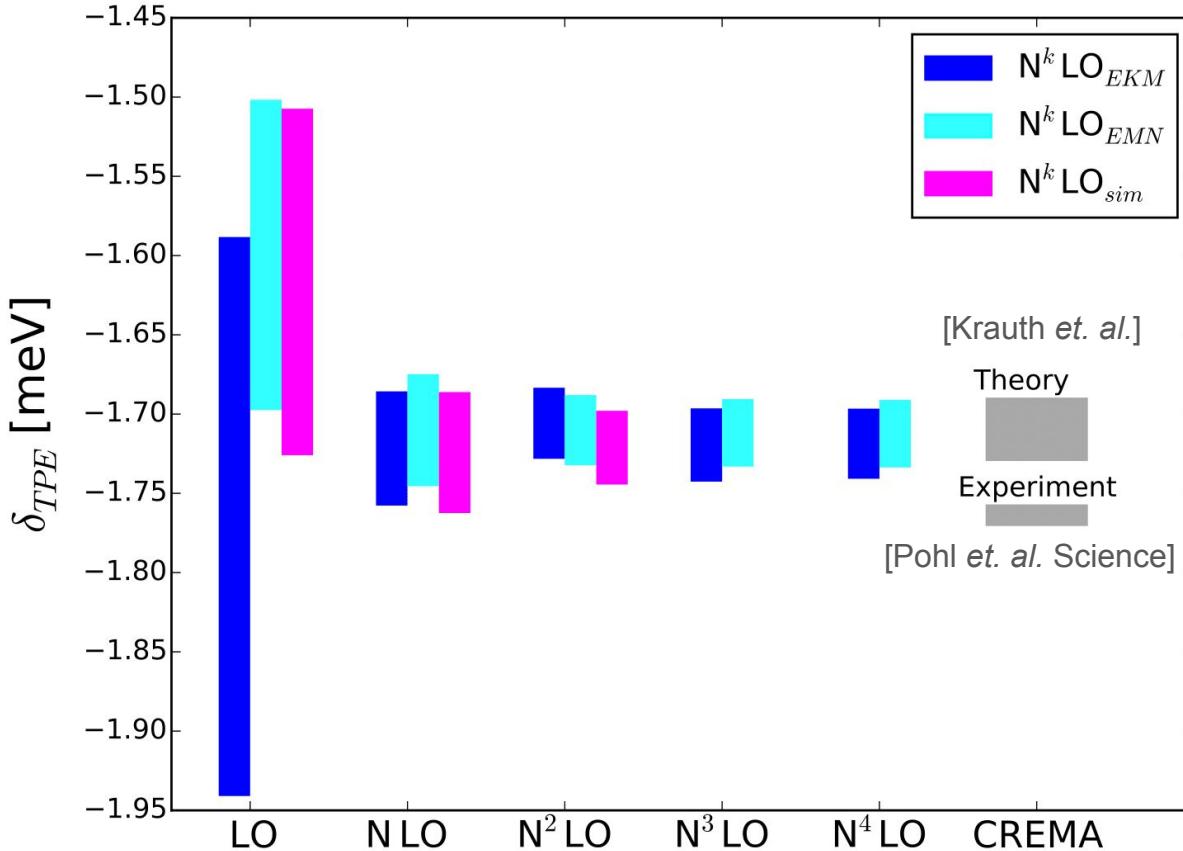
The total uncertainty



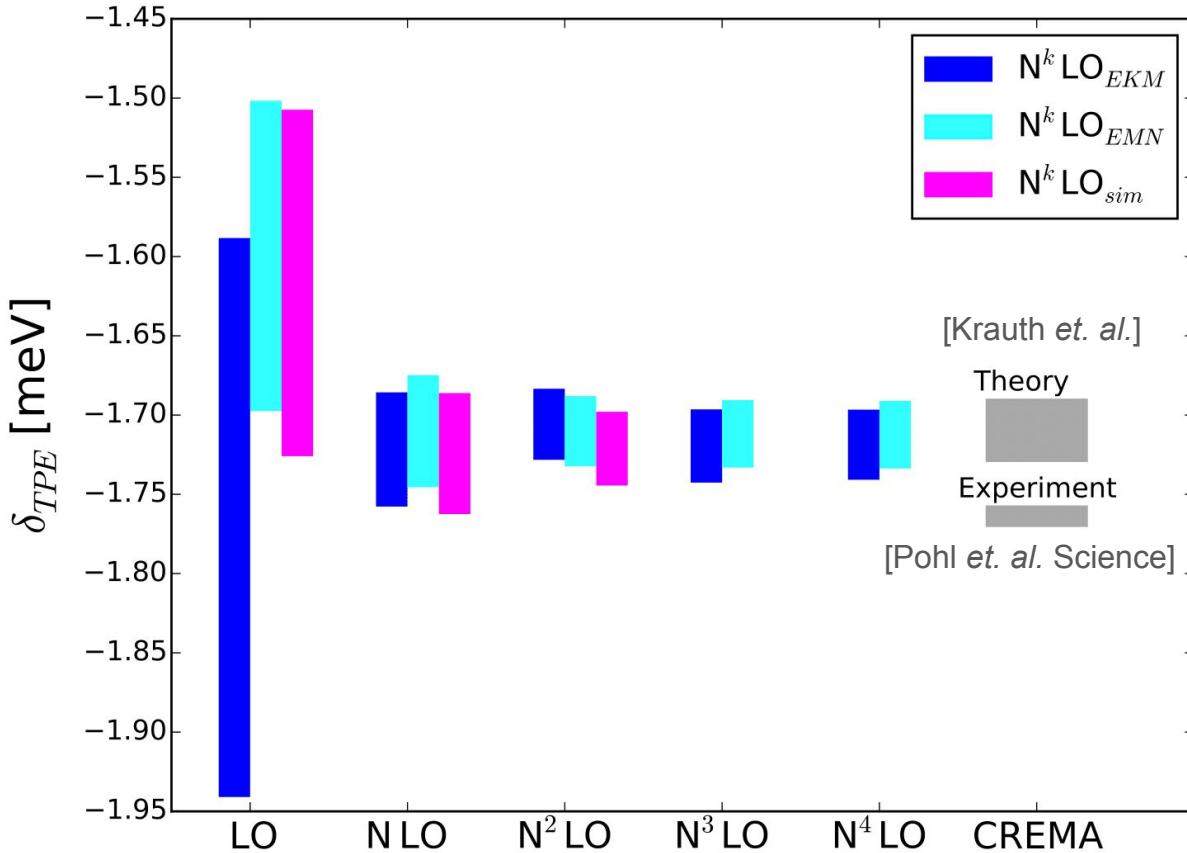
The total uncertainty



The total uncertainty



The total uncertainty



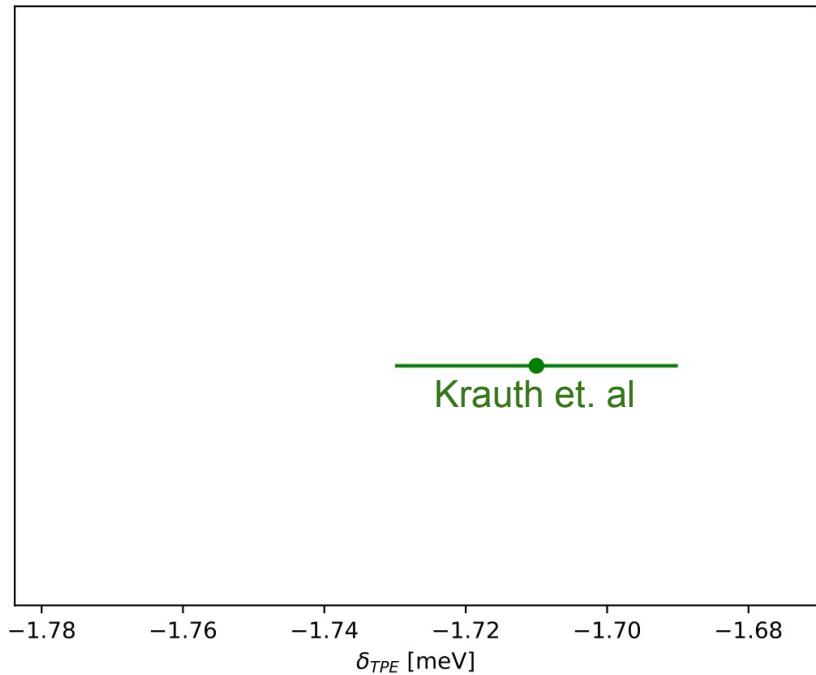
Contribution	Uncertainty in meV
Nuclear physics (syst)	+0.008 -0.011
