Electromagnetic properties of nuclei: from few- to many-body systems

Lecture 12

Muonic Atoms

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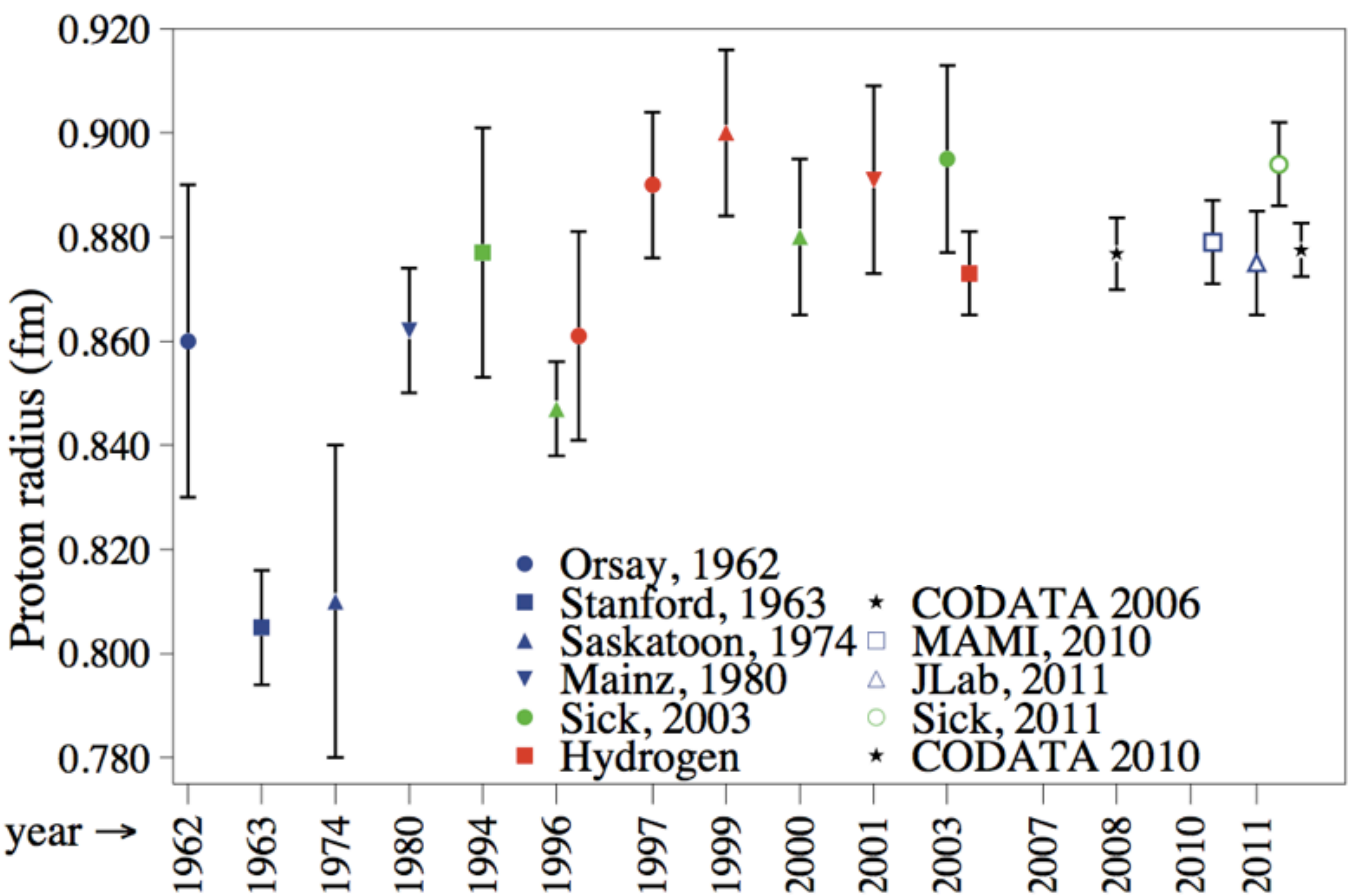
November 24th, 2017

Lecture series for SFB 1245
TU Darmstadt

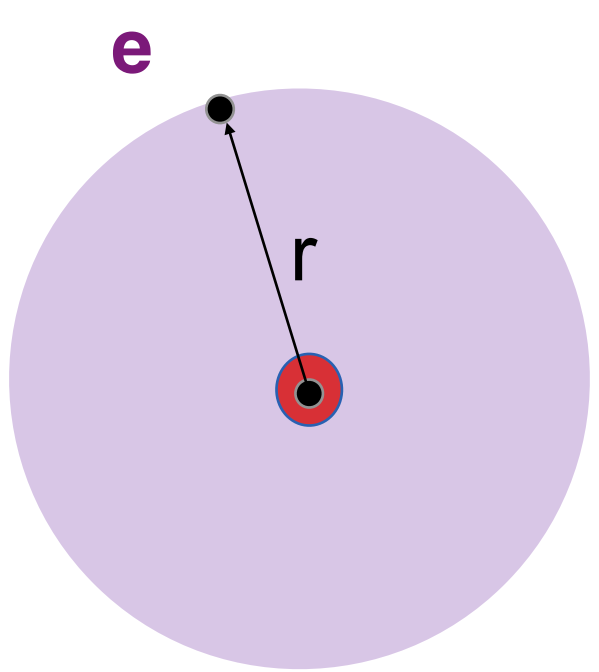
Proton-radius puzzle

The Proton Charge Radius



Pic from Pohl *et al.*, Ann.Rev.Nucl.Part.Sci. 63 (2013)

$$r_p^2 = -6 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0}$$



CODATA2010:
r_p= 0.8775(51)fm

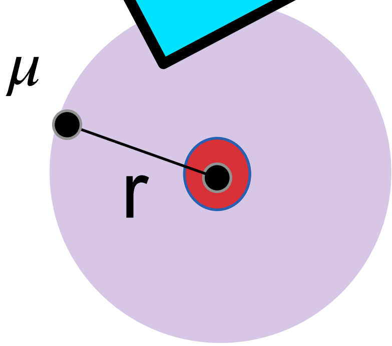
Mohr, *et al.* Rev. Mod Phys. (2012)

