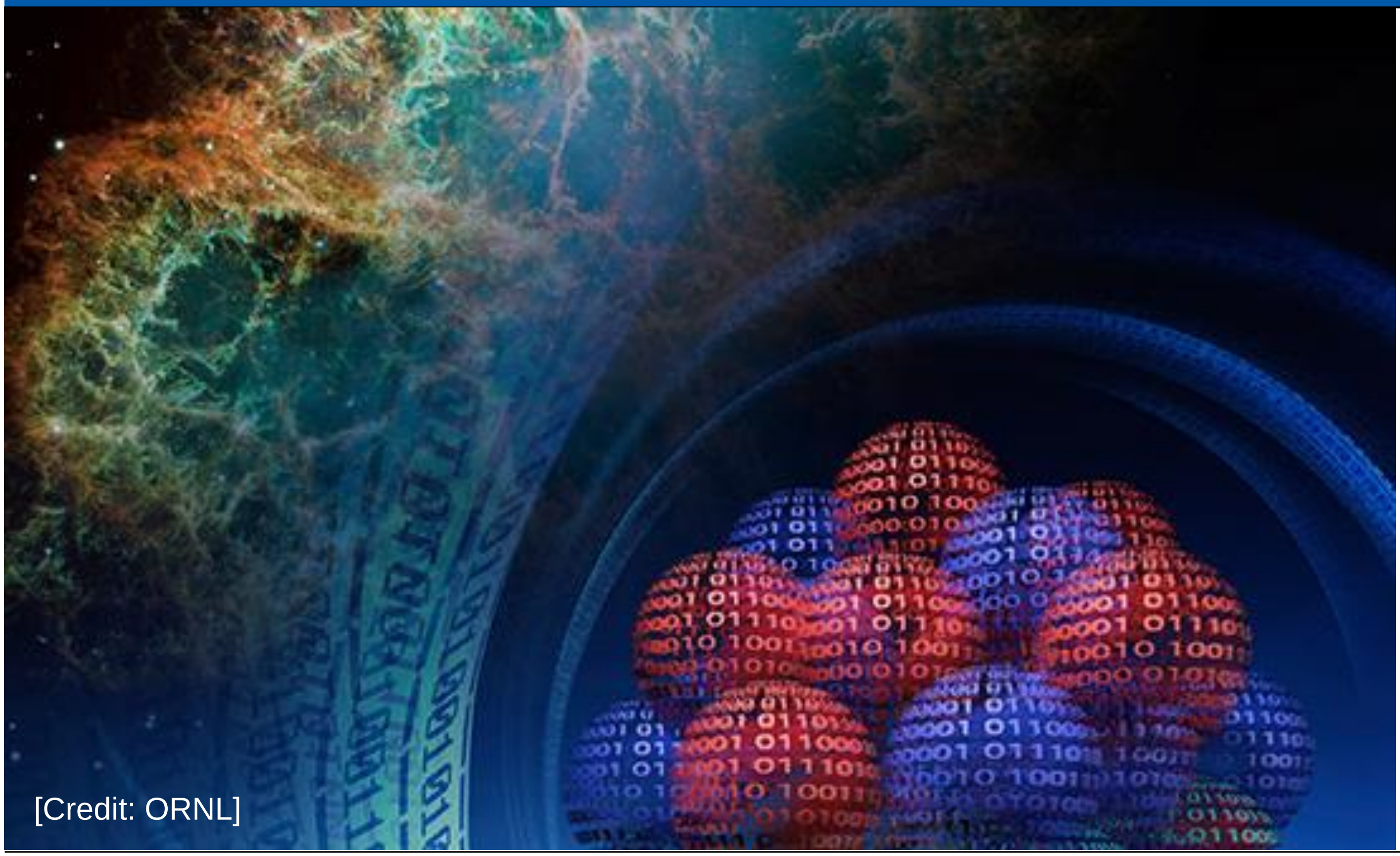
Nuclear Equation of State from *ab-initio* point-of-view

TECHNISCHE UNIVERSITÄT DARMSTADT

Christian Drischler

Topical Lecture Week 2017

February 21, 2017


[Credit: ORNL]

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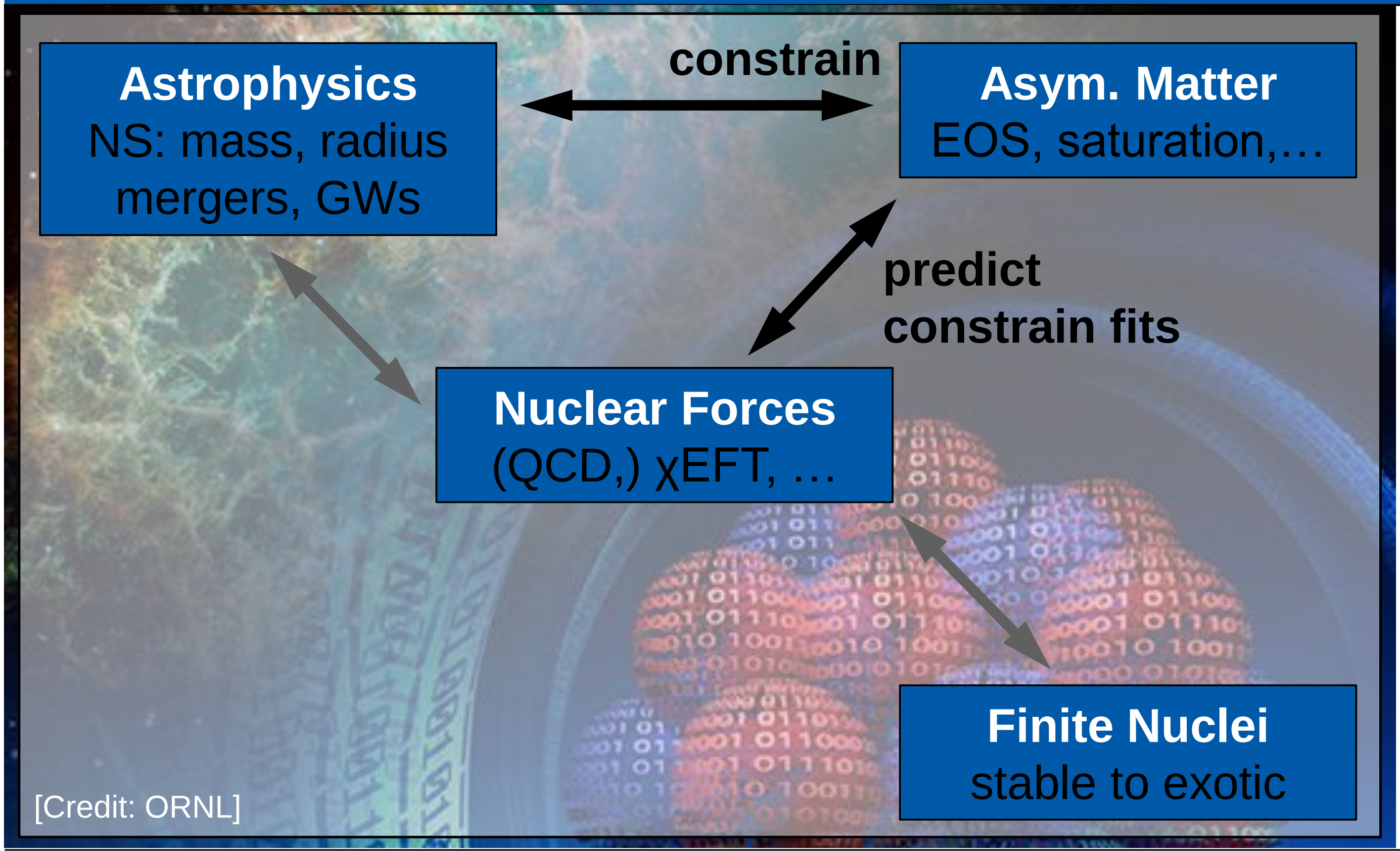

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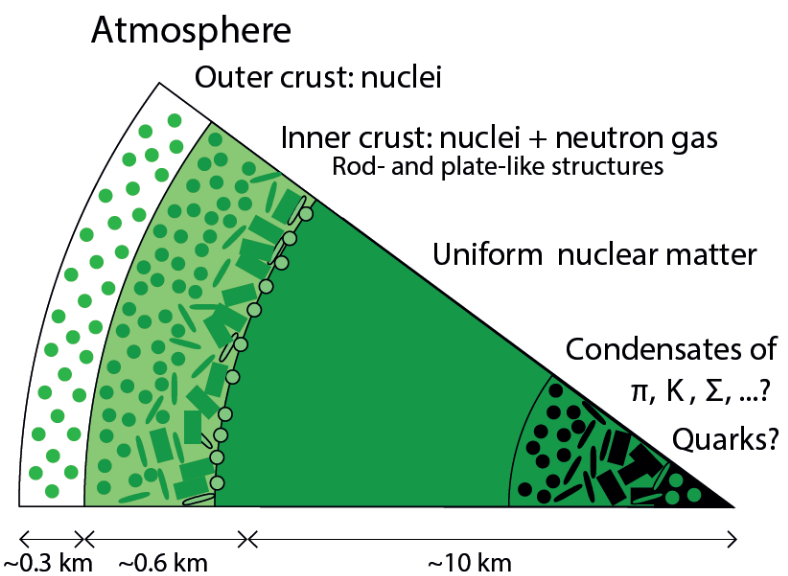
Nuclear Equation of State from *ab-initio* point-of-view