Nuclear physics (stat)	± 0.001
η -expansion	± 0.005
Single-nucleon	± 0.0102
Atomic physics	± 0.0172
Total	+0.022 -0.024

$$\delta_{TPE} = -1.715 \text{ meV}$$

Summary

Krauth et. al. [2016]

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$



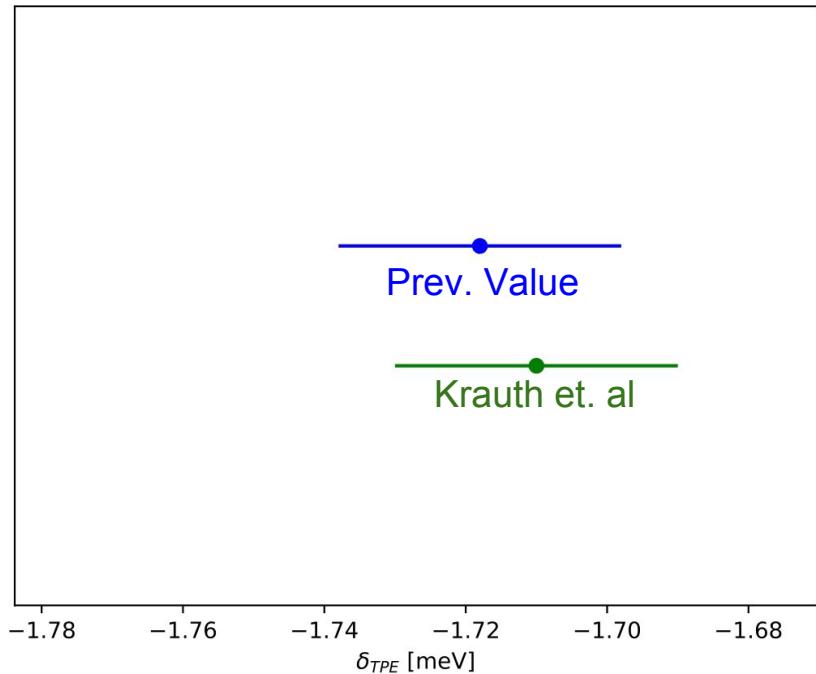
Summary

Krauth et. al. [2016]