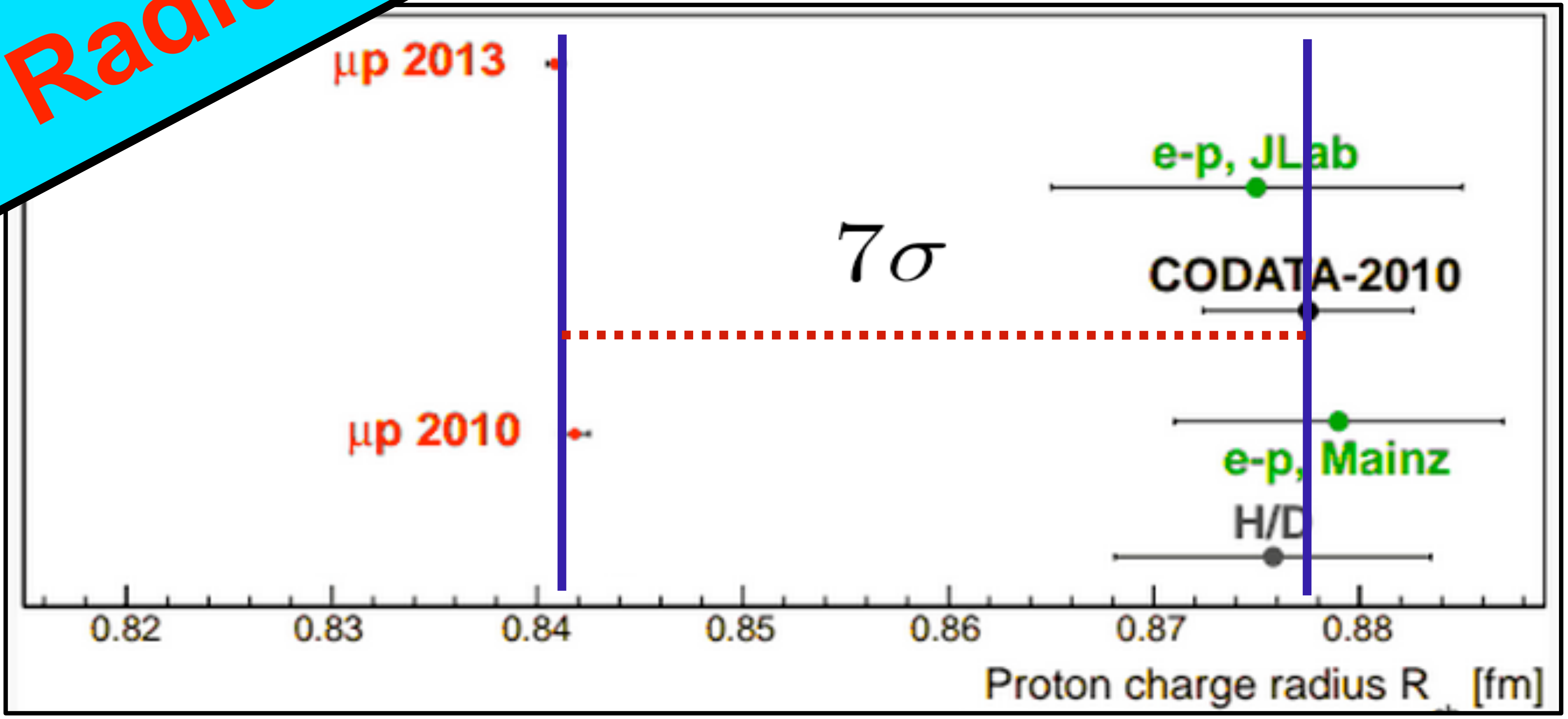
How Small is the Proton?

In 2010, proton charge radius was determined in muonic Hydrogen at PSI from spectroscopy measurements of th amb-shift:

Pohl et al r_p= 0.84184(67) fm
Ant 013): r_p= 0.84087(39) fm

Proton Radius Puzzle!





Origin of the discrepancy?

- Experimental results may be wrong:

"Multiple independent electron-proton experiments agree, and the muonic hydrogen experiment looks more convincing than any of the electron-proton experiments"
Pohl, Gilman, Miller, Pachucki, Ann.Rev.Nucl.Part.Sci. 63 (2013)

Electron scattering experiments are done at finite Q^2 , maybe not small enough

Dispersion analysis:
global fit of n and p give r_p= 0.84(1) with
$\chi^2 \approx 2.2$

Lorenz, Hammer, Meissner, EPJA (2012)

- Exotic hadronic structures?

Birse, McGovern EPJA (2012) vs Miller PLB (2013)

- New physics beyond standard model?

New force carrier, e.g. dark photon, that couples differently with e and μ
Yavin, Pospelov, Carlson etc...

New Experiments to Shed Light on the Puzzle

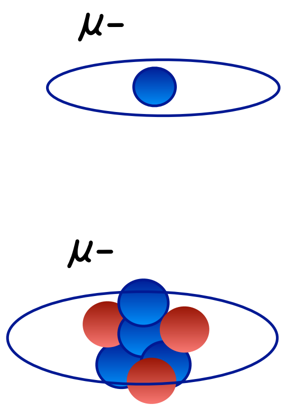
- Higher precision electron scattering experiments

  Q^2 from 10^{-4} GeV^2 to 10^{-2} GeV^2

  Efforts also in Mainz

- MUSE collaboration
  measure $e^{\pm}p$ and $\mu^{\pm}p$ to reduce systematic errors

- CREMA collaboration currently analyzing Lamb shift data in light muonic atoms: Deuterium, Helions



See talk by Randolf Pohl

Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2 \rangle + \delta_{\rm TPE}$$

in a $Z\alpha$ expansion up to 5th order

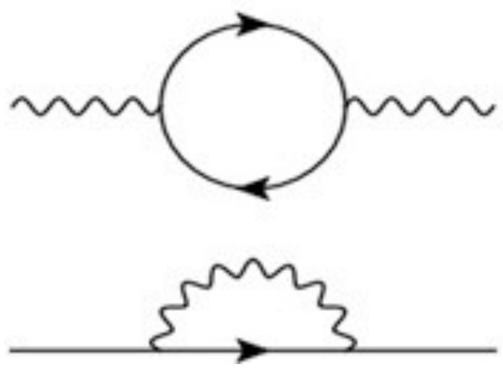
Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E_{2S-2P} = \boxed{\delta_{\mathrm{QED}}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

- QED corrections

vacuum polarizations
lepton self energy
relativistic recoil



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- Nuclear structure corrections

Elastic corrections: Finite size

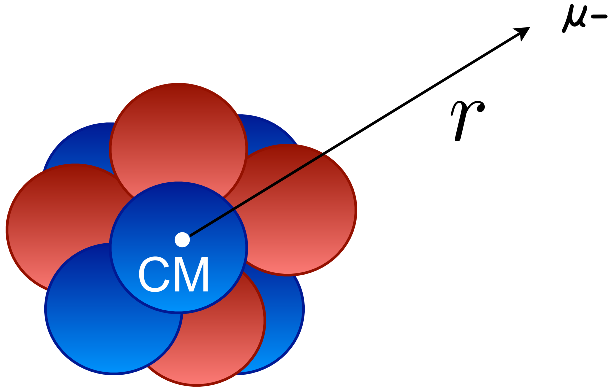
Charge Radius From the Lamb-Shift

Extracting the radius from measurements requires theoretical input

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \boxed{\delta_{\mathrm{TPE}}}$$

- Nuclear structure corrections

Contains elastic terms, such as the Zemach moment
and inelastic terms such as the nuclear polarization



Dipole excitation

Stronger Coulomb - reduced energy

Charge Radius From the Lamb-Shift

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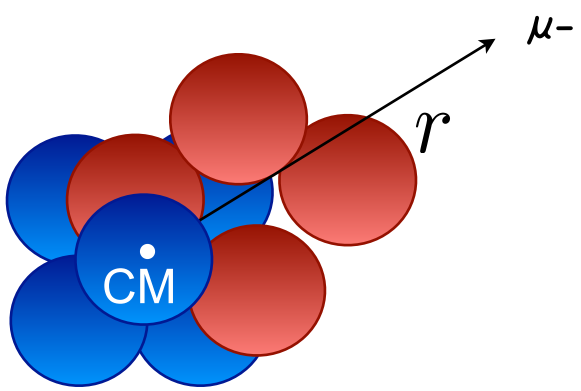


