Electromagnetic properties of nuclei: from few- to many-body systems

Lecture 1

Electromagnetic Processes

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Lecture series for SFB 1245
TU Darmstadt

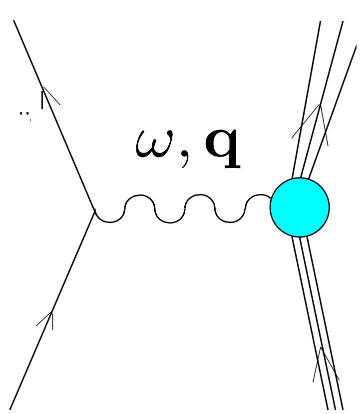
Electromagnetic probes

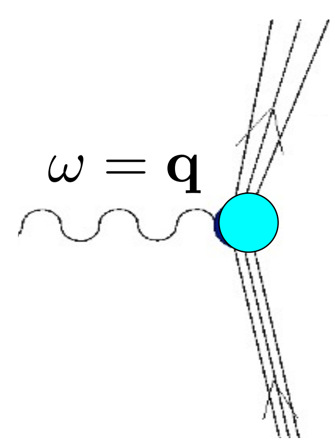
"*With the electromagnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself*"
[De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto \left| \langle \Psi_f | J^\mu | \Psi_0 \rangle \right|^2$$

Can be treated perturbatively $\alpha = \dfrac{1}{137} \ll 1$

Use Born approximation ⇒ single photon exchange





Electromagnetic probes

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[De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Can be used to:

- Understand the nuclear dynamics
- Understand the structure of the nucleon

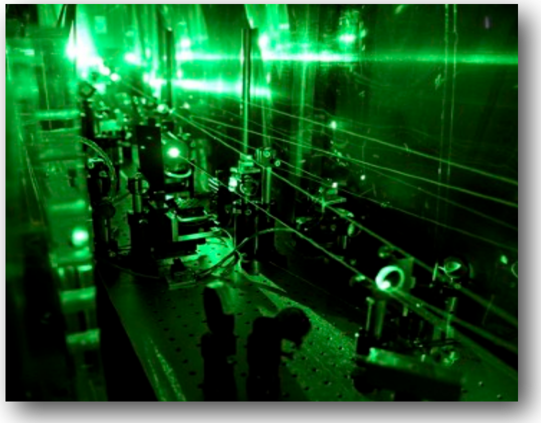
Electromagnetic probes

Provide important informations in other fields of physics, where nuclear physics plays a crucial role:

- Astrophysics

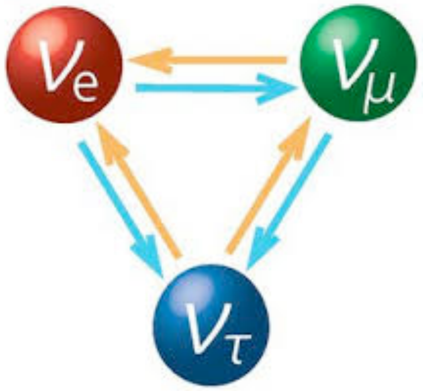
Example: radiative capture reactions

- Atomic physics



Example: muonic atoms

- Particle physics

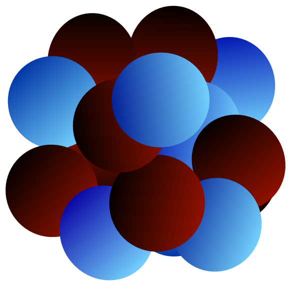


Example: neutrino experiment extending electromagnetic to electro-weak

Electromagnetic probes

Here we will consider a study of electromagnetic observables in nuclei described in the ab-initio approach

Ab-initio approach



- Start from neutrons and protons as building blocks (centre of mass coordinates, spins, isospins)

- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons

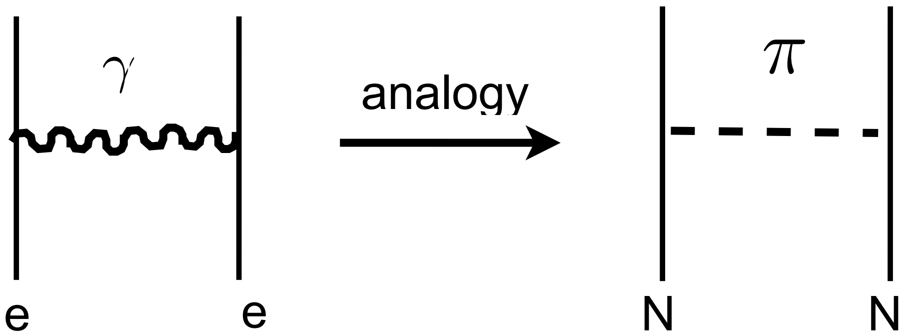
$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$

- Find numerical solutions with no approximations or controllable approximations

- Calibrate the force and then calculate A-body nuclei to compare with experiment or to provide predictions for observables which are hard or impossible to measure

The nuclear hamiltonian



$r \sim \frac{1}{m}$

one-pion exchange potential (OPE)

Hideki Yukawa
Nobel prize in 1949

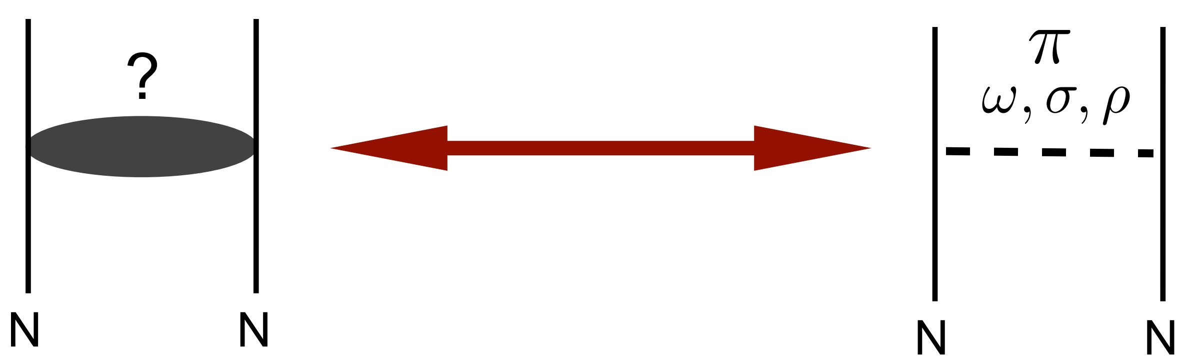
electromagnetic force:
infinite range ⇨ exchange of massless particle