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

Previous Value [2014,2016]

$$\delta_{TPE}(\text{Prev Work}) = -1.718(22) \text{ meV}$$



Summary

Krauth et. al. [2016]

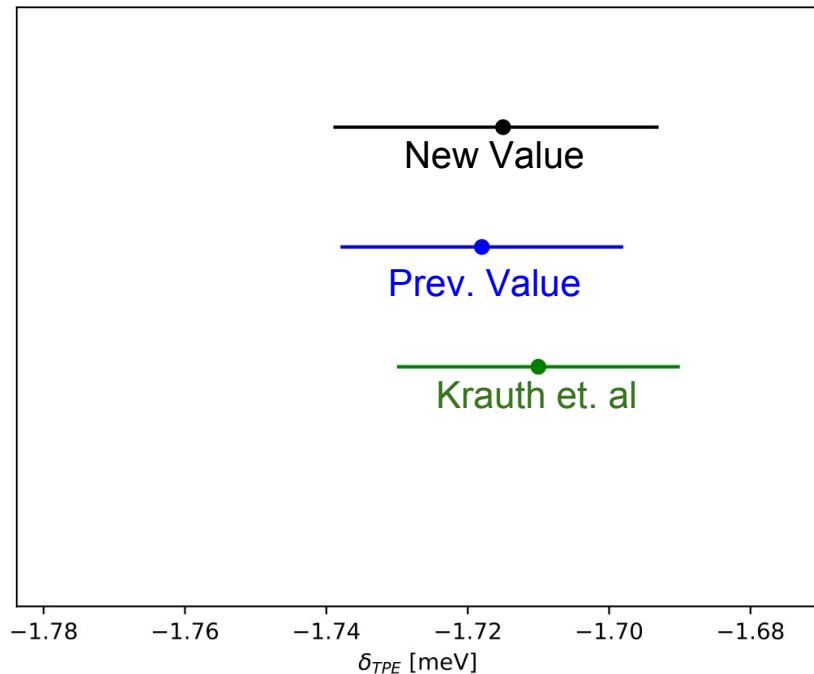
$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

Previous Value [2014,2016]

$$\delta_{TPE}(\text{Prev Work}) = -1.718(22) \text{ meV}$$

New value [2017]

$$\delta_{TPE} = -1.715^{+22}_{-24} \text{ meV}$$



Summary

Krauth et. al. [2016]

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

Previous Value [2014,2016]

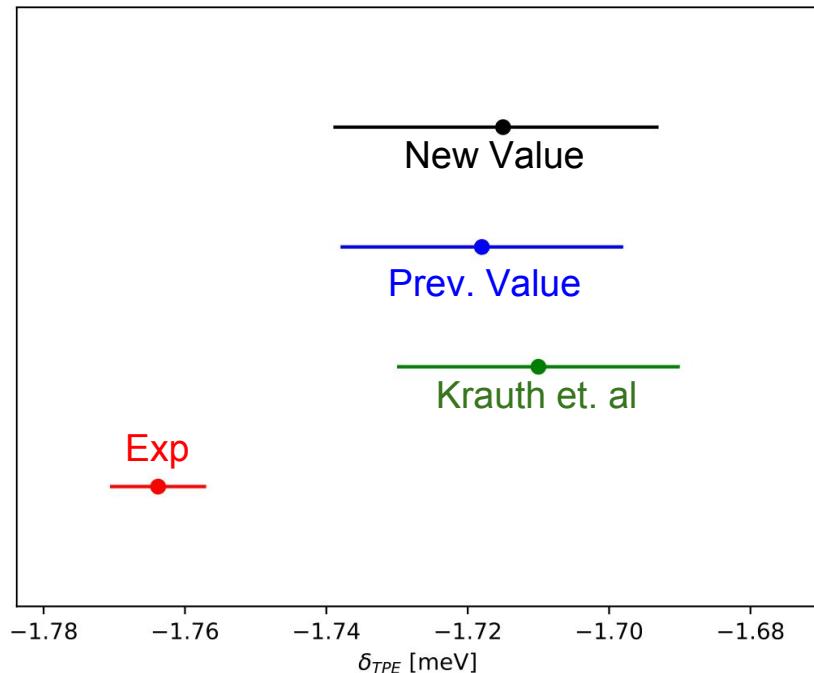
$$\delta_{TPE}(\text{Prev Work}) = -1.718(22) \text{ meV}$$