Dipole excitation

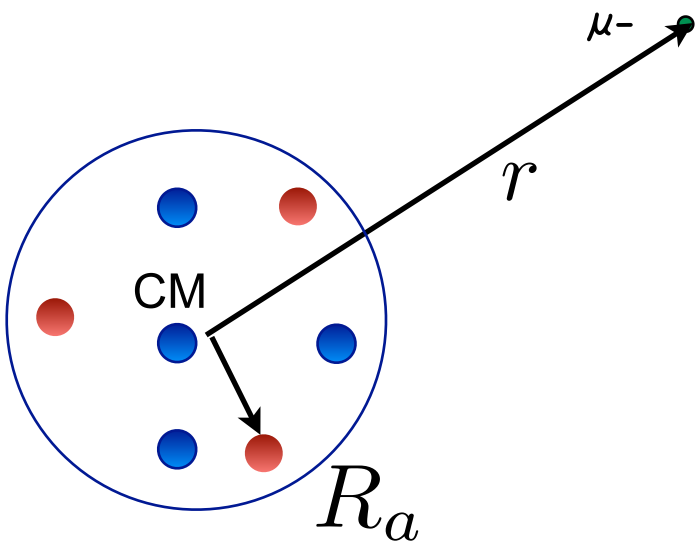
Stronger Coulomb - reduced energy

The distorted charge distribution follows the orbiting μ like a "tide"

The Muonic Atom System

$$H = H_N + H_\mu + \Delta V$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



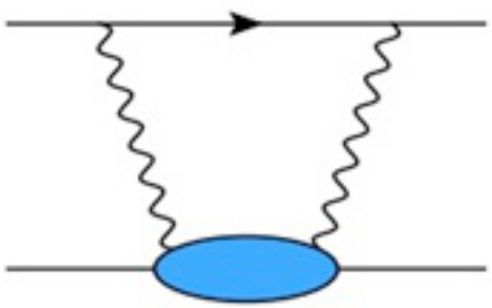
NB: H_N stands for nuclear hamiltonian, not normal ordered hamiltonian

Perturbative potential: correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{|\mathbf{r}|} - \frac{1}{|\mathbf{r} - \mathbf{R}_a|} \right) = \sum_a^Z \alpha \Delta V(\mathbf{r}, \mathbf{R}_a)$$

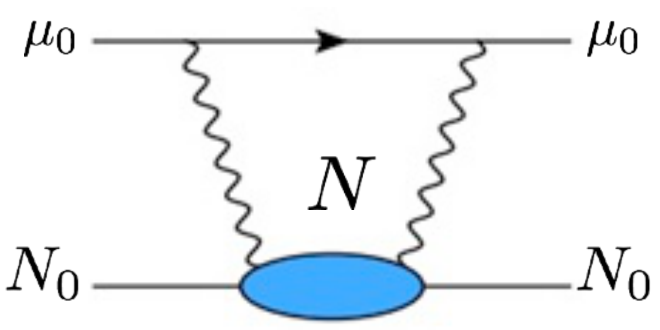
Elaborate on it a bit more

Using perturbation theory at second order one obtains the expression for up to order $(Z\alpha)^5$



The Muonic Atom System

Second order perturbation theory provides the correction to the energy of the lepton as



$$\omega_N = E_N - E_{N_0}$$

$$\delta_{\rm TPE} = \langle N_0 \mu_0 | \Delta V \, G \, \Delta V | N_0 \mu_0 \rangle$$

$$G = \frac{1}{E_{N_0} + \epsilon_{\mu_0} - H_N - H_\mu}$$

Green's function in the non-relativistic limit

Focus e.g. on inelastic part: $1 - |N_0\rangle\langle N_0| = \sum_{N \neq N_0} |N\rangle\langle N|$

$$\delta_{\rm TPE} \to - \sum_{ab}^{Z} \sum_{N \neq N_0} \int d^3r d^3r' \langle N_0 | \Delta V(\mathbf{r}, \mathbf{R}_a) | N \rangle \langle \mu_0 | \mathbf{r} \rangle$$
$$\langle \mathbf{r} | \frac{1}{H_\mu + \omega_N - \epsilon_{\mu_0}} | \mathbf{r}' \rangle \langle \mathbf{r}' | \mu_0 \rangle \langle N | \Delta V(\mathbf{r}', \mathbf{R}_b) | N_0 \rangle$$

Position of nucleon inside the nucleus

Used $H_N |N\rangle = E_N |N\rangle$

The Muonic Atom System

Now we would like to rewrite this energy correction in a different way

$$\delta_{\rm TPE} \to - \sum_{a b}^{Z} \sum_{N \neq N_0} \int d^3r d^3r' \langle N_0 | \Delta V(\mathbf{r}, \mathbf{R}_a) | N \rangle \langle \mu_0 | \mathbf{r} \rangle$$
$$\langle \mathbf{r} | \frac{1}{H_\mu + \omega_N - \epsilon_{\mu_0}} | \mathbf{r}' \rangle \langle \mathbf{r}' | \mu_0 \rangle \langle N | \Delta V(\mathbf{r}', \mathbf{R}_b) | N_0 \rangle$$

Introducing the transition proton density distribution

$$\rho_N(\mathbf{R}) = \langle N | \frac{1}{Z} \sum_a^Z \delta(\mathbf{R} - \mathbf{R}_a) | N_0 \rangle$$

so that the Coulomb matrix element becomes

$$\sum_b^Z \langle N | \Delta V(\mathbf{r}', \mathbf{R}_b) | N_0 \rangle = \int d\mathbf{R}' \sum_b \langle N | \delta(\mathbf{R}' - \mathbf{R}_b) | N_0 \rangle \Delta V(\mathbf{r}', \mathbf{R})$$
$$= Z \int d\mathbf{R}' \rho_N(\mathbf{R}') \Delta V(\mathbf{r}', \mathbf{R}')$$

Finally:

$$\delta_{\rm TPE} \to - \sum_{N \neq N_0} \int d\mathbf{R} d\mathbf{R}' \rho_N^*(\mathbf{R}) P(\mathbf{R}, \mathbf{R}', \omega_N) \rho_N(\mathbf{R}')$$

Muonic matrix element

Muonic matrix element

$$P(\mathbf{R},\mathbf{R}',\omega_N) = -Z^2 \int d^3r d^3r' \Delta V(\mathbf{r},\mathbf{R}) \langle \mu_0|\mathbf{r}\rangle \langle \mathbf{r}| \frac{1}{H_\mu + \omega_N - \epsilon_{\mu_0}} |\mathbf{r}'\rangle \langle \mathbf{r}'|\mu_0\rangle \Delta V(\mathbf{r}',\mathbf{R}')$$

