Electromagnetic properties of nuclei: from few- to many-body systems

Lecture 8

Few-body methods - Applications

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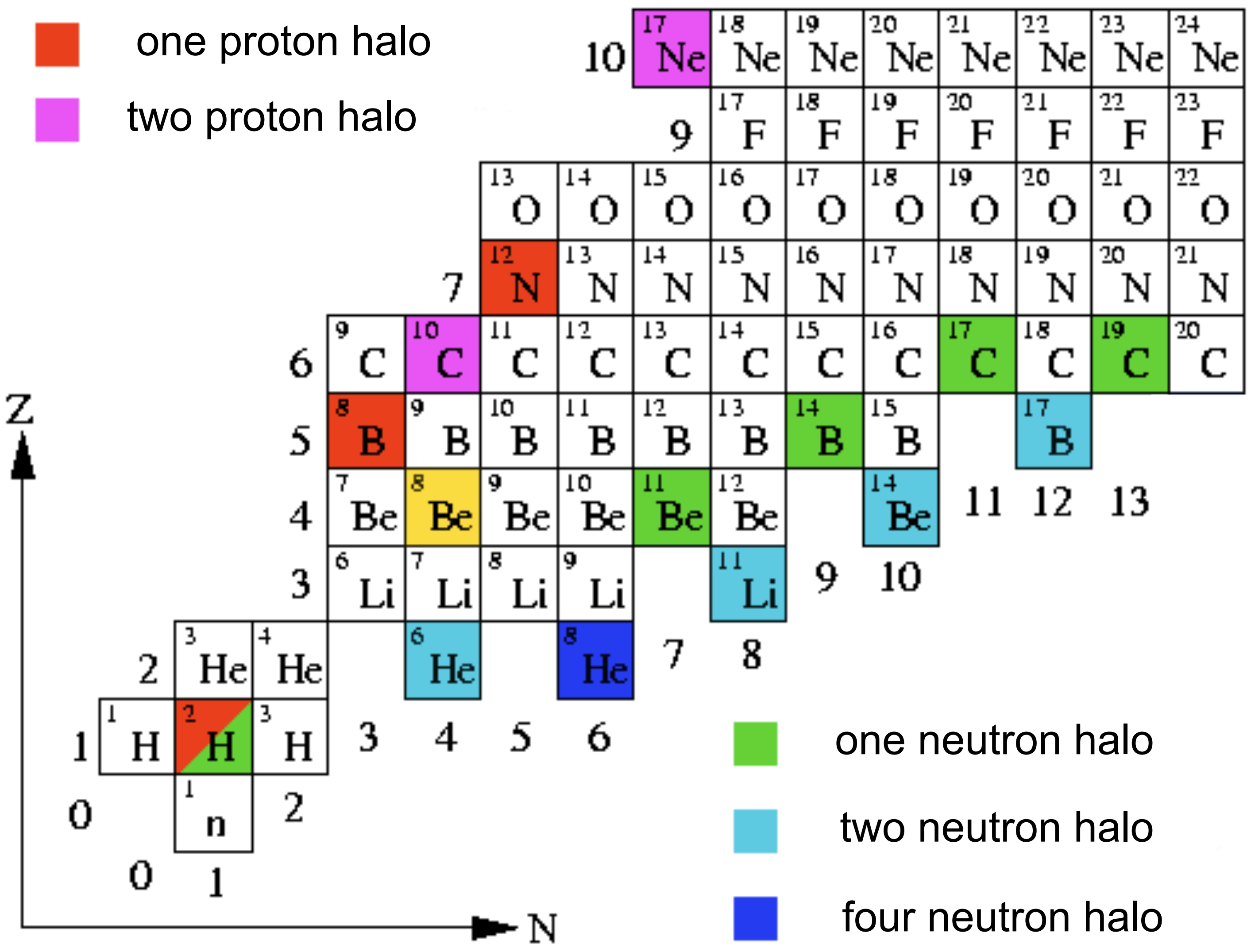
November 22nd, 2017

Lecture series for SFB 1245
TU Darmstadt

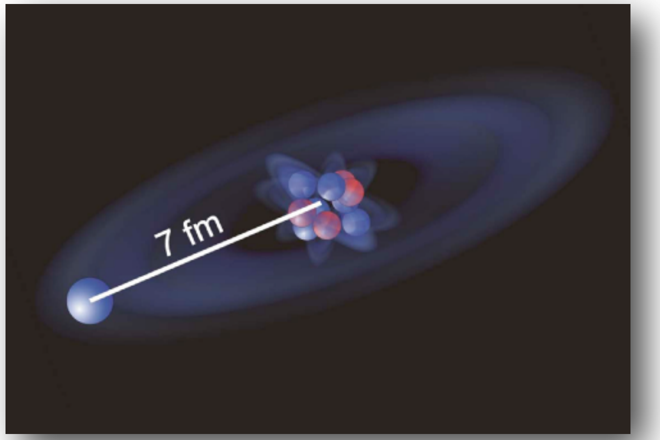
Physics cases of interest:

- **Halo nuclei**
- **Form factors**
- **Muonic atoms**

Halo nuclei



- Exotic nuclei
  with an interesting structure



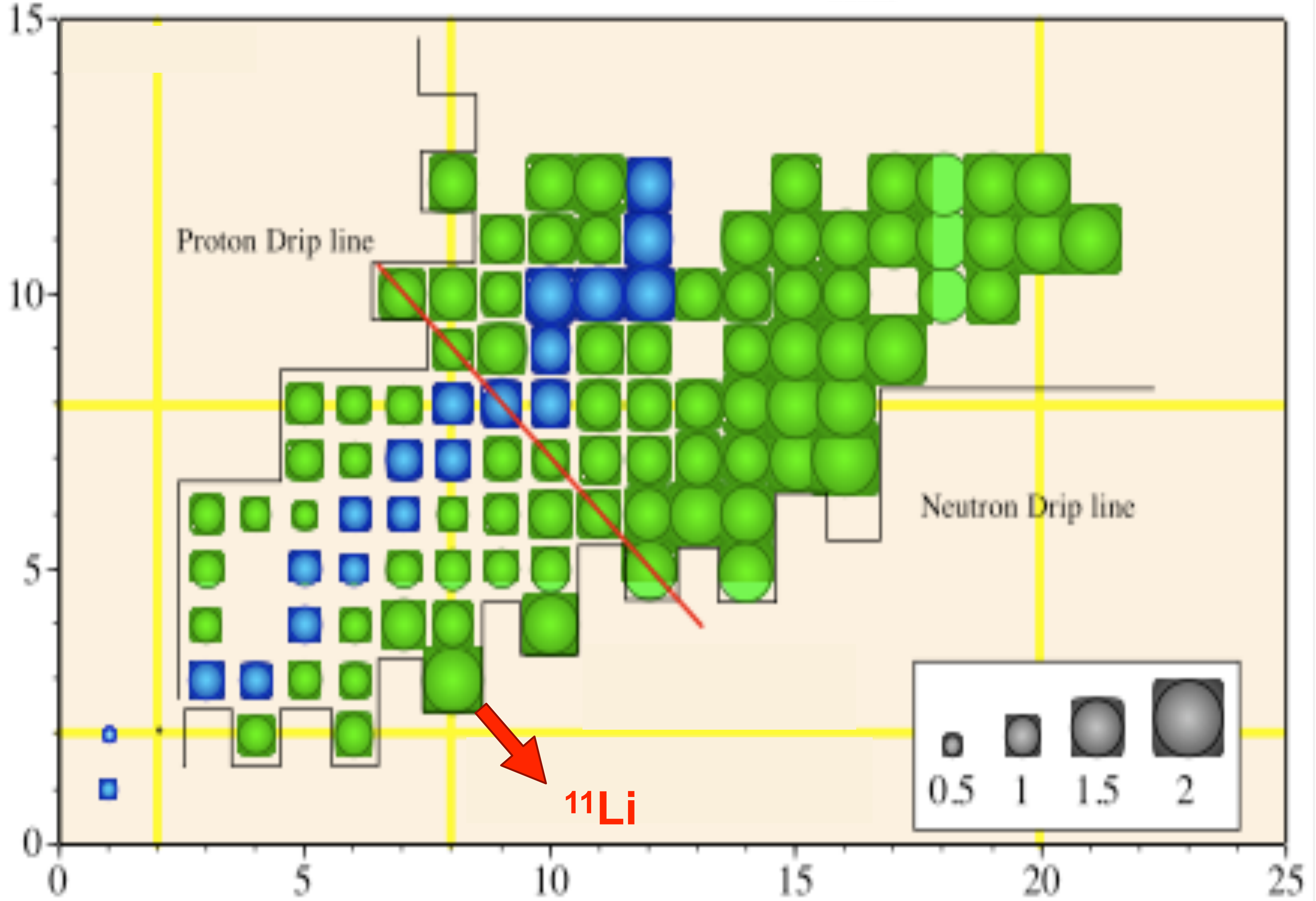
- Neutron halos:
  Large n/p ratio (neutron-rich)

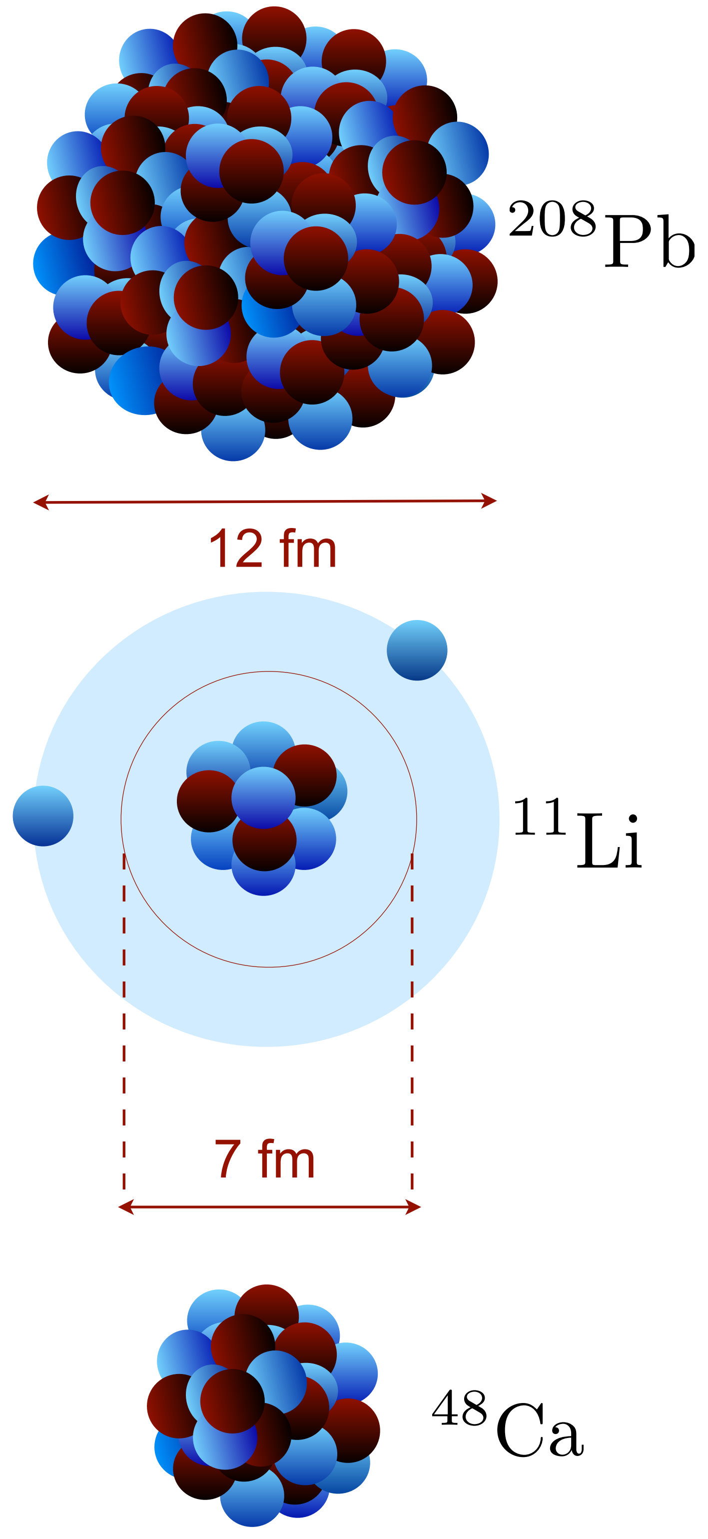
Halo	n/p
^{6}He	2
^{8}He	3
^{11}Li	2.66
^{12}C	1

Halo nuclei

- Large size

Nuclear radius for stable nuclei: $R_N \sim r_0 A^{1/3}$ with $r_0 \sim 1.2$ fm

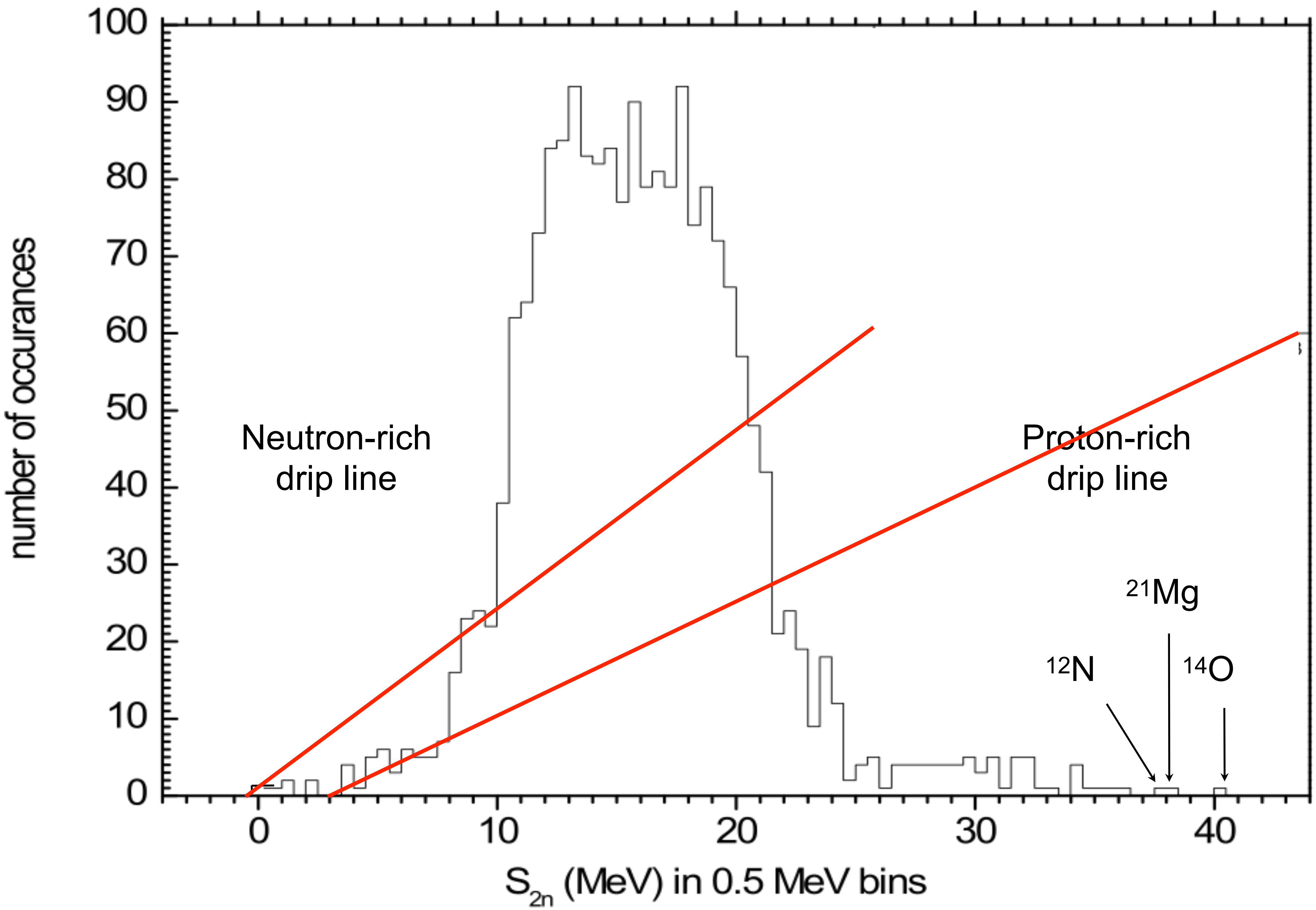




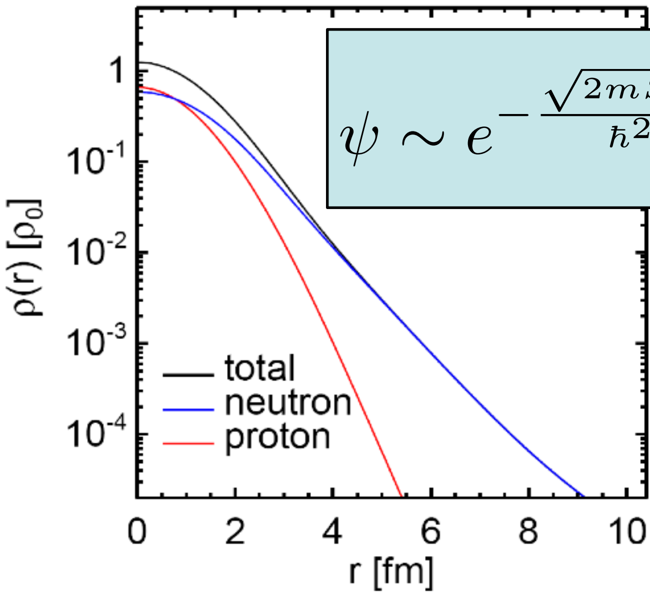
Halo nuclei

- Small nucleon(s) separation energies

$$S_{2n} = BE(Z,N) - BE(Z,N-2)$$



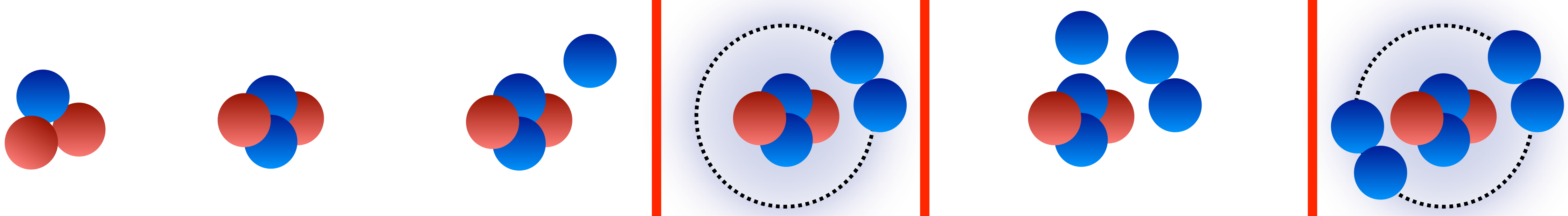
Long tail in the w.f.



$$\psi \sim e^{-\frac{\sqrt{2mS_{2n}}}{\hbar^2}r}$$

The helium isotope chain

^3He	^4He	^5He	^6He	^7He	^8He	....
bound	bound	unbound	bound	unbound	bound	
			halo		halo	



Borromean system

lives 806 ms

Most exotic nucleus "on earth"

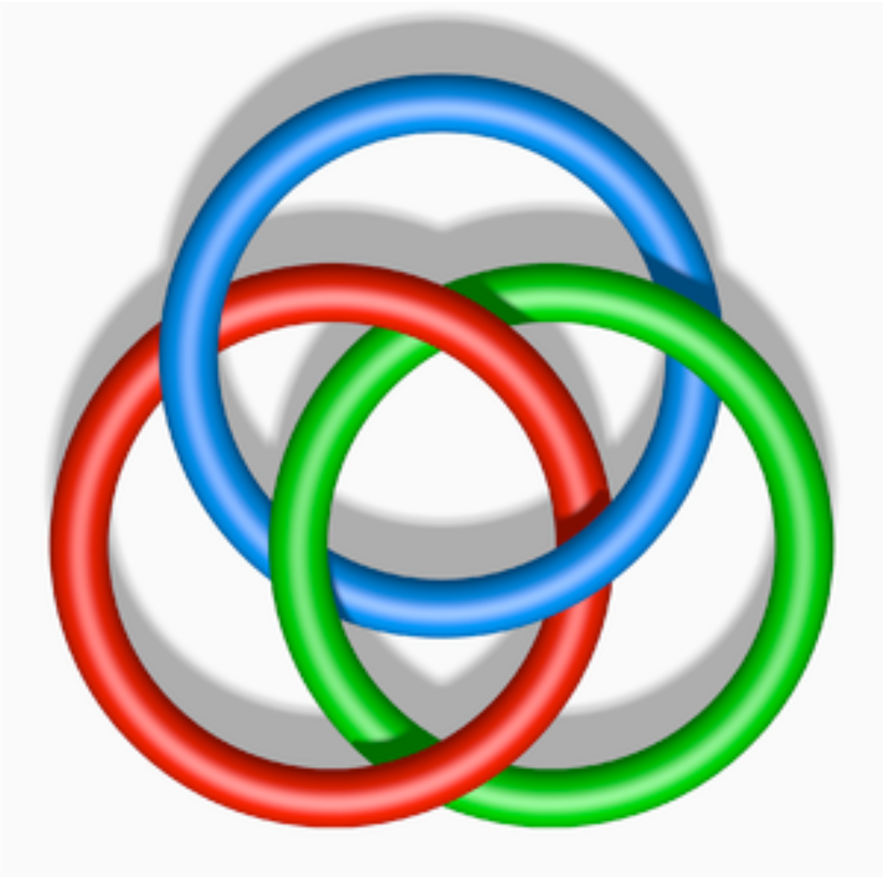
$$\frac{N}{Z} = 3$$

lives 108 ms

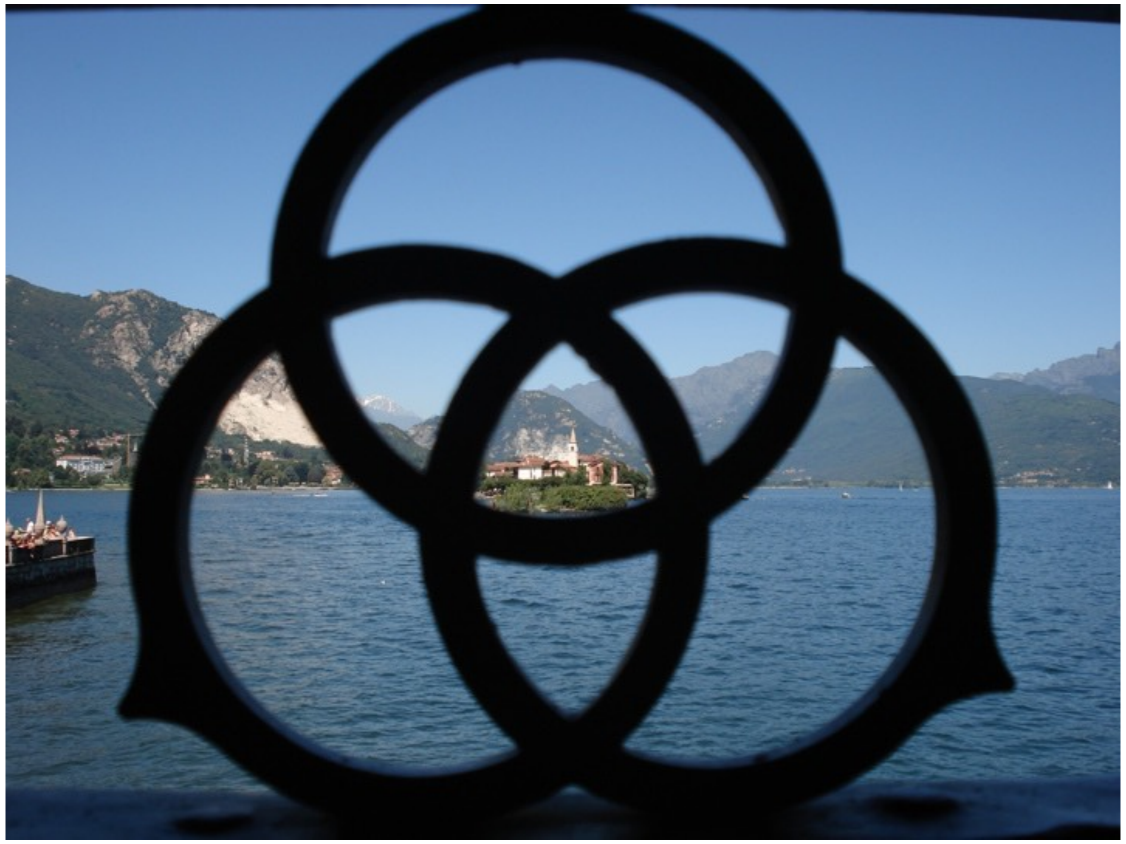
Even if they are exotic short lived nuclei, they can be investigated experimentally. From a comparison of theoretical predictions with experiment we can test our knowledge on nuclear forces in the neutron rich region

Borromean Nuclei

Named after Borromean rings by M.V. Zhukov *et al*., Phys. Rep **231**, 151 (1993)



Isola Bella, Lago Maggiore, Italia



pic credit P.Capel

Halo Nuclei - Experiment

New Era of Precision Measurements for masses and radii

- Masses (and thus binding energies) are measured with Penning traps

TITAN TRIUMF

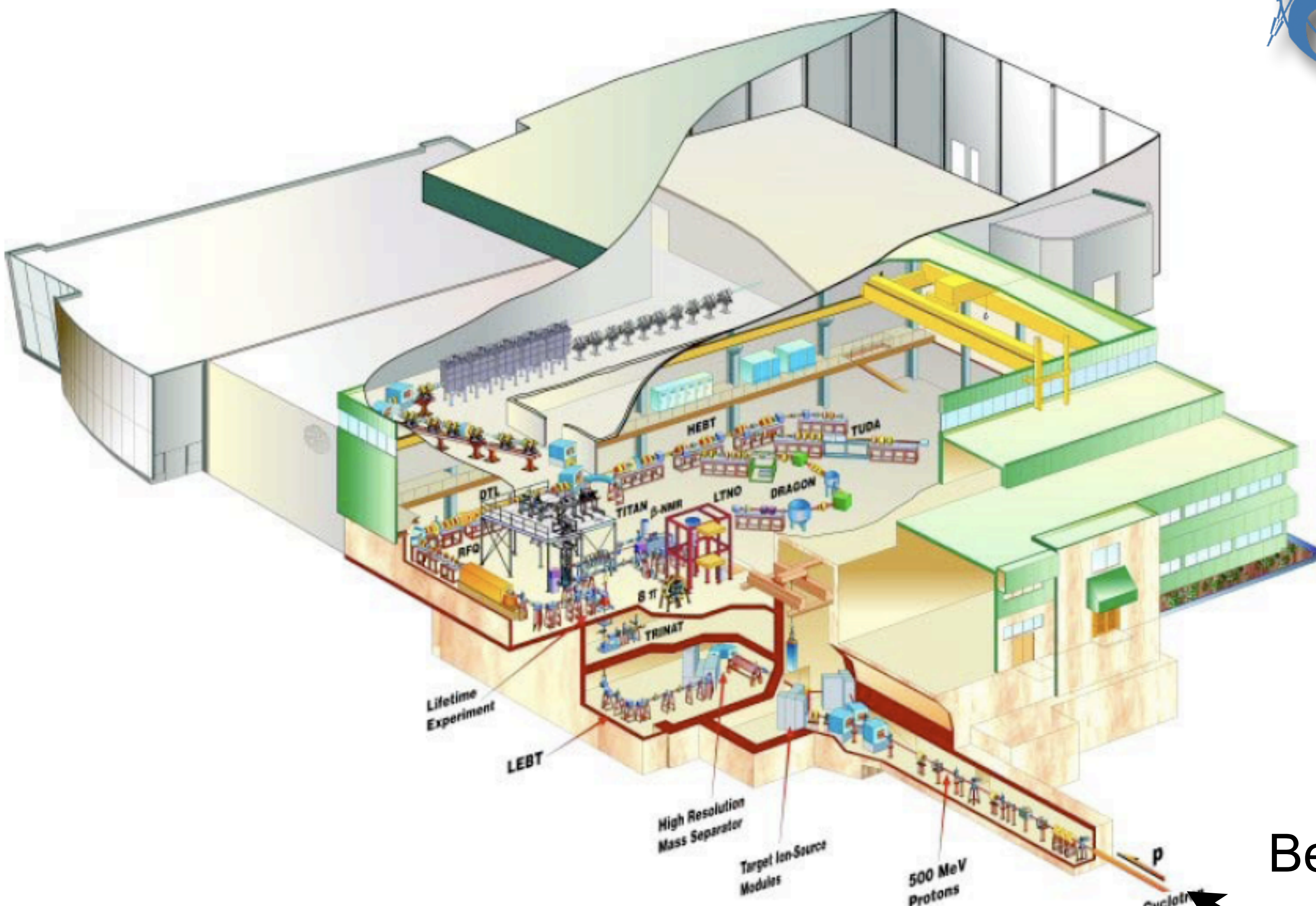
Can reach a relative precision of 10^{-8}

- Charge radii are measured with Laser Spectroscopy

ARGONNE
GANIL
ISOLDE

Halo Nuclei - Experiment

TRIUMF



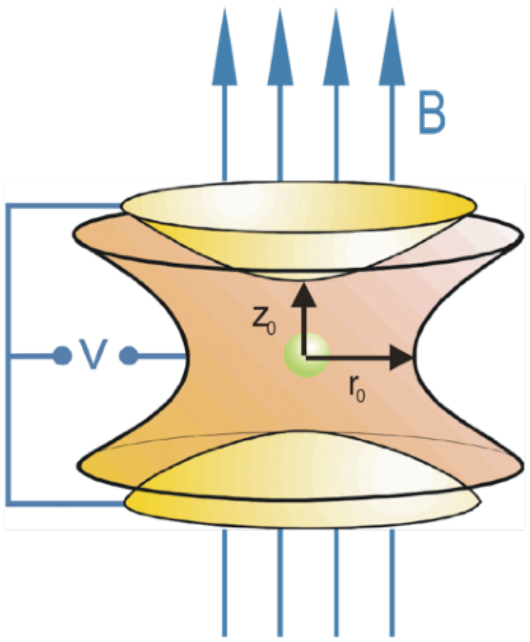
Beam

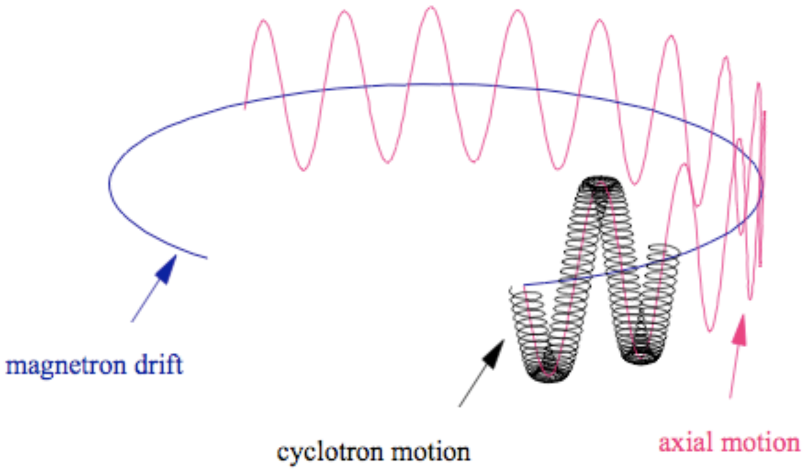
Cyclotron

Halo Nuclei - Experiment

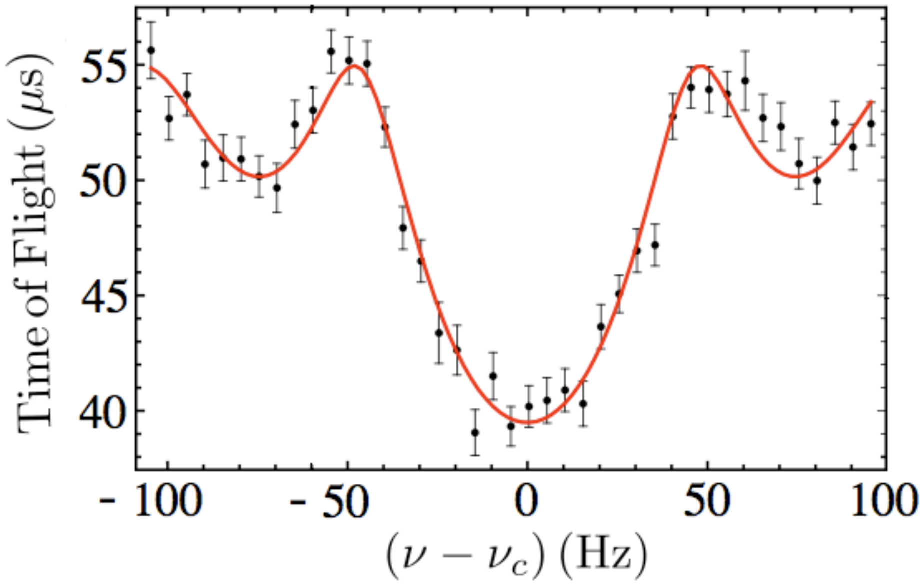
Penning Trap TITAN TRIUMF

Superposition of strong homogeneous magnetic field and a weak electrostatic quadrupole electric field





ion eigenmotions are known



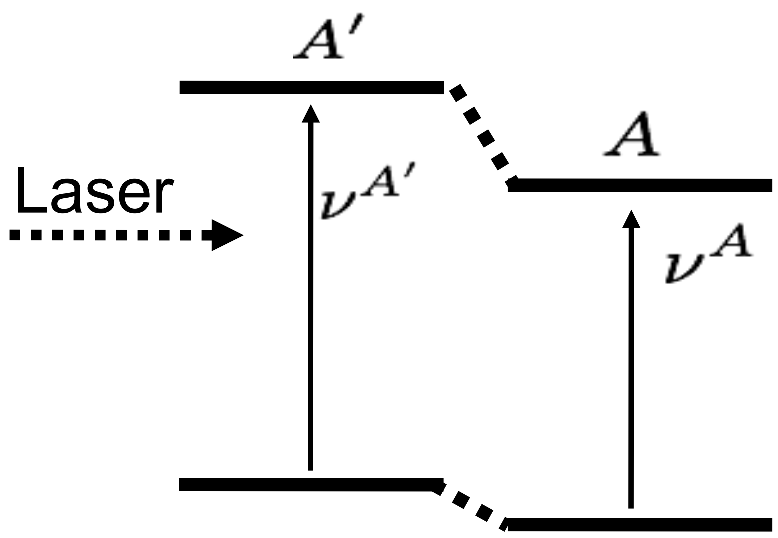
$$\nu_c = \frac{1}{2\pi}\frac{qB}{m} \qquad \frac{\delta m}{m} = \frac{\delta \nu_c}{\nu_c} = \frac{\delta \nu_c 2\pi m}{qB}$$

$$\delta \nu_c \sim \frac{1}{T}\frac{1}{\sqrt{N}}$$

$$\frac{\delta m}{m} = \frac{m}{qTB\sqrt{N}}$$

known from the beam

Laser Spectroscopy for radii



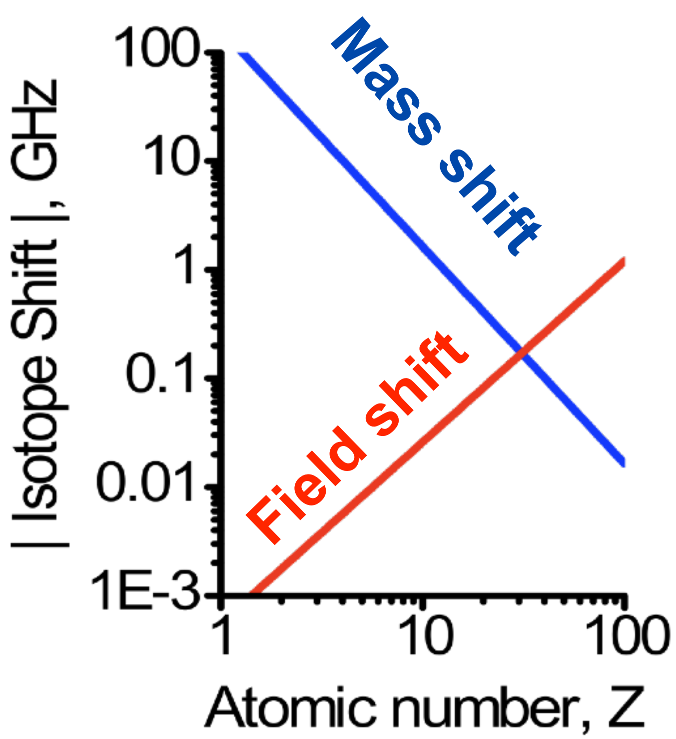
Experiment

$$\overbrace{\delta\nu^{A,A'} = \nu^{A'} - \nu^{A}}^{\text{Experiment}}$$

Theory: from precise atomic structure calculations

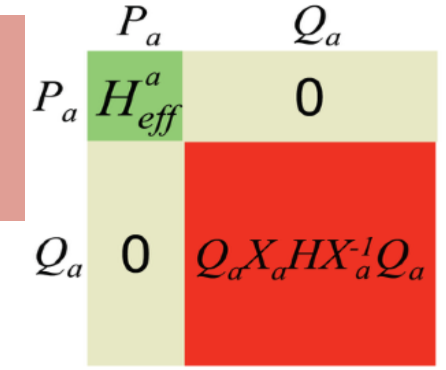
$$= \underbrace{\overbrace{\delta\nu^{mass}_{A,A'} + K}^{\text{Theory}}}_{\text{Mass shift}} \underbrace{\delta\langle r^2\rangle_{AA'}}_{\text{Field shift}}{}^{\text{ch}}$$

Mass shift — Field shift



- Mass shifts dominates for light nuclei
- Nuclear masses are input for calculations of K ➡ can be the largest source of systematic errors if not known precisely
- Precise mass measurements are key for a better determination of radii

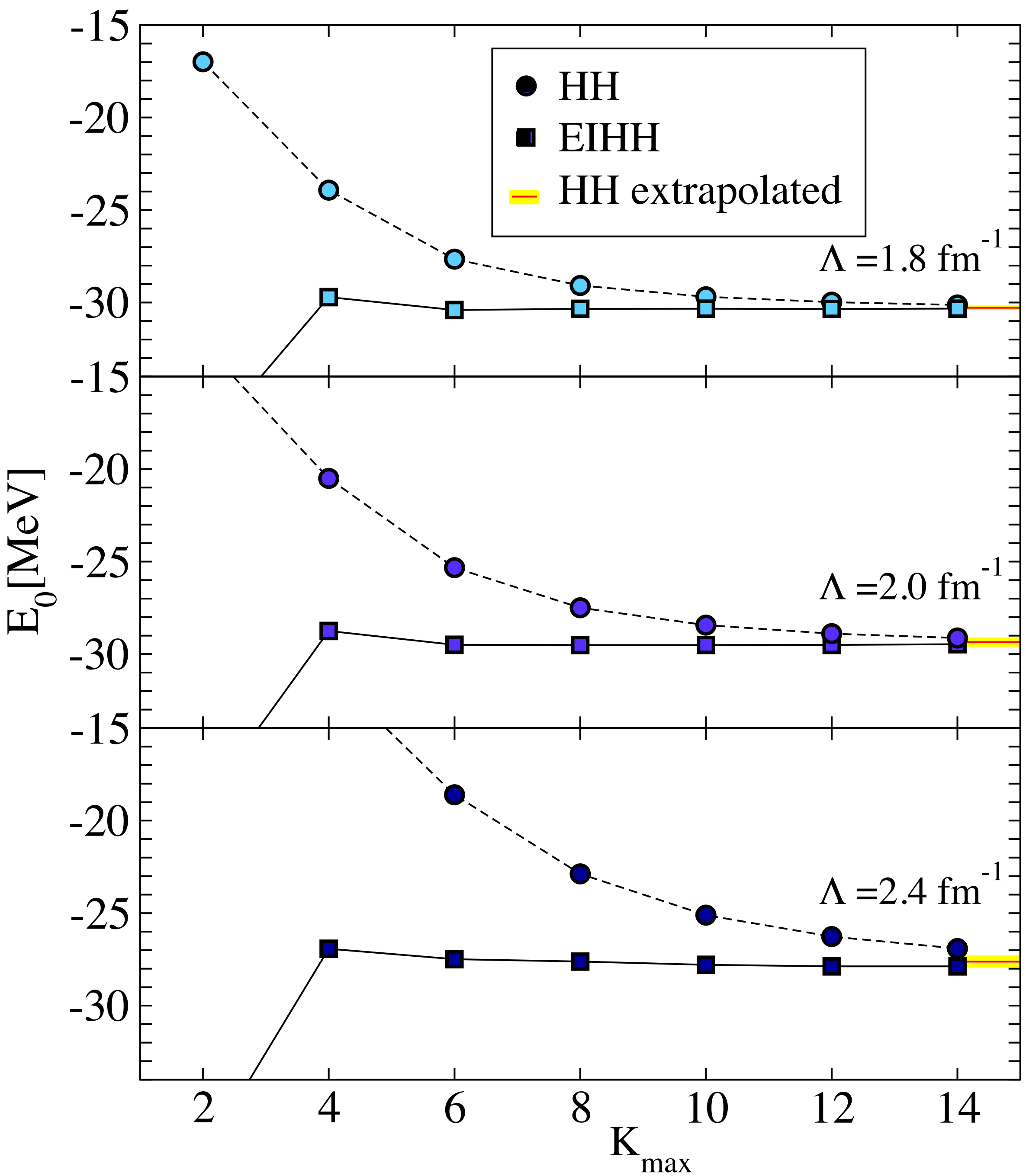
^{6}He from HH

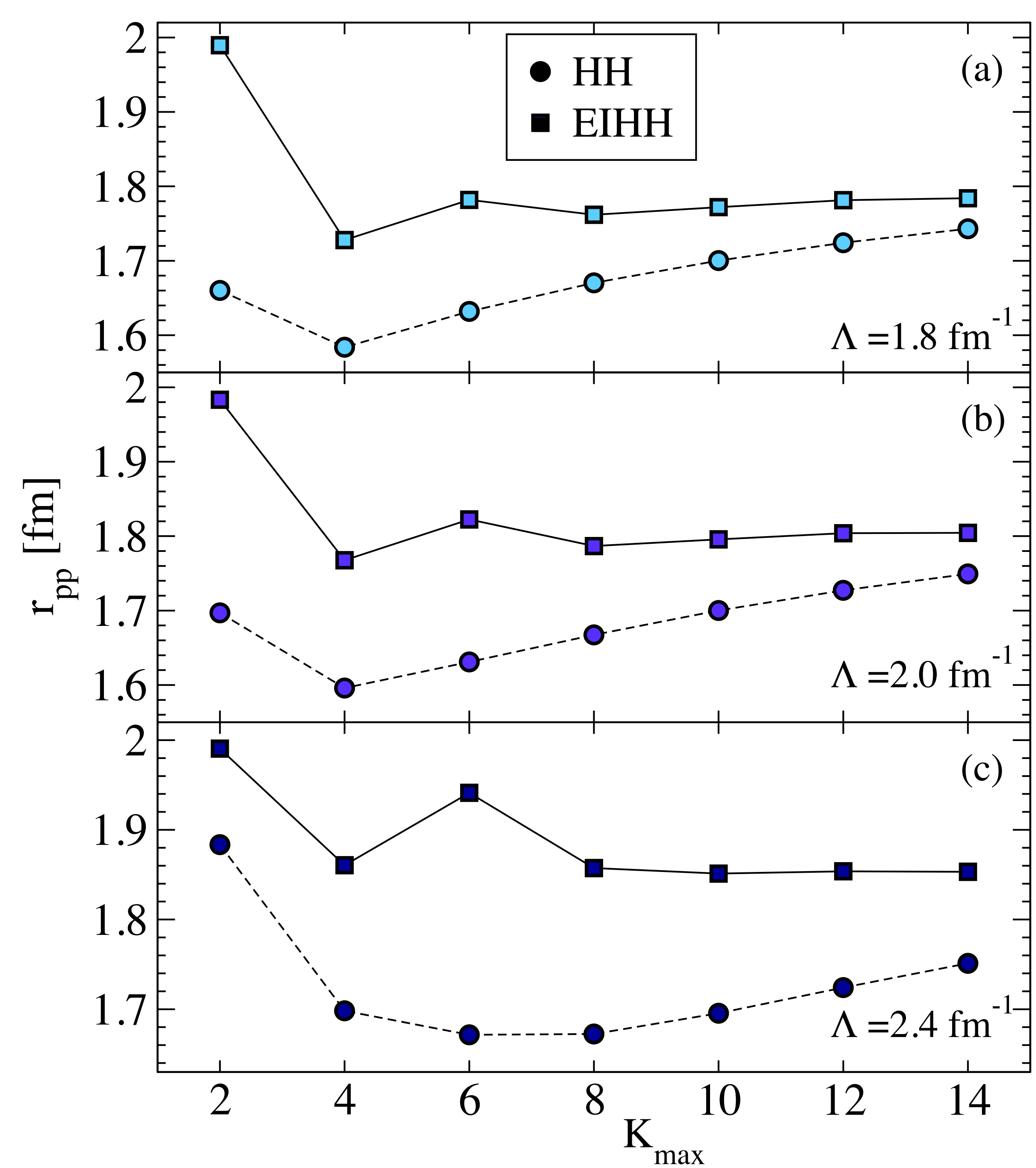


S.B. *et al.*, PRC 86, 034321 (2012)

$V_{\rm low\ k}$ from N^{3}LO (500 MeV)

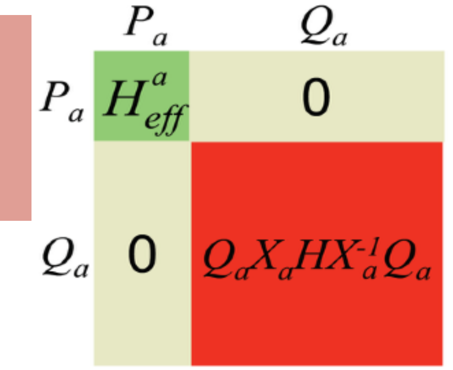
Extrapolated $E(K_{max}) = E^{\infty} + Ae^{-BK_{max}}$



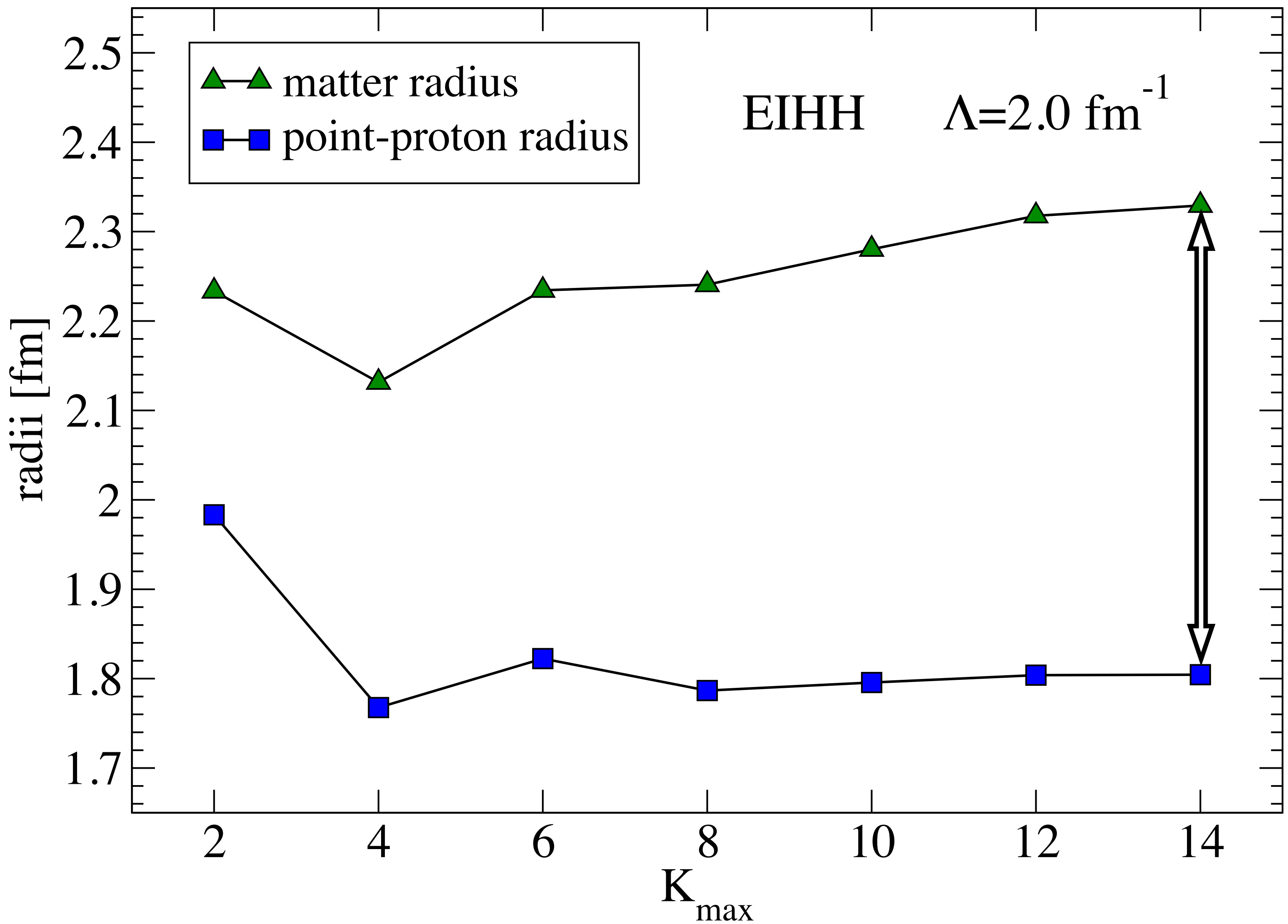


- EIHH agrees with extrapolated HH results
- EI is key to reach a reliable convergence of radii

^{6}He from HH



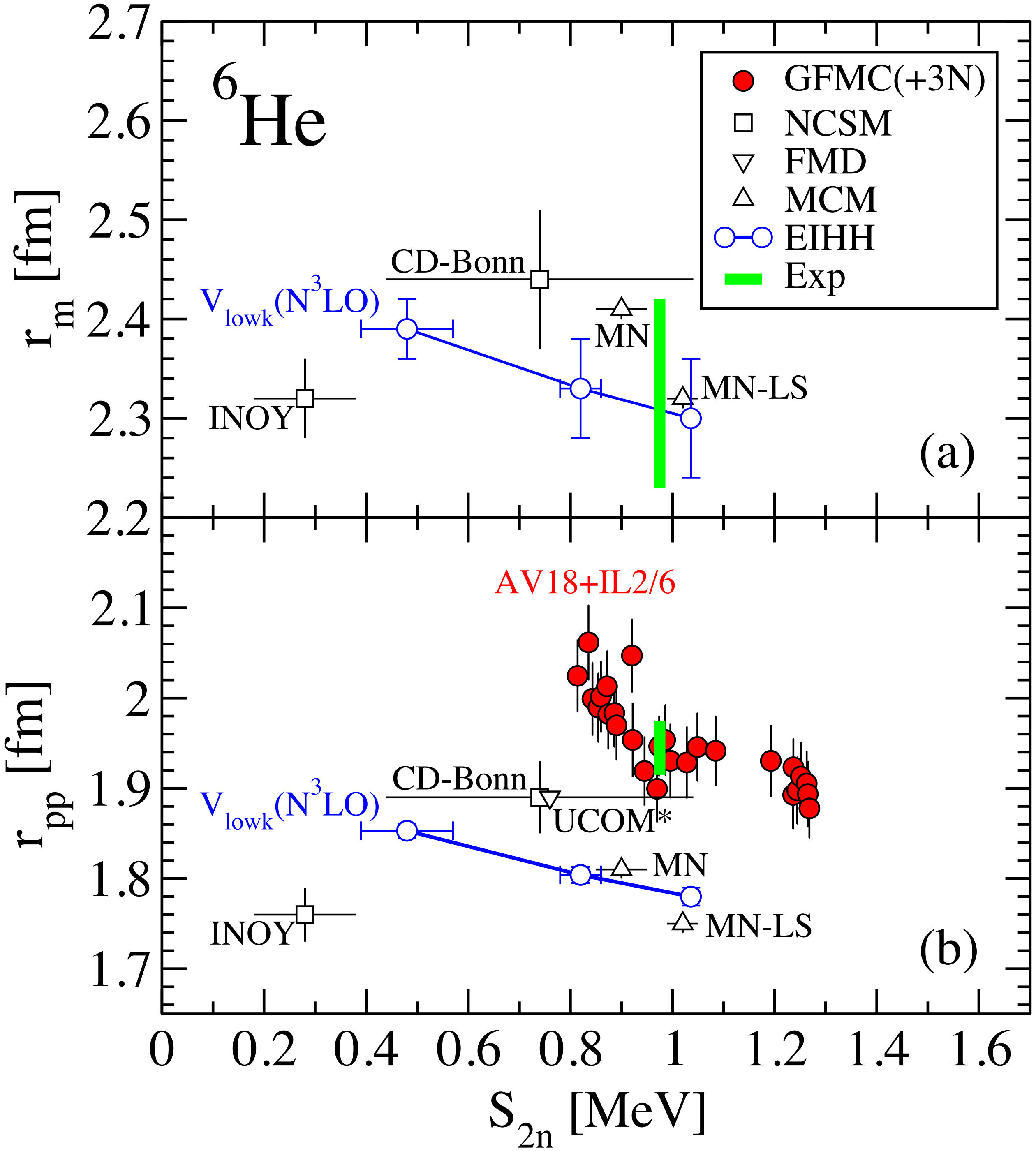
S.B. *et al.*, PRC 86, 034321 (2012)



- Point-Proton radii converge better and are smaller than matter radii ⇨ halo structure

Theory vs Experiment

Phys. Rev. Lett. 108, 052504 (2012)



(a) Experimental matter radius relatively uncertain

(b) Experimental charge radius well constrained

Relativistic corrections

$$r_{\rm pp}^2 = r_{\rm c}^2 - R_p^2 - \frac{N}{Z} R_n^2 - \frac{3}{4M_p^2} - r_{\rm so}^2$$

0.877(7) fm from electron scattering and H spectroscopy

0.84184(67) from spectroscopy of muonic hydrogen

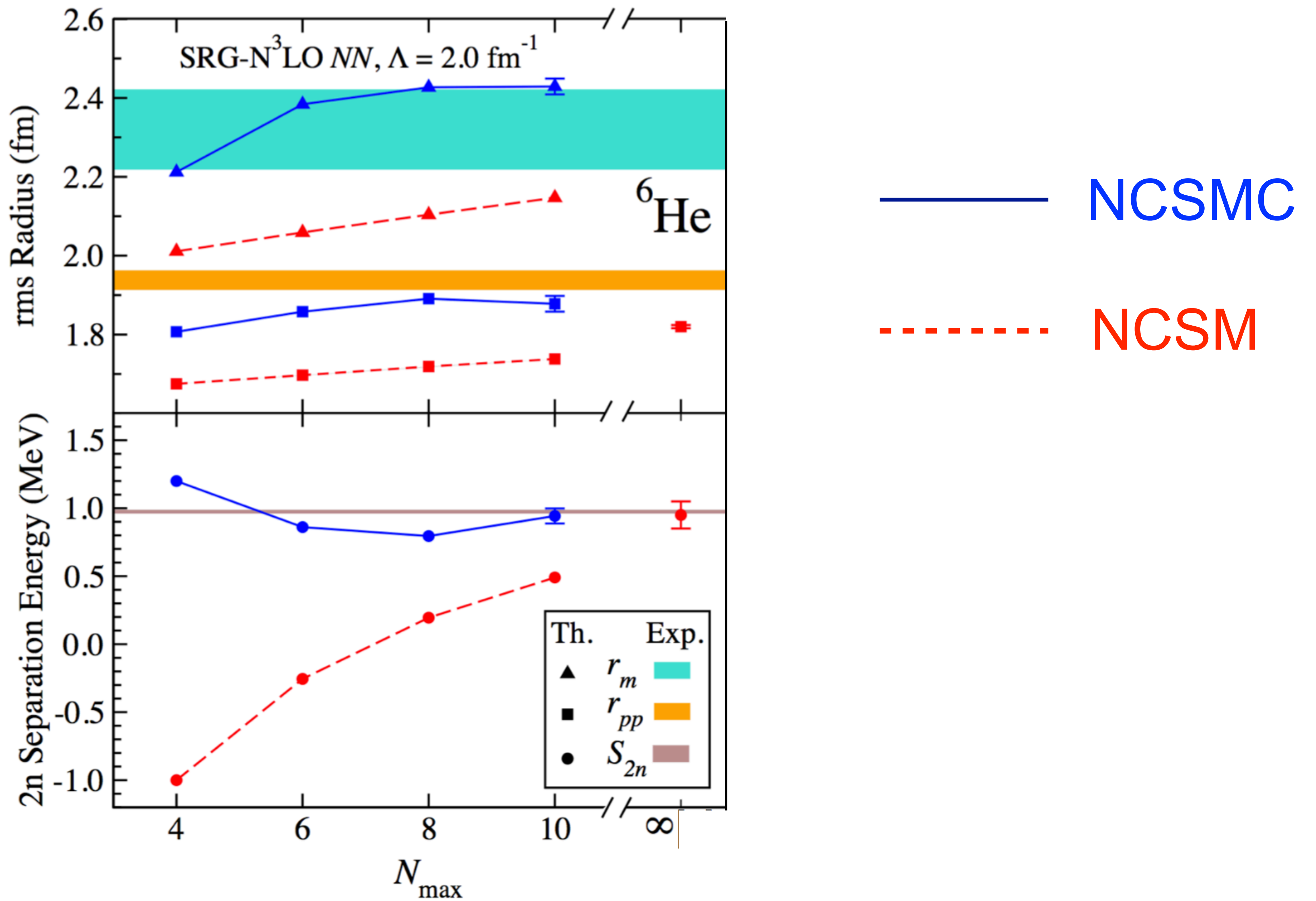
Calculated ab-initio ~-0.082 fm^2

- It is important to compare observable together
- Correlation between radii and separation energy

Update on other methods

NCSM/NCSMC

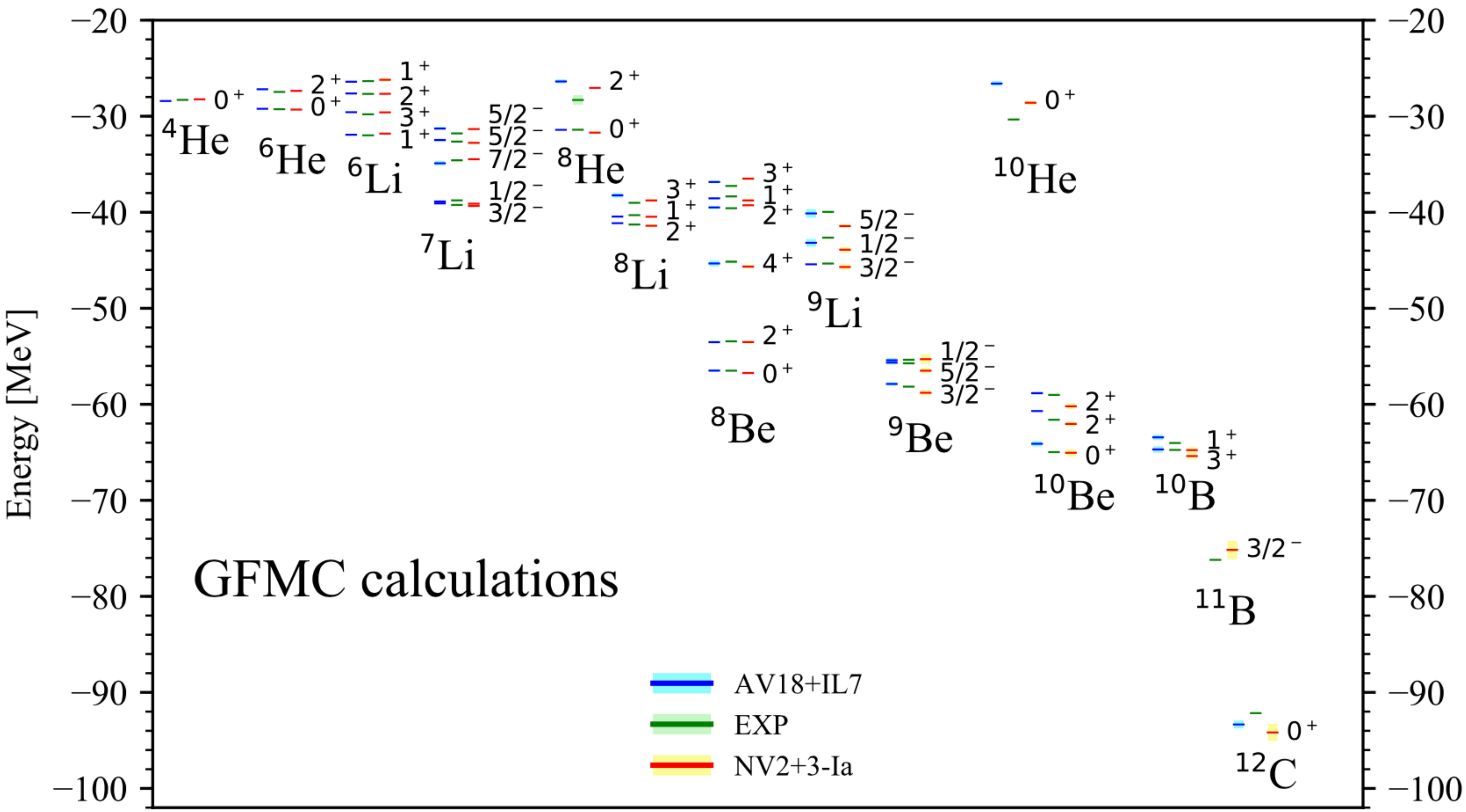
Romero-Redondo et al., Phys. Rev. Lett. 117, 222501 (2016)



No three-body forces included also here

GFMC

Only method with three-body forces on ^{6}He

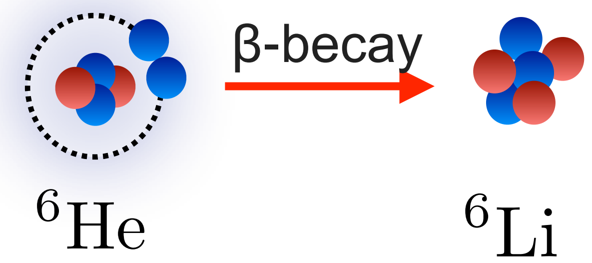


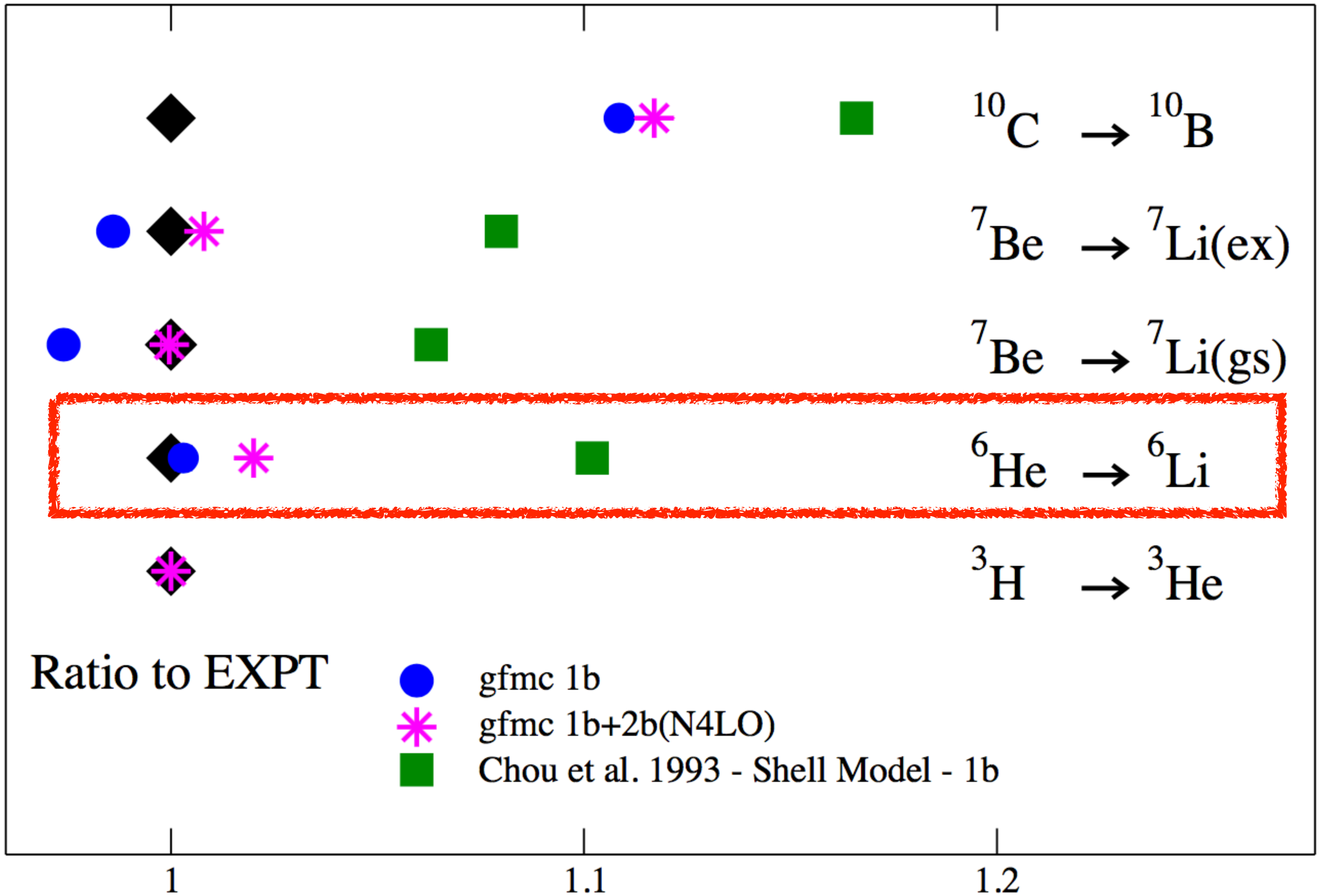
M. Piarulli et al, arXiv:1707.02883

Update on other methods

GFMC

Only method with three-body forces on ^6He

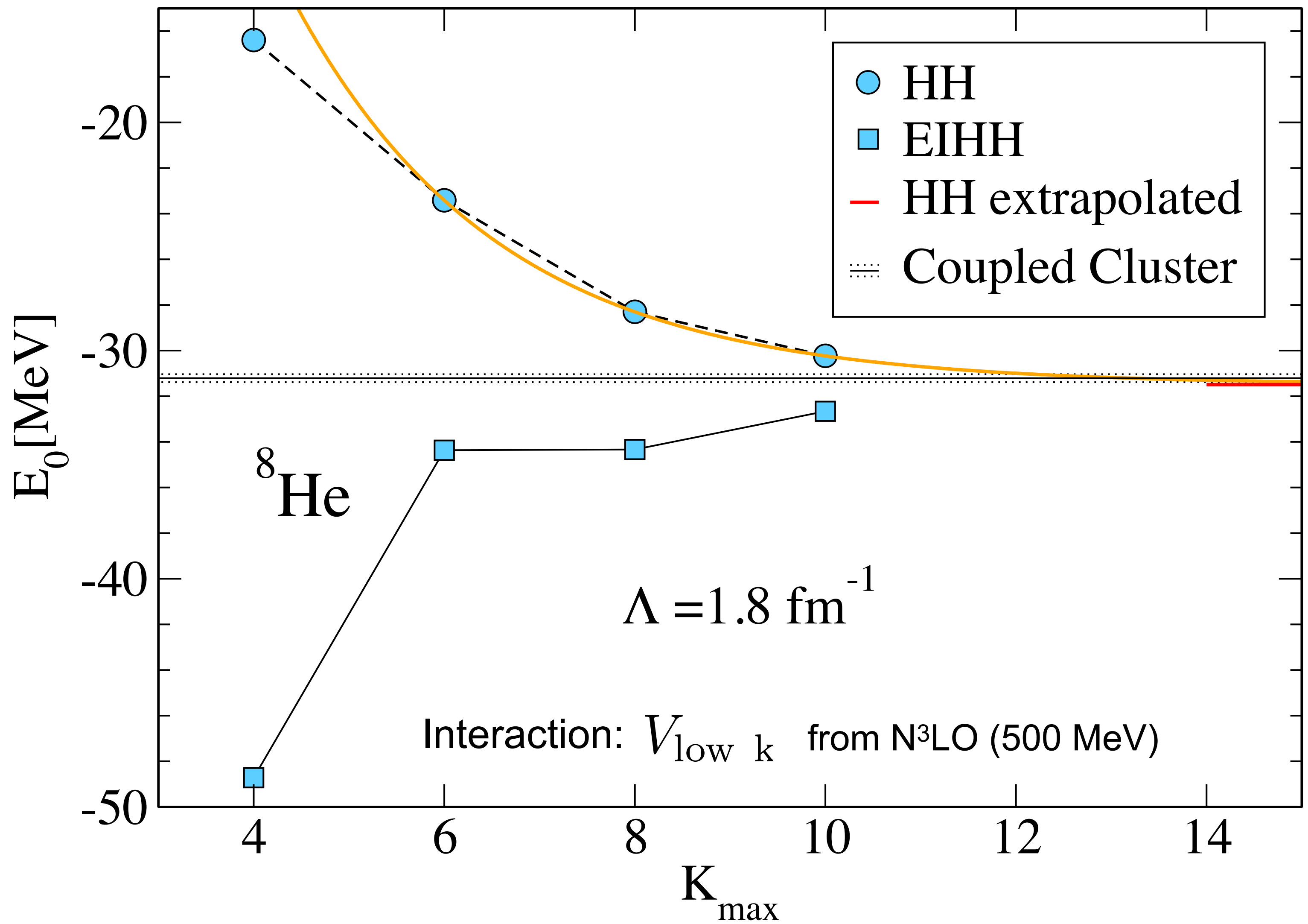




S. Pastore et al, 1709.03592

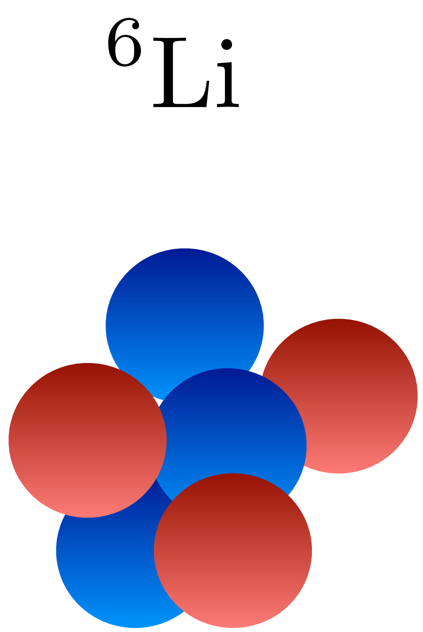
^{8}He from HH?

S.B. *et al.*, PRC 86, 034321 (2012)



- Difference between HH and EIHH is about 2.4 MeV
- EIHH seems less effective than for ^{6}He
- Extrapolating HH results get $E_\infty = -31.49\text{MeV}$ comparable to Coupled cluster

Do halo nuclei respond differently to em probes?



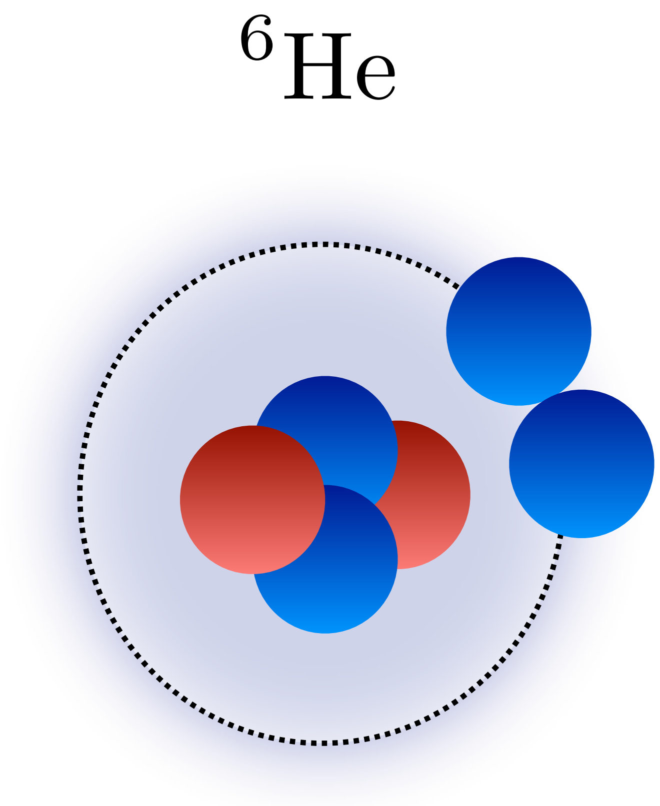
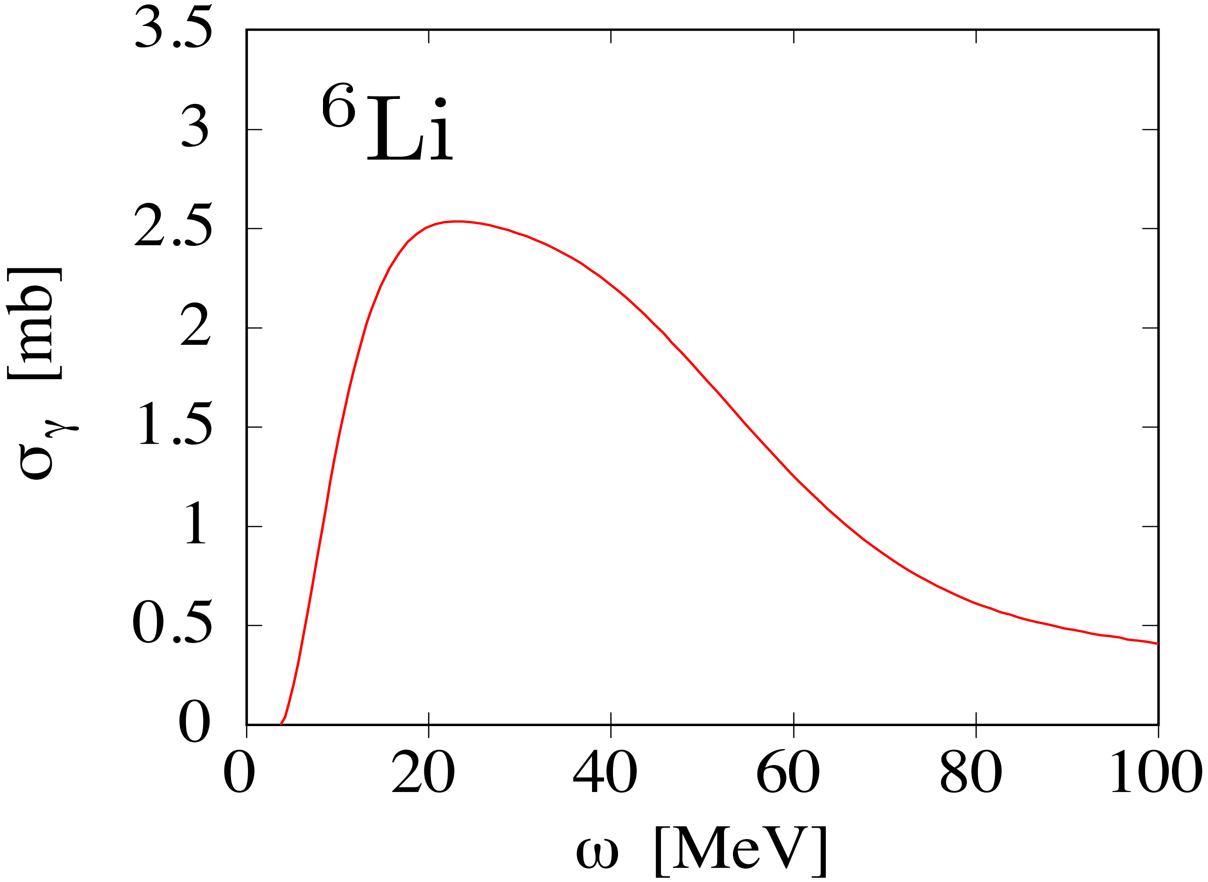


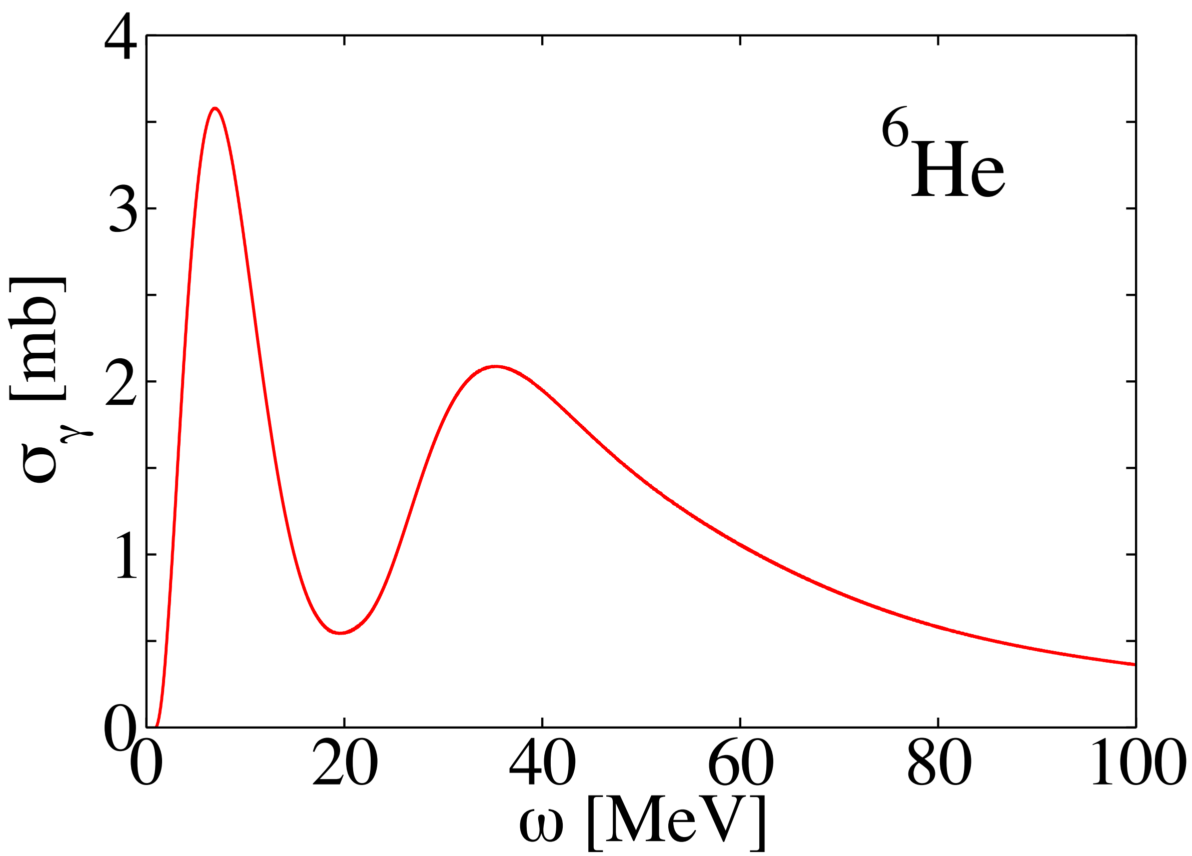
Photo-absorption reaction

S.Bacca et al, PRL **89** 052502 (2002)

$$\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R^{E1}(\omega)$$

AV4' potential





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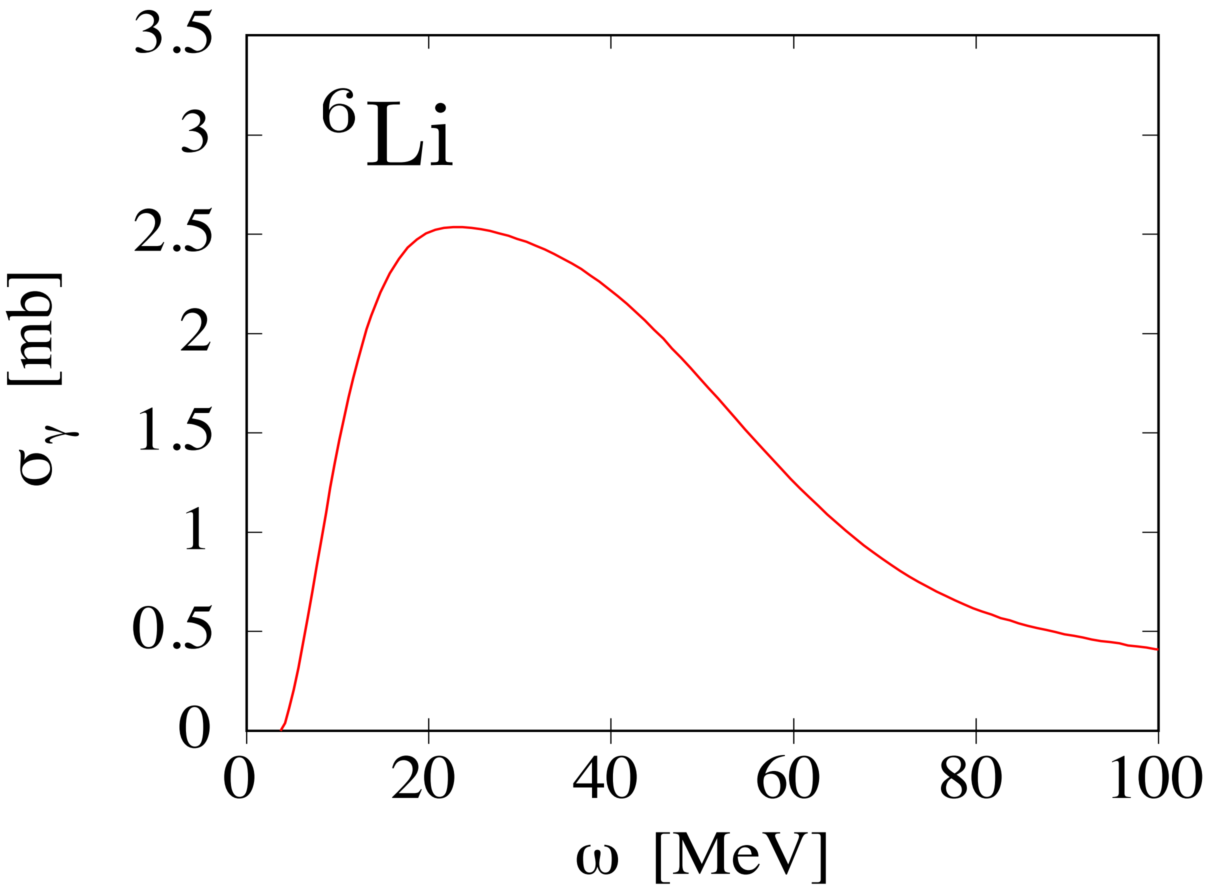
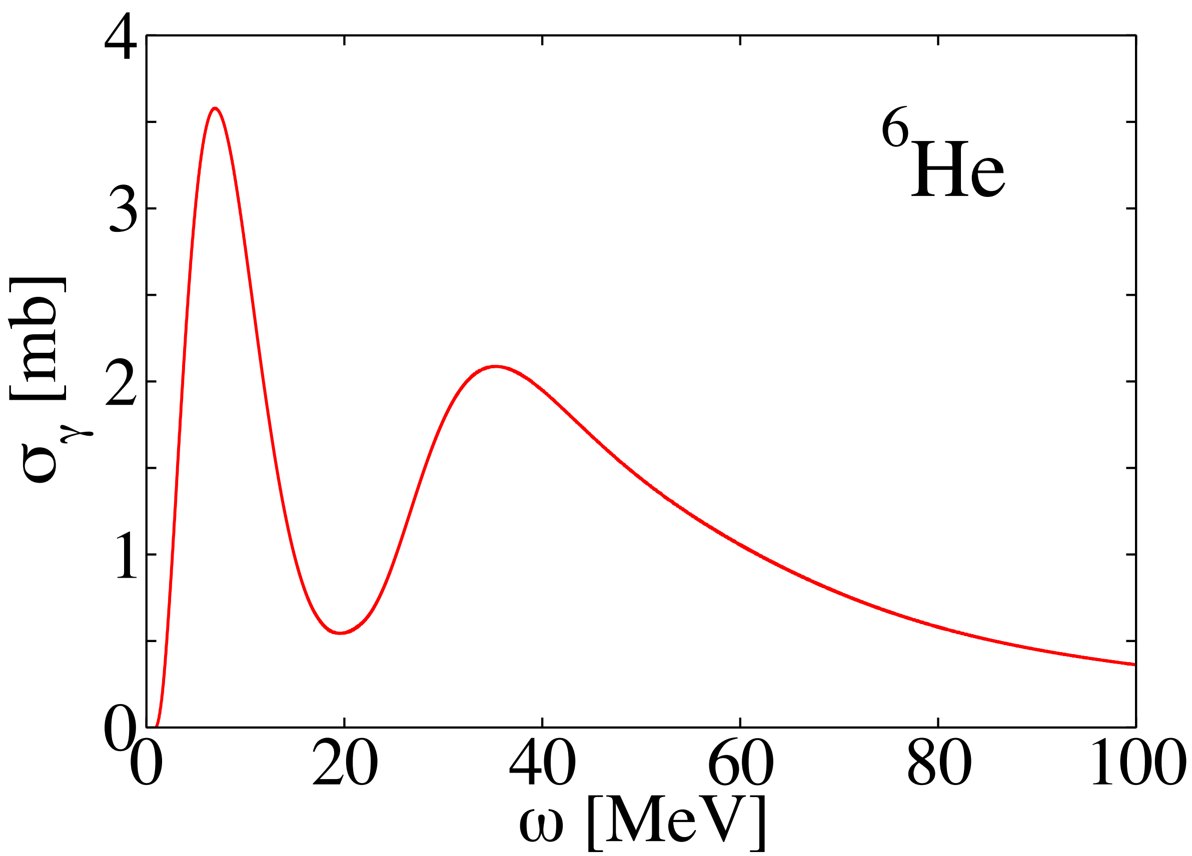
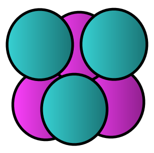




Giant Dipole Resonance

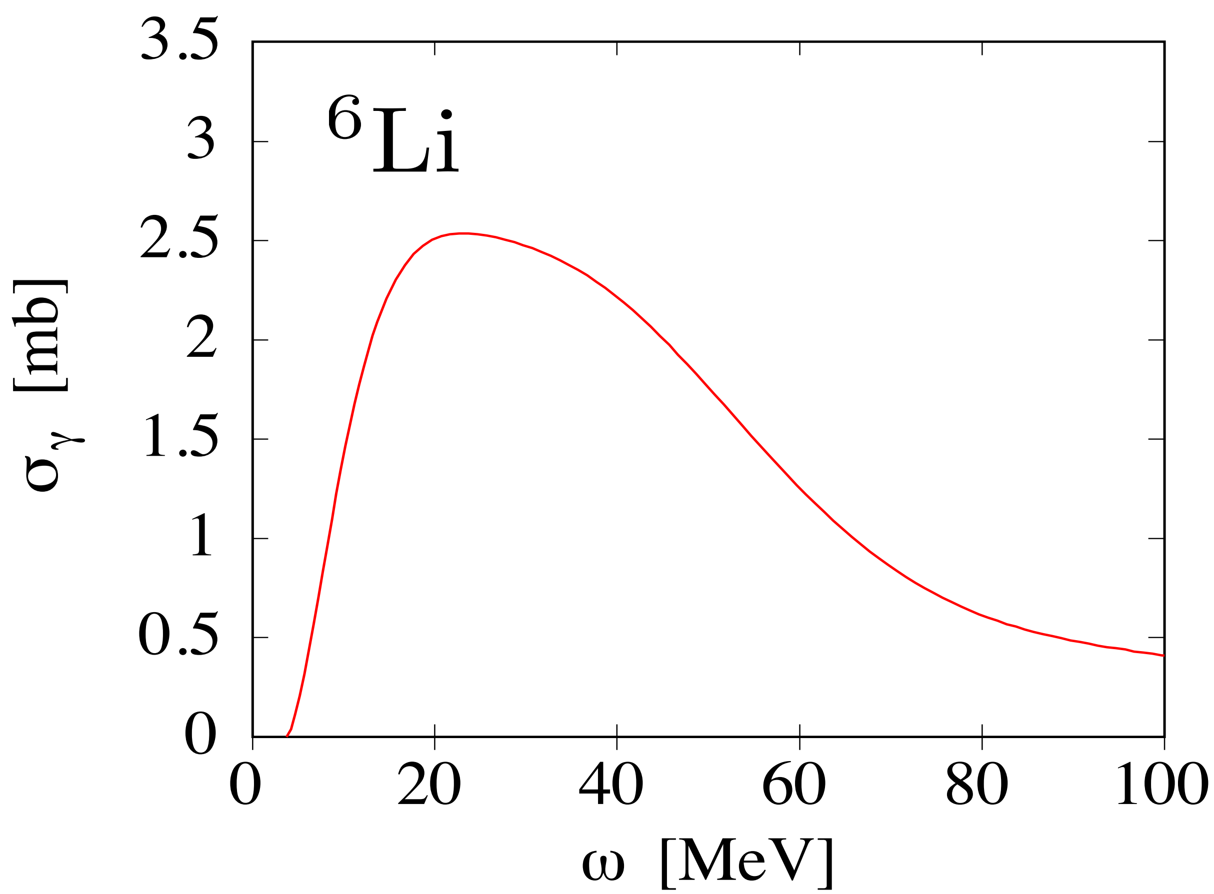


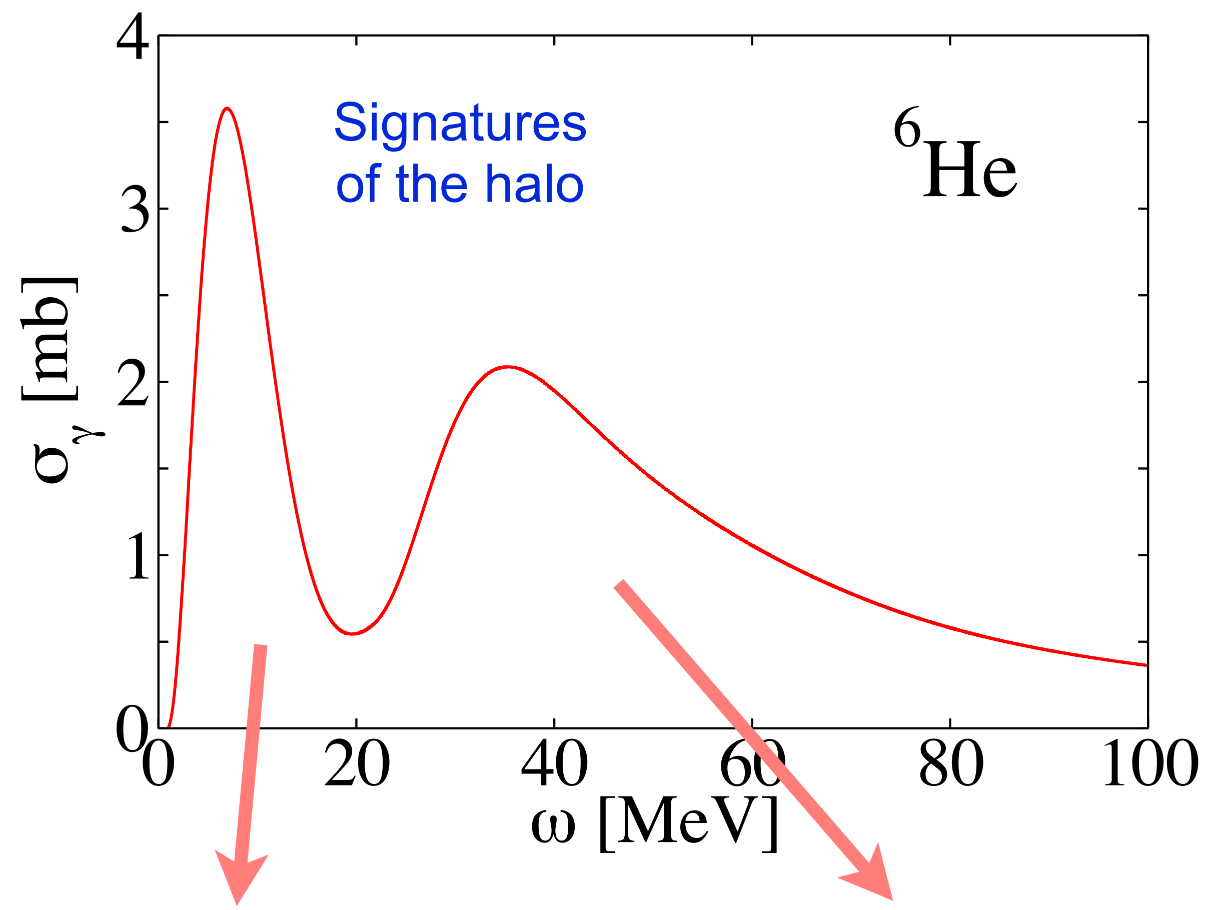
protons ⟷ neutrons

Photo-absorption reaction

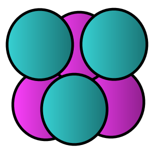
S.Bacca et al, PRL **89** 052502 (2002)

$$\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R^{E1}(\omega)$$ AV4' potential



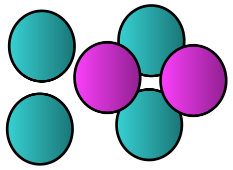


Giant Dipole Resonance



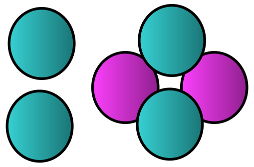
protons ⟷ neutrons

Soft-dipole Mode



neutron halo ⟷ α-core

Giant Dipole Mode



neutrons ⟷ protons

Photo-absorption reaction

S.Bacca et al, PRL **89** 052502 (2002)

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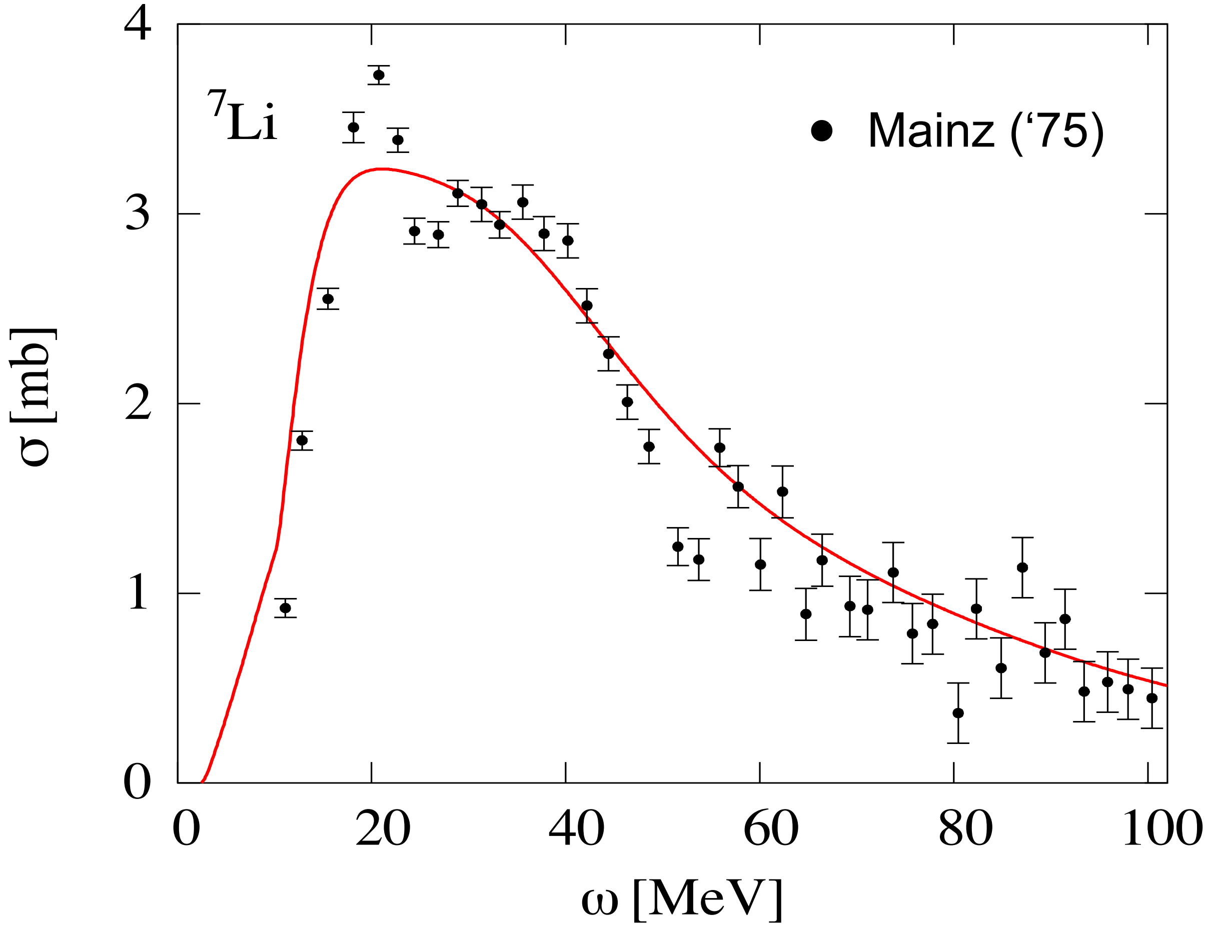
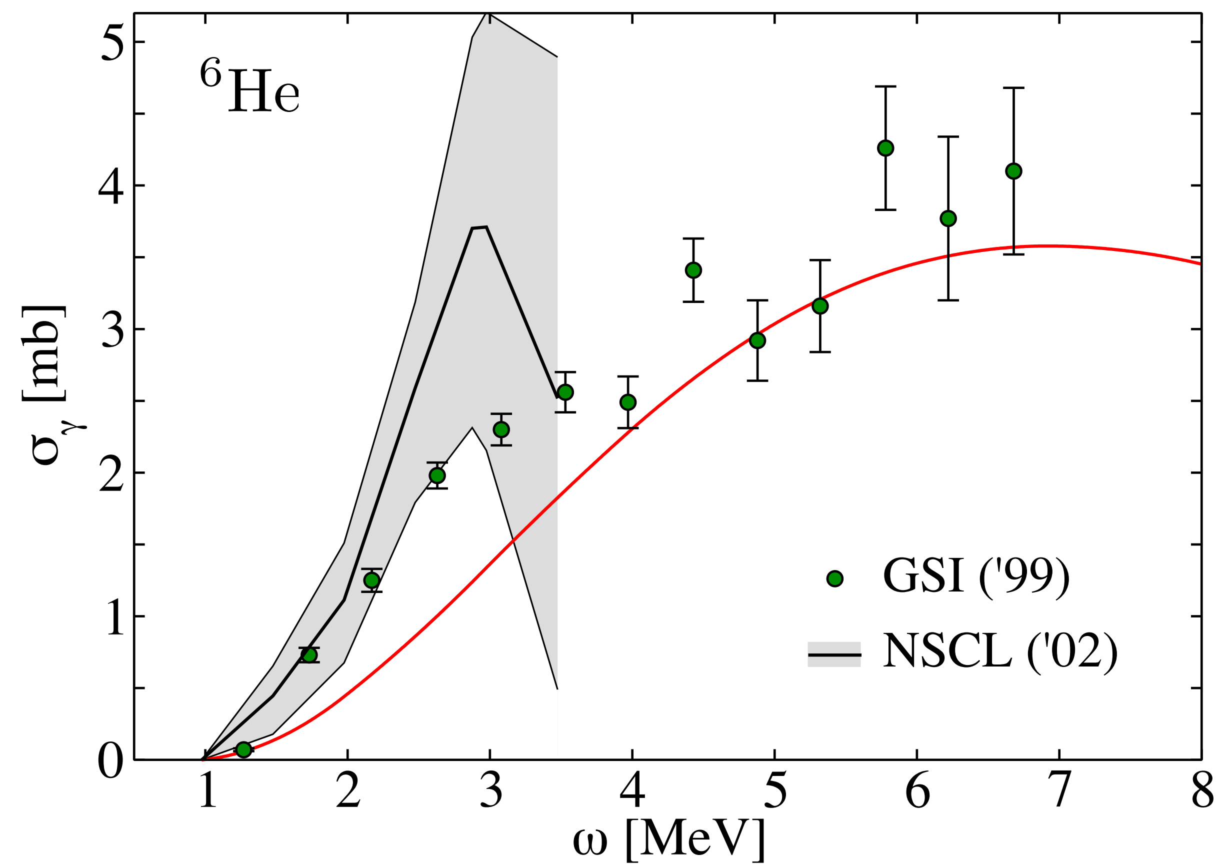
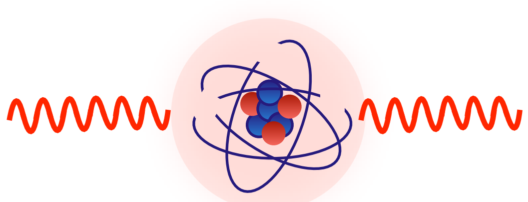
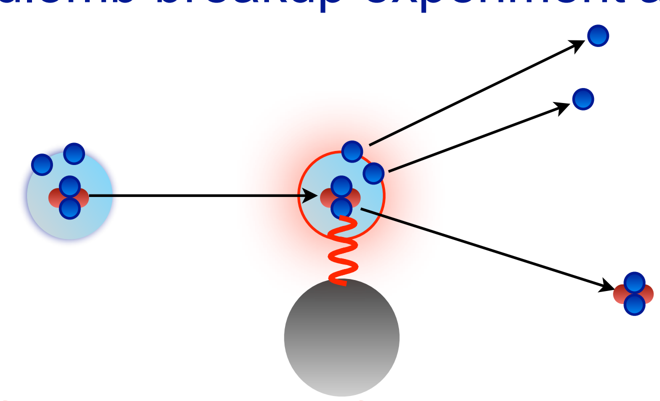




Target absorption experiment with real photons



Coulomb breakup experiment at RIBs



See talk by Christopher Lehr

Photo-absorption reaction

S.Bacca et al, PRL **89** 052502 (2002)

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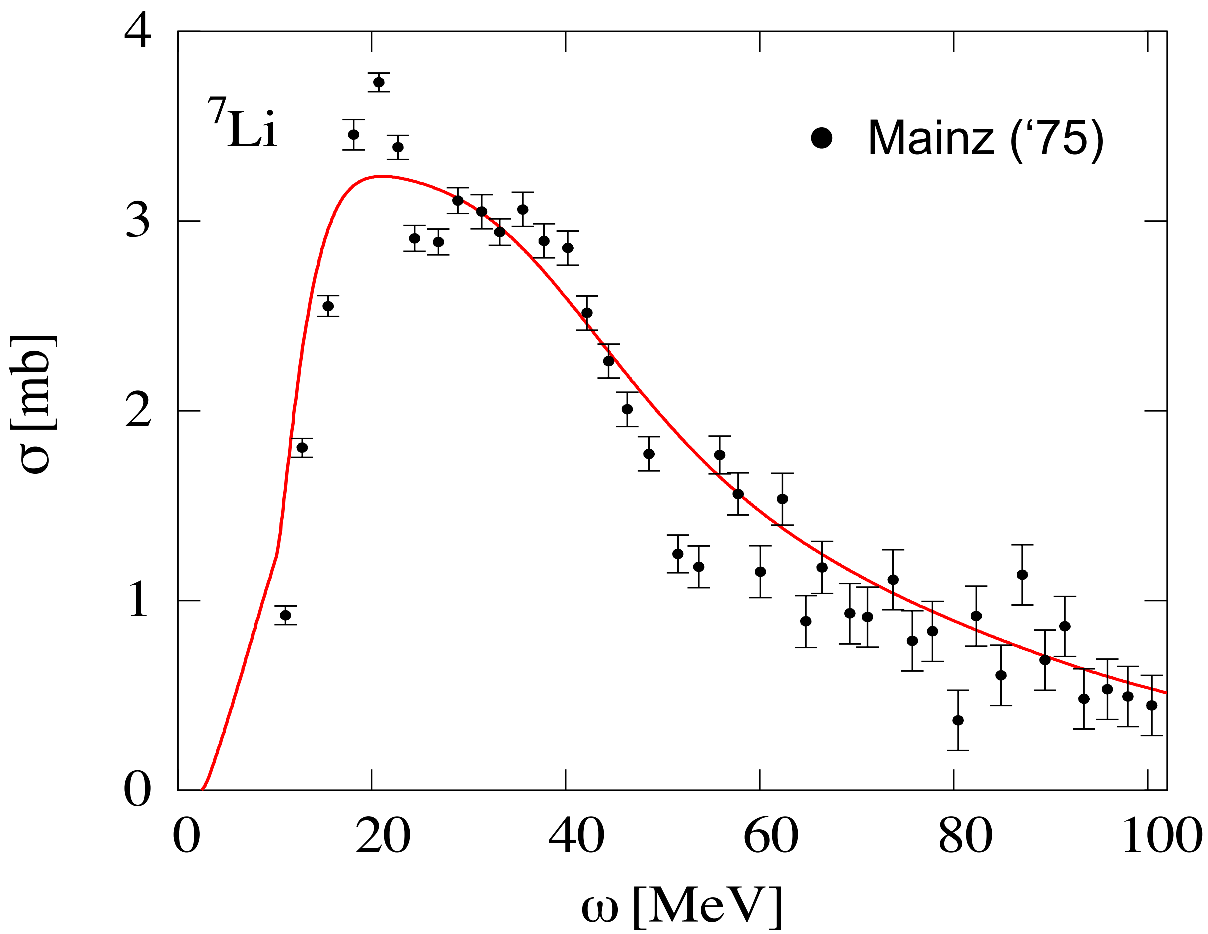
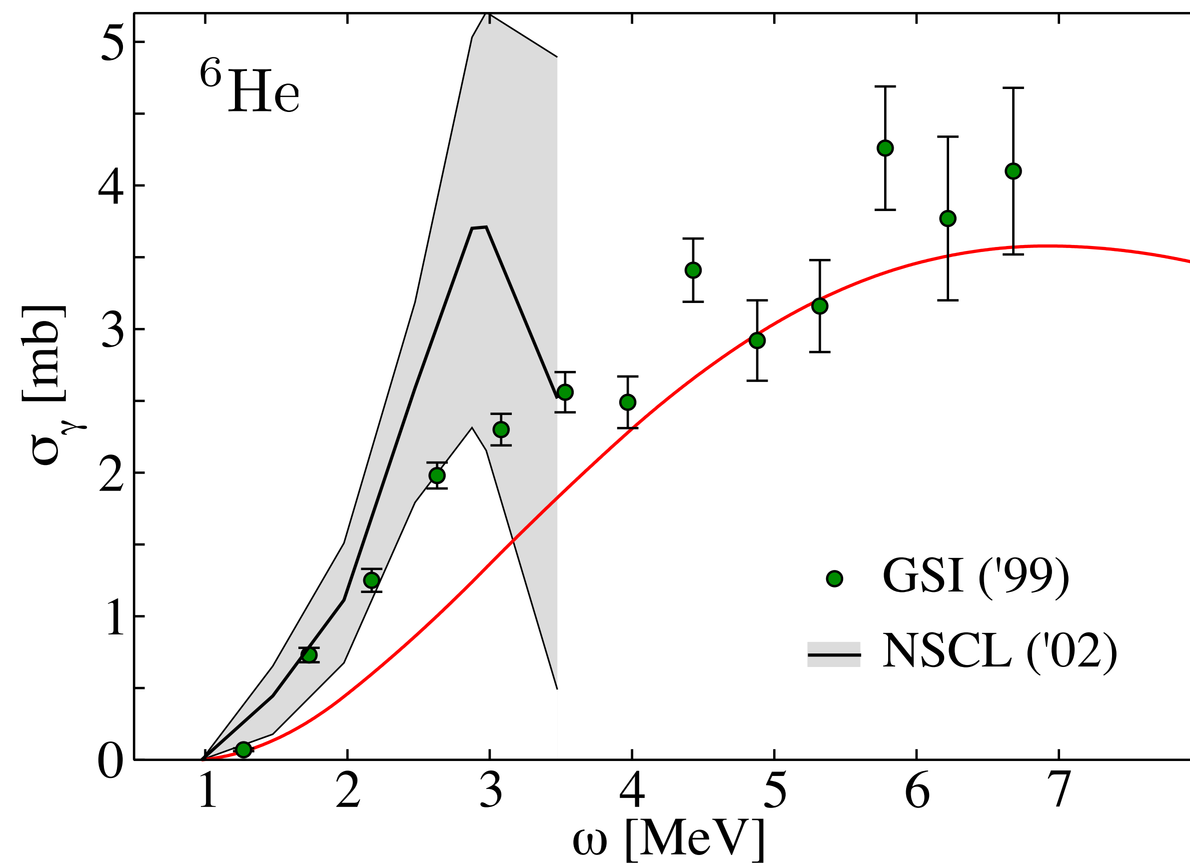




⇒ Overall good agreement!

⇒ Theory misses strength on threshold!

$$V_{NN} = V_C(r) + V_S(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_T(r)\vec{\tau}_1 \cdot \vec{\tau}_2 + V_{ST}(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 \; \vec{\tau}_1 \cdot \vec{\tau}_2$$

Spin-orbit might be important!

Could be revisited with modern interactions

Physics cases of interest:

- **Halo nuclei**
- **Electron scattering**
- **Muonic atoms**

Elastic Electron Scattering

$$R_L(\omega,\mathbf{q}) = \sum_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ ⟵ charge operator

$$R_T(\omega,\mathbf{q}) = \sum_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ ⟵ current operator

$\omega = 0$ No energy transfer, only momentum

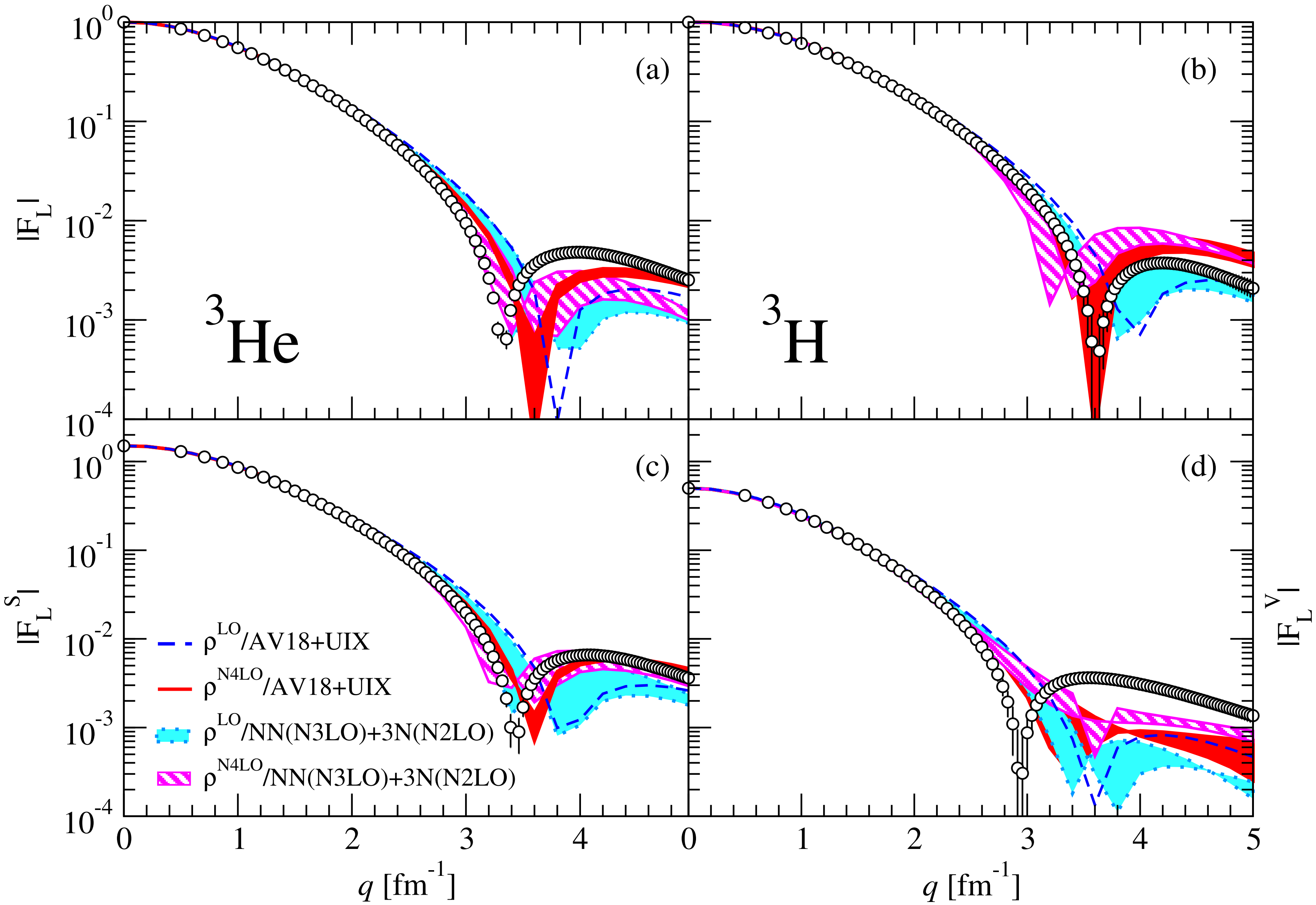
$f = 0$ Nucleus stays in ground-state

Form factors

Elastic Electron Scattering

From S. B. and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014)

Work by Piarulli, Schiavilla, Marcucci, …



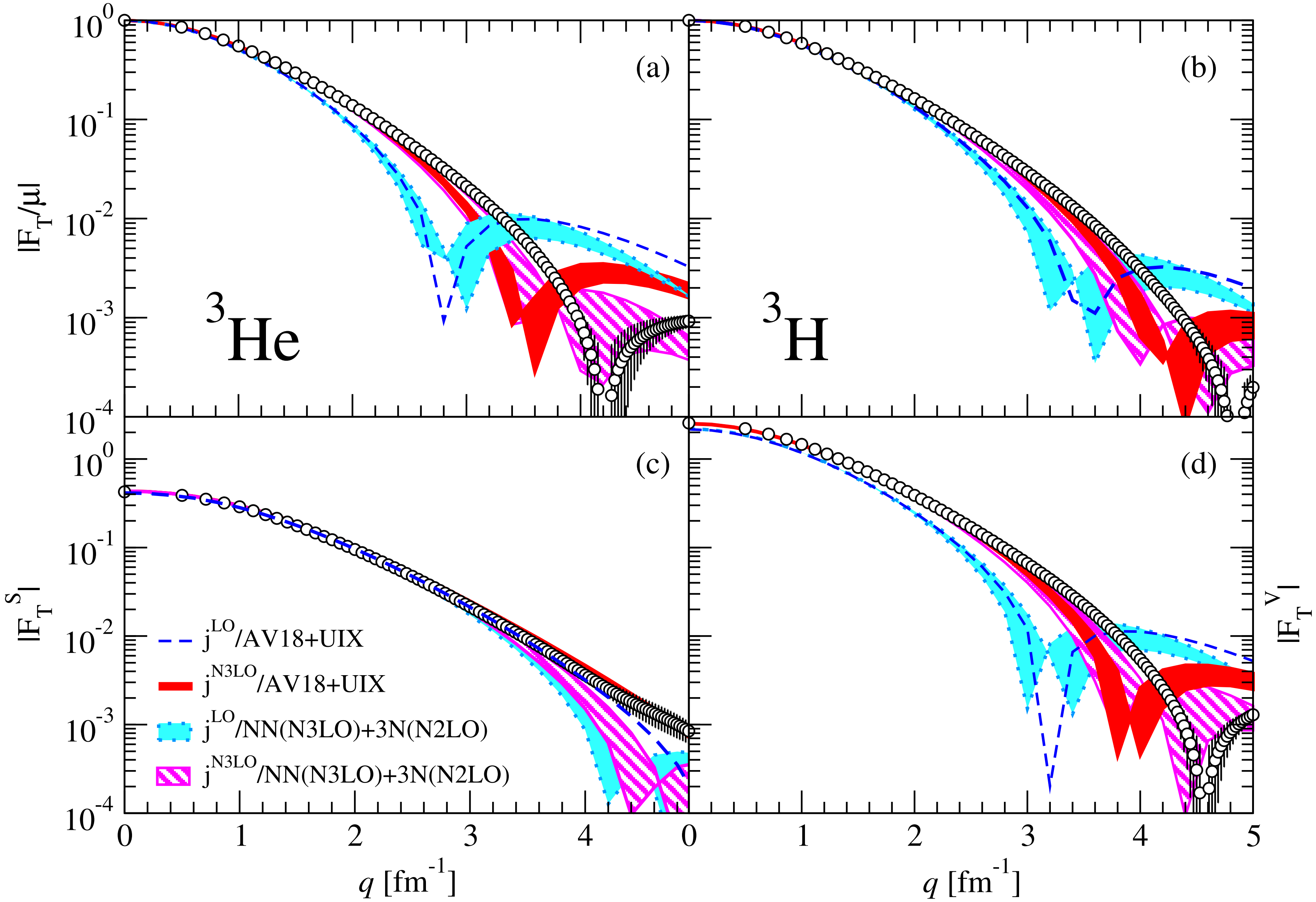
$$F_L^{S,V}(q) = \frac{1}{2}[2F_L(q,^3\mathrm{He}) \pm F_L(q,^3\mathrm{H})]$$

Traditional nuclear physics and chiral EFT agree
Two-body currents not important in longitudinal FF

Elastic Electron Scattering

From S. B. and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014)

Work by Piarulli, Schiavilla, Marcucci, …



$$F_T^{S,V}(q) = \frac{1}{2}[\mu(^3\text{He})F_T(q,^3\text{He}) \pm \mu(^3\text{H})F_T(q,^3\text{H})]$$

Traditional nuclear physics and chiral EFT slightly different
Two-body currents important in transverse/magnetic FF

Inelastic Electron Scattering

$\omega \neq 0$ Energy and momentum transferred

$f \neq 0$ Nucleus does not stay in ground-state

$^{3,4}He$ → First step:
Study $R_L(\omega, \mathbf{q})$ (many-body operators negligible)
to investigate the effect of final state interaction and 3NF

Inelastic Electron Scattering

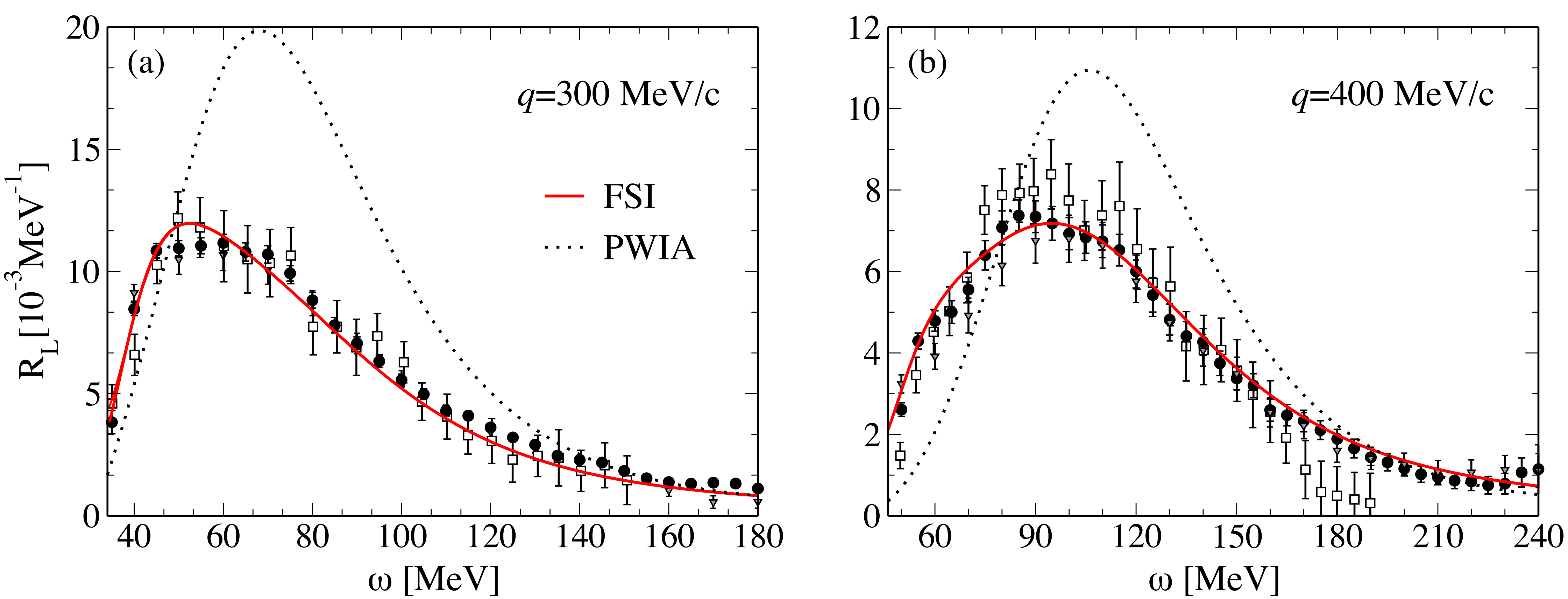
$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f \left|\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle\right|^2 \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

$$\rho(\mathbf{q}) = \sum_k^A e^{i\mathbf{q}\cdot\mathbf{r}'_k}\,\frac{1+\tau_k^3}{2} = \sum_J^\infty C_J^S(\mathbf{q}) + C_J^V(\mathbf{q})$$

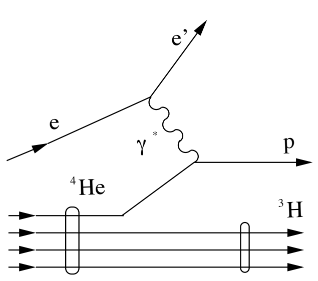
- Calculate every multipole on a grid of q
- Multipole expansion converges with finite number of multipoles
- Solve LIT equation for every multipole
- Invert LIT for every multipole and sum equiv to invert sum of LITs

Final state interaction

S.B. *et al.*, PRL **102**, 162501 (2009)



PWIA



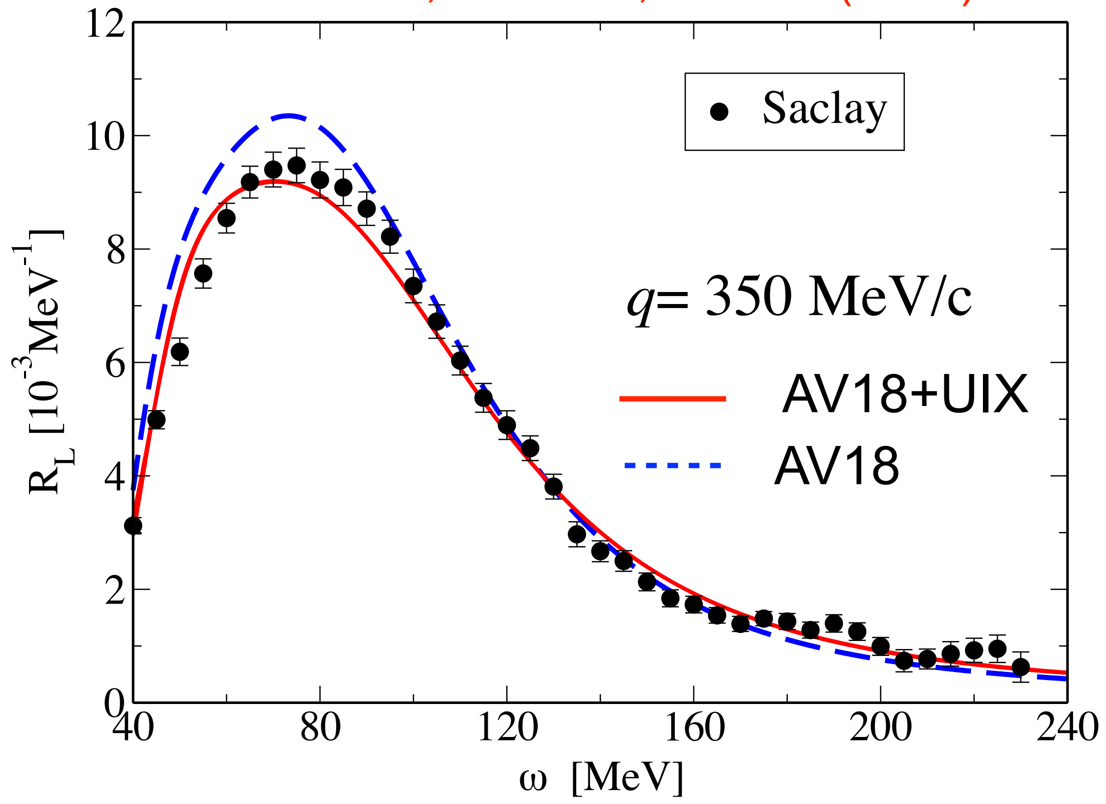
Full FSI:

AV18+UIX

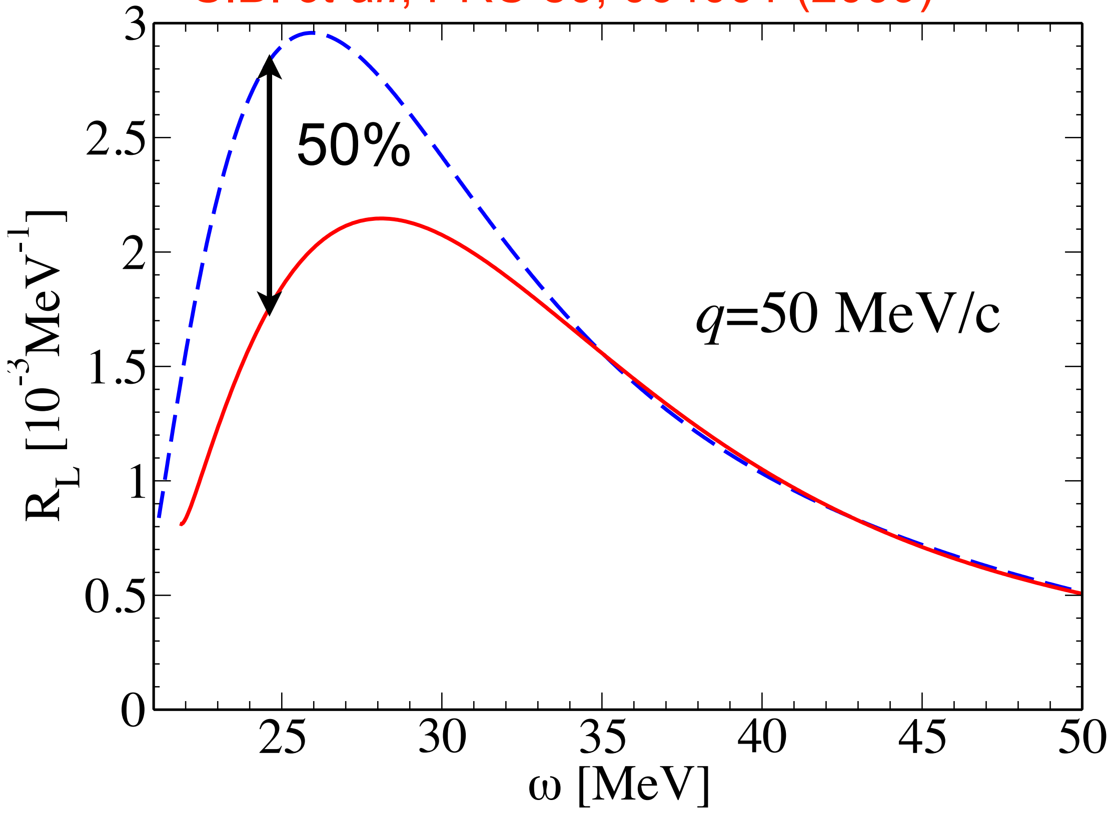
Strong effect of FSI: known form Carlson and Schiavilla PRL 68 (1992) and PRC 49 R2880 (1994) but now we can look at the energy dependence of FSI

Effects of nuclear Hamiltonians

S.B. *et al.*, PRL **102**, 162501 (2009)



S.B. *et al.*, PRC **80**, 064001 (2009)



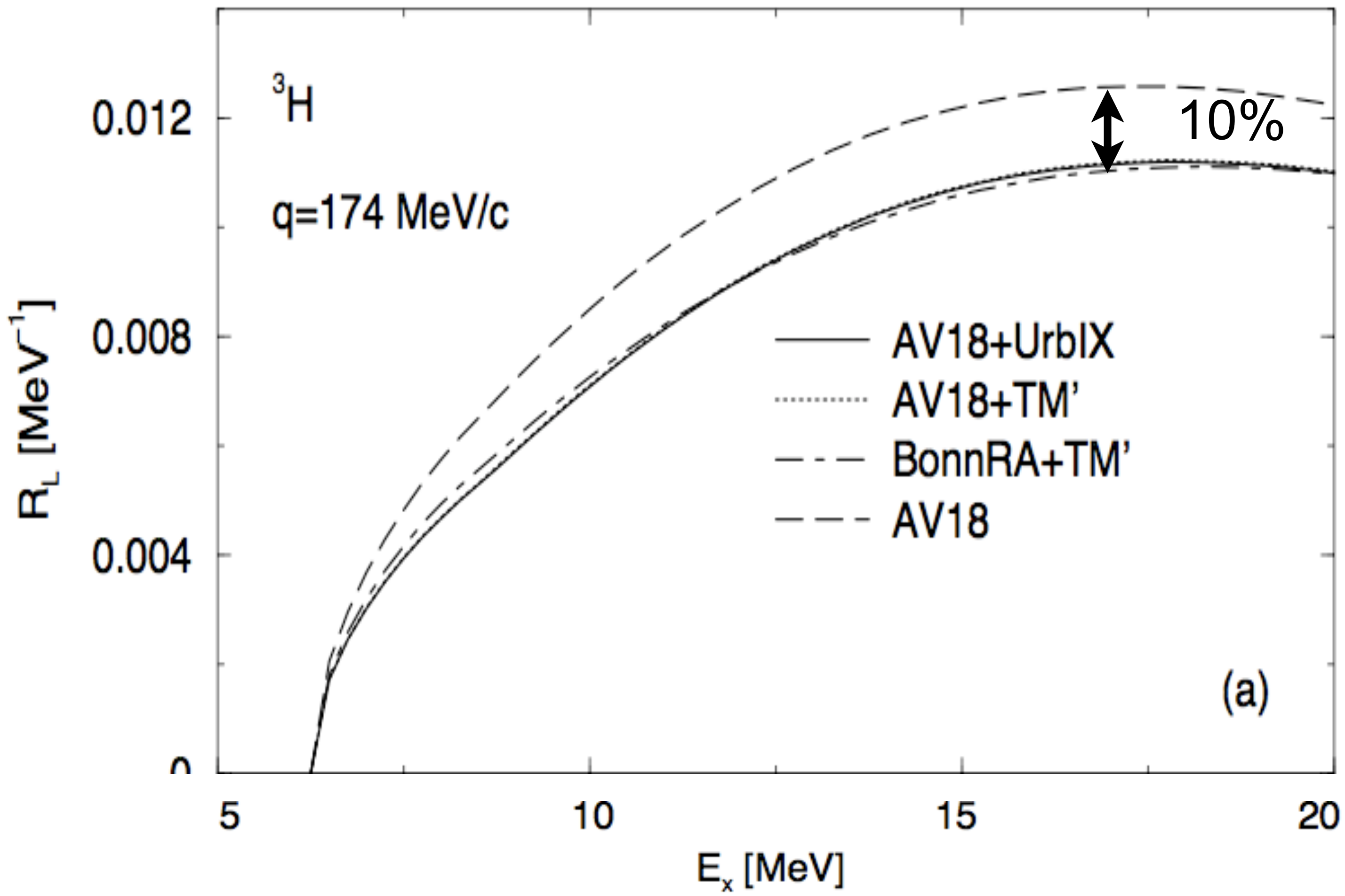
➡ Comparison with experiment improves with 3NF and at low q the reduction of the peak is up to 50%

Note:
Structure of the response close to threshold is not considered here.
This is the response 1-2 MeV above threshold.

Effects of nuclear Hamiltonians

A=3

Efros al., PRC 69 (2004) 044001



→ 3NF effects are weaker than for ^{4}He

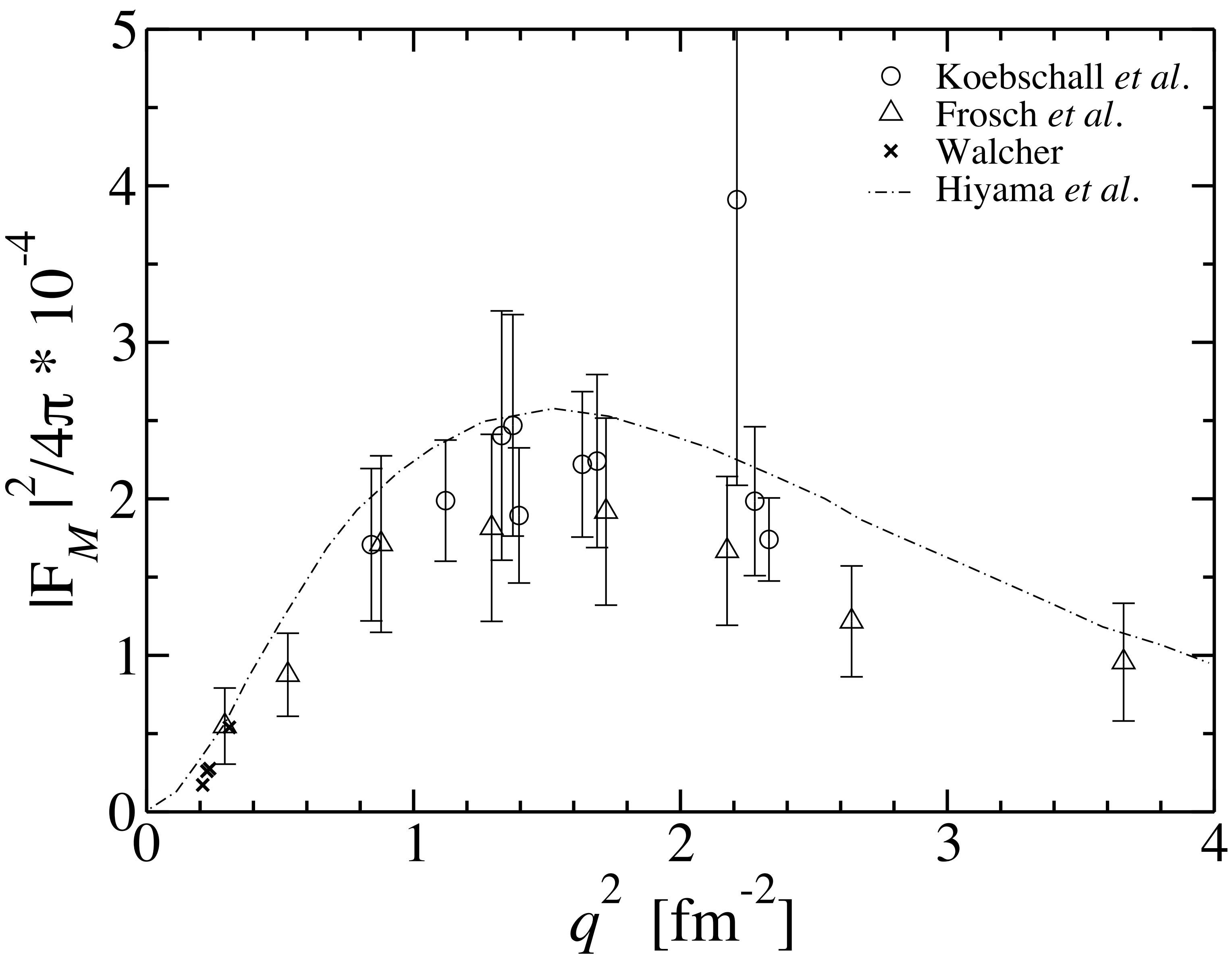
Monopole Resonance $^4He(e,e')0^+$

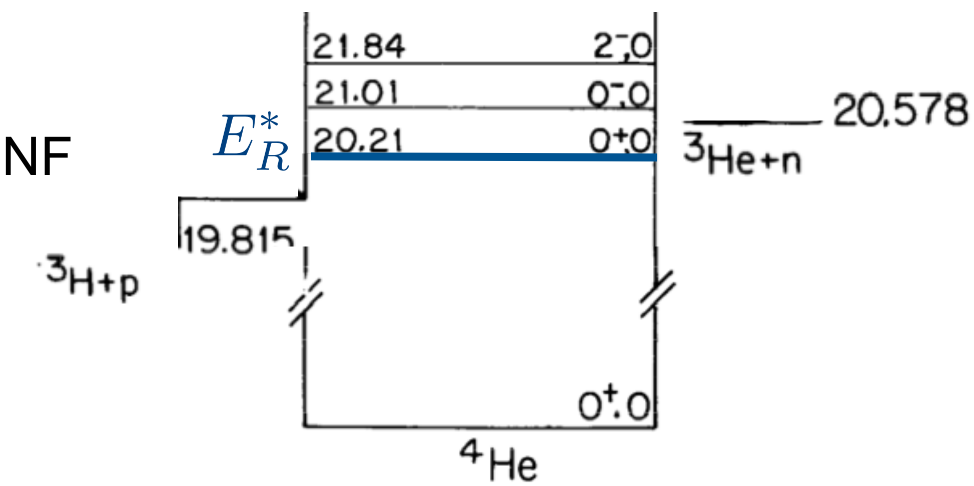
Resonant Transition Form Factor

$0_1^+ \longrightarrow 0_2^+$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2}\int d\omega R^{\rm res}_{\mathcal{M}}(q,\omega)$$

First ab-initio calculation: Hiyama *et al.*, PRC **70** 031001 (2004)
obtained good description of data with phenomenological central 3NF





$\Gamma_R = 270 \pm 70$ keV

AV8' + central 3NF

E_0 = -28.44 MeV

E_0^{exp}= -28.30 MeV

Monopole Resonance $^4He(e,e')0^+$

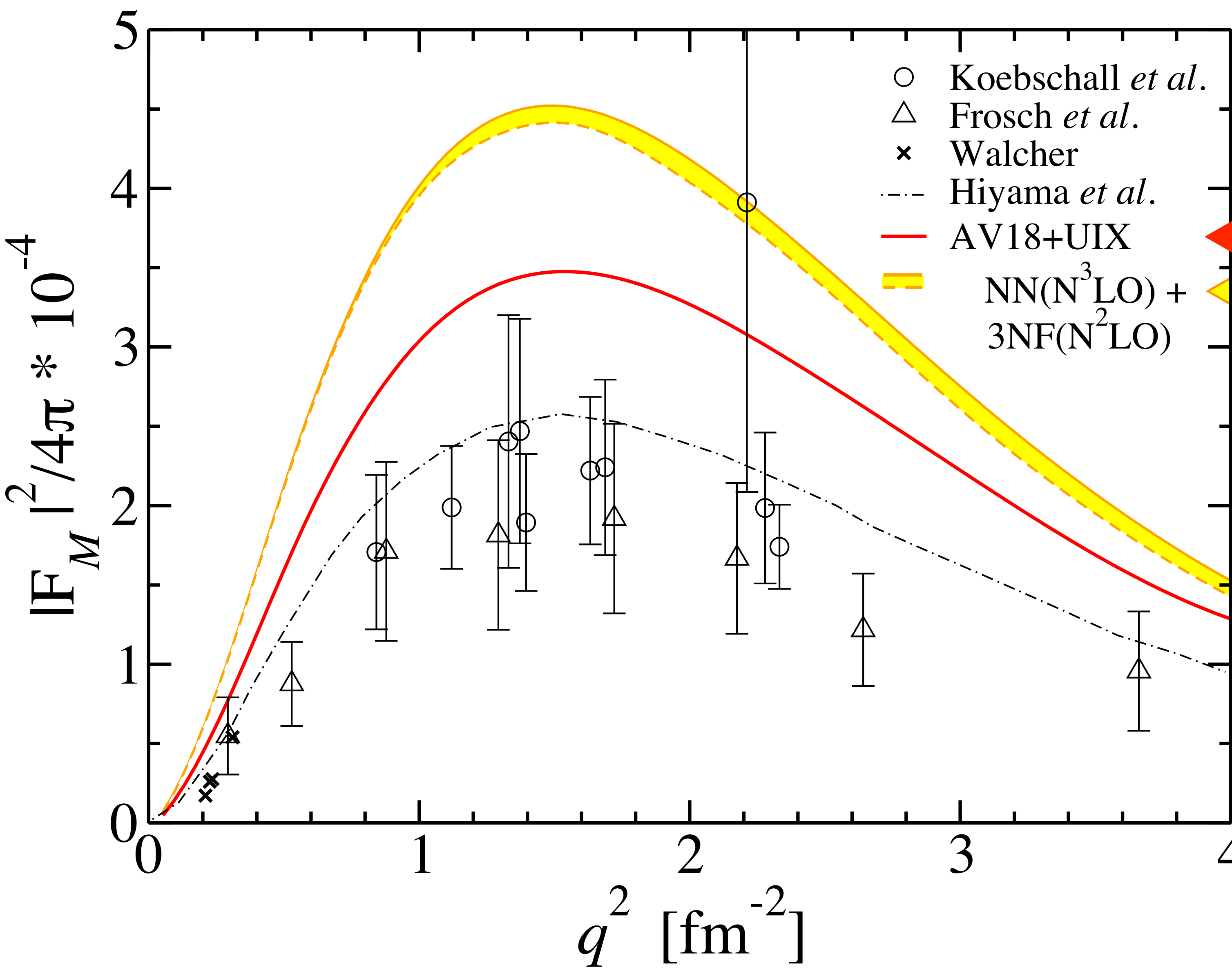
Resonant Transition Form Factor

$0_1^+ \longrightarrow 0_2^+$

$$|F_{\mathcal{M}}(q)|^2 = \frac{1}{Z^2}\int d\omega R_{\mathcal{M}}^{\rm res}(q,\omega)$$

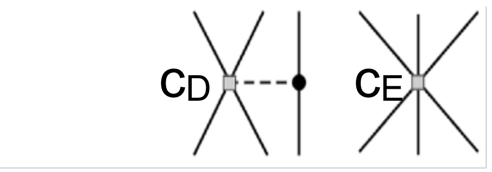
First ab-initio calculation with realistic three-nucleon forces and with the Lorentz Integral Transform method

S.B. *et al.*, PRL **110**, 042503 (2013)



conventional forces

EFT forces



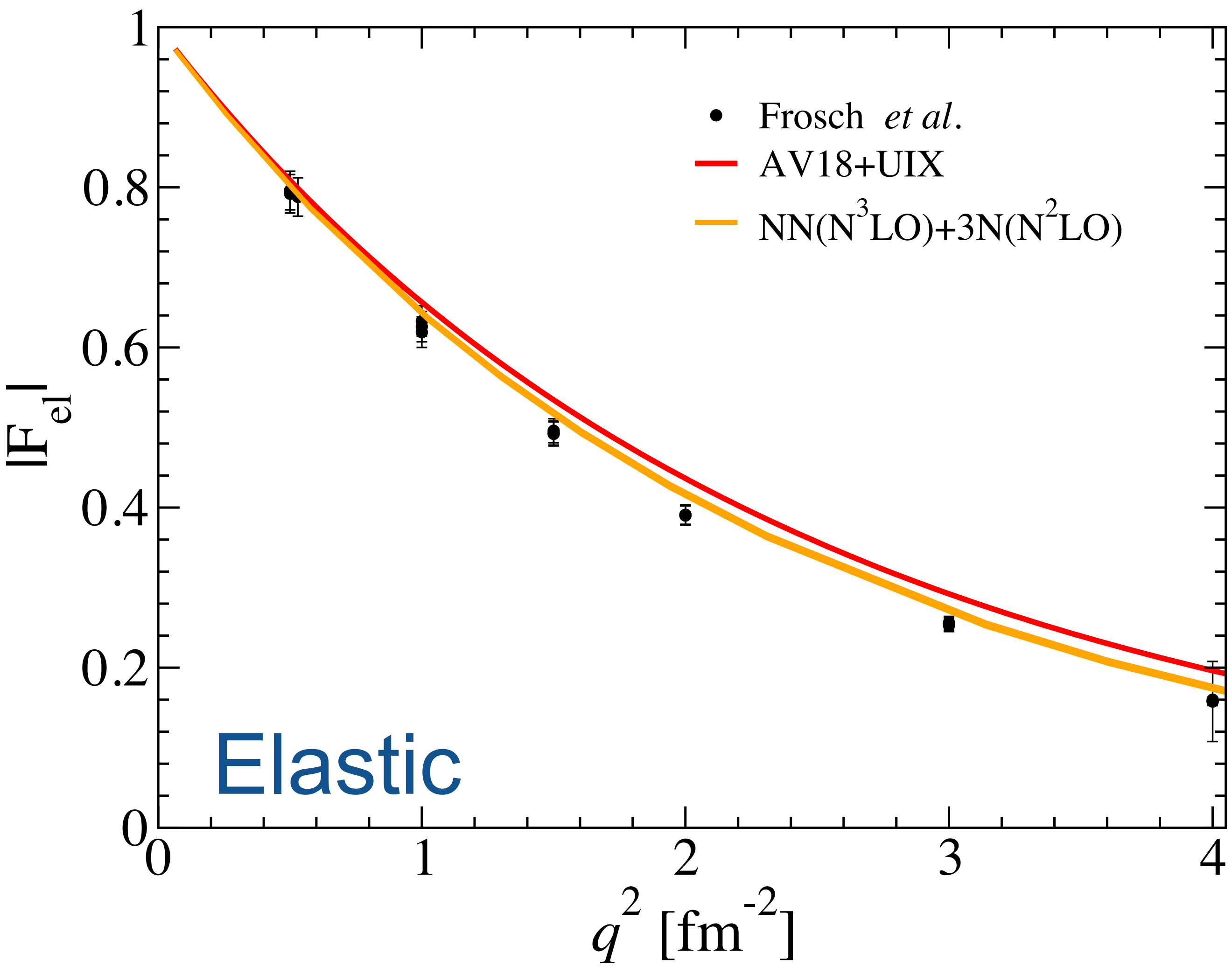
Two-parameterization of the 3NF from EFT:

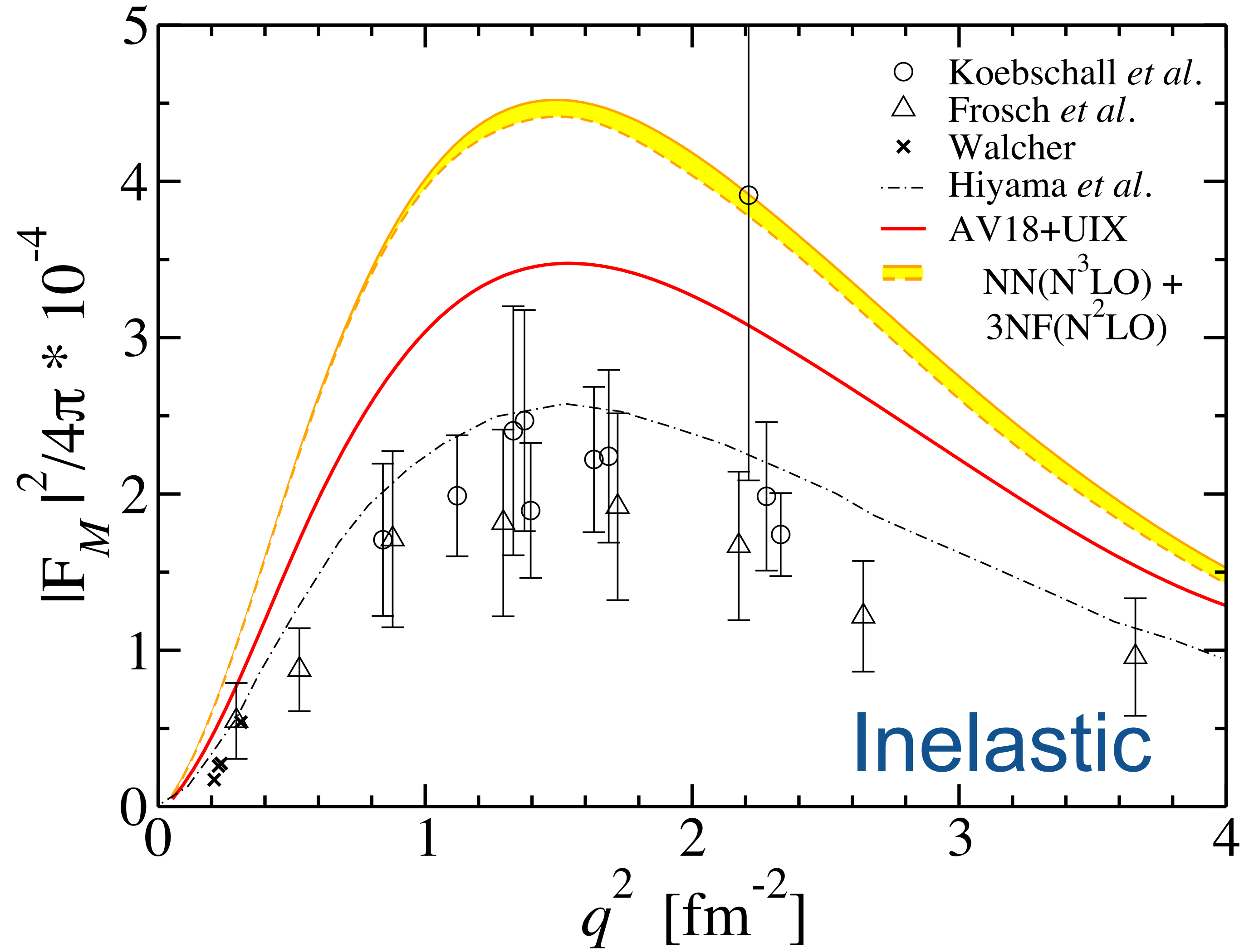
- c_D=1 c_E=-0.029 ^{3}H g.s. energy
  Navratil, FBS **41** (2007) 117-140
- c_D=-0.2 c_E=-0.205 ^{3}H g.s.energy and beta-decay
  Gazit *et al.*, PRL **103** (2009) 102502

Pronounced sensitivity to the input Hamiltonian

Sensitivity to Nuclear Hamiltonians

S.B. *et al.*, PRL **110**, 042503 (2013)



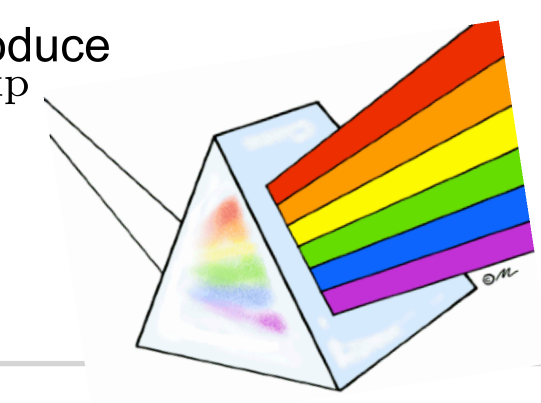


→ The inelastic monopole resonance acts as a prism to nuclear Hamiltonians.

AV8' + central 3NF	E_0 = -28.44 MeV
AV18+UIX	E_0 = -28.40 MeV
NN(N³LO)+3NF(N²LO)	E_0 = -28.357 MeV
	E_0^{exp} = -28.30 MeV

H

reproduce E_0^{exp}

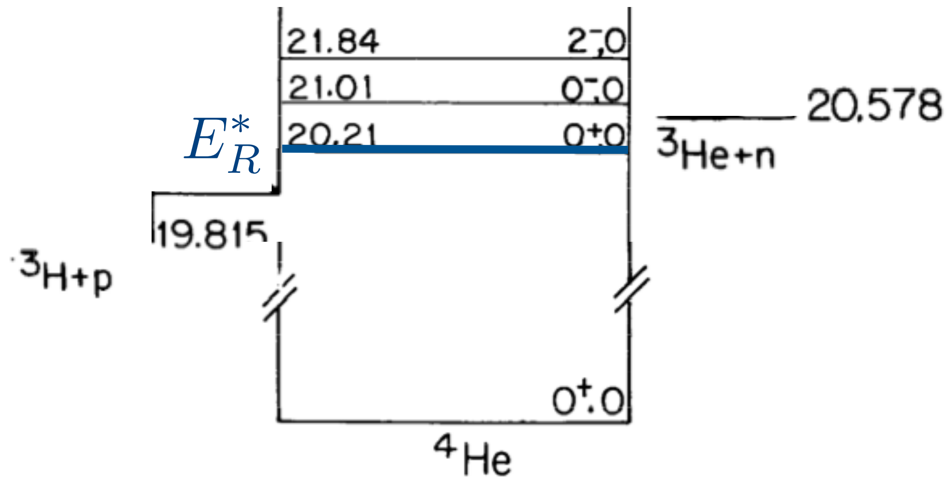


spectrum in $|F_{\mathcal{M}}|^2$

Sensitivity to Nuclear Hamiltonians

- Location of the resonance?

AV8' + central 3NF E_R^* = 20.25 MeV
AV18+UIX E_R^* = 21.00(20) MeV
NN(N^{3}LO)+3NF(N^{2}LO) E_R^* = 21.01(30) MeV



E_R^* = 20.21 MeV

The "realistic Hamiltonians" fail to reproduce the correct position of the 0^+_2 resonance

More theoretical work needed to understand this.

- This be measured again: S-Dalinac, MAMI, MESA

Inelastic Electron Scattering

^{3}He → Study $R_T(\omega, \mathbf{q})$ and two-body currents effects

Two-body currents

MEC: meson exchange currents enhance strength by factor 2

Different calculations are within experimental error bars