New value [2017]

$$\delta_{TPE} = -1.715^{+22}_{-24} \text{ meV}$$

Experimental

$$\delta_{TPE}(\text{Exp}) = -1.7638(68) \text{ meV}$$



Summary

Krauth et. al. [2016]

$$\delta_{TPE}(\text{Krauth et al.}) = -1.710(20) \text{ meV}$$

Previous Value [2014,2016]

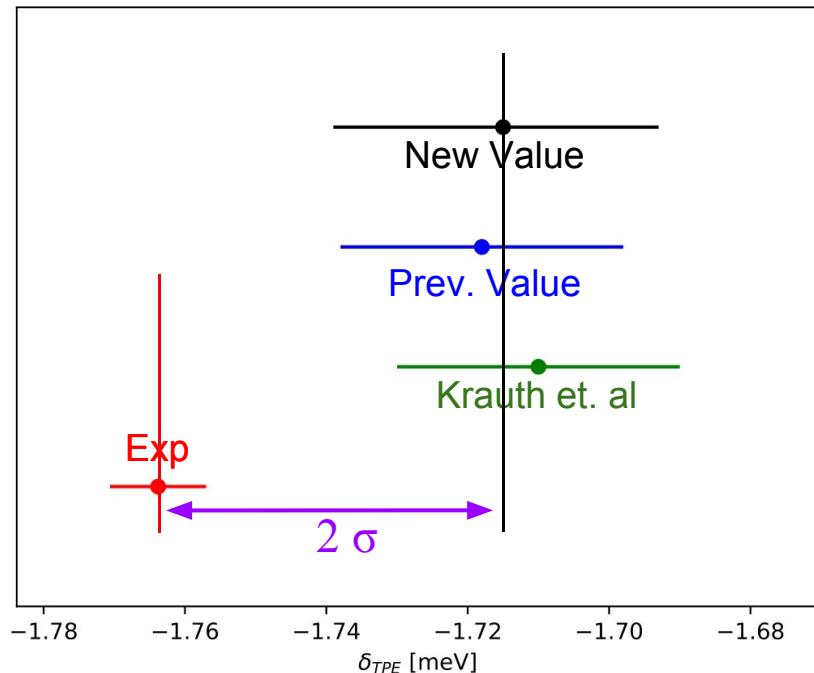
$$\delta_{TPE}(\text{Prev Work}) = -1.718(22) \text{ meV}$$

New value [2017]

$$\delta_{TPE} = -1.715^{+22}_{-24} \text{ meV}$$

Experimental

$$\delta_{TPE}(\text{Exp}) = -1.7638(68) \text{ meV}$$



Limitations of η expansion

- In the eta-expansion, higher order terms are very difficult $O(\eta^3)$

Limitations of η expansion

- In the eta-expansion, higher order terms are very difficult $O(\eta^3)$
- Eta expansion uncertainty dominates in A=3 systems

	2H	3H	3He
$O(\eta^3)$	0.4	2.0	2.0
$O(\alpha^6)$	1	1.5	1.5

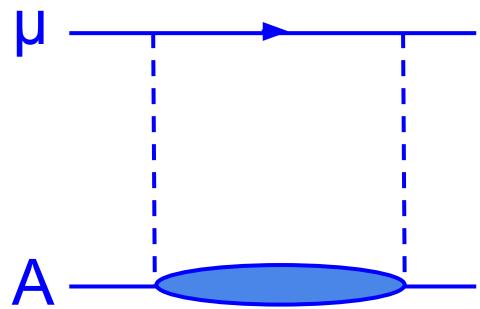
Limitations of η expansion

- In the eta-expansion, higher order terms are very difficult $O(\eta^3)$
- Eta expansion uncertainty dominates in A=3 systems

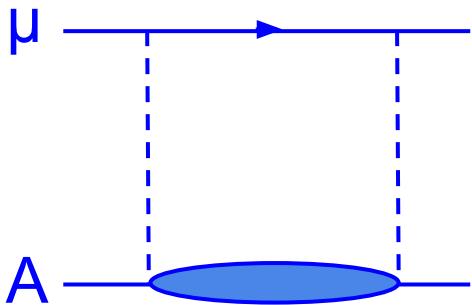
	2H	3H	3He
$O(\eta^3)$	0.4	2.0	2.0
$O(\alpha^6)$	1	1.5	1.5

- Can we avoid the eta-expansion?