Where the lepton wave function is

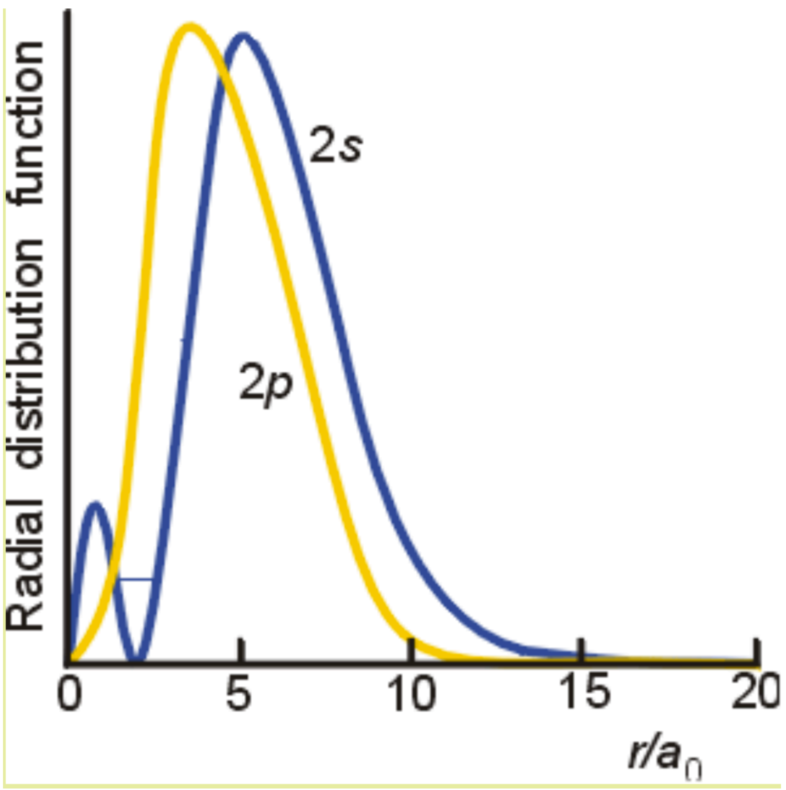
$$\langle \mathbf{r}|\mu_0\rangle \equiv \phi_{n\ell m}(\mathbf{r}) = \sqrt{\frac{4\pi}{2\ell+1}}\, \phi(0)\; R_{n\ell}\left(\frac{m_r Z\alpha r}{n}\right) Y_{\ell m}(\hat{r})$$

with energy

$$\epsilon_n = -\frac{m_r Z^2 \alpha^2}{2n^2}$$

Lepton wave function at the centre of the nucleus

Only in S-state $\ell = 0$



Nuclear structure corrections will take place only where the nucleus and lepton wave functions overlap

Muonic matrix element

$$P(\mathbf{R},\mathbf{R}',\omega_N) = -Z^2 \int d^3r d^3r' \Delta V(\mathbf{r},\mathbf{R}) \langle\mu_0|\mathbf{r}\rangle \langle\mathbf{r}| \frac{1}{H_\mu + \omega_N - \epsilon_{\mu_0}} |\mathbf{r}'\rangle \langle\mathbf{r}'|\mu_0\rangle \Delta V(\mathbf{r}',\mathbf{R}')$$

Where the lepton wave function is

$$\langle\mathbf{r}|\mu_0\rangle \equiv \phi_{n\ell m}(\mathbf{r}) = \sqrt{\frac{4\pi}{2\ell+1}}\,\phi(0)\; R_{n\ell}\left(\frac{m_r Z\alpha r}{n}\right) Y_{\ell m}(\hat{r})$$

with energy

$$\epsilon_n = -\frac{m_r Z^2 \alpha^2}{2n^2}$$

Lepton wave function at the centre of the nucleus

Appears twice: → $\phi^2(0) = (m_r Z\alpha)^3/8\pi$

Total $(Z\alpha)^5$

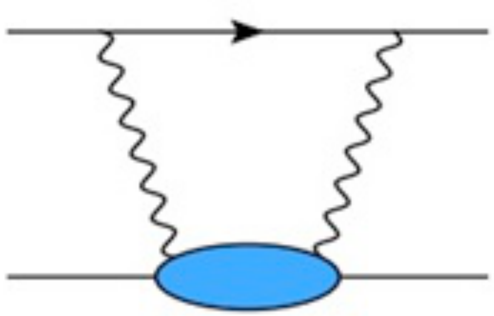
Coulomb also appars twice: $(Z\alpha)^2$

Lepton propagator

$$\frac{1}{H_\mu + \omega_N - \epsilon_{\mu_0}}$$

$\omega_N >> \epsilon_{\mu_0}$ → $\epsilon_{\mu_0} \to 0$

$$H_\mu \to \frac{p^2}{2m_r}$$



Neglect Coulomb
in the intermediate state:
Add it later as Coulomb correction

Muonic matrix element

… putting all together

$$P(\mathbf{R},\mathbf{R}',\omega_N) = -\phi^2(0)Z^2\int d^3r d^3r' \Delta V(\mathbf{r},\mathbf{R})\langle\mathbf{r}|\frac{1}{\frac{q^2}{2m_r}+\omega_N}|\mathbf{r}'\rangle\Delta V(\mathbf{r}',\mathbf{R}')$$

$$= -\phi^2(0)Z^2\int \frac{d^3q}{(2\pi)^3} d^3r d^3r' \Delta V(\mathbf{r},\mathbf{R}) e^{i\mathbf{q}\cdot\mathbf{r}}\frac{1}{\frac{q^2}{2m_r}+\omega_N} e^{-i\mathbf{q}\cdot\mathbf{r}'}\Delta V(\mathbf{r}',\mathbf{R}')$$

$$= -\phi^2(0)Z^2\int \frac{d^3q}{(2\pi)^3} \Delta\tilde{V}(\mathbf{q},\mathbf{R})\frac{1}{\frac{q^2}{2m_r}+\omega_N}\Delta\tilde{V}^*(\mathbf{q},\mathbf{R}')$$

with $\Delta\tilde{V}(\mathbf{q},\mathbf{R}) = \int d^3r V(\mathbf{r},\mathbf{R}) e^{i\mathbf{q}\cdot\mathbf{r}} = \frac{4\pi\alpha}{q^2}\left(1-e^{i\mathbf{q}\cdot\mathbf{R}}\right)$

→

$$P(\mathbf{R},\mathbf{R}',\omega_N) = -\phi^2(0)Z^2\int \frac{d^3q}{(2\pi)^3}\left(\frac{4\pi\alpha}{q^2}\right)^2 \frac{1}{\frac{q^2}{2m_r}+\omega_N}\boxed{\left(1-e^{i\mathbf{q}\cdot\mathbf{R}}-e^{-i\mathbf{q}\cdot\mathbf{R}'}+e^{i\mathbf{q}\cdot(\mathbf{R}-\mathbf{R}')}\right)}$$

Muonic matrix element

$$P(\mathbf{R},\mathbf{R}',\omega_N) =$$

$$-\phi^2(0)Z^2\int\frac{d^3q}{(2\pi)^3}\left(\frac{4\pi\alpha}{q^2}\right)^2\frac{1}{\frac{q^2}{2m_r}+\omega_N}\boxed{\left(1-e^{i\mathbf{q}\cdot\mathbf{R}}-e^{-i\mathbf{q}\cdot\mathbf{R}'}+e^{i\mathbf{q}\cdot(\mathbf{R}-\mathbf{R}')}\right)}$$