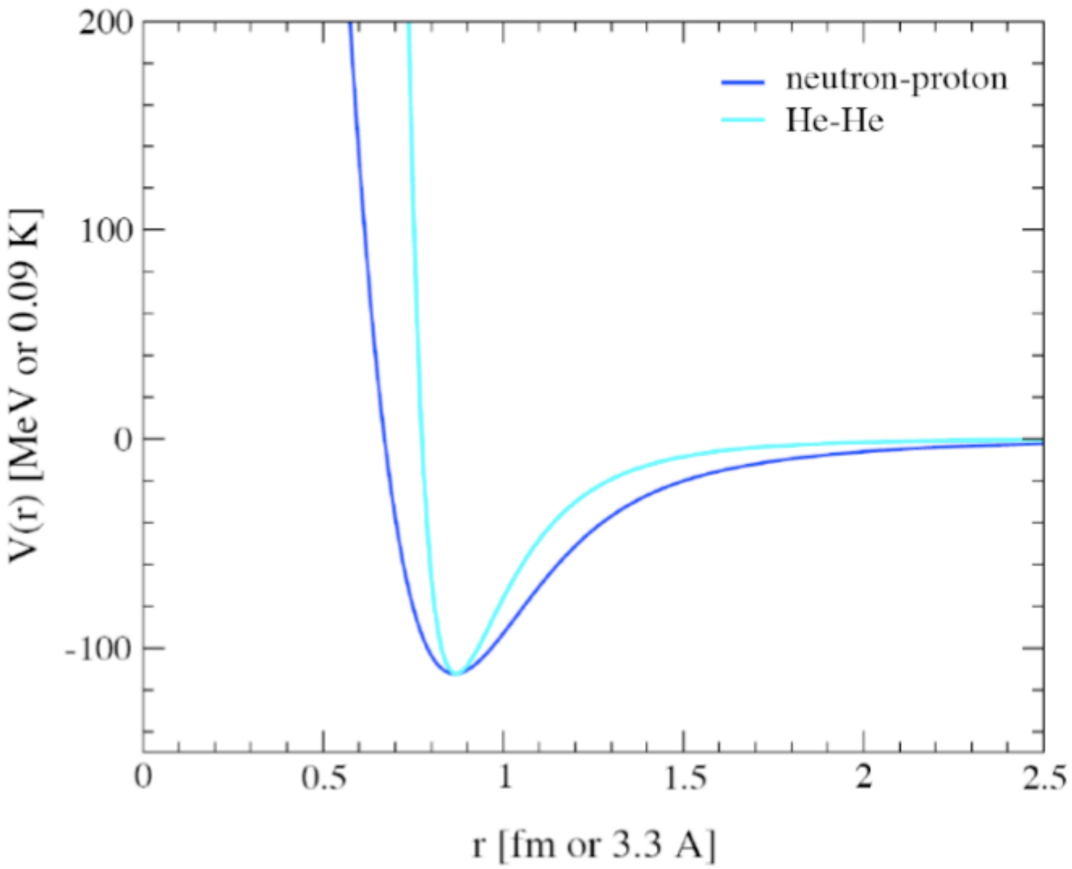
NN force:
finite range ⇨ exchange of massive particle

Realistic NN potentials: fit to NN scattering data with $\chi^2 \approx 1$

Phenomenological models

Meson exchange models





Similar to the potential between two He atoms (liquid Helium) or between two molecules (van der Waals forces)

Three-body forces

Are two-body forces enough?

Some Hamiltonians do not reproduce the right binding energy of nuclei with two-nucleon forces

^{4}He

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

Exp. -28.296 MeV

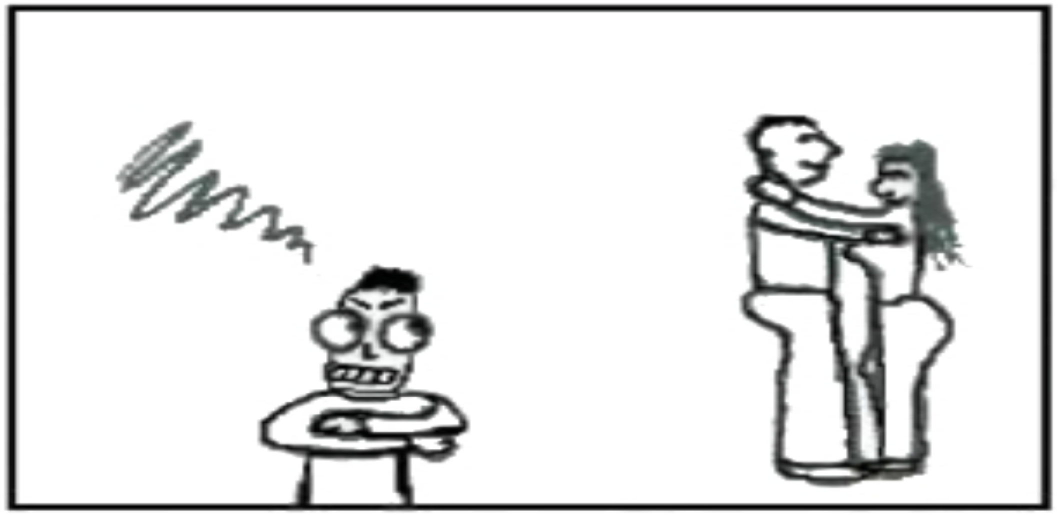
Need to add three-body forces!