η -less expansion

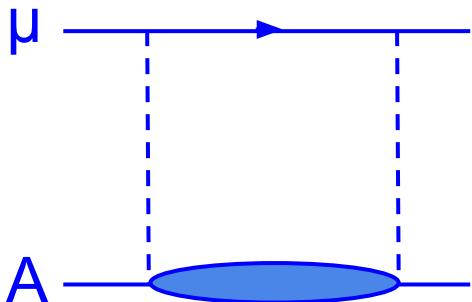


η -less expansion



$$P_{NR} = -2m_r\phi^2(0)Z^2 \int \frac{d^3q}{(2\pi)^3} \left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{q^2 + 2m_r\omega_N} (1 - e^{i\mathbf{q}\cdot\mathbf{R}})(1 - e^{-i\mathbf{q}\cdot\mathbf{R}'})$$

η -less expansion



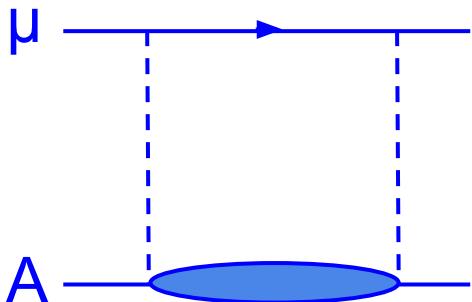
$$P_{NR} = -2m_r\phi^2(0)Z^2 \int \frac{d^3q}{(2\pi)^3} \left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{q^2 + 2m_r\omega_N} (1 - e^{i\mathbf{q}\cdot\mathbf{R}})(1 - e^{-i\mathbf{q}\cdot\mathbf{R}'})$$

- Insert intermediate states

$$\sum_{N \neq N_0} |N\rangle\langle N|$$

A red arrow points from the term $\sum_{N \neq N_0} |N\rangle\langle N|$ up towards the η -less expansion equation above it.

η -less expansion



$$P_{NR} = -2m_r\phi^2(0)Z^2 \int \frac{d^3q}{(2\pi)^3} \left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{q^2 + 2m_r\omega_N} (1 - e^{i\mathbf{q}\cdot\mathbf{R}})(1 - e^{-i\mathbf{q}\cdot\mathbf{R}'})$$

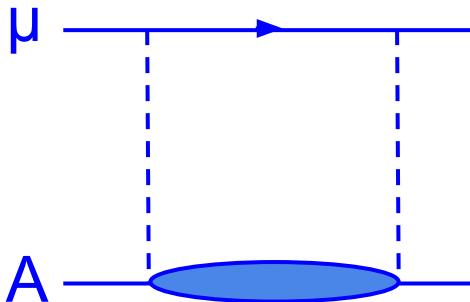
- Insert intermediate states

$$\sum_{N \neq N_0} |N\rangle\langle N|$$

↑
↑

$$\delta_{NR}^A = -8(Z\alpha)^2 |\phi(0)|^2 \int dq \int d\omega K_{NR}(q, \omega) S_L(q, \omega)$$

η -less expansion



$$P_{NR} = -2m_r\phi^2(0)Z^2 \int \frac{d^3q}{(2\pi)^3} \left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{q^2 + 2m_r\omega_N} (1 - e^{i\mathbf{q}\cdot\mathbf{R}})(1 - e^{-i\mathbf{q}\cdot\mathbf{R}'})$$

- Insert intermediate states

$$\sum_{N \neq N_0} |N\rangle\langle N|$$

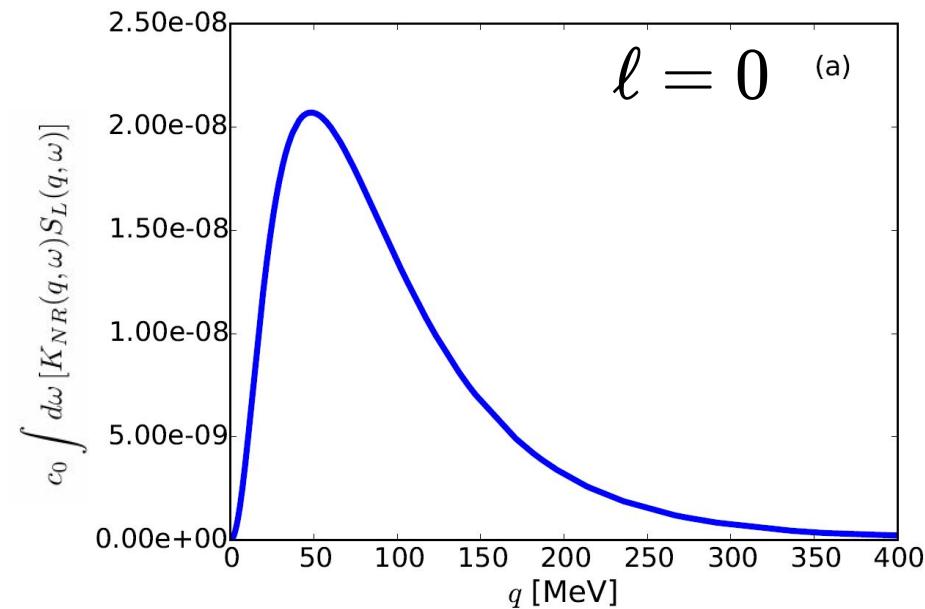
A red arrow points from the term $(1 - e^{-i\mathbf{q}\cdot\mathbf{R}'})$ in the equation above to this expression.

$$\delta_{NR}^A = -8(Z\alpha)^2 |\phi(0)|^2 \int dq \int d\omega K_{NR}(q, \omega) S_L(q, \omega)$$