We first integrate in the angular part $\hat{q}$

$$\int d\hat{q}\; e^{i\mathbf{q}\mathbf{R}} = 2\pi\int_0^\pi d\theta\;\sin\theta\; e^{iqR\cos\theta}$$

$$= -2\pi\left[\frac{e^{iqR\cos\theta}}{iqR}\right]_0^\pi = 4\pi\frac{\sin(qR)}{qR}$$

The $\square$ part of the integrand can be written as

$$1-\frac{\sin(qR)}{qR}-\frac{\sin(qR')}{qR'}+\frac{\sin(q|\mathbf{R}-\mathbf{R}'|)}{q|\mathbf{R}-\mathbf{R}'|}$$

Muonic matrix element

Looking at these terms more carefully, we realize that some of them go to zero when taking matrix element between N and N_0, due to orthogonality between them

$$1 - \frac{\sin(qR)}{qR} - \frac{\sin(qR')}{qR'} + \frac{\sin(q|\mathbf{R}-\mathbf{R}'|)}{q|\mathbf{R}-\mathbf{R}'|}$$

0 0 0

See pg 14

$$\langle N_0|f(\mathbf{R})|N\rangle\langle N|f(\mathbf{R}')|N_0\rangle \rightarrow \langle N_0|f(\mathbf{R})|N\rangle\langle N|N_0\rangle = 0$$

Only term that survives is

$$P = -16 m_r \alpha^2 Z^2 \phi^2(0) \int_0^\infty \frac{dq}{q^2} \frac{1}{q^2 + 2m_r\omega_N} \left(\frac{\sin(q|\mathbf{R}-\mathbf{R}'|)}{q|\mathbf{R}-\mathbf{R}'|} - 1 \right)$$

To subtract the constant divergent term in the limit R-R'$\longrightarrow$ 0

Muonic matrix element

$$P = -16 m_r \alpha^2 Z^2 \phi^2(0) \int_0^\infty \frac{dq}{q^2} \frac{1}{q^2 + 2m_r\omega_N} \left(\frac{\sin(q|\mathbf{R}-\mathbf{R}'|)}{q|\mathbf{R}-\mathbf{R}'|} - 1 \right)$$

This is the formula which we first integrate over q (Mathematica)

$$P = -\frac{2\pi\alpha^2 Z^2 \phi^2(0)}{m_r\omega_N^2} \frac{1}{|\mathbf{R}-\mathbf{R}'|} \left[e^{-\sqrt{2m_r\omega_N}|\mathbf{R}-\mathbf{R}'|} - 1 + \sqrt{2m_r\omega_N}|\mathbf{R}-\mathbf{R}'| - m_r\omega_N|\mathbf{R}-\mathbf{R}'|^2 \right]$$

Then we expand the exponent in multipoles for small $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$

To obtain a tractable formula

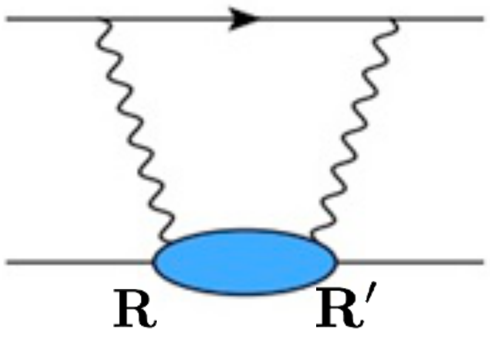
$$P \simeq \frac{m_r^3 (Z\alpha)^5}{12} \sqrt{\frac{2m_r}{\omega}} \left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4} |\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10} |\boldsymbol{R}-\boldsymbol{R}'|^4 \right]$$

Contributions to $\delta_{\rm TPE}$

NB: $\omega_N = \omega$

Non relativistic terms - summarizing

- Take non-relativistic kinetic energy in muon propagator
- Neglect Coulomb force in the intermediate state
- Expand the muon matrix elements in $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'|$



$$P \simeq \frac{m_r^3(Z\alpha)^5}{12}\sqrt{\frac{2m_r}{\omega}}\left[|\boldsymbol{R}-\boldsymbol{R}'|^2 - \frac{\sqrt{2m_r\omega}}{4}|\boldsymbol{R}-\boldsymbol{R}'|^3 + \frac{m_r\omega}{10}|\boldsymbol{R}-\boldsymbol{R}'|^4\right]$$

$\delta^{(0)}$ $\delta^{(1)}$ $\delta^{(2)}$

★ $|\boldsymbol{R}-\boldsymbol{R}'|$ "virtual" distance traveled by the proton between the two-photon exchange

★ Uncertainty principle $|\boldsymbol{R}-\boldsymbol{R}'| \sim \frac{1}{\sqrt{2m_N\omega}}$

★ $\sqrt{2m_r\omega}|\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} = 0.17$ e.g. for μ-^{4}He

Contributions to $\delta_{\rm TPE}$

NB: $R(\omega) = S(\omega)$

Non relativistic terms

★ $\delta^{(0)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^2$

dominant term, related to the energy-weighted integral of the dipole response function

$$\delta^{(0)}_{D1} = -\frac{2\pi m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega \sqrt{\frac{2m_r}{\omega}} S_{D_1}(\omega)$$

★ $\delta^{(1)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^3$

contains a part that cancels the Zemach moment elastic contribution

cf. Pachucki (2011)
Friar (2013)

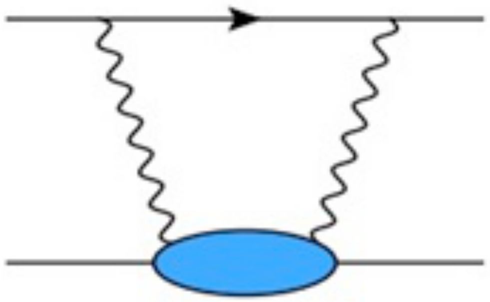
★ $\delta^{(2)} \propto |\boldsymbol{R} - \boldsymbol{R}'|^4$

leads to energy-weighted integrals of three different response functions

$S_{R^2}(\omega), S_Q(\omega), S_{D1D3}(\omega)$

Contributions to $\delta_{\rm TPE}$

Relativistic terms



- Take the relativistic kinetic energy in muon propagator
- Separate in longitudinal and transverse term
- Related to the dipole response function

$$\delta^{(0)}_{L(T)} = \frac{2m_r^3}{9}(Z\alpha)^5 \int_{\omega_{\rm th}}^{\infty} d\omega\, K_{L(T)}\left(\frac{\omega}{m_r}\right)\, S_{D_1}(\omega)$$

Coulomb term

- Consider the Coulomb force in the intermediate states
- Naively it is a $\delta^{(0)}_C \sim (Z\alpha)^6$ corrections, but actually logarithmically enhanced

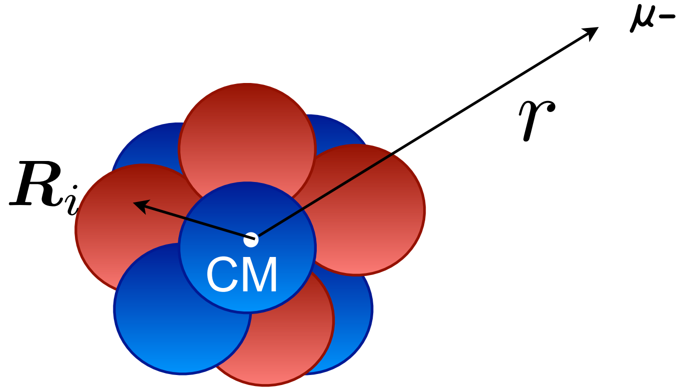
$$\delta^{(0)}_C \sim (Z\alpha)^5 \log(Z\alpha)$$