Motivation: Neutron Stars



Lattimer, Prakash, Science **304**, 536 (2004)

neutron stars are of extremes:

- $R \sim (10 - 14)$ km, $M \sim 2\, M_{\text{sun}}$
- most densest objects we observe

outer core: $n \sim n_0$

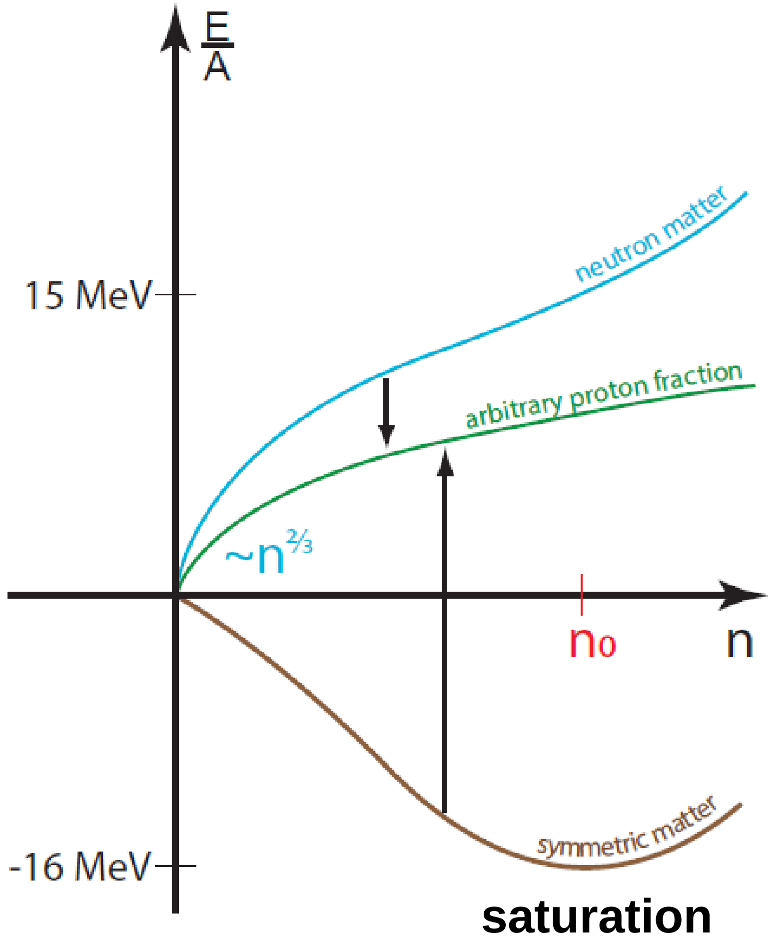
- homogeneous, infinite nuclear matter

nuclear matter: well-suited system to apply/check

- nuclear forces
- many-body approaches

Nuclear Equation of State from *ab-initio* point-of-view

Landscape of Nuclear Matter



neutron matter ($Z = 0$):

Hebeler *et al.*, Astrophys. J., **773**, 11 (2013)
Krüger *et al.*, PRC **88**, 025802 (2013)
Gezerlis *et al.*, PRL **111**, 032501 (2013)
Roggero *et al.*, PRL **112**, 221103 (2014)
Wlazłowski *et al.*, PRL **113**, 182503 (2014)
Lynn *et al.*, PRL **116**, 062501 (2016)
Dyhdalo *et al.*, PRC **94**, 034001 (2016)
...

symmetric matter ($N = Z$):

Hebeler *et al*. PRC **83**, 031301(R) (2011)
Holt, Kaiser, Weise, PPNP **73** 35 (2013)
Coraggio, Holt *et al*. PRC **89**, 044321 (2014)
Wellenhofer *et al.*, PRC **89**, 064009 (2014)
Carbone *et al.*, PRC **90** 054322 (2014)
Coraggio, Holt *et al.*, PRC **89**, 044321 (2014)
...

applications to neutron stars??

Nuclear Equation of State
from *ab-initio* point-of-view

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Neutron Stars in β Equilibrium: 0 < *x* << 0.5

Such calculations are more involved: less symmetries

$$x = \frac{n_p}{n_p + n_n} \qquad \text{or,} \qquad \beta = \frac{n_n - n_p}{n_n + n_p} \qquad \text{with} \quad \beta = 1 - 2x$$

Obtaining the equation of state:

- parametrizations (fits to PNM plus empirical properties)
- empirically constrained coefficients

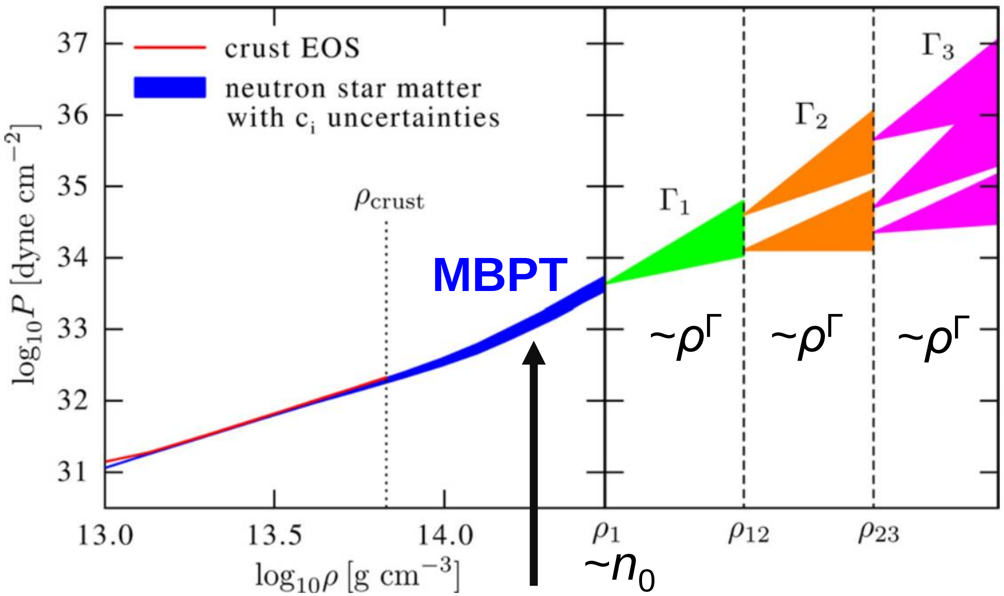
$$\frac{E}{A}(\beta, n) \overset{\text{Taylor}}{=} \sum_{i,j} C_{ij}\, \beta^i \left(\frac{n - n_0}{n_0}\right)^j$$

e.g., C_{00} ~ –16 MeV, C_{20} ~ 31 MeV, …

Nuclear Equation of State from *ab-initio* point-of-view

Constraining Neutron Stars

Hebeler *et al.*, Astrophys. J., **773**, 11



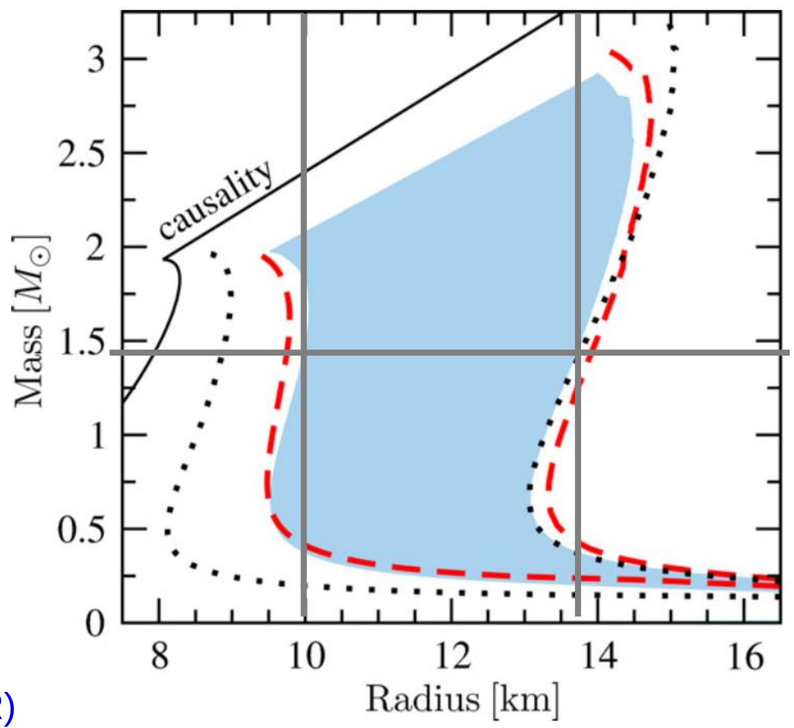
Neutron matter extrapolated to β equilibrium

TOV Eqs.