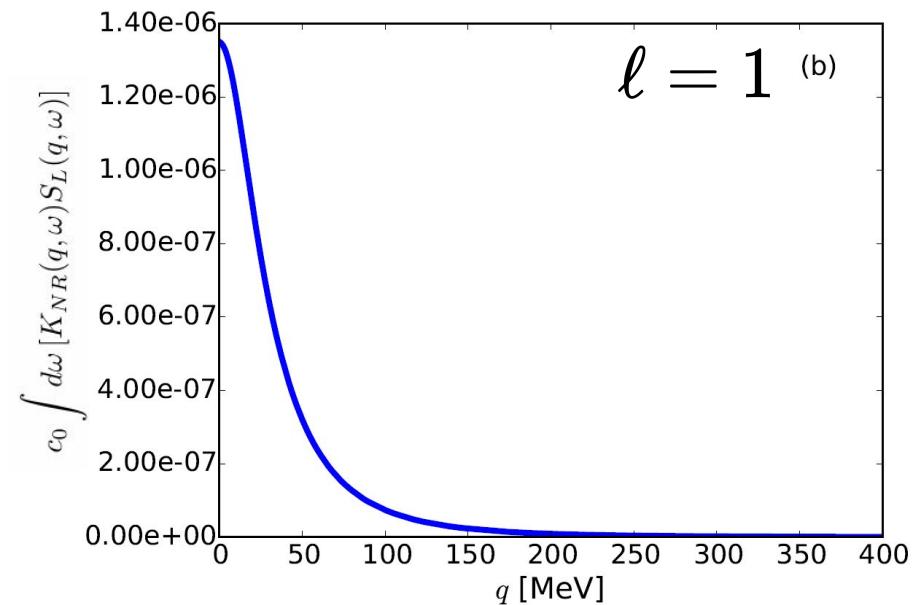
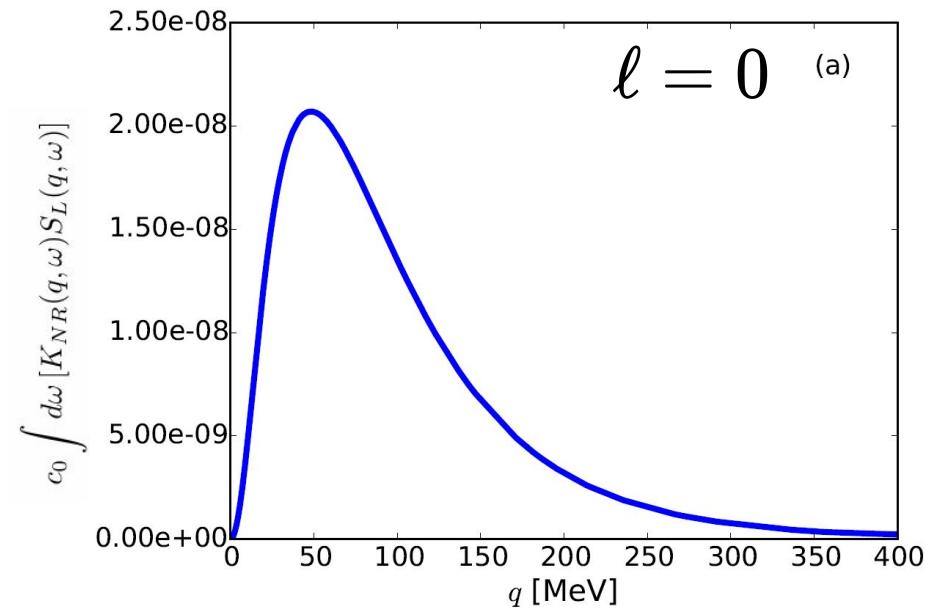
- Full treatment

$$\delta_{TPE}^A = -8(Z\alpha)^2 |\phi(0)|^2 \int dq \int d\omega [K_L(q, \omega) S_L(q, \omega) + K_T(q, \omega) S_T(q, \omega) + K_S(q, \omega) S_T(0, \omega)]$$

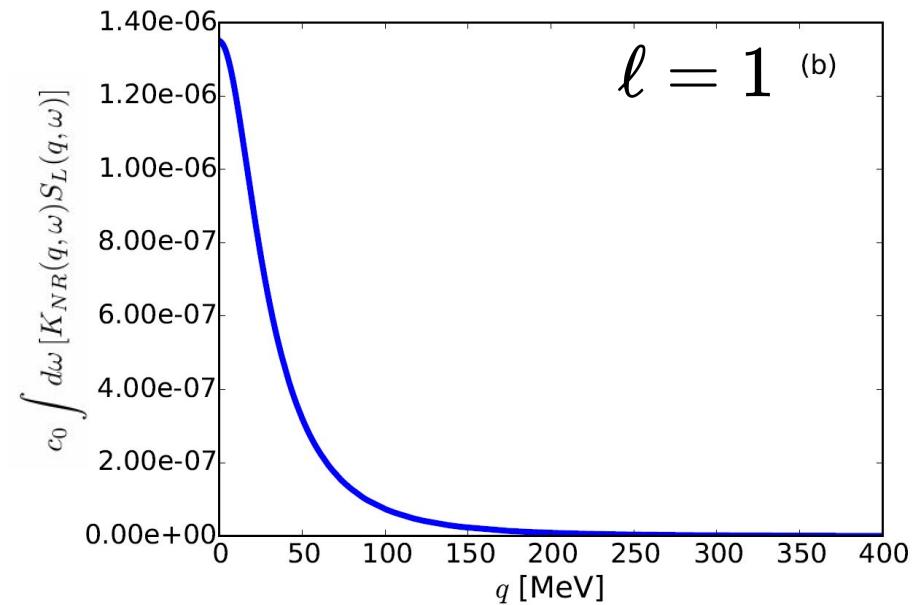
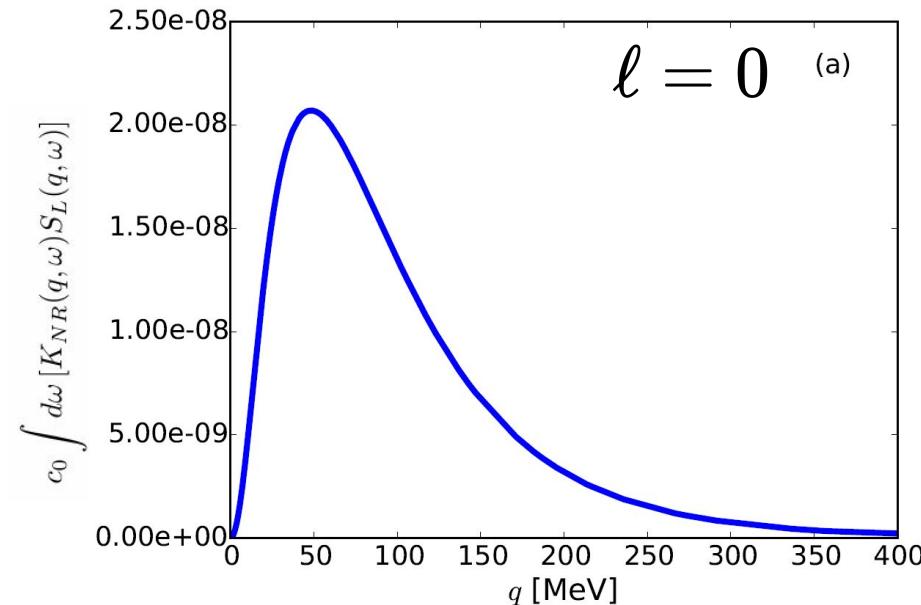
η -less expansion



η -less expansion

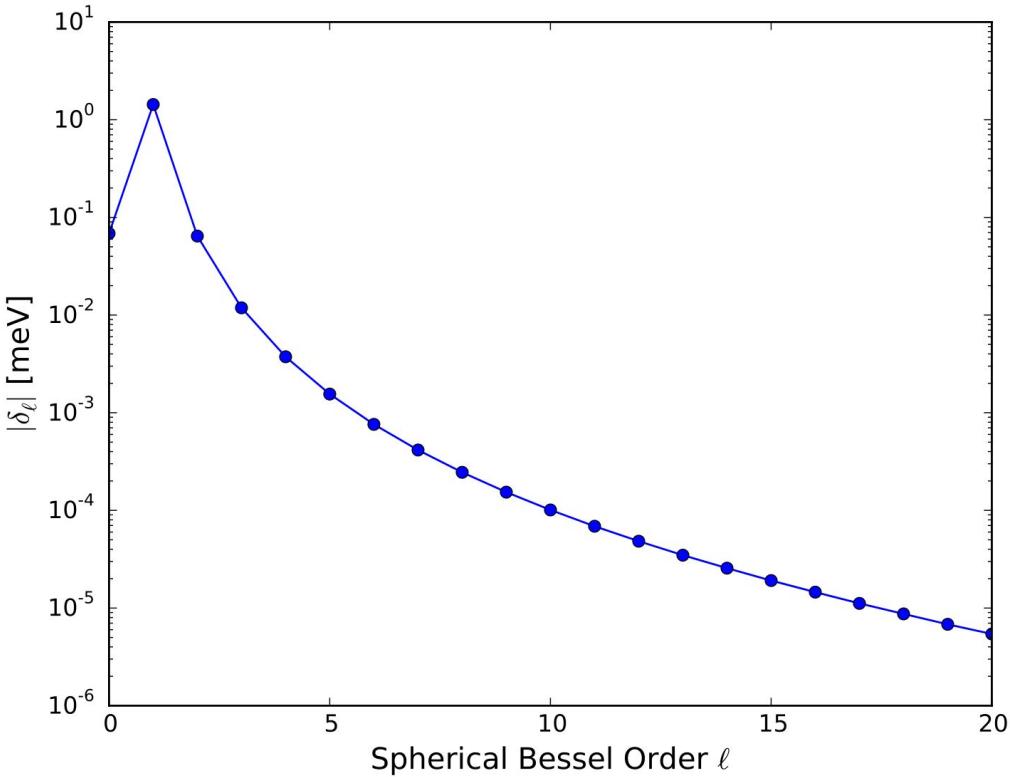


η -less expansion



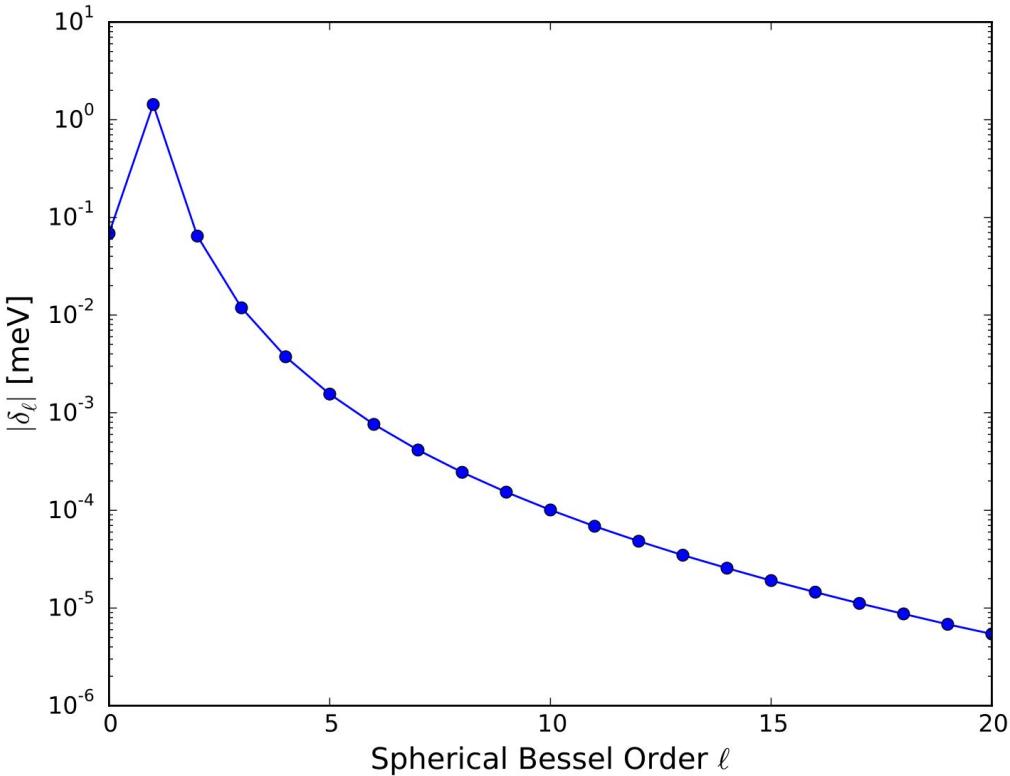
$N^3\text{LO-EM}$	$\delta_{\ell=0}^{(0)}$	$\delta_{\ell=1}^{(0)}$	$\delta_{\ell=2}^{(0)}$	$\delta_{\ell=3}^{(0)}$	$\delta_{\ell=4}^{(0)}$	$\sum \delta_{\ell}^{(0)}$
[meV]	-0.069	-1.436	-0.064	-0.012	-0.004	-1.585
[meV]	[PLB 2014., Proc. H.H.I 2016]				-1.590	

η -less expansion



ℓ	δ_ℓ [meV]
0	-6.856010106985817E-002
1	-1.43618685244822
2	-6.442225521652188E-002
3	-1.186645711405751E-002
4	-3.741888013064462E-003
5	-1.552676561183561E-003
6	-7.602058875707891E-004
7	-4.151874435608740E-004
8	-2.454274817754850E-004
9	-1.537989305734875E-004
10	-1.010484483922072E-004
11	-6.881334910974345E-005
12	-4.837697294962907E-005
13	-3.480144716570742E-005
14	-2.561972635803522E-005
15	-1.913783414930239E-005
16	-1.455885571724146E-005
17	-1.117441231993074E-005
18	-8.713757524696711E-006
19	-6.827246811968316E-006
20	-5.428475472307609E-006
Sum	-1.5882493506924E+00

η -less expansion



η -expansion Result:

$$N^3 LO_{EM} - 1.590 \text{ [meV]}$$

ℓ	δ_ℓ [meV]
0	-6.856010106985817E-002
1	-1.43618685244822
2	-6.442225521652188E-002
3	-1.186645711405751E-002
4	-3.741888013064462E-003
5	-1.552676561183561E-003
6	-7.602058875707891E-004
7	-4.151874435608740E-004
8	-2.454274817754850E-004
9	-1.537989305734875E-004
10	-1.010484483922072E-004
11	-6.881334910974345E-005
12	-4.837697294962907E-005
13	-3.480144716570742E-005
14	-2.561972635803522E-005
15	-1.913783414930239E-005
16	-1.455885571724146E-005
17	-1.117441231993074E-005
18	-8.713757524696711E-006
19	-6.827246811968316E-006
20	-5.428475472307609E-006
Sum	-1.5882493506924E+00

Outlook

Uncertainty Analysis:

- Reduce atomic physics uncert. $O(\alpha^6)$

Outlook

Uncertainty Analysis:

- Reduce atomic physics uncert. $O(\alpha^6)$

Etaless Expansion

- Implement transverse corrections in A=2
- Apply formalism to A=3 systems
- Extend formalism for HFS