Friar (1977), Pachucki (2011)

- Related to the dipole response function

Contributions to $\delta_{\rm TPE}$

Finite Nucleon Size Corrections



- In point nucleon limit $\Delta V = -\alpha \sum_i^Z \frac{1}{|\boldsymbol{r} - \boldsymbol{R}_i|}$

- Consider finite nucleon size by including charge distributions

$$\Delta V = -\alpha \sum_i^Z \int d\boldsymbol{R}' \, \frac{n_p(\boldsymbol{R}' - \boldsymbol{R}_i)}{|\boldsymbol{r} - \boldsymbol{R}'|} - \alpha \sum_j^N \int d\boldsymbol{R}' \, \frac{n_n(\boldsymbol{R}' - \boldsymbol{R}_j)}{|\boldsymbol{r} - \boldsymbol{R}'|}$$

- When you do the Fourier transform, you get $n_p(q^2)$, $n_n(q^2)$

Low-q approximation of the nucleon form factors

$$n_p(q^2) = G_p^E(q) \simeq 1 - \frac{\langle r_p^2 \rangle}{6} q^2$$

$$n_n(q^2) = G_n^E(q) \simeq -\frac{\langle r_n^2 \rangle}{6} q^2$$

Charge Radius From the Lamb-Shift

The accuracy of the extracted radius depends on the accuracy of $\delta_{\rm TPE}$

$$\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2 \rangle + \delta_{\rm TPE}$$

Roughly: 95% 4% 1%

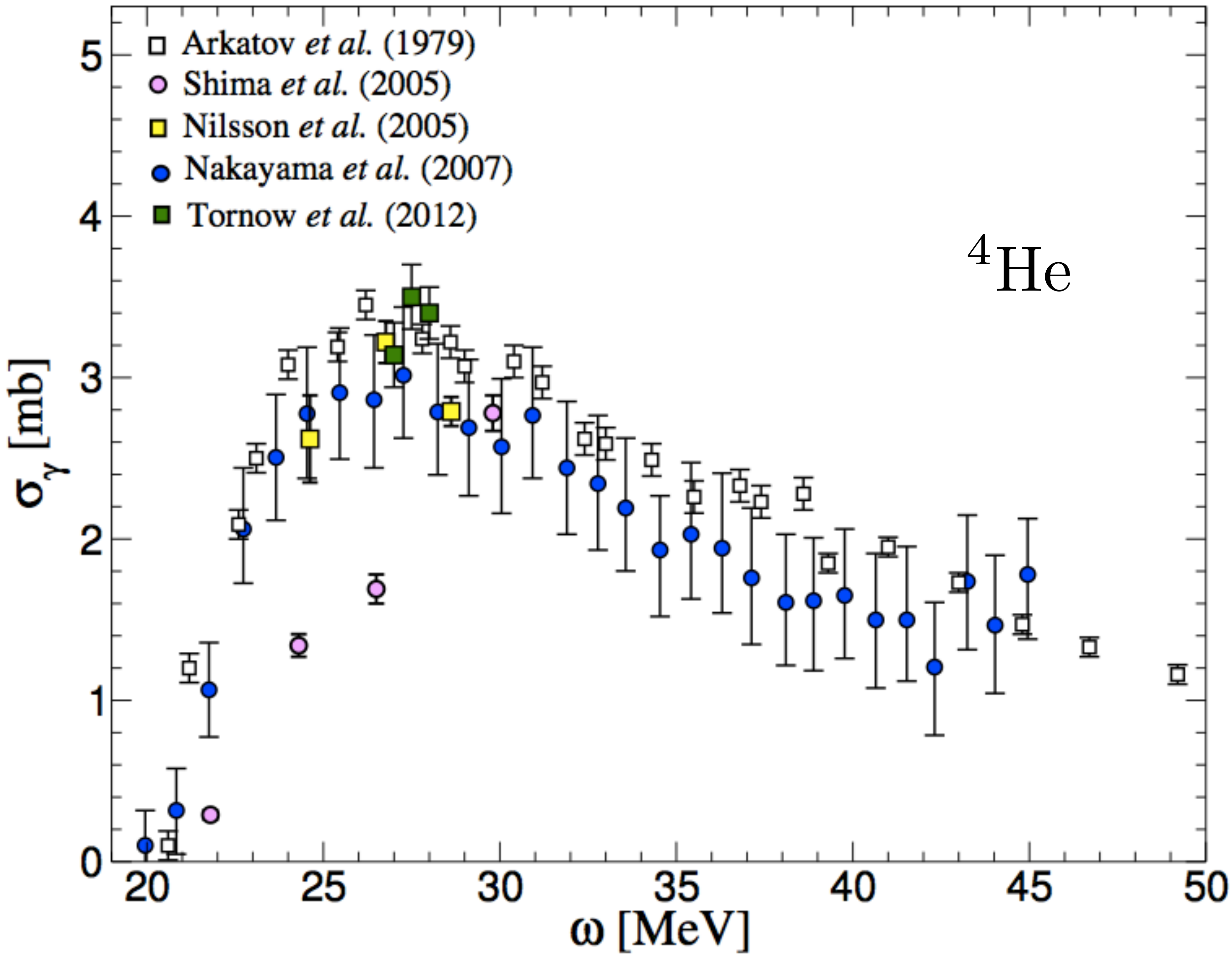
The Lamb-shift is measured with an accuracy of 1-10 μeV
QED corrections are very well known
Even if TPE is the smallest term, it needs to be know quite accurately to be able to exploit the experimental precision

To estimate the nuclear TPE information on the excitations of the nucleus are needed
→ nuclear response function

$$S_{\hat{O}}(\omega) = \frac{1}{2j_0+1} \sum_{N \neq N_0, J} |\left\langle N_0 J_0 || \hat{O} || N J \right\rangle|^2 \delta(\omega - E_N + E_0)$$