+ $M_{max} \geq 1.97\ M_{sun}$

+ causality

Mass-Radius Relation



Improvements needed:

- calculate asym. matter directly
- higher orders in the **chiral** and perturbative **expansion**

for QMC, see also: Gandolfi *et al.*, Phys. Rev. C **85**, 032801(R)

Nuclear Equation of State from *ab-initio* point-of-view

Motivation: Infinite Matter

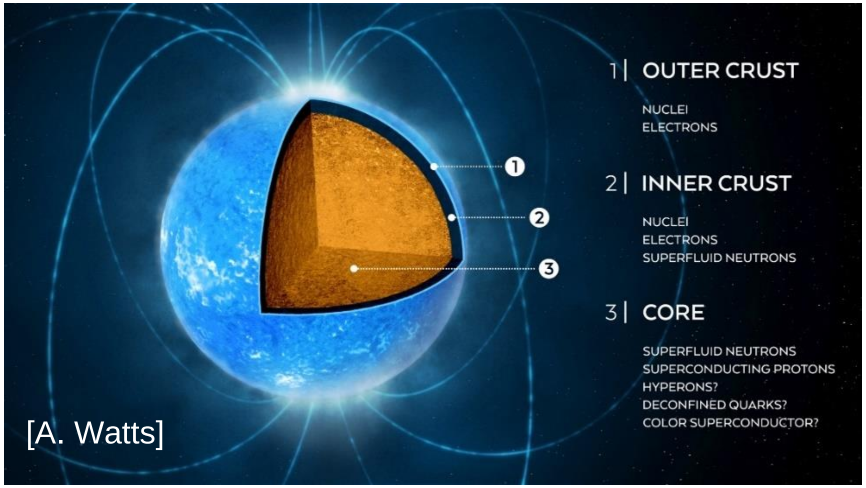
see also: Wellenhofer *et al.*, PRC **93**, 055802

density
proton fraction
temperature

Energy per particle: $\frac{E}{A}(n, x, T)$

based on **chiral effective field theory** (EFT):

- direct determination of astrophysical quantities: sym. energy, …
- ideal to **test** (and to improve) **nuclear forces** ~n_0
- constrain neutron-star EOS: mass-radius relations, …



[A. Watts]

$$S_2(n) = \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\frac{E}{A}(n,\beta)\bigg|_{\beta=0}$$

$$L = 3n_0 \frac{\partial}{\partial n} S_2(n)\bigg|_{n=n_0}$$

Nuclear Equation of State from *ab-initio* point-of-view

Chiral Effective Field Theory

Nuclear Matter interacts via the Strong Interaction

(not considering Coulomb)

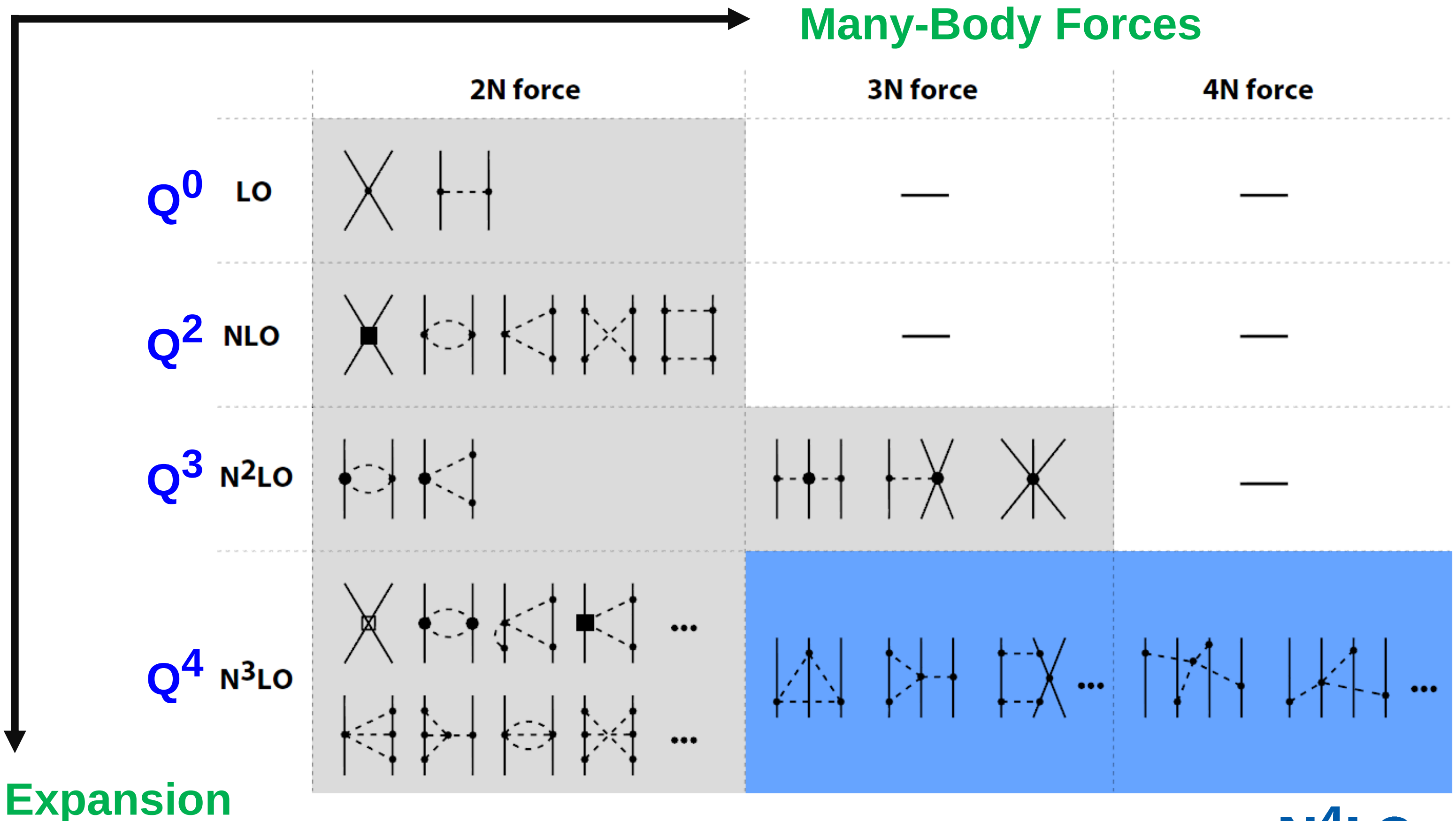
- fundamental theory is known
- QCD is non-perturbative at low energies of interest
- **modern approach**: chiral EFT
    - relevant degrees of freedom instead of quarks/gluons
    - use e.g., **nucleons** and **pions**
    - pion exchanges and short-range contact interactions
    - expand most general Lagrangian in powers of $Q = max(p,m_{\pi}) / \Lambda_{b} \sim 1/3$

Weinberg, Phys. Lett. B **251**, 288 (1990)
Weinberg, NP B **363**, 3 (1991)
Weinberg, Phys. Lett. B **295**, 114 (1992)

Nuclear Equation of State from *ab-initio* point-of-view

Hierarchy of Nuclear Forces in Chiral EFT

see: Epelbaum *et al.*, PRL **115**, 122301



... and ongoing work at N^{4}LO ...

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, ...

Nuclear Equation of State from *ab-initio* point-of-view

3N forces beyond Hartree-Fock?

CD, Hebeler, Schwenk, PRC **93**, 054314

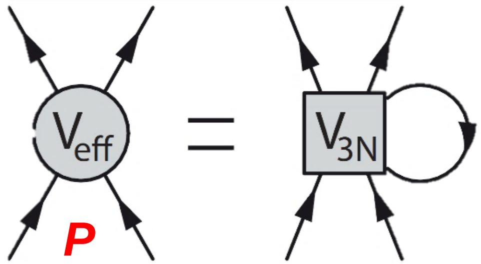
Effective NN potentials

by summing *one* particle over the occupied states of the Fermi sea

» dominant 3N contributions

Holt *et al.*, PRC **81**, 024002
Hebeler *et al.*, PRC **82**, 014314

so far: only N^2LO 3N and ***P*** = 0



some more applications:
Wellenhofer *et al.*, PRC **92**, 015801
Holt *et al.*, Progr. Part. Nucl. Phys. **73**, 35
Hebeler *et al.*, Ann. Rev. Nucl. Part. Sci. **65**, 457–84
...

Improved Method

- applicable to all nuclear forces
- N^3LO 3N forces due to recent PW decomposition

Hebeler *et al.*, PRC **91**, 044001

towards *consistent* N^3LO calculations

Nuclear Equation of State from *ab-initio* point-of-view

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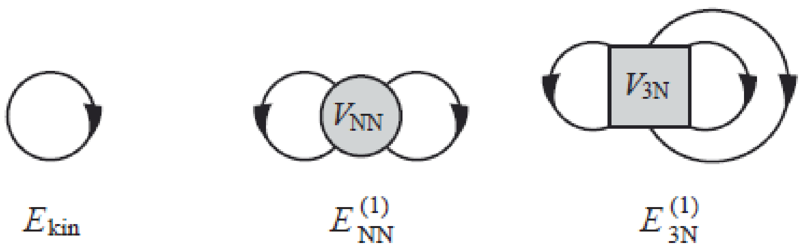
MBPT Diagrams

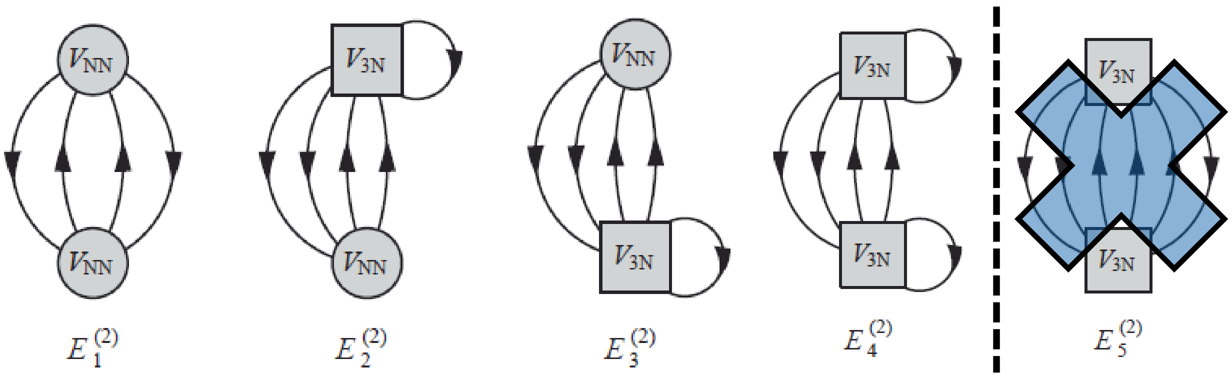
Hebeler *et al.*, Phys. Rev. C **82**, 014314

Normal-Ordered NN level $\bar{V}_{\mathrm{as}} = V_{\mathrm{NN}} + \xi \, V_{\mathrm{eff}}$

Initial NN Forces
Normal-Ordered 3N Forces
combinatorial factor



Hartree-Fock



Second Order

+ Higher Orders

Nuclear Equation of State from *ab-initio* point-of-view

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Outline

0 **Improved Normal-Ordering Method**

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1 **Isospin-Asymmetric Nuclear Matter**

2 **Many-Body Convergence?**

3 **BCS Pairing Gaps in Neutron Matter**

$$S_2(n) = \frac{1}{2} \frac{\partial^2}{\partial \beta^2} \frac{E}{A}(n, \beta)\Big|_{\beta=0}$$

$$L = 3n_0 \frac{\partial}{\partial n} S_2(n)\Big|_{n=n_0}$$

see also:
Vidaña *et al.*, PRC **80**, 045806
CD *et al.*, PRC **89**, 025806
Drews, Weise, PRC **91**, 035802
Wellenhofer *et al.*, PRC **93**, 055802

CD, Hebeler, Schwenk, PRC **93**, 054314.

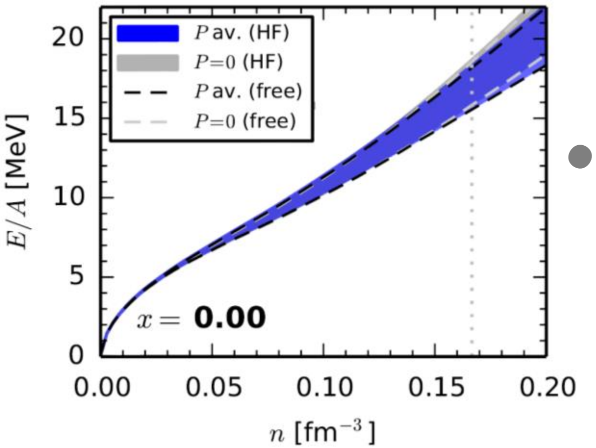
ISOSPIN-ASYMMETRIC NUCLEAR MATTER

Objectives: equation of state, saturation point, incompressibility, symmetry energy

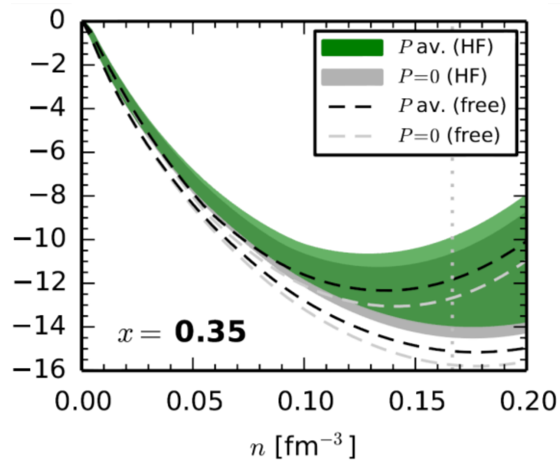
Nuclear Equation of State from *ab-initio* point-of-view

Equation of State

CD, Hebeler, Schwenk, PRC **93**, 054314



Neutron Matter



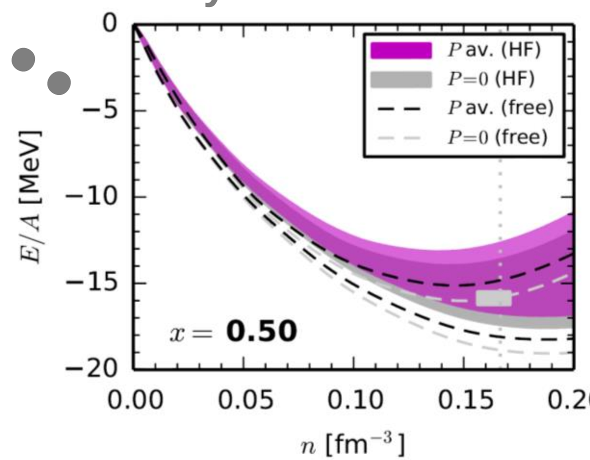
11 proton fractions

***x* = 0.0, 0.05, ..., 0.5**

up to second order

$$x = \frac{n_p}{n_n + n_p}$$

Symmetric Matter



Uncertainty bands: Hebeler *et al.*, PRC **83**, 031301(R)

- 7 Hamiltonians: evolved N^{3}LO NN + bare N^{2}LO 3N
- different combinations of λ/Λ_{3N}
- c_D, c_E fit *only* to few-body data
- free and Hartree-Fock spectrum

Nuclear Equation of State from *ab-initio* point-of-view

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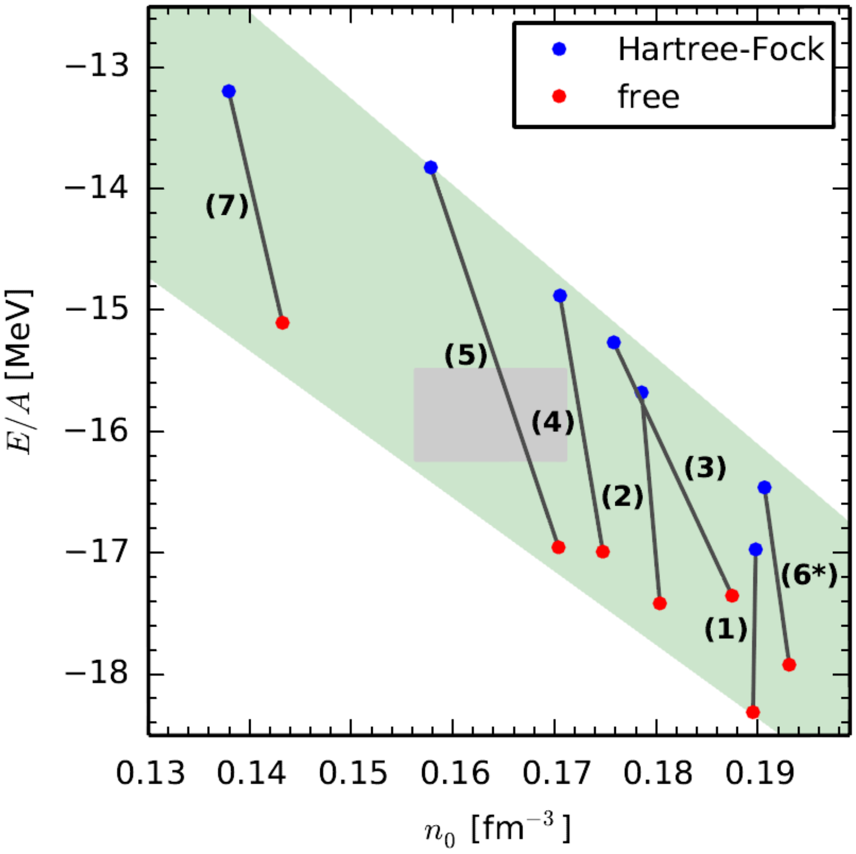
Saturation Properties

CD, Hebeler, Schwenk, PRC **93**, 054314



Coester-like correlation

- covers the empirical range due to 3N contributions

Coester *et al.*, PRC **1**, 769

empirical saturation point:

max. range of 14 EDF's

Dutra *et al.*, PRC **85**, 035201
Kortelainen *et al.*, PRC **89**, 054314

$$n_0 = (0.138 - 0.193)\ \mathrm{fm}^{-3}$$
$$K = (182 - 254)\ \mathrm{MeV}$$

Nuclear Equation of State from *ab-initio* point-of-view

Symmetry Energy and Slope Parameter

see also: Hagen *et al.*, Nat. Phys. **12**, 186

standard expansion: $\beta = 1 - 2x$

$$\frac{E}{A}(n,\beta) = \frac{E_{\rm SNM}(n)}{A} + S_2(n)\beta^2 + \ldots$$

$$S_2(n) = S_v + \frac{L}{3}\left(\frac{n-n_0}{n_0}\right) + \cdots$$

tight constraints

$$S_v = (30.9 \pm 1.4)\ \mathrm{MeV}$$

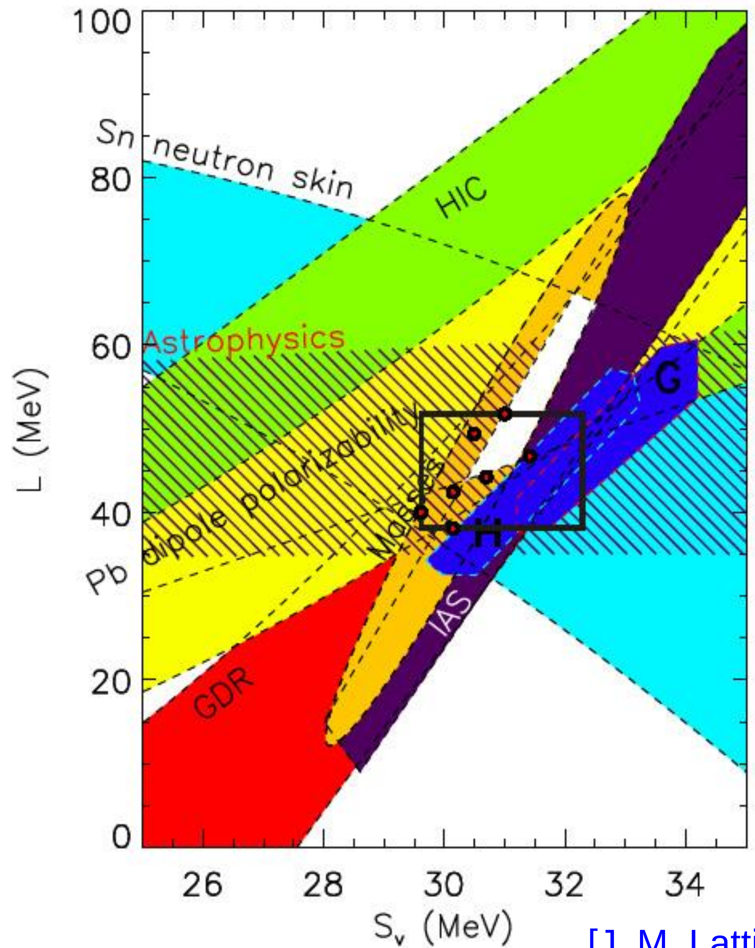
$$L = (45.0 \pm 7.1)\ \mathrm{MeV}$$

in **agreement** with emp. extractions

Lattimer, Lim, Astrophys. J. **771**, 51

quadratic expansion is **reliable**;
but nonanalytical quartic term: $\beta^4 \ln|\beta|$

Kaiser, PRC **91**, 065201
Wellenhofer *et al.*, PRC **93**, 055802



[J. M. Lattimer]

Nuclear Equation of State from *ab-initio* point-of-view

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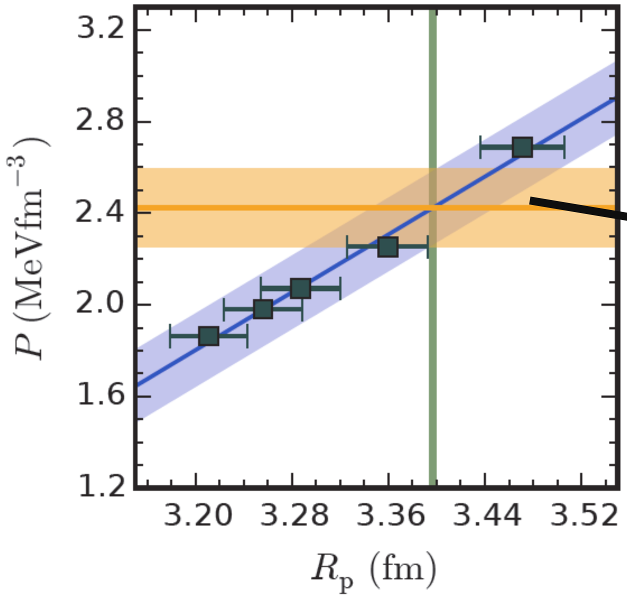
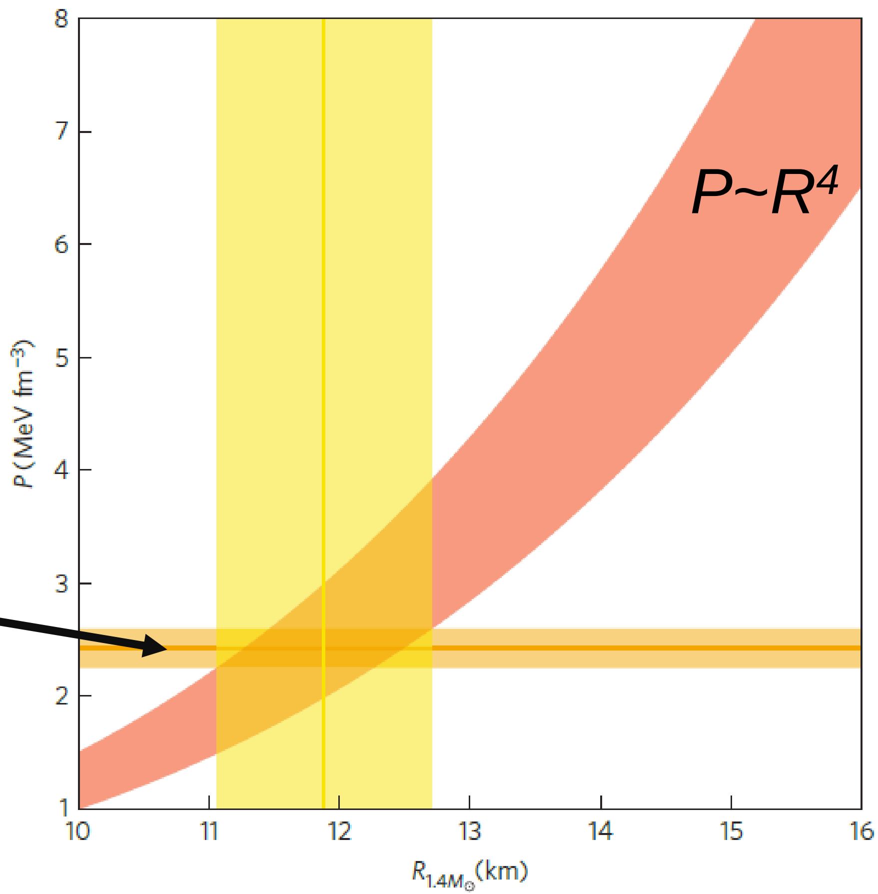
Radius Estimate for a 1.4 M_{sun} Neutron Star

Hagen *et al.*, Nat. Phys. **12**, 186

Universal (empirical) relation by Lattimer, Prakash

Pressure constrained by CC calculations of ^{48}Ca radii and measurement

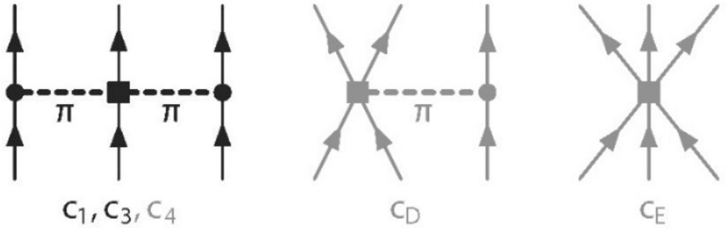




Lattimer, Prakash, APJ, **550**, 426
Lattimer, Lim, Astrophys. J. **771**, 51

see also:
Dickhoff, Barbieri, Prog. Part. Nucl. Phys. **52**, 377
Rios *et al.*, PRC **79**, 025802
Krüger *et al.*, PRC **88**, 025802
Tews *et al.*, PRC **93**, 024305

Neutron Matter:



CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307.

MANY-BODY CONVERGENCE?

Objectives: test many-body convergence
study impact of N^3LO 3N forces

Nuclear Equation of State from *ab-initio* point-of-view

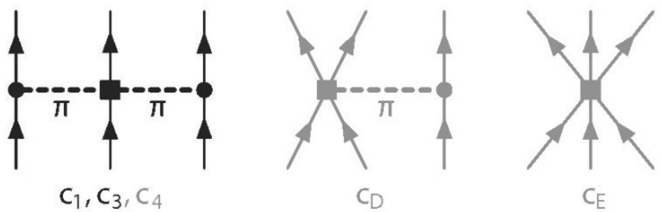
Testing Many-Body Convergence

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307

- consistent N^{3}LO NN/3N forces
- **finite proton fractions** need reliable fits of c_D, c_E at N^{3}LO

Golak *et al.*, Eur. Phys. J. A **50** 177

Neutron Matter



Uncertainty bands

- use always c_i's recommended for N^{3}LO calculations
- plus many-body uncertainty

Krebs *et al.*, PRC **85**, 054006

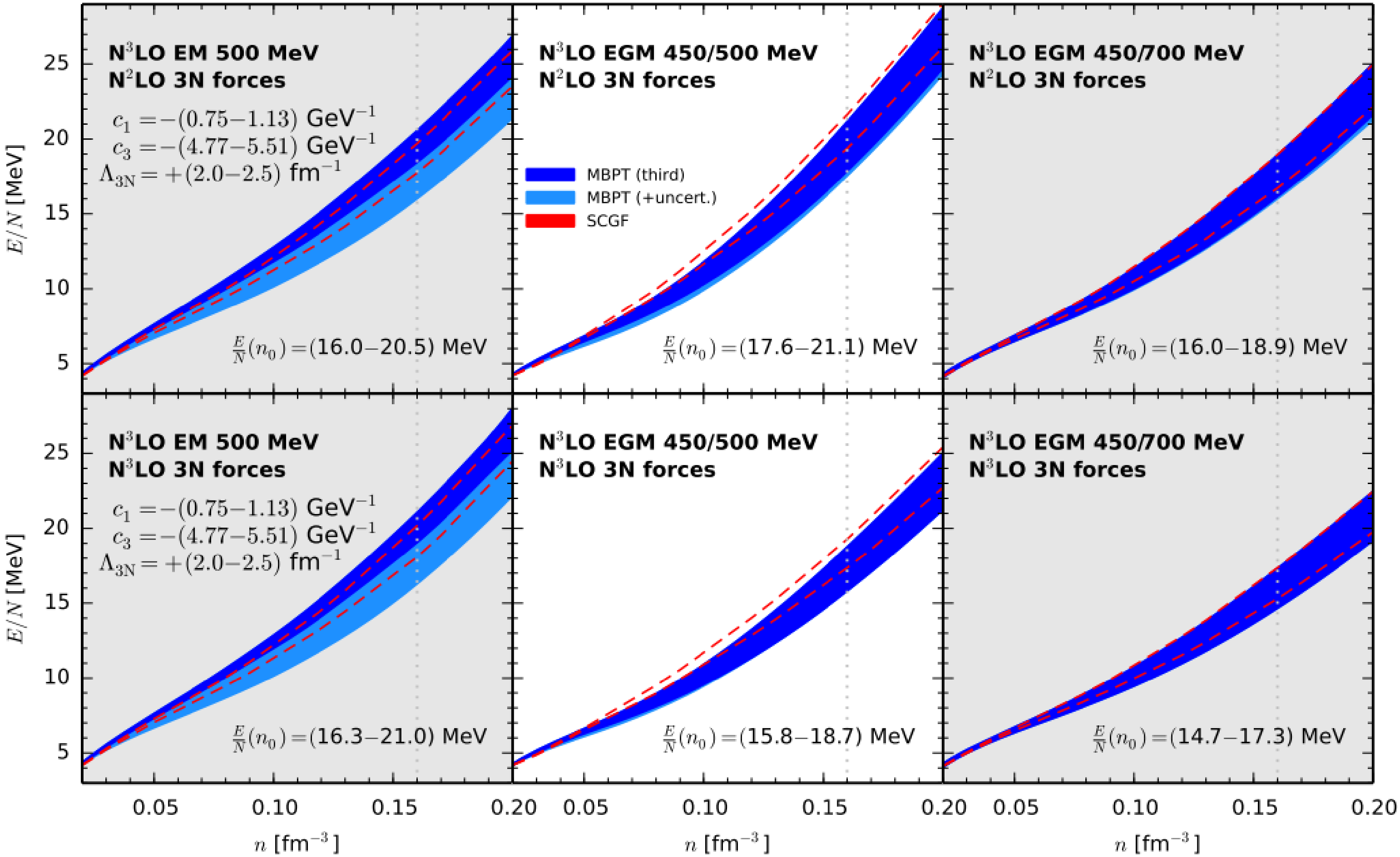
MBPT	SCGF Method
Improved Normal-Ordering Method	
up to third order	**nonperturbative**
free vs. HF spectrum	full spectral function
T=0 MeV	Extrapolated to T=0 MeV

see also: Carbone *et al.*, PRC **90** 054322

Nuclear Equation of State from *ab-initio* point-of-view

MBPT vs. SCGF Method

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



N^{2}LO 3N

N^{3}LO 3N

Nuclear Equation of State from *ab-initio* point-of-view

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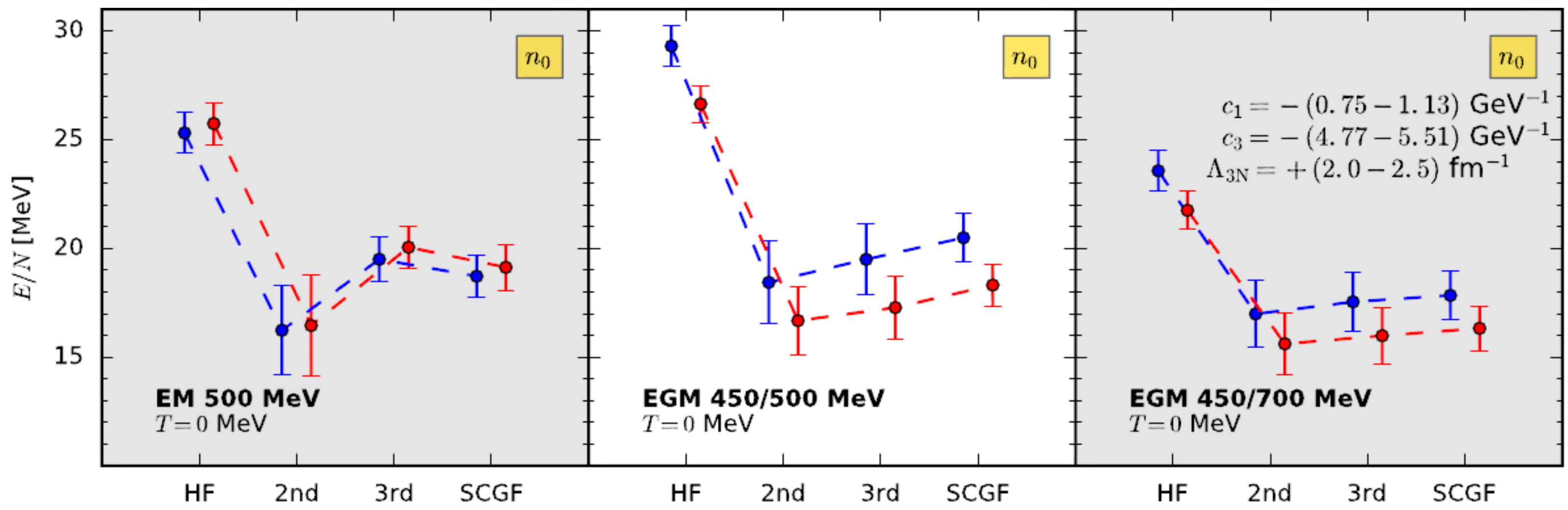
Testing Many-Body Convergence

CD, Carbone, Hebeler, Schwenk, PRC **94**, 054307



Order-by-order analysis: (at saturation density)

- attractive second vs. repulsive third order
- MBPT **well converged** for EGM potentials (small third order)
- EM 500 MeV is less perturbative (larger third order)
- small energy shift due to N^3LO 3N w.r.t. N^2LO 3N contributions

see also:
Srinivas, Ramanan, PRC **94**, 064303
Ding *et al.*, PRC **94**, 025802
Maurizio *et al.*, PRC **90**, 044003
Page *et al.*, "Novel Superfluids", Oxford University Press

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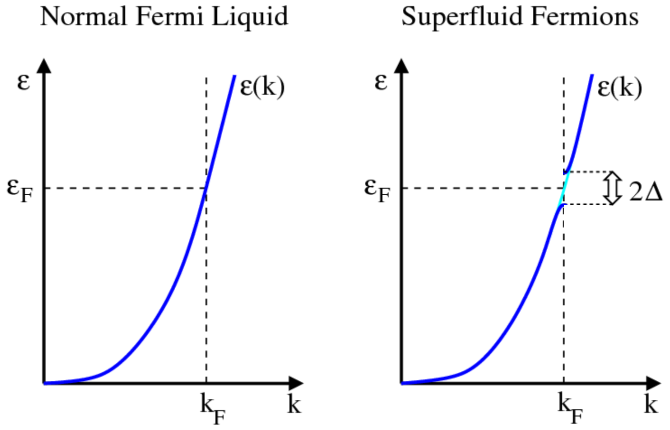

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302.

BCS PAIRING GAPS IN NEUTRON MATTER

Objectives: study subleading 3N contributions
recent (semi-)local NN potentials, new uncertainties

Nuclear Equation of State from *ab-initio* point-of-view

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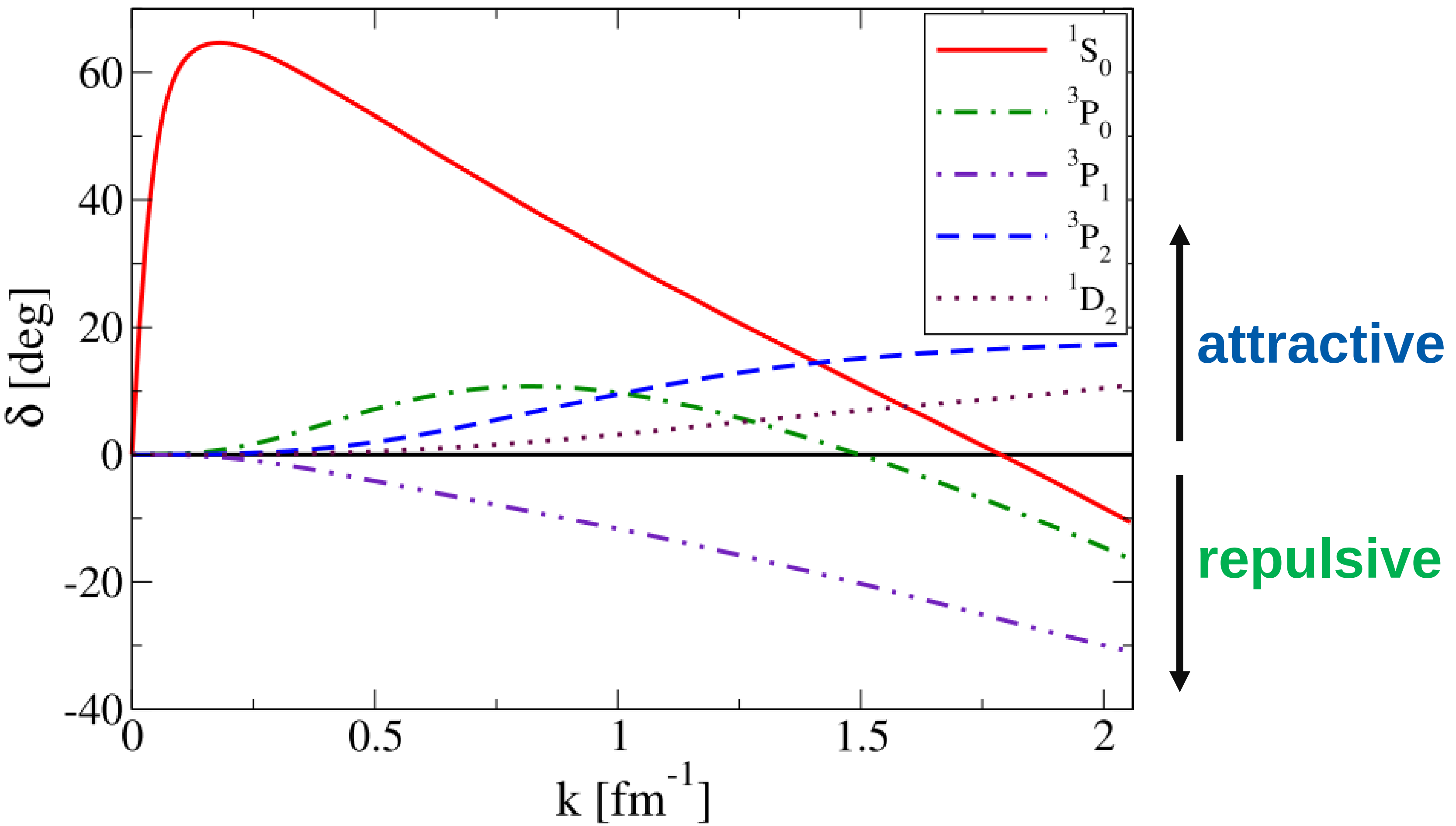
Attractive Interactions: Phase Shifts



Nuclear Equation of State from *ab-initio* point-of-view

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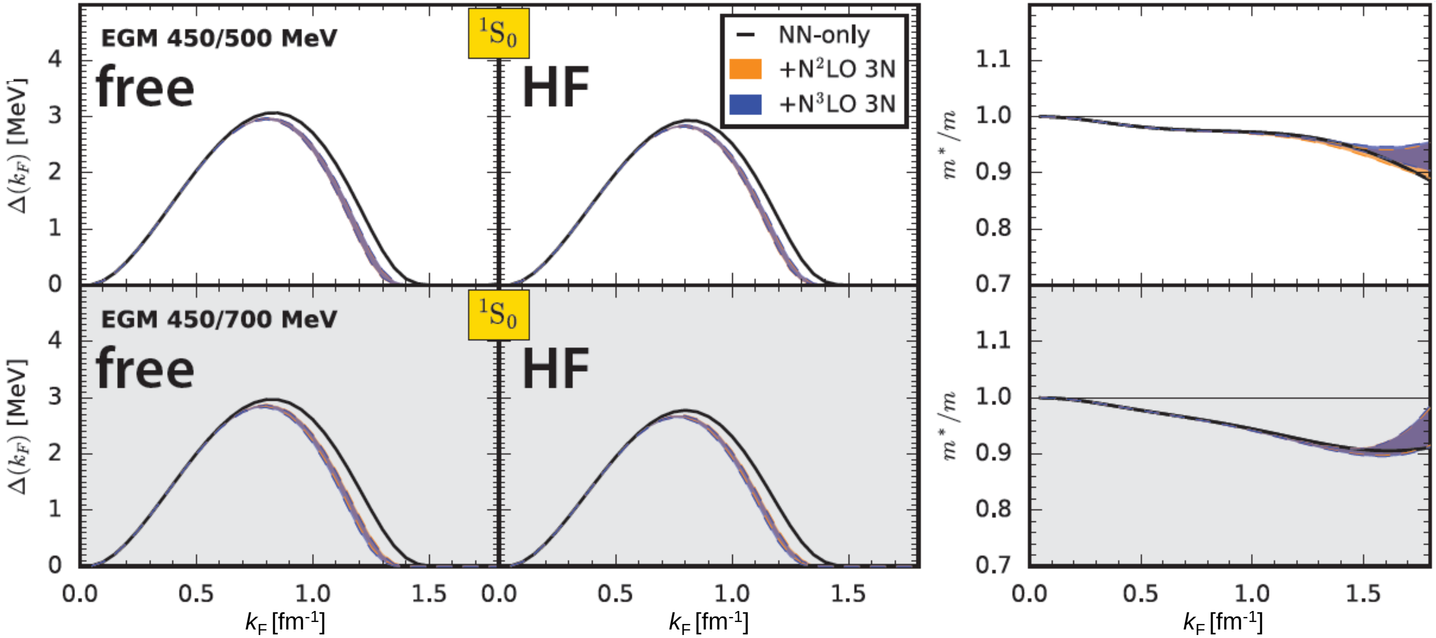
Pairing Gaps: 3N forces in $^1S_0$

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302



- uncertainties: 3N parameter variation (recommended values)
- **pairing gap at low densities**
  - universal gaps: strongly constrained by phase shifts
  - **small 3N contributions**: only small suppression for $k_F > 0.8$ fm^{-1}
  - almost independent of the energy spectrum

Nuclear Equation of State from *ab-initio* point-of-view

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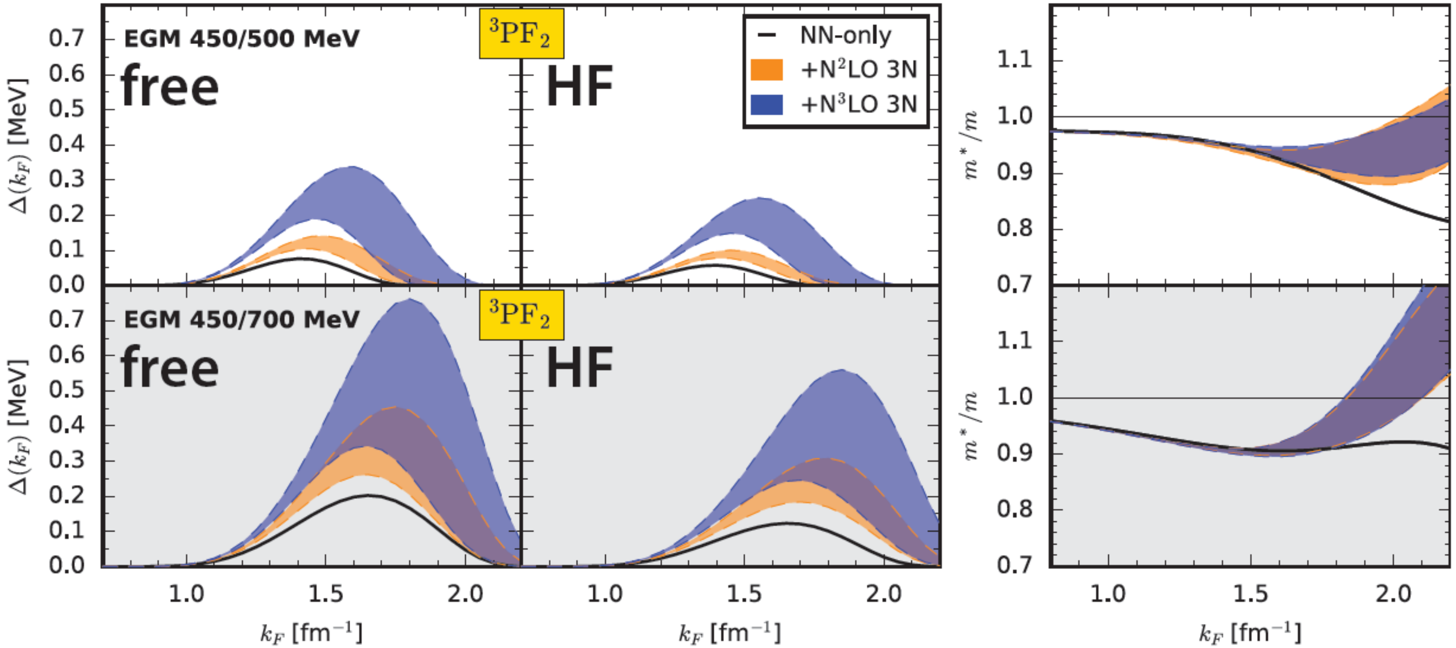
Pairing Gaps: 3N forces in $^3P_2$–$^3F_2$

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302

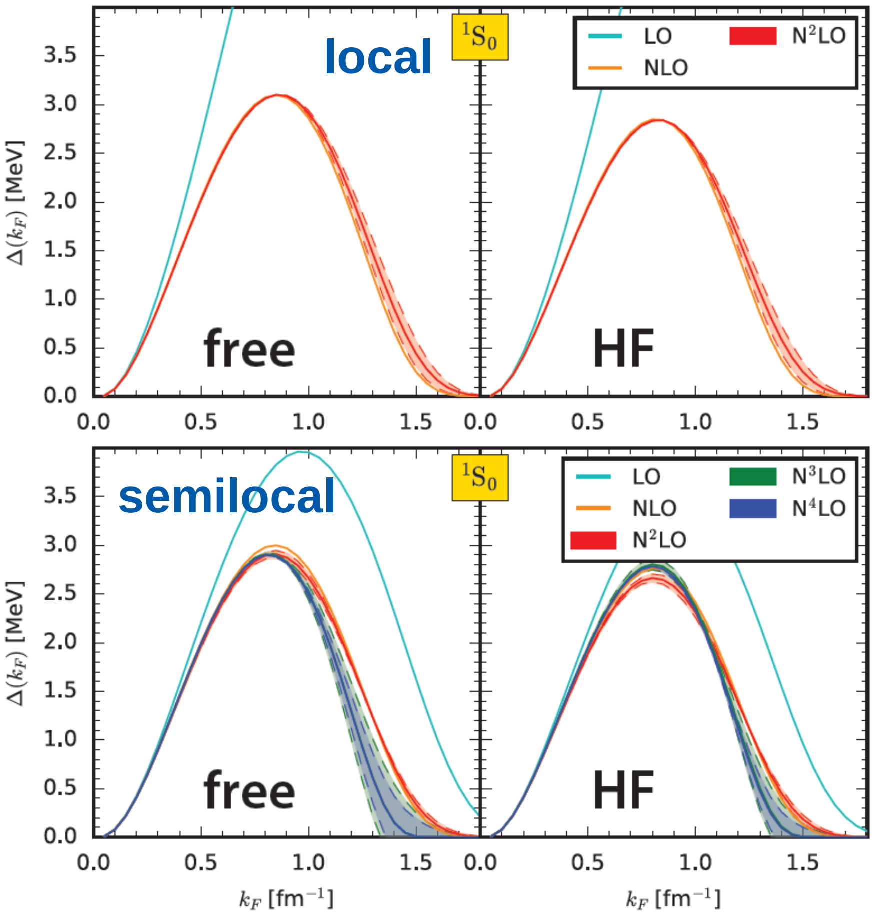


- uncertainties: 3N parameter variation (recommended values)
- **pairing gap at high densities**
  - **3N forces add attraction**: larger max. gap and closure at higher densities
  - effective masses are enhanced due to 3N forces
  - chiral EFT still efficient at $k_F > 2$ fm^{-1}**?**

Nuclear Equation of State from *ab-initio* point-of-view

(Semi-)Local NN: $^1S_0$ channel

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302



local and semilocal NN forces:

- up to N^2LO and N^4LO
- R_0 = 0.9, 1.0, 1.1 and, 1.2 fm

new uncertainties (Epelbaum *et al.*)

order-by-order analysis in the chiral expansion (LO neglected)

findings: $Q(k_\mathrm{F}) = \max\left(\frac{p}{\Lambda_b}, \frac{m_\pi}{\Lambda_b}\right)$

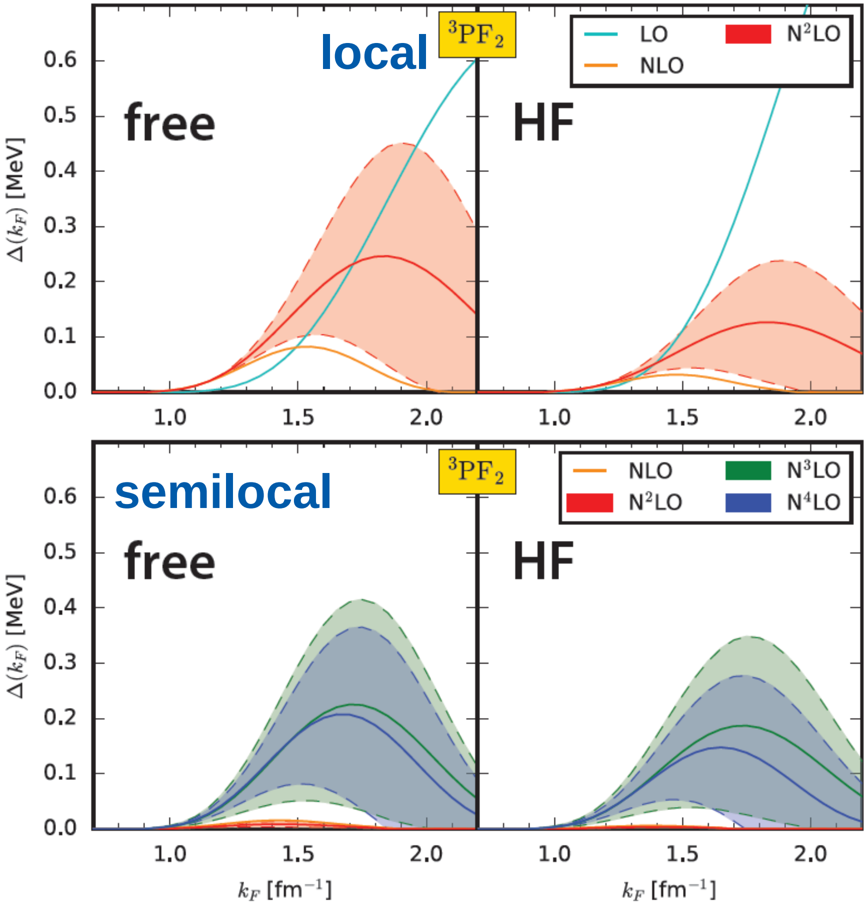
- at NLO and beyond **gaps agree** up to $k_\mathrm{F} \sim (0.6{-}0.8)\ \mathrm{fm}^{-1}$
- sensitivity to spectrum is again small

Gezerlis *et al.*, PRC **90**, 054323
Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

Nuclear Equation of State from *ab-initio* point-of-view

(Semi-)Local NN: $^3P_2$–$^3F_2$ channel

CD, Krüger, Hebeler, Schