Electromagnetic properties of nuclei: from few- to many-body systems

Lecture 2

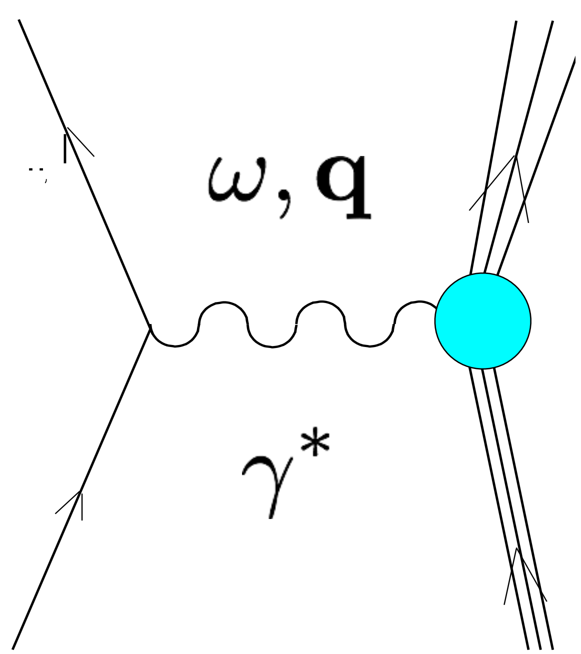
Electromagnetic Processes

Sonia Bacca

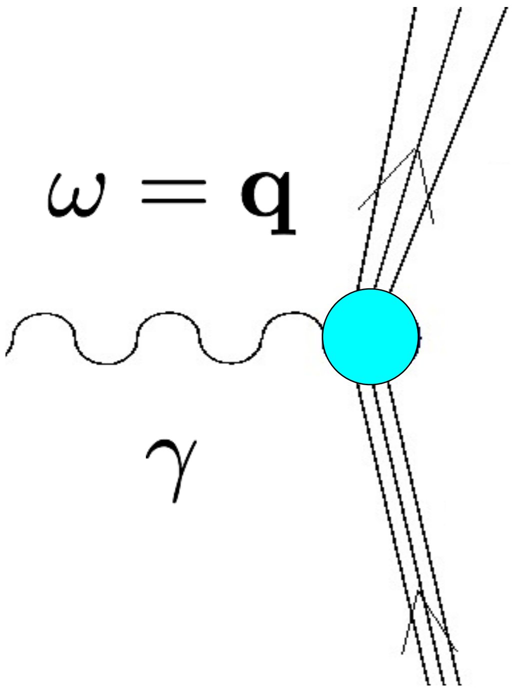
November 21st, 2017

Lecture series for SFB 1245
TU Darmstadt

Electromagnetic probes



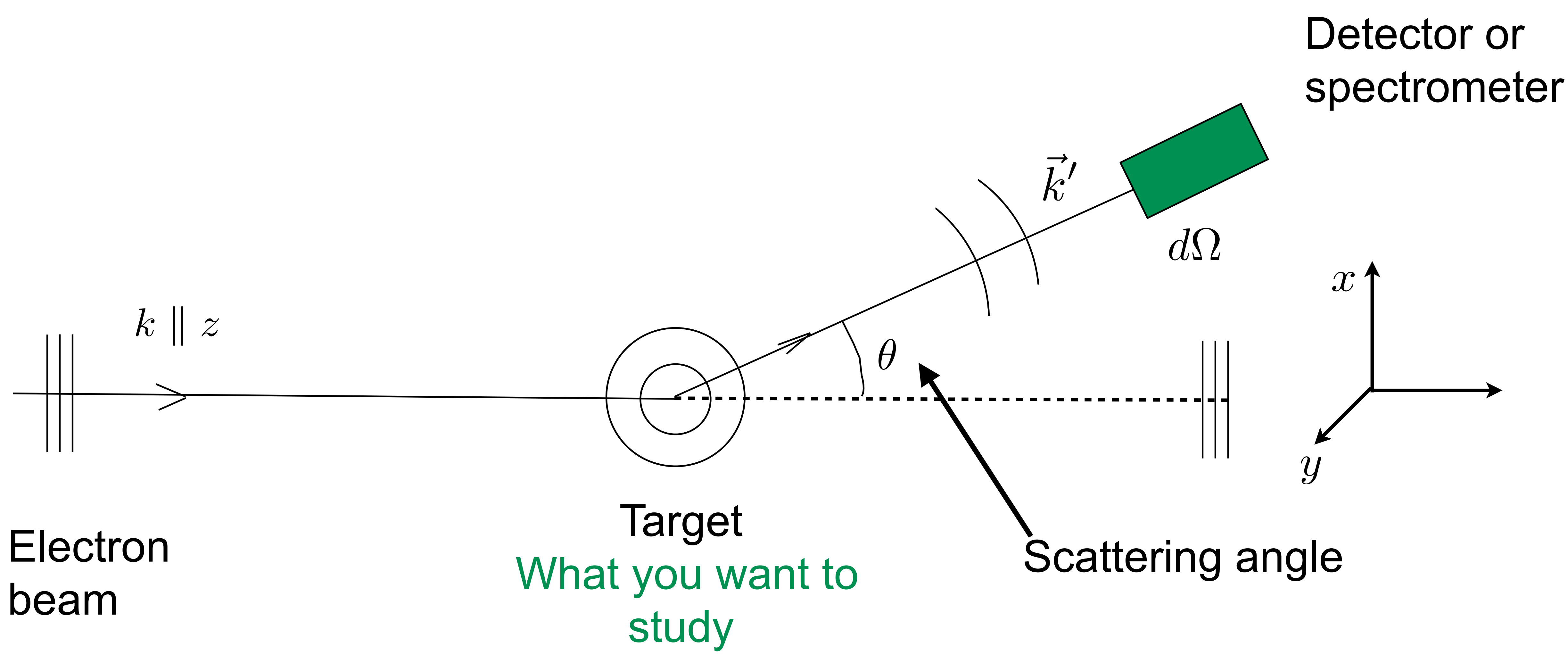
Electron scattering



Photoabsorption

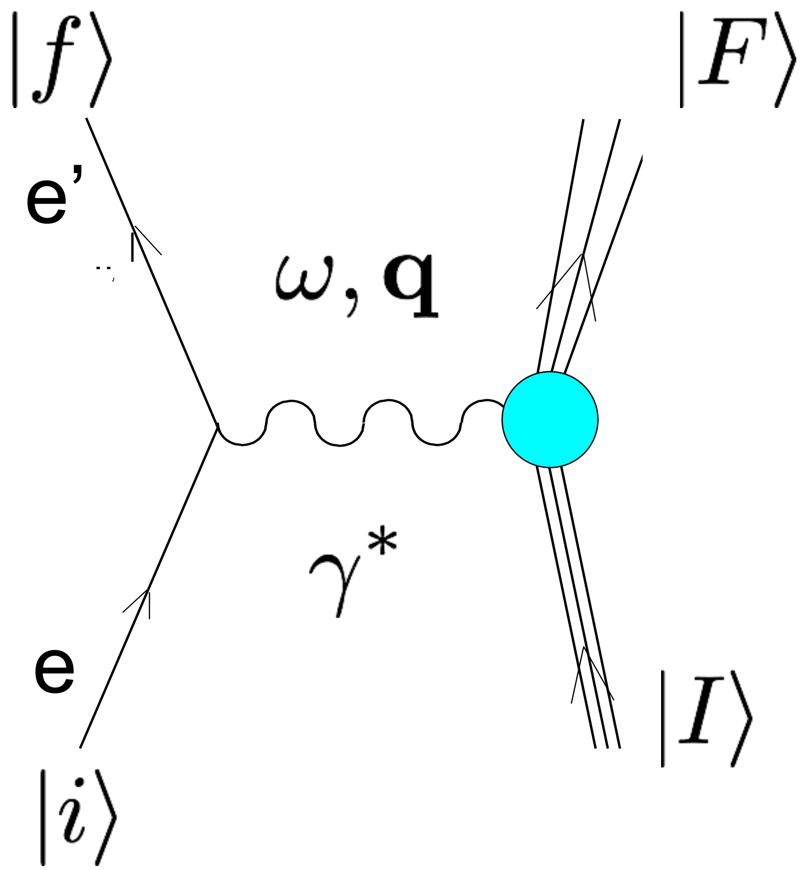
The real photon case can be seen as a special case of the virtual photon

Electron scattering



You can vary the beam energy and the scattering angle

Electron scattering



$$|i\rangle \qquad k^\mu = (\epsilon, \mathbf{k})$$

$$|f\rangle \qquad k'^\mu = (\epsilon', \mathbf{k}')$$

$\omega = \epsilon - \epsilon'$ Energy transfer

$\mathbf{q} = \mathbf{k} - \mathbf{k}'$ Momentum transfer

$$\epsilon = \sqrt{\mathbf{k}^2 + m^2} \simeq |\mathbf{k}|$$

$$\epsilon' = \sqrt{\mathbf{k}'^2 + m^2} \simeq |\mathbf{k}'|$$

$$\Rightarrow \quad \omega^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2|\mathbf{k}||\mathbf{k}'|$$

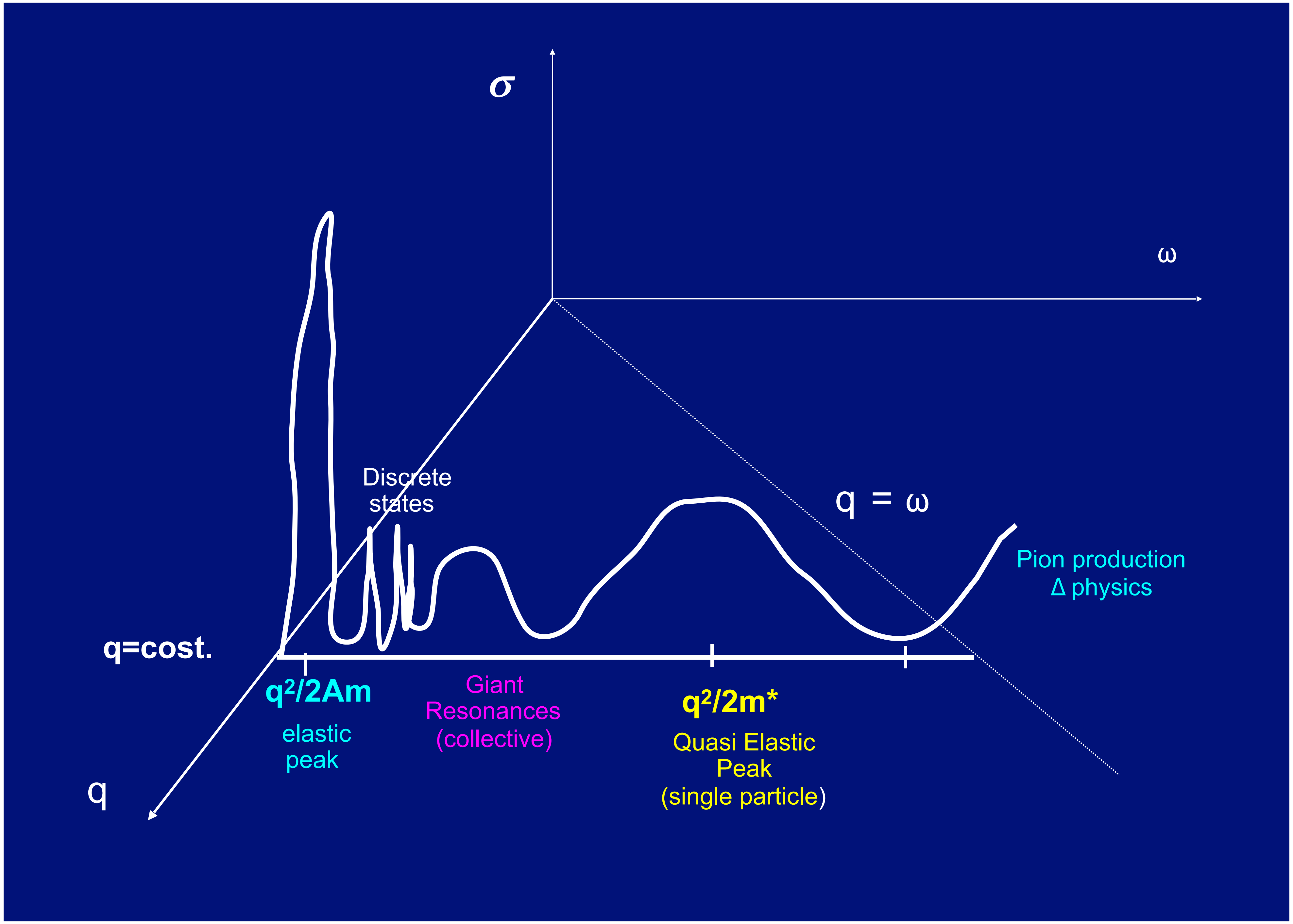
$$|\mathbf{q}|^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2\,\mathbf{k}\cdot\mathbf{k}'$$

$$|\mathbf{q}|^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2\,|\mathbf{k}||\mathbf{k}'|\cos\theta$$

$$\Rightarrow \quad \mathbf{q}^2 \geq \omega^2$$

Electron scattering

$$q^2 = \epsilon^2 + \epsilon'^2 - 2\ \epsilon\epsilon' \cos\theta$$



Inclusive electron scattering

(e,e')