What is the origin of three-body forces?

Nucleons are effective degrees of freedom

"The three-body force is a force that does not exist in a two-nucleon system, but appears in a system with three objects or more" $A \geq 3$

As an analogy, if we identify nucleons with human beings and forces with emotions, then jealousy is a good example of a three-body force

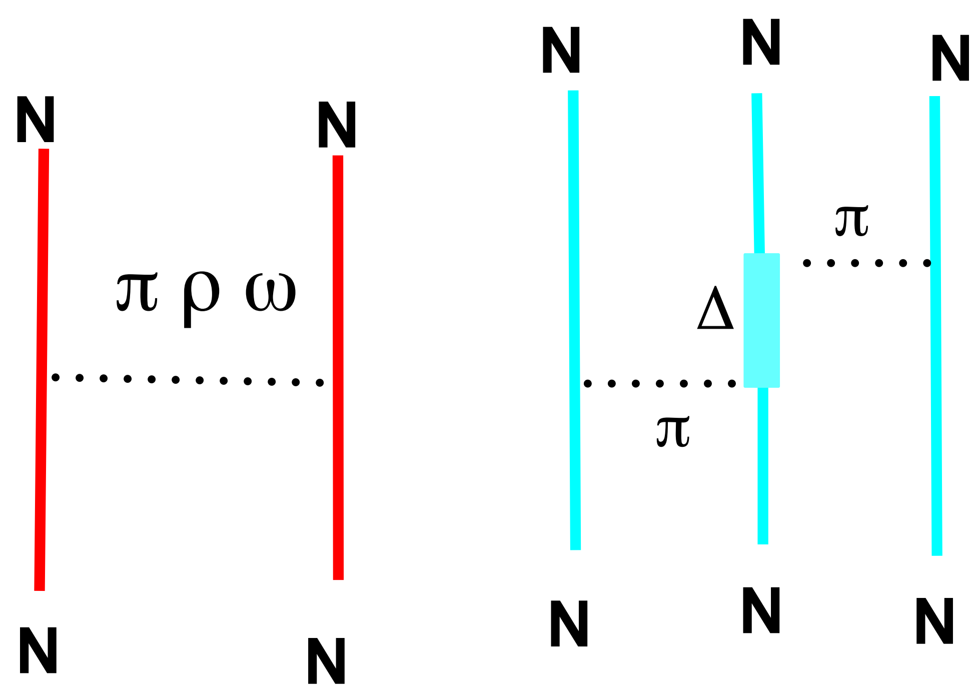


From N. Kalantar, FM50

Three-body forces

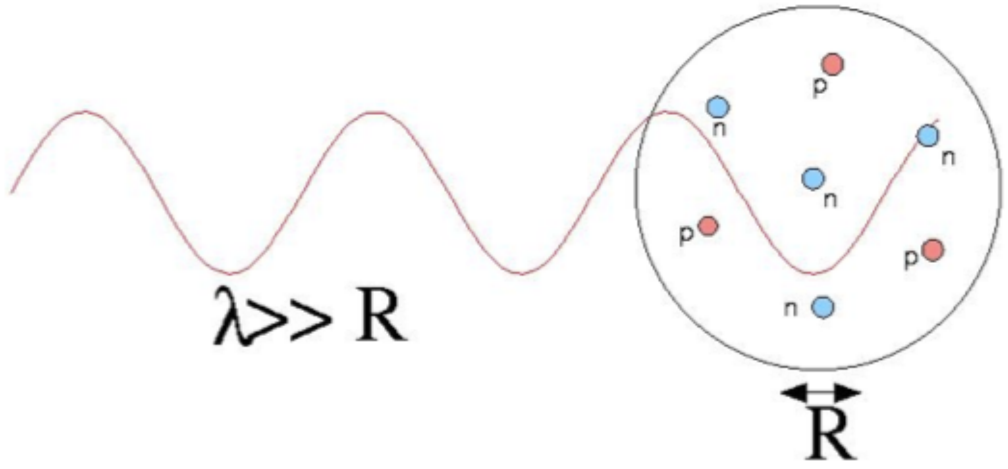
Phenomenological three-body forces

$$H = T + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} + \ldots.$$



Adds operators of more complicated structure and **one additional parameter fitted on triton binding energy**

Chiral effective field theory



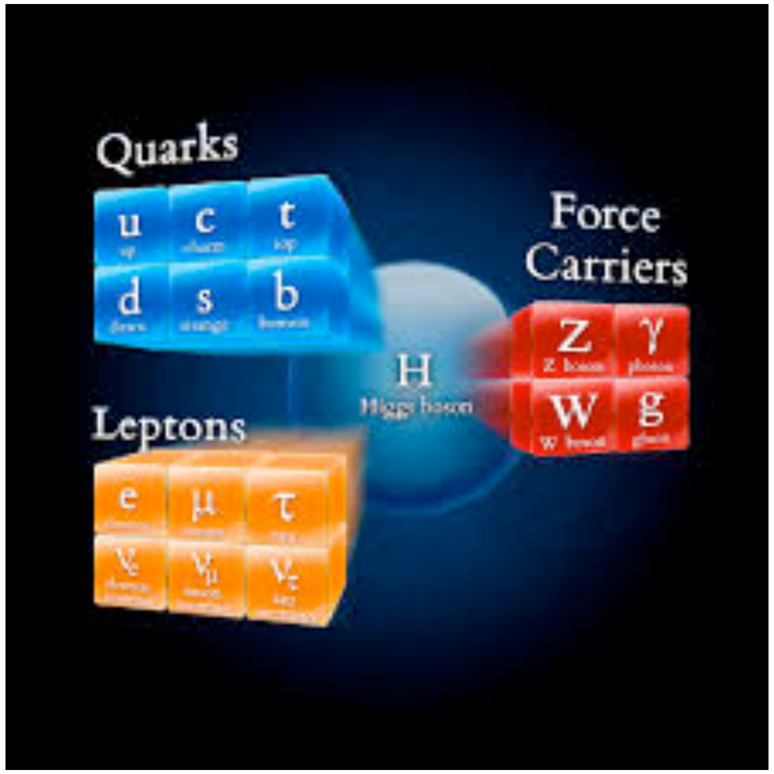
Limited resolution at low energy

Separation of scales

$$\frac{1}{\lambda} = Q \ll \Lambda_b = \frac{1}{R}$$

Effective field theory:
Focus on the relevant degrees of freedom at the energy of interest
but construct a theory which preserves the symmetries of the fundamental theory