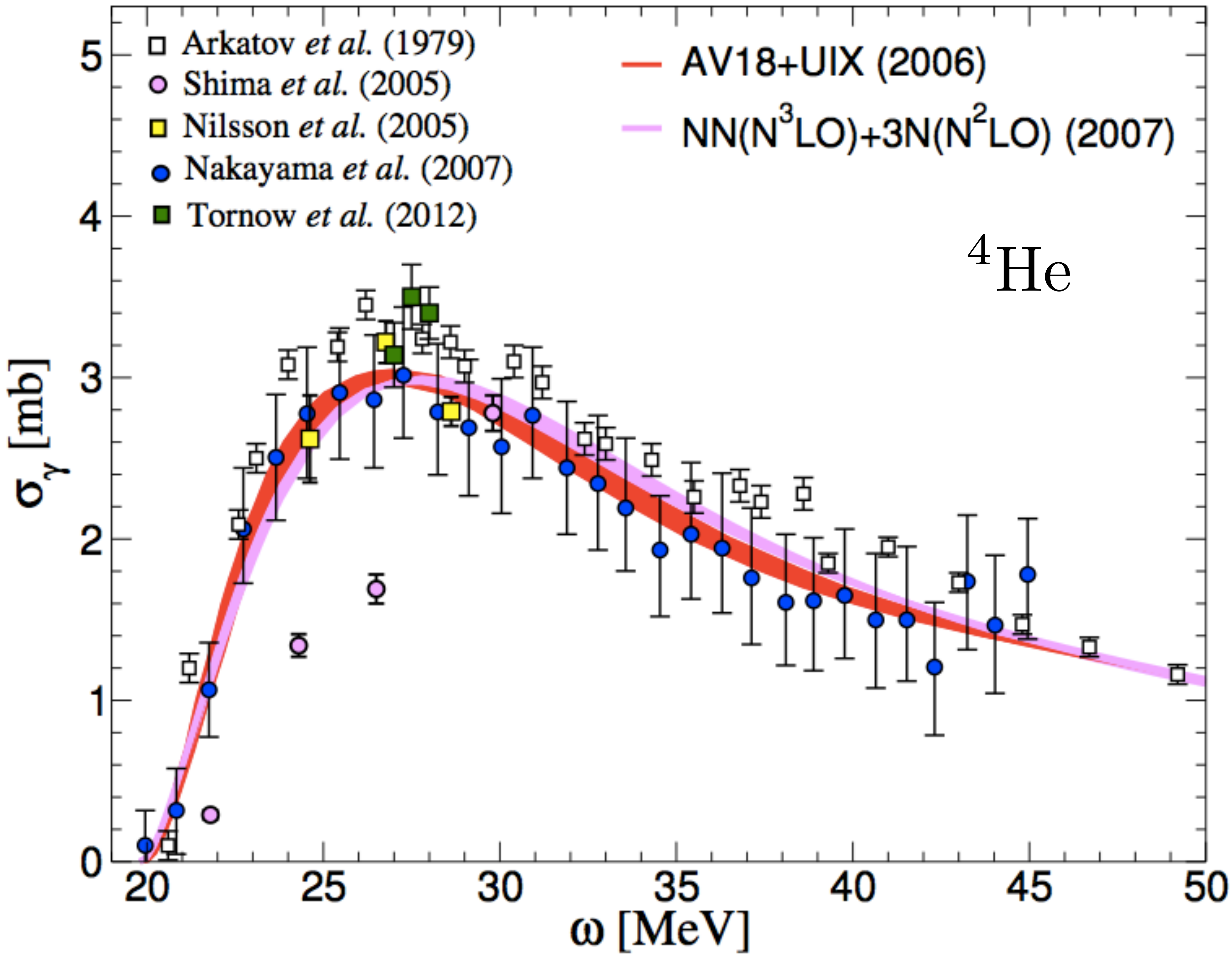
- Extract it from data
- Theoretically calculate it

Extracting TPE from data



Typically 20% of error on δ_{TPE}

Extracting TPE from data



Using ab initio theory we can be much more precise on response functions

Calculate TPE: previous work

- Simple potential models

  μ-^{12}C (Square-well) Rosenfelder '83

  μ-D (Yamaguchi) Lu & Rosenfelder '93

- From experimental photo-absorption cross section

  μ-^{4}He Bernabeu & Karlskog '74; Rinker'76; Friar '77 — 20% Uncertainty

  μ-D Carlson, Gorchtein, Vanderhagen 2014 — 7% Uncertainty

- Zero-range expansion (pion-less EFT)

  μ-D Friar 2013 — Accuracy roughly estimated ~ 2%

- State-of-the-art potentials

  μ-D (AV14) Leidemann & Rosenfelder '95
  (AV18) Pachucki 2011

  Accuracy < 2% or less

Ab-initio Theory Tools

- Use few-body techniques: HH

- Realistic potentials from phenomenology (traditional) or from chiral effective field theory

- LIT to deal with the continuum problem

What can nuclear theory tell you?

Provide the so far most accurate $\delta_{\rm TPE}$

CREMA collaboration experimental program at PSI (Switzerland) ⇒ see talk by R. Pohl

				Uncertainty
μD (Science 2016)	-1.727(20) meV	⇒	PLB **736**, 334 (2014)	1%
μ^4He^+ (analyzing data)	-9.58(38) meV	⇒	PRL **111**, 143402 (2013)	6%
μ^3He^+ (analyzing data)	-15.46(39) meV	⇒	PLB **755**, 380 (2016)	3%
μ^3H (impossible/possible?)	-0.767(25) meV	⇒	PLB **755**, 380 (2016)	
μ^6Li^{2+}, μ^7Li^{2+} (future)				

$$\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2\rangle + \boxed{\delta_{\rm TPE}}$$

- No matter what the nature of the puzzles is, in order to extract radii from muonic atom measurements, nuclear structure calculations will always be needed

TPE Corrections in $\mu^4\mathrm{He}^+$

Ji et al., PRL **111**, 143402 (2013)

[meV]	AV18/UIX	χEFT★
$\delta^{(0)}$	-3.743	-3.981
$\delta^{(1)}$	0.741	0.809
$\delta^{(2)}$	0.077	0.101
δ_{NS}	0.517	0.530
δ_{TPE}	-2.408	-2.542

- Systematic convergence from $\delta^{(0)}$ to $\delta^{(2)}$
- The difference between the two potentials is 5.5%
- Uncertainty from nuclear physics

$$\frac{5.5\%}{\sqrt{2}} \rightarrow \pm 4\%$$

★ C_D=1 and C_E=-0.029

The work is not yet finished ...