Only final electron is detected

Relatively simple kind of experiment

Energy

Kinematical regions corresponding to different processes:

- elastic peak ($\omega = \frac{|Q^2|}{2M_A}$): all the energy transferred to the nucleus converts into recoil and the nucleus remains in its ground state $\rightarrow$ nuclear g.s. properties (charge and current distribution)
- excitation of low-lying discrete inelastic states
- collective excitations of the nucleus (giant resonances, like the dipole giant resonance, oscillation of proton against neutrons)
- quasielastic peak (QEP) ($\omega \simeq \frac{|Q^2|}{2m_N}$): the dominant process is the scattering off one nucleon $\rightarrow$ single particle properties (momentum and energy distributions of nuclei inside the nucleus)
- Δ peak ($\omega \simeq \frac{|Q^2|}{2m_N} + \frac{m_\Delta^2 - m_N^2}{2m_N}$) and pion production

….

Inclusive electron scattering

Inclusive cross section A(e,e')X

$$\sigma \sim |\langle F\, f \,|\, J_\mu A^\mu \,|\, I\, i \rangle|^2\, \delta(\omega - E_F + E_I)$$

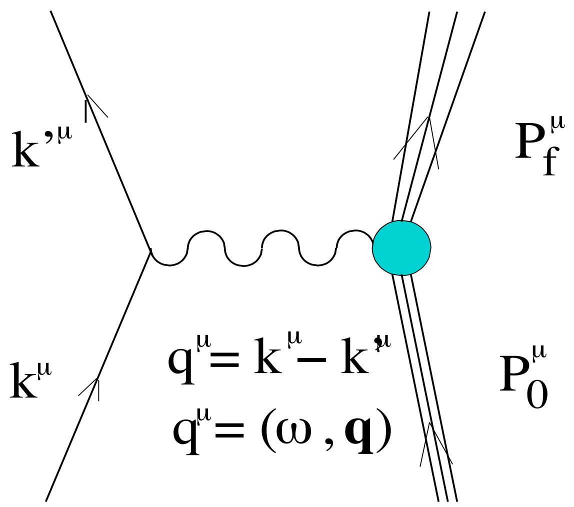
experiment

External field: electron
Known!

Hadronic properties. Unknown.
HERE IS THE INTERESTING PHYSICS!!!

Work out: the lepton and nuclear information can be separated in the cross section

Inclusive electron scattering



$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

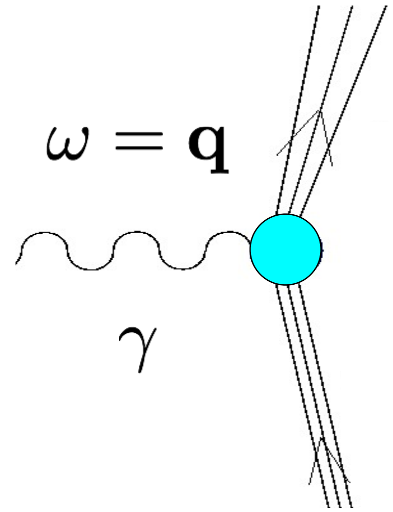
with $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$ and θ scattering angle

and σ_M Mott cross section

Longitudinal part:
Photon polarization along momentum axis

Transverse part:
Photon polarization transverse to momentum axis

Photoabsorption



$\omega = \mathbf{q}$

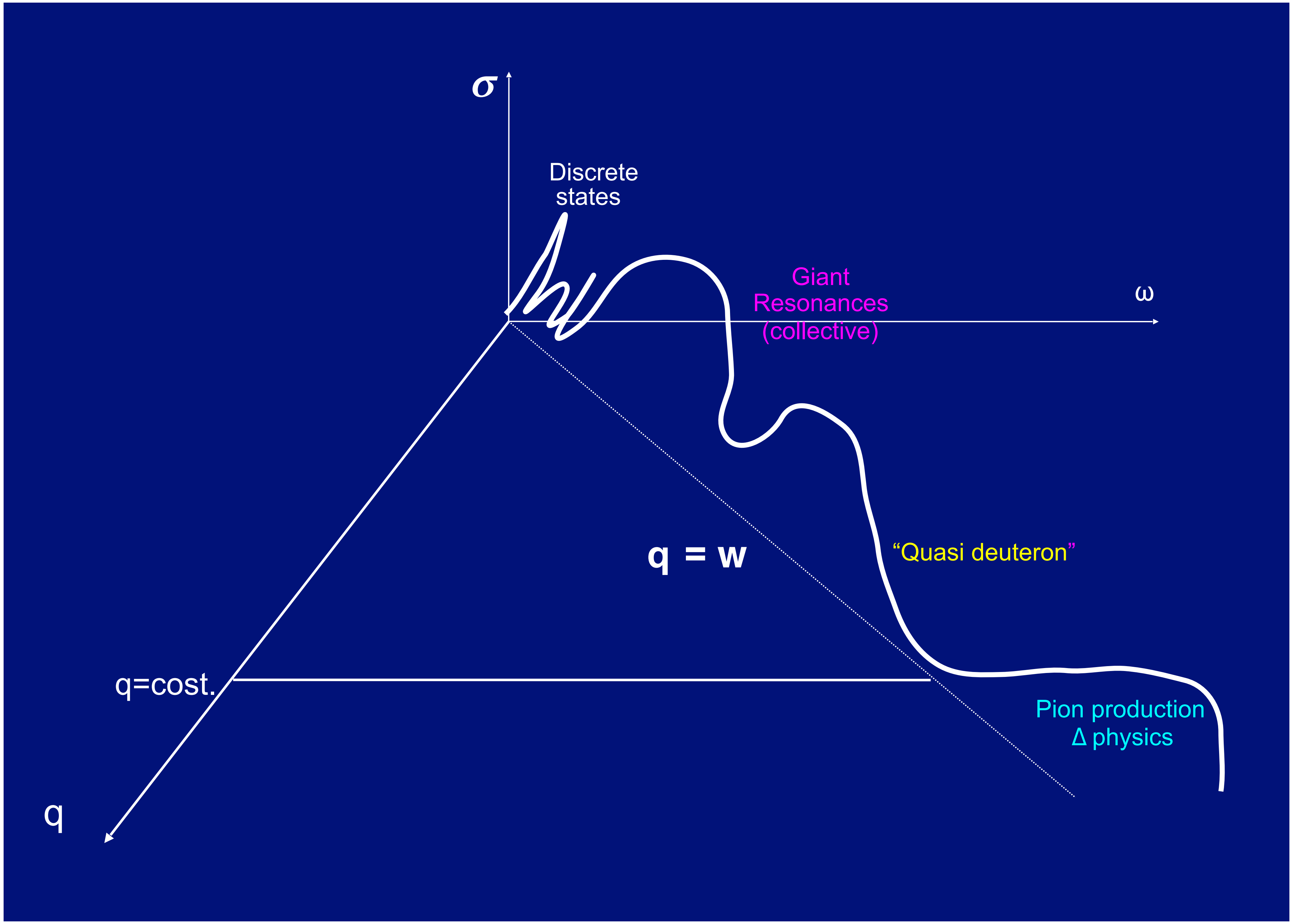
$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

Longitudinal part:
Photon polarization along
momentum axis

Transverse part: $\omega = \mathbf{q}$
Photon polarization
transverse to momentum axis

Photoabsorption

Real photons



Response Functions

Inclusive cross section A(e,e')X

Nuclear part

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ ← charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ ← current operator

$$|F\rangle \to |\Psi_f\rangle$$

$$|I\rangle \to |\Psi_0\rangle$$

Change of notation for hadronic states

Electron scattering

Inclusive cross section A(e,e')X

Nuclear part

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

current operator

$$\rho(\mathbf{q}) = \int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\rho(\mathbf{x})$$

$$\mathbf{J}(\mathbf{q}) = \int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\mathbf{J}(\mathbf{x})$$

These are intended as operators

Electron scattering

Inclusive cross section A(e,e')X

Nuclear part

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ ← charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ ← current operator

Longitudinal part $\Rightarrow$ information on the charge density

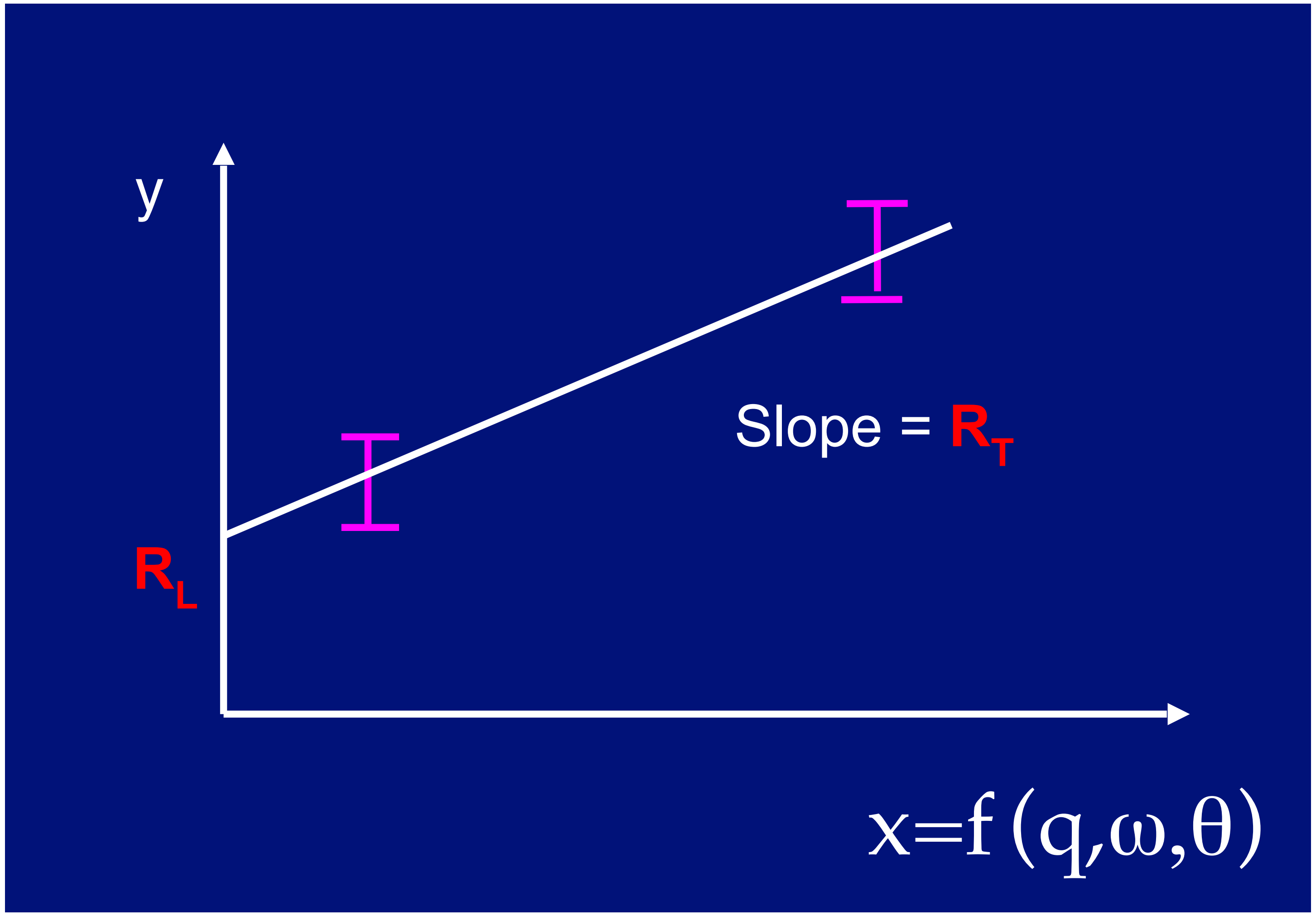
Transverse part $\Rightarrow$ information on the current density

Rosenbluth separation

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

$$\frac{d^2\sigma}{d\Omega d\omega} / \sigma_M / Q^4 / \mathbf{q}^4 = R_L + R_T f(q, \omega, \theta)$$

$$y = a + bx$$



Elastic scattering

$\omega = 0$ No energy transfer, only momentum

$f = 0$ Nucleus stays in ground-state

$$R_L(\omega, \mathbf{q}) \rightarrow F_L(q) \rightarrow |\langle \Psi_0 | \rho(q) | \Psi_0 \rangle|^2$$

Charge distribution: sensitive to protons

$$R_T(\omega, \mathbf{q}) \rightarrow F_T(q) \rightarrow |\langle \Psi_0 | \mathbf{J}_T(q) | \Psi_0 \rangle|^2$$

Current distribution: sensitive also to neutrons because of magnetic moment and spin currents

Form factors

Nuclear charge radius

How do you measure the nuclear charge radius?