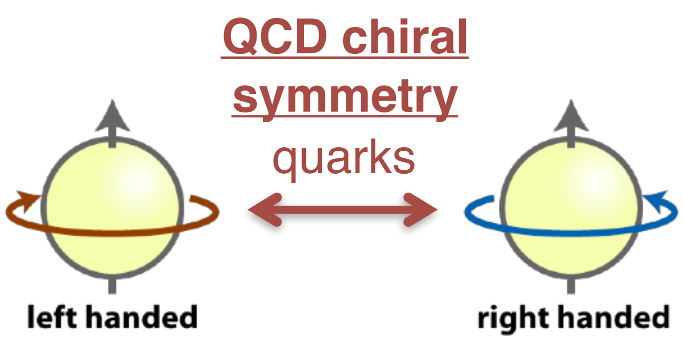
Chiral effective field theory



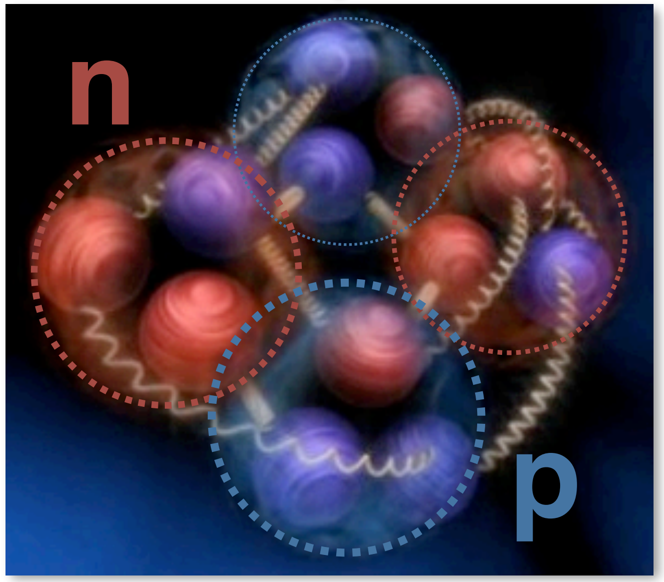
Quark/gluon (high energy) dynamics

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}_L i \gamma_\mu D^\mu q_L + \bar{q}_R i \gamma_\mu D^\mu q_R - \bar{q} \mathcal{M} q$$

In the limit of vanishing quark masses the QCD Lagrangian is invariant under chiral symmetry