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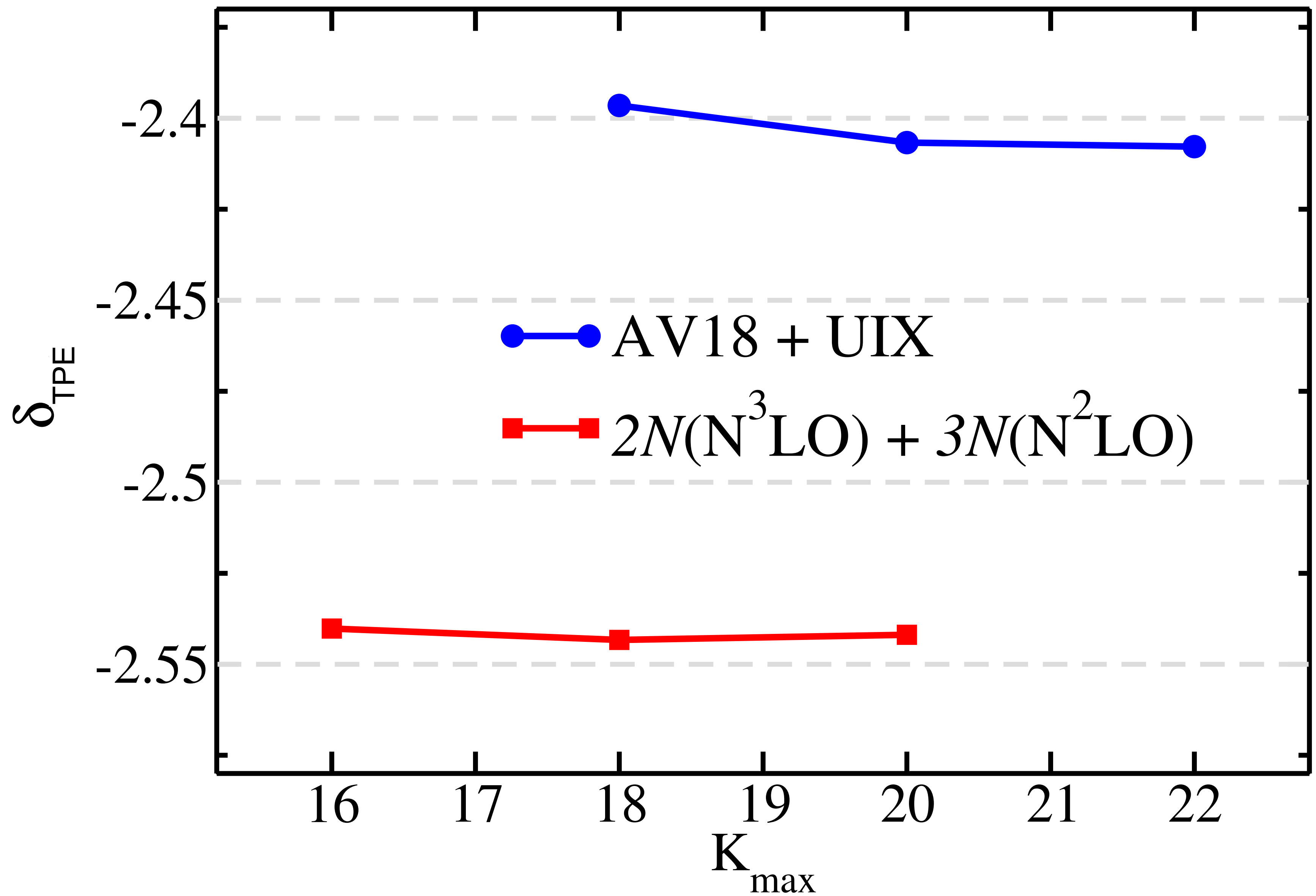
Numerical accuracy → HH expansion



0.4%

0.2%

TPE corrections in $\mu^4\mathrm{He}^+$

Ji et al., PRL **111**, 143402 (2013)

Atomic Physics Uncertainty

- $(Z\alpha)^6$ effects (beyond second order perturbation theory)
- Relativistic and Coulomb effects to multipoles other than dipole
- Higher order nuclear size effects

Combined they give an additional 3-4 %

JG|U

TPE corrections in $\mu\,^4\mathrm{He}^+$

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Error Budget

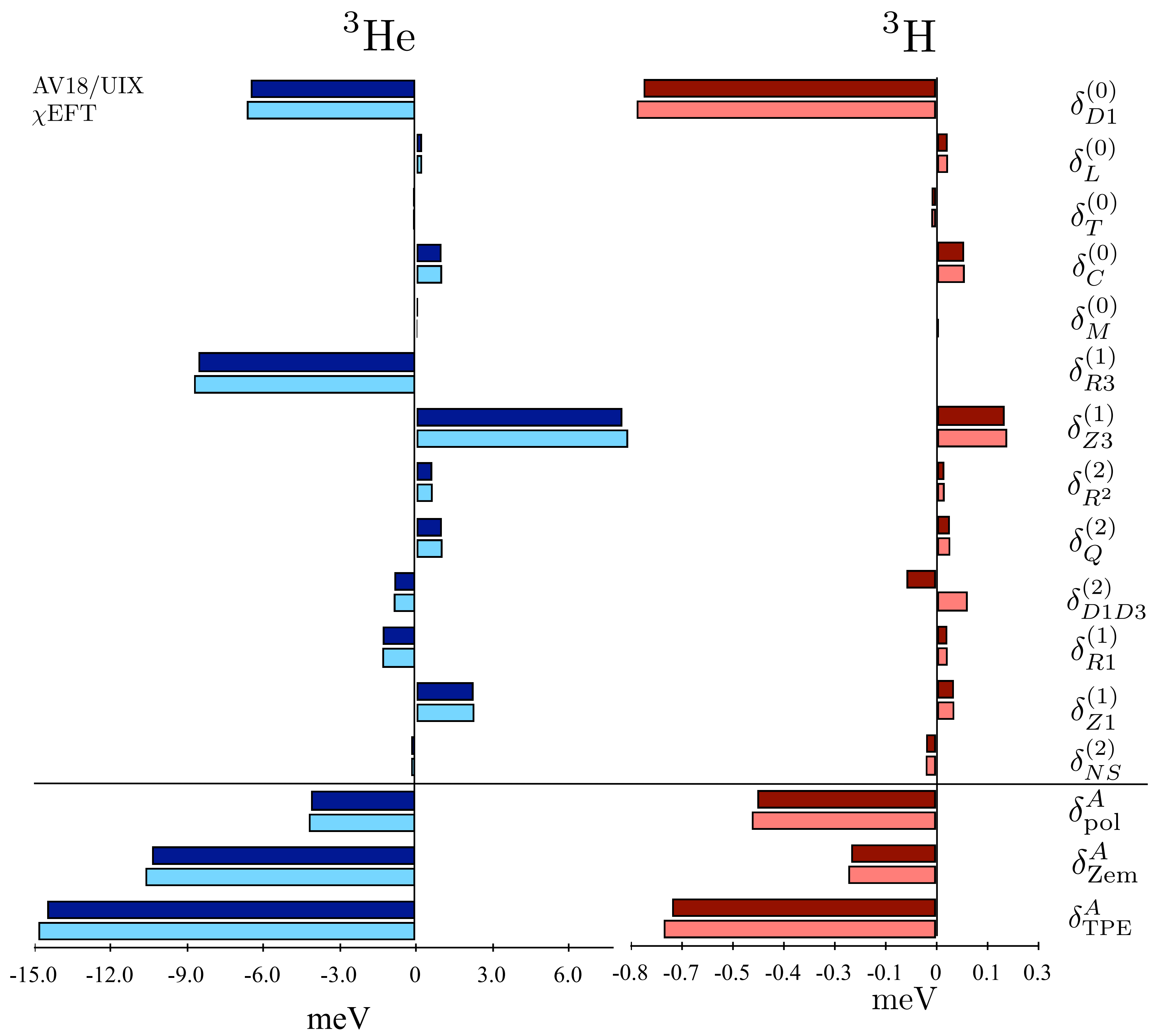
Nuclear Physics	4%
Numerical Accuracy	0.4%
Atomic Physics	4%
Total	6%

- Dramatic improvement from pervious work based on experimental data

Bernabeu & Karlskog ’74; Rinker’76; Friar ‘77 20% Uncertainty

TPE Corrections in A=3

Nevo Dinur et al., PLB **755**, 380 (2016)



Possible Refinements

To further understand the uncertainty in nuclear physics

- Use chiral EFT at different orders to track the convergence
- At a fixed order vary the cutoff to assess the theoretical error
- Use regression analysis to pin down statistical errors

We will first apply this analysis to μD   see talk by O.J.Hernandez