From elastic electron scattering off a nucleus

$$\frac{d^2\sigma}{d\Omega} = \sigma_M \left[|F_L(q^2)|^2 + \left(\frac{1}{2} - \tan^2\frac{\theta}{2} \right) |F_T(q^2)|^2 \right]$$

⇒ Rosenbluth separation to obtain the longitudinal or charge form factor

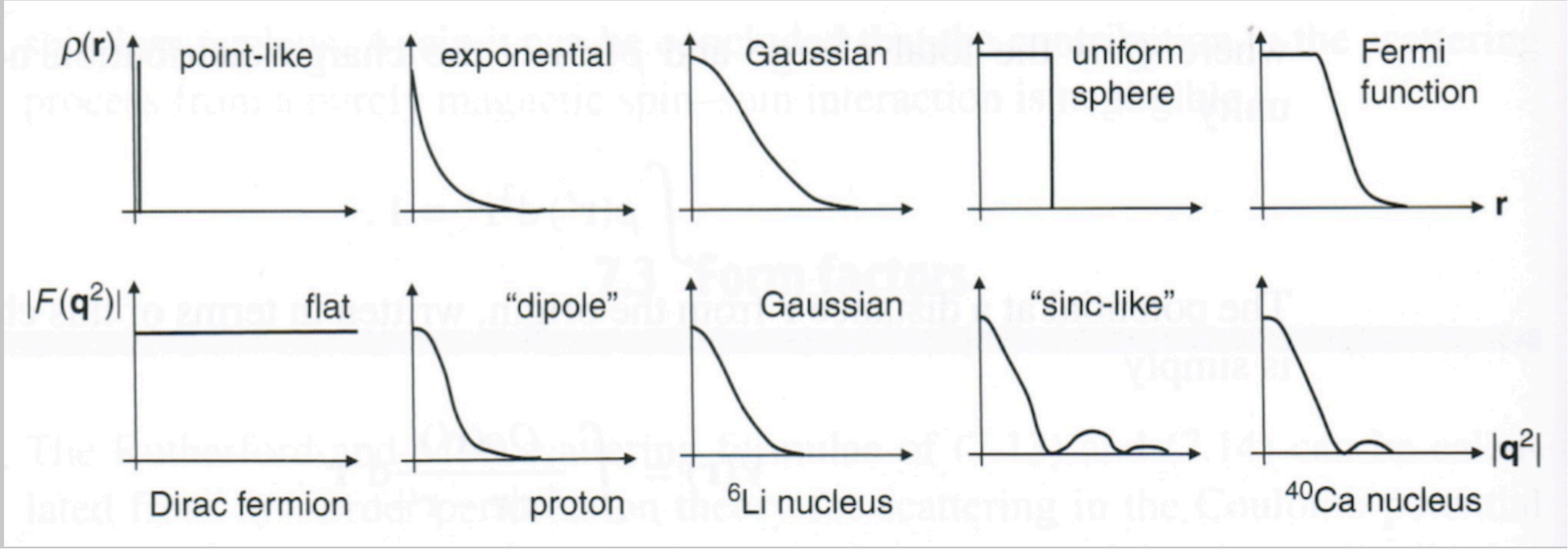
$$|F_L(q^2)|^2 = |F(q^2)|^2 \rightarrow F(q^2)$$

Nuclear charge radius

Fourier transform of the charge distribution

$$F(q^2) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \rho(\mathbf{x})$$

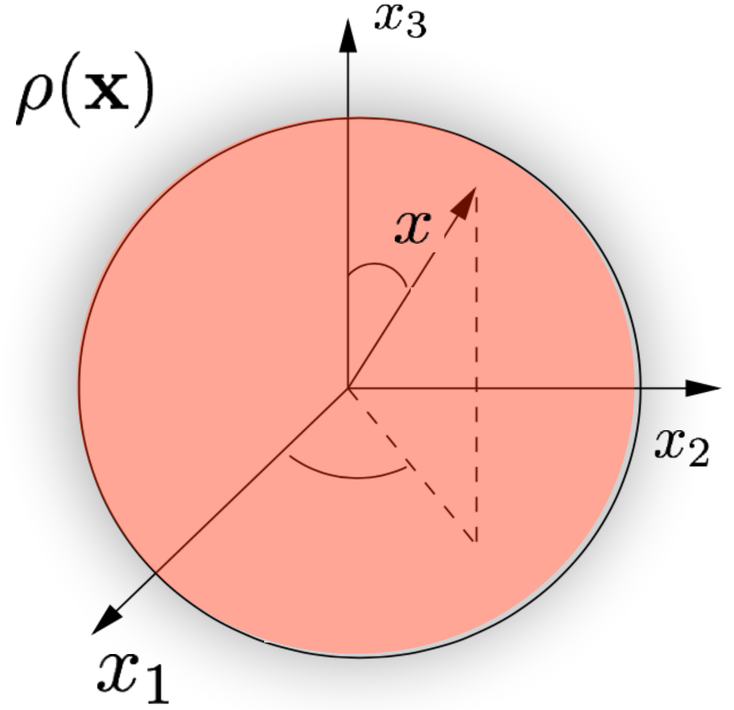
⚠ To be intended as the expectation value of the the charge operator in coordinate space, not an operator here



Nuclear charge radius

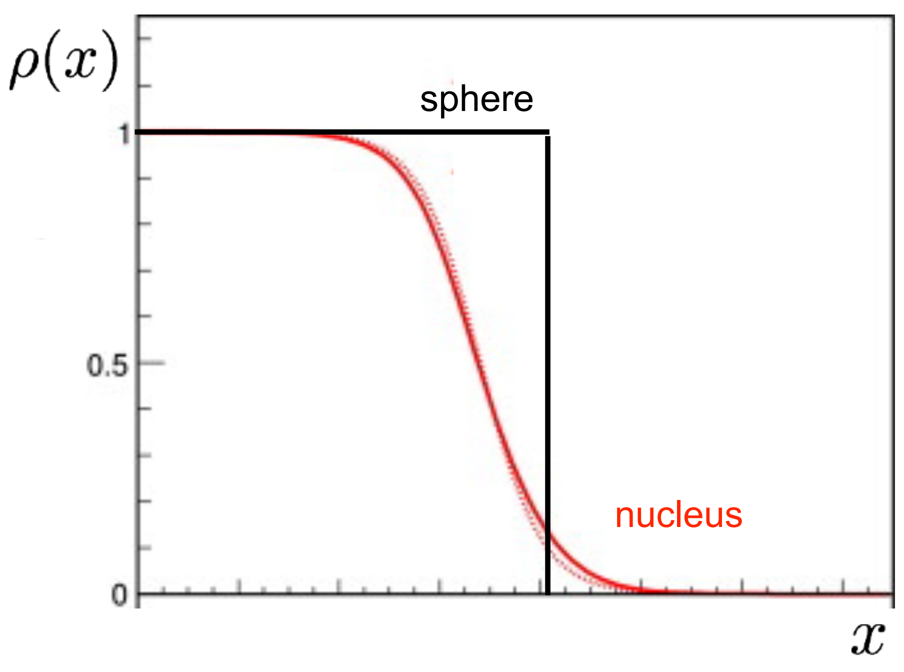
Fourier transform of the charge distribution

$$F(q^2) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \rho(\mathbf{x})$$



Assuming spherical symmetry, only interesting part is the radial dependence

$$\begin{aligned} F(q^2) &= \int dx x^2 \rho(x) \int d\phi d\theta e^{iqx\cos\theta} \\ &= 2\pi \int dx x^2 \rho(x) \int d\theta e^{iqx\cos\theta} \\ &= \frac{4\pi}{q} \int dx x \rho(x) \sin qx \end{aligned}$$



Nuclear charge radius

Now consider low-q limit

$$F(q^2) = \frac{4\pi}{q}\int dx x\rho(x)\sin qx = \frac{4\pi}{q}\int dx x\rho(x)\left(qx - \frac{(qx)^2}{3!}\right) + \dots$$

$$= 4\pi\int dx x^2\rho(x) - \frac{q^2}{6}4\pi\int dx x^4\rho(x) + \dots$$

$$= Z(1 - \frac{q^2}{6}\langle r^2\rangle + \dots)$$

Nuclear charge radius

Now consider low-q limit

$$F(q^2) = \frac{4\pi}{q}\int dx x \rho(x) \sin qx = \frac{4\pi}{q}\int dx x \rho(x) \left(qx - \frac{(qx)^2}{3!}\right) + \dots$$

$$= 4\pi \int dx x^2 \rho(x) - \frac{q^2}{6} 4\pi \int dx x^4 \rho(x) + \dots$$

$$= Z(1 - \frac{q^2}{6}\langle r^2\rangle + \dots)$$

Typically F is then normalized to 1, i.e. divided by Z

$$\langle r^2\rangle = -6 \frac{dF(q^2)}{dq^2}\bigg|_{q^2=0}$$

If you measure the form factor at low q and then take the derivative with respect to q^2 you obtain the charge radius.

Nuclear charge radius

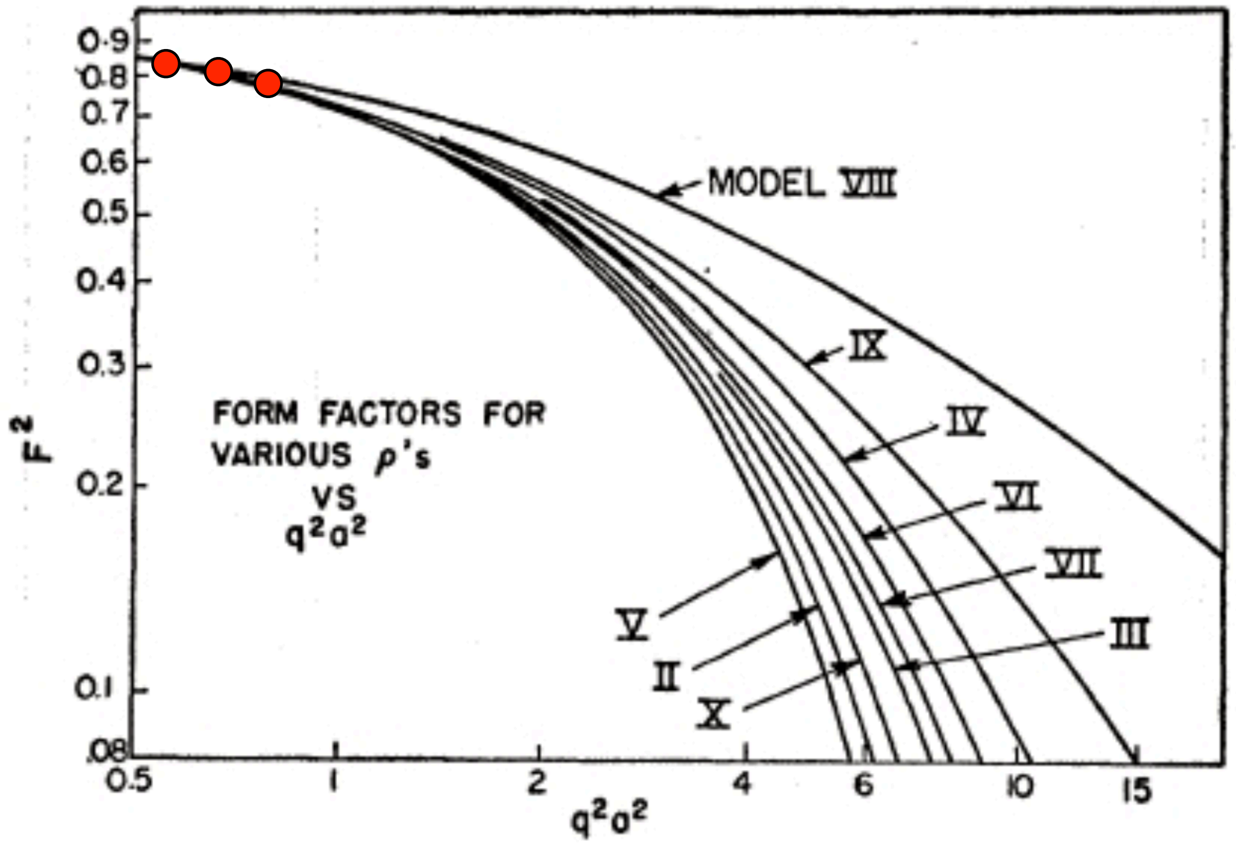
$$\langle r^2 \rangle = -6 \frac{dF(q^2)}{dq^2}\bigg|_{q^2=0}$$

How small does q have to be?

Formula is valid only if q is small

$$F(q^2) = Z(1 - \frac{q^2}{6}\langle r^2 \rangle + \dots)$$

For low q, we do not expect the electron to penetrate the nucleus. Therefore, all spherically symmetric charge distribution are going to appear similar from the outside. As a result, the scattering will be similar as well.



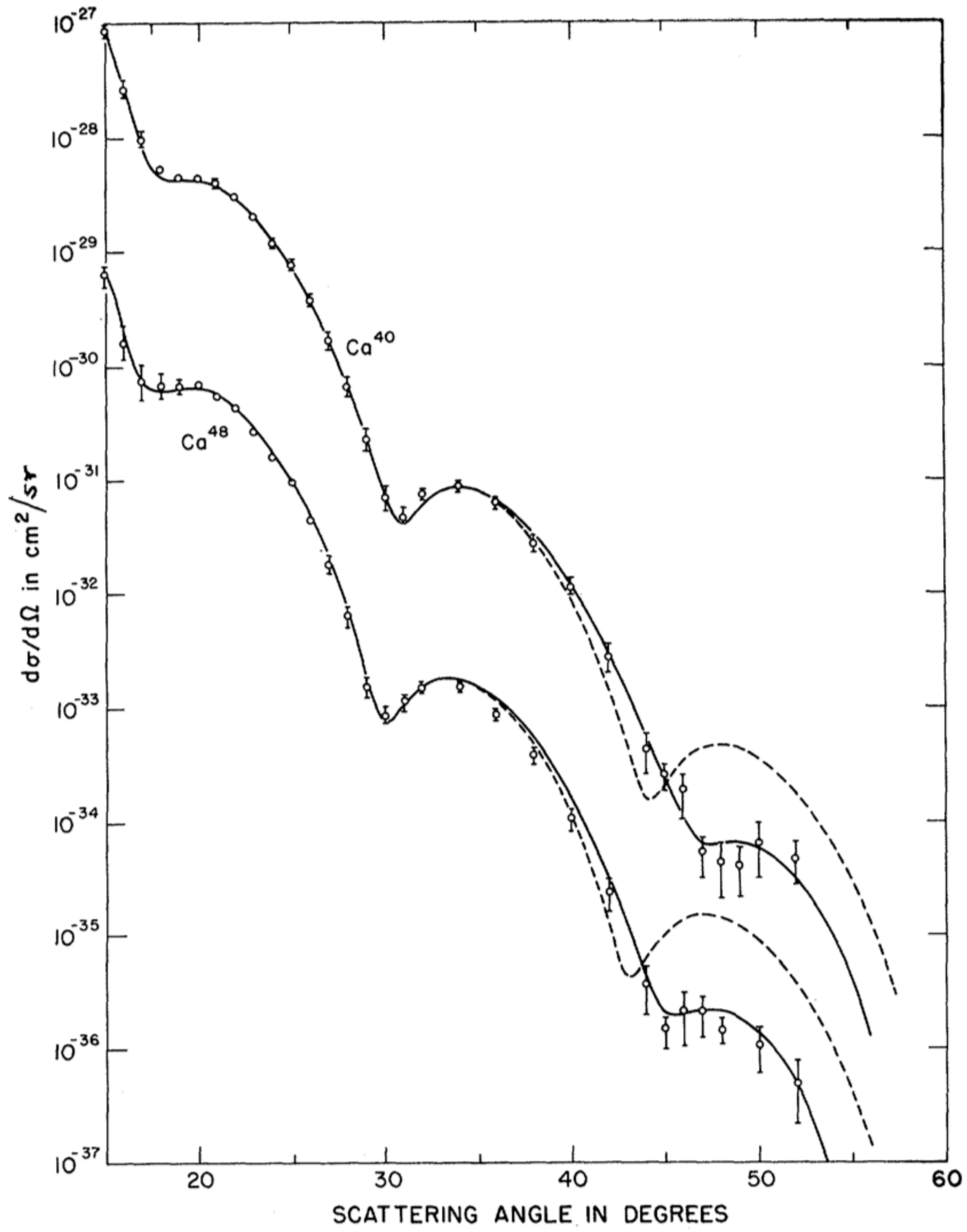
Fourier transform of various spherical shapes

Nuclear charge radius

Better to do an ansatz of the radial shape (parameters), do the Fourier transform and then fit the parameters to the experimental form factor

$$\rho(r) = eZ \left(\frac{b^2}{2\pi}\right)^{3/2} e^{-b^2 r^2/2}$$

$$\rho(r) = \frac{\rho_1}{exp\left[(r-c)/z_1\right] + 1}$$



Nuclear charge radius

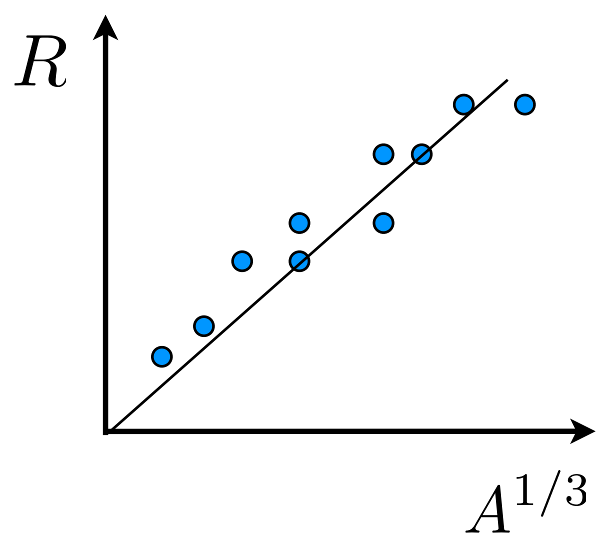
From electron scattering you obtain the charge radius and the charge distributions

$$R = \sqrt{\langle r^2 \rangle}$$

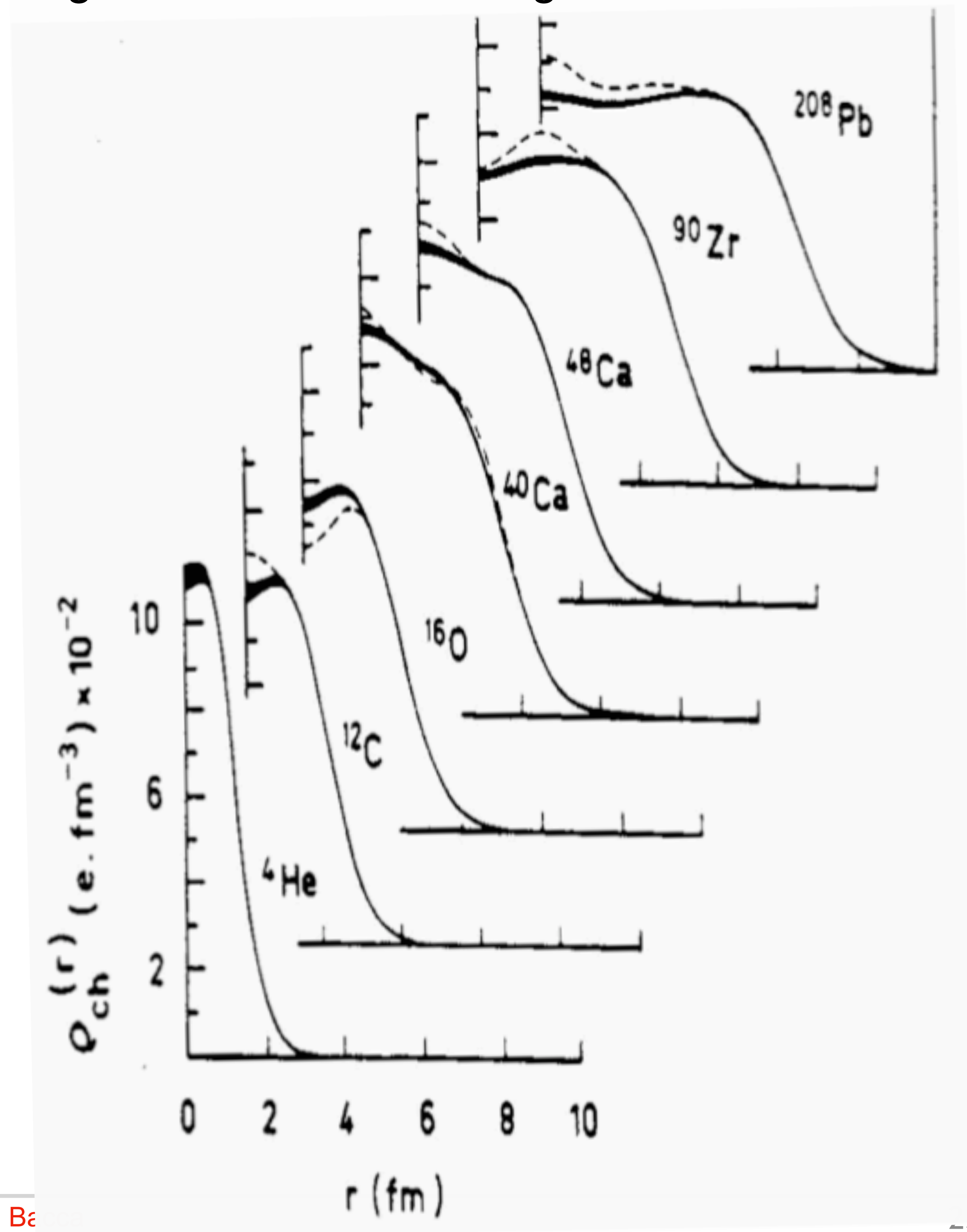
stable nuclei

$$R \simeq r_0 A^{1/3}$$

$$r_0 = 1 - 1.25\text{fm}$$



Behaviour that is disobeyed by exotic nuclei, such as halo nuclei



Elastic scattering

Physics studies:

Compare between N=Z and Neutron-rich isotopes

how does the charge distribution change because of the presence of the NEUTRONS?

how does the current distribution change as a function of the number of NEUTRONS?

how does the current distribution change because of two-body currents?

which two-body currents are relevant in one case and in the other?

Inelastic scattering

$\omega \neq 0$	Energy and momentum transferred
$f \neq 0$	Nucleus does not stay in ground-state

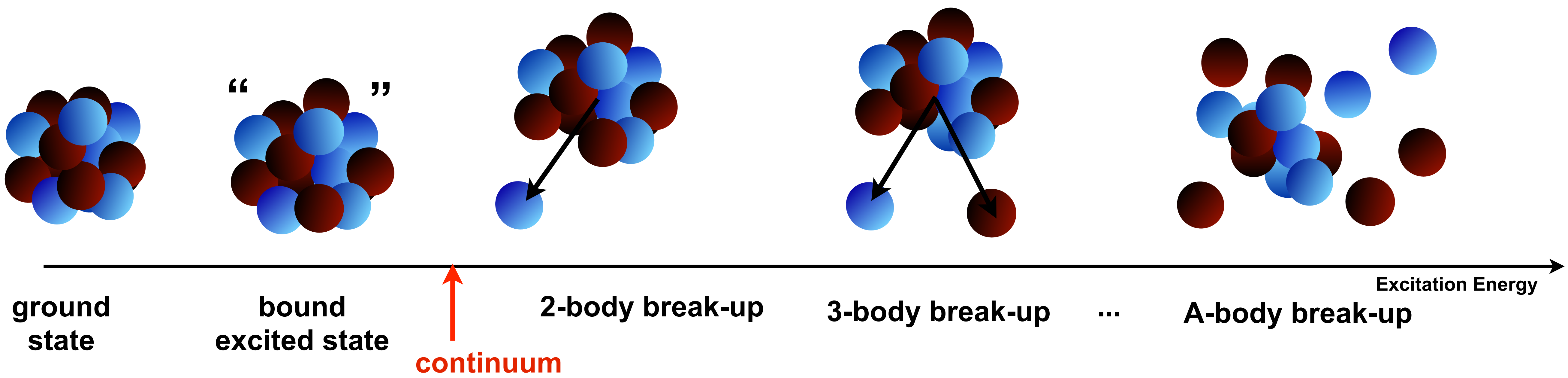
Much richer!

Information is about both ground state and excited states

Not only "static" properties but also
"dynamical" properties e.g. collective motions

More complicated, though.
Need to be able to calculate final states also in the continuum

Inelastic scattering



$$R(\omega) \propto |\langle \Psi_f | \mathcal{O} | \Psi_0 \rangle|^2$$

Exact knowledge limited in energy and mass number

We will discuss this important issue later