Chiral symmetry is explicit and spontaneous broken



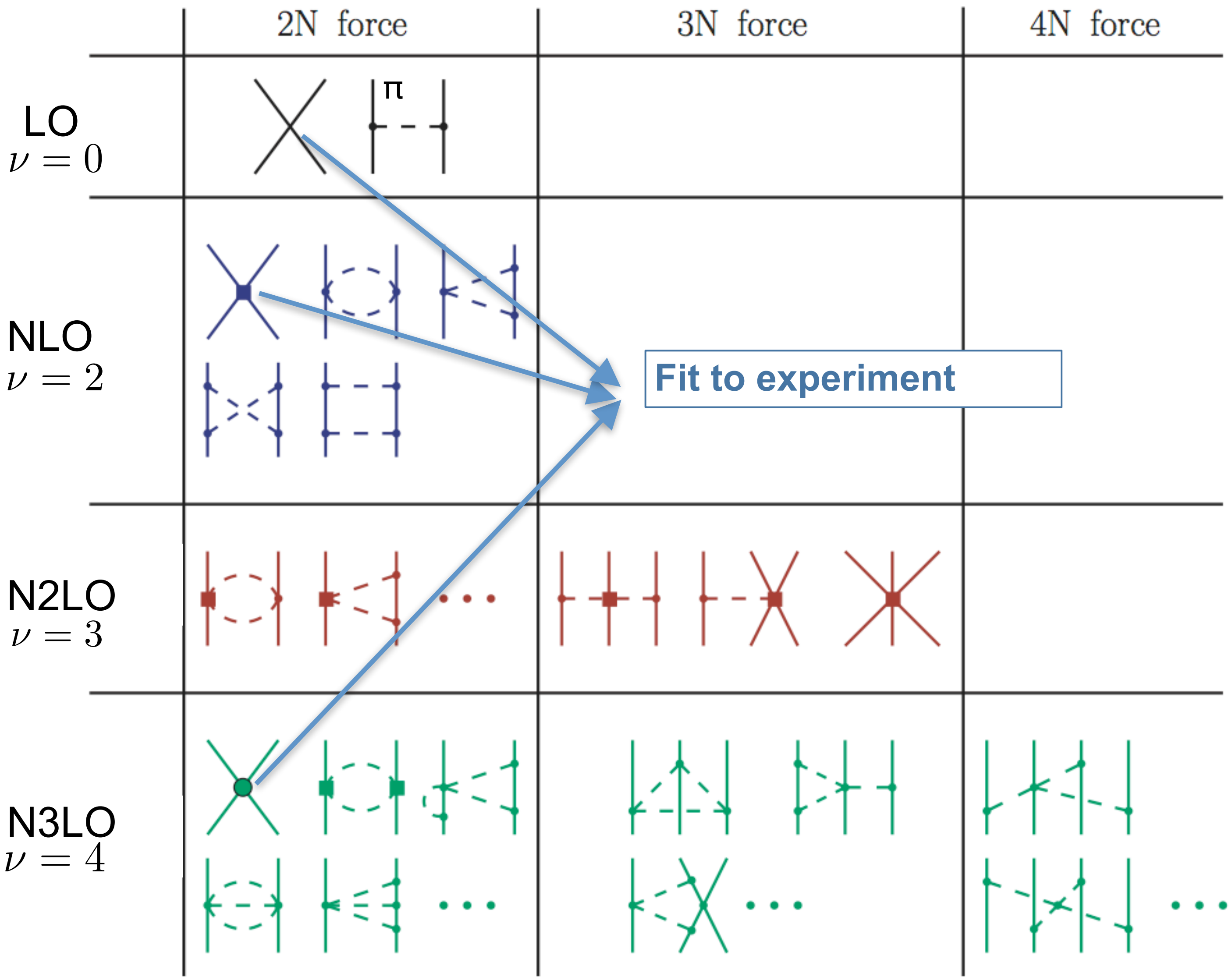
Nucleon/pion (low energy) dynamics

$$\mathcal{L}_{eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \ldots$$

Compatible with explicit and spontaneous **chiral symmetry breaking**

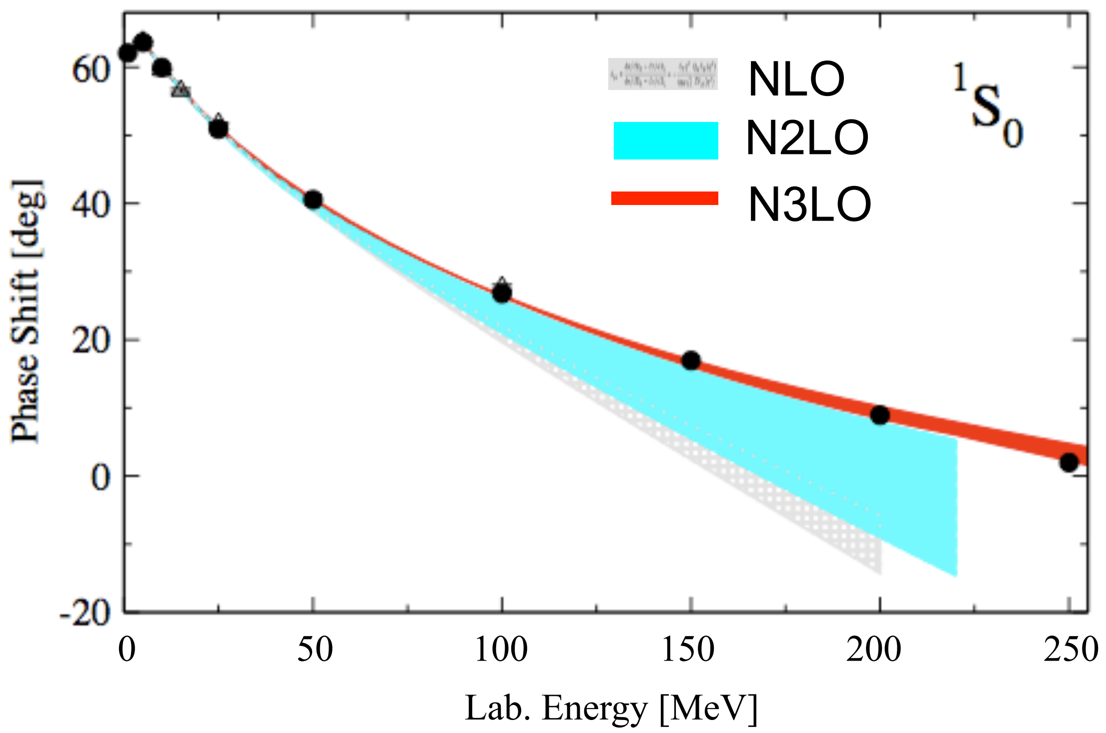
Chiral effective field theory

Weinberg, van Kolck, Epelbaum, Meissner, Machleidt



Details of short distance physics not resolved, but captured in low energy constants (LEC)

LEC fit to experiment - NN sector -

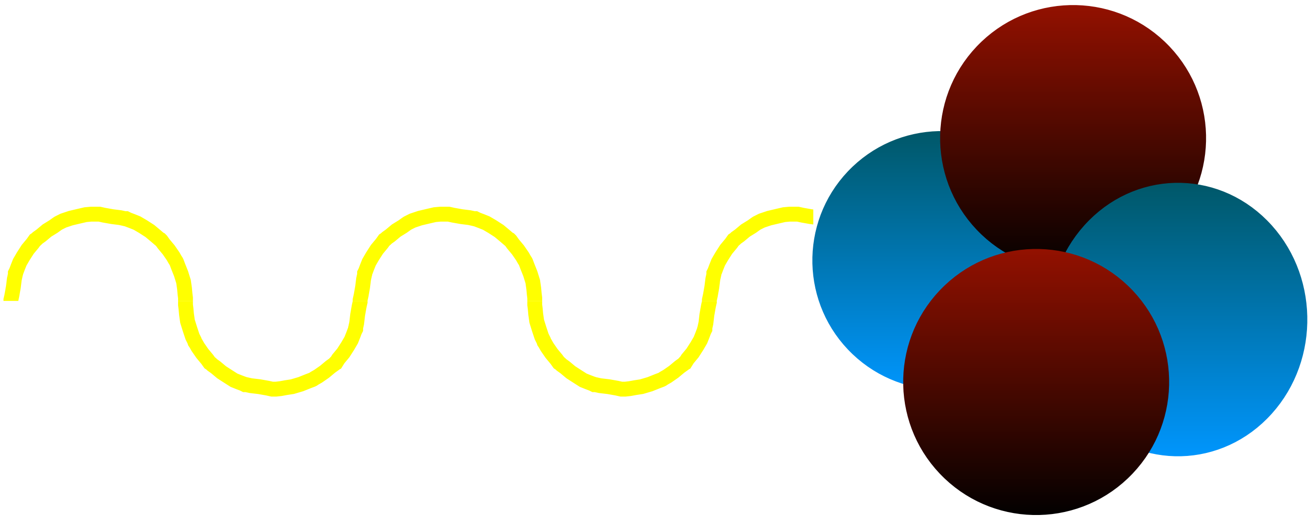


Epelbaum *et al.* (2009)

Systematic expansion $\mathcal{L} = \sum_{\nu} c_{\nu} \left(\frac{Q}{\Lambda_b} \right)^{\nu}$

LEC fit to experiment - 3N sector -
Using A=3 data or including other A>3 nuclei

Coupling to the em field



Coupling to the em field

Let us visualize the electromagnetic perturbation in a nucleus by first considering the simple case of a point particle carrying charge e and no magnetic moment (for now)

In the absence of an em field, the hamiltonian would be just the kinetic energy

$$H_0 = \frac{\mathbf{p}^2}{2m}$$

In the presence of an em field, the momentum is modified by the following transformation

$$\mathbf{p} \rightarrow \mathbf{p} - \frac{e}{c}\mathbf{A}$$

Where $\mathbf{A}$ is the vector potential for the electromagnetic field

The hamiltonian for the charged particle now take the form

$$H' = \frac{(\mathbf{p} - \frac{e}{c}\mathbf{A})^2}{2m}$$

Coupling to the em field

$$H' = \frac{(\mathbf{p} - \frac{e}{c}\mathbf{A})^2}{2m}$$

$$H' = \frac{\mathbf{p}^2}{2m} - \frac{e}{mc}\mathbf{p}\cdot\mathbf{A} + \frac{1}{2m}\frac{\mathbf{A}^2 e^2}{c^2} \rightarrow 0$$

The interaction hamiltonian at leading order is

$$H_{int} = -\frac{e}{mc}\mathbf{p}\cdot\mathbf{A}$$

and if we define the em current as $\mathbf{J} = \mathbf{v} = \frac{\mathbf{p}}{m}$ for a unit of charge,

The interaction hamiltonian becomes

$$H_{int} = -\frac{e}{c}\mathbf{J}\cdot\mathbf{A}$$

Coupling to the em field

Now, if we have don't have a point-like particle, but a spacial charge distribution

$$e \to e\rho(\mathbf{x})$$

Then you have to integrate on that spacial distribution

$$H_{int} = -\frac{e}{c} \int d^3x \; \mathbf{J}(\mathbf{x}) \cdot \mathbf{A}(\mathbf{x})$$

Finally, in a quantum field theory language, you will use the four-vector notation and set e=c=1

$$H_{int} = \int d^3x \; J_\mu(\mathbf{x}) A^\mu(\mathbf{x})$$

perturbative interaction hamiltonian

$$J_\mu(\mathbf{x}) = (\rho(\mathbf{x}), -\mathbf{J}(\mathbf{x})) \qquad A^\mu(\mathbf{x}) = (\phi(\mathbf{x}), \mathbf{A}(\mathbf{x}))$$

Fermi golden rule

This perturbative hamiltonian will act on the nucleus and perturbed it.
Any transition rate generated by the electromagnetic perturbation can be calculated starting from the Fermi golden rule (perturbation theory)

Frauenfelder Henley, Chapter 10

$$w_{I,F} = \frac{2\pi}{\hbar} |\langle F | H_{int} | I \rangle|^2 \rho(E)$$

Density of states

Final state

Initial state

We now understand that the cross section of an em process will involve the calculation of

$$\sigma^{em} \sim |\langle F| \; \rho \;\; \text{or} \;\; \mathbf{J} \; |I\rangle|^2 \rho(E)$$

Nuclear charge and current operators

As the nuclear potential admit an expansion into many-body operators, also the electromagnetic charge and current operators do

$$\rho = \rho_{(1)} + \rho_{(2)} + \cdots = \sum_{i}^{A} \rho_i + \sum_{i<j}^{A} \rho_{ij} + \ldots$$ charge operator

$$\mathbf{J} = \mathbf{J}_{(1)} + \mathbf{J}_{(2)} + \cdots = \sum_{i}^{A} \mathbf{J}_i + \sum_{i<j}^{A} \mathbf{J}_{ij} + \ldots$$ current operator

Indexes are running one the number of nucleons, in general.
NB: neutrons, even though do not have a total charge, they have a charge distribution and a magnetic moment

Nuclear charge and current operators

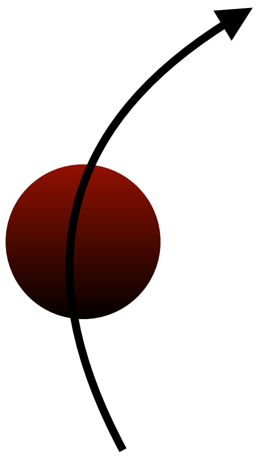
One-body part, for point-like objects

$$\rho_{(1)}(\mathbf{x}) = e \sum_i^A \frac{1+\tau_i^3}{2} \, \delta(\mathbf{x} - \mathbf{r}_i)$$

NB: isospin operator effectively projects on protons

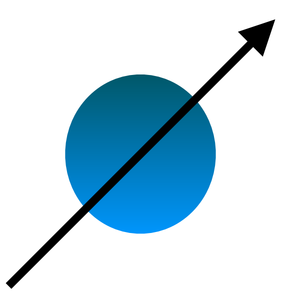
Convection current

$$\mathbf{J}^c_{(1)}(\mathbf{x}) = \frac{e}{2m} \sum_i^A \frac{1+\tau_i^3}{2} \{\mathbf{p}_i, \delta(\mathbf{x} - \mathbf{r}_i)\}$$



Spin current

$$\mathbf{J}^s_{(1)}(\mathbf{x}) = i \frac{e}{2m} \sum_i^A \mu_i \frac{1+\tau_i^3}{2} \boldsymbol{\sigma}_i \times [\mathbf{p}_i, \delta(\mathbf{x} - \mathbf{r}_i)]$$



The use of one-body operators only is called Impulse Approximation

Continuity Equation

Since the Hamiltonian is invariant under a gauge transformation, then according to the Noether theorem, there is a conserved quantity, the electromagnetic current.

The electromagnetic current conservation reads:

$$\partial^\mu J_\mu(\mathbf{x}) = 0$$

Continuity equation

Which can be written as $\frac{\partial}{\partial t}\rho(\mathbf{x}) - \boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x}) = 0 \Rightarrow \frac{\partial}{\partial t}\rho(\mathbf{x}) = \boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x})$

The continuity equation can also be written in momentum space

$$q^\mu J_\mu(\mathbf{q}) = 0$$

$$\omega\rho(\mathbf{q}) - \mathbf{q}\cdot\mathbf{J}(\mathbf{q}) = 0 \Rightarrow \omega\rho(\mathbf{q}) = \mathbf{q}\cdot\mathbf{J}(\mathbf{q})$$

we will use this later on

Charge and current operators are not independent on each other

Continuity Equation

We shall write the continuity equation yet in a another way starting from

$$\frac{\partial}{\partial t}\rho(\mathbf{x}) = \boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x})$$

And by realizing that operators always need to be sandwiched with w.f. in expectation values, e.g.

$$\langle\ \rho\ \rangle(t) = \langle\Psi(\mathbf{x},t)|\ \rho\ |\Psi(\mathbf{x},t)\rangle$$

$$\frac{d}{dt}\langle\ \rho\ \rangle = \frac{1}{i}\langle[\rho,H]\rangle + \langle\ \frac{\partial}{\partial t}\rho\ \rangle \quad = 0$$

$$\langle\ \frac{\partial}{\partial t}\rho\ \rangle = -\frac{1}{i}\langle[\rho,H]\rangle = \frac{1}{i}\langle[H,\rho]\rangle = -i\langle[H,\rho]\rangle$$

Equating and omitting the bra-kets

$$\Rightarrow \quad \boxed{\boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x}) = -i[H,\rho(\mathbf{x})]}$$

Charge and current operators are not independent on each other and they are related to the nuclear hamiltonian

Continuity Equation

$$\nabla \cdot \mathbf{J}(\mathbf{x}) = -i[H, \rho(\mathbf{x})]$$

Now, if we take the one-body charge and current operators, we will see that

$$\nabla \cdot \mathbf{J}_{(1)}(\mathbf{x}) = -i\,[T, \rho_{(1)}(\mathbf{x})]$$

But $[V, \rho_{(1)}(\mathbf{x})] \neq 0$

Thus, there has to be a two-body current operator such that

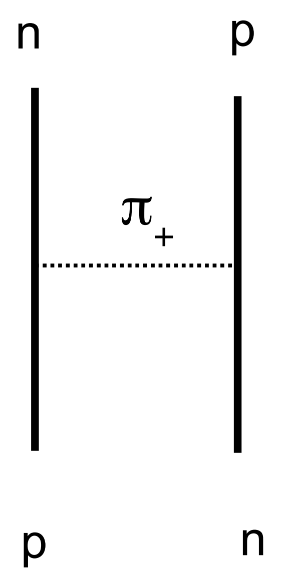
$$\nabla \cdot \mathbf{J}_{(2)}(\mathbf{x}) = -i[V, \rho_{(1)}(\mathbf{x})]$$

NB: The continuity equation constraints only the divergence of the current, not the curl
So you can always add an arbitrary part of the current so that the curl is zero
Unless you rely on a microscopic theory to guide you in the construction
of the current

Two-body currents

Clearly these two-body currents are related to the fact that nucleons are not free particles but interact with each other.

They interact mostly via a one-pion exchange



If the pion is charged, it can interact with a photon

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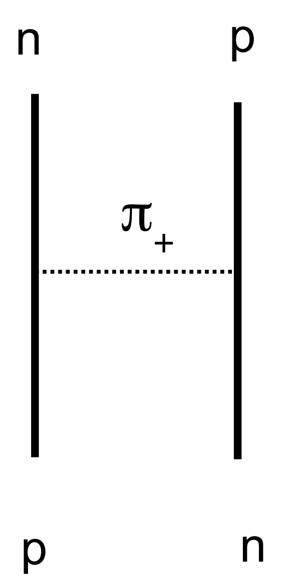
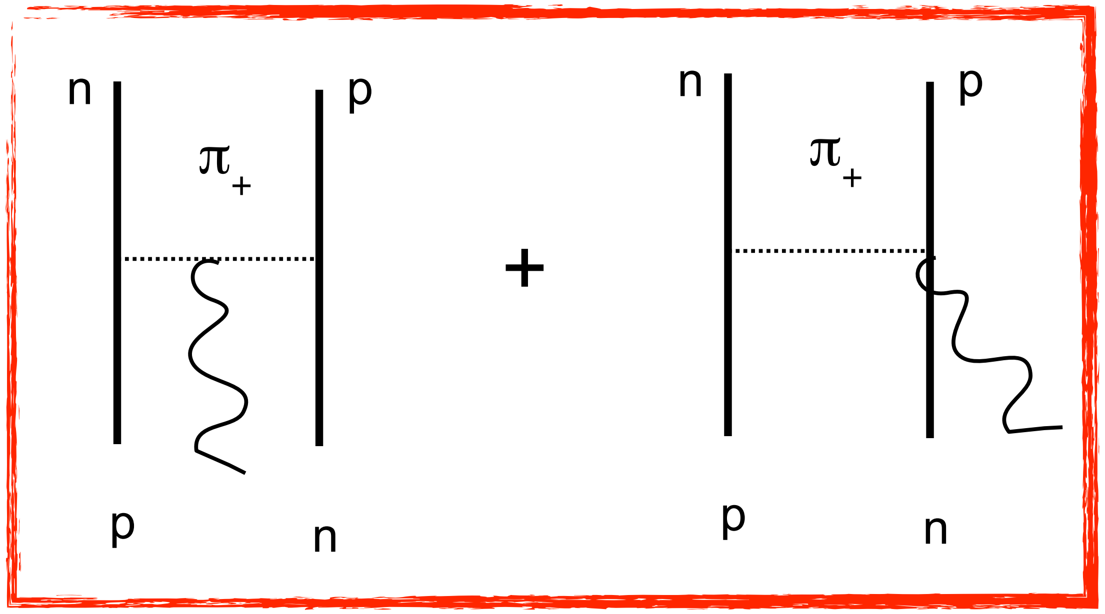


If the pion is charged, it can interact with a photon

Diagram of a two-body current



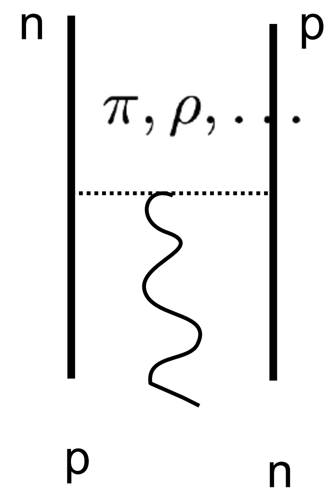
Leading term of two-body currents OPE

Two-body currents

Clearly these two-body currents have to do with subnuclear degrees of freedom

In the past, meson-exchange theory could be used to construct currents

Meson-exchange currents



Today, this is done in the language of chiral effective field theory

Pastore, Schiavilla, Epelbaum….

Operator	LO	NLO	N2LO	N3LO	N4LO
$\mathbf{J}$	$\nu = -2$ IA(NR)	$\nu = -1$ OPE	$\nu = 0$ IA(RC)	$\nu = 1$ OPE(LECs) TPE(LECs) CT(LECs)	
ρ	$\nu = -3$ IA(NR)	$\nu = -2$ —	$\nu = -1$ IA(RC)	$\nu = 0$ OPE	$\nu = 1$ TPE

Power counting
for charge and currents

$$eQ^{\nu}$$

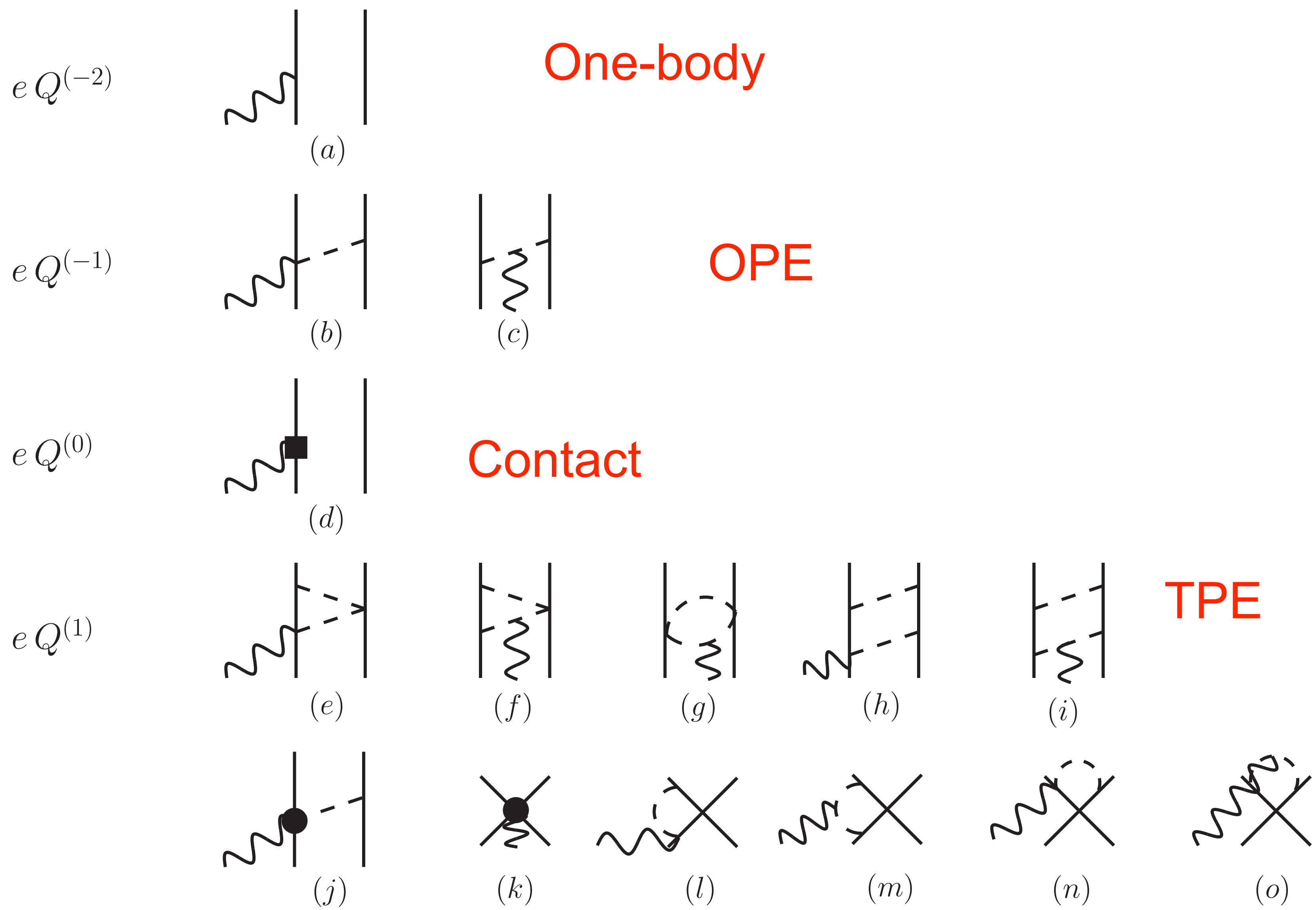
NR: non relativistic
RC: relativistic correction

S. Bacca and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

Two-body currents

S. Bacca and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

Various contributions to the current operator J



Two-body currents

How do we get a handle on these currents?

By studying electromagnetic observables

Em probes can be used to:

and subnuclear

- Understand the nuclear dynamics
- Understand the structure of the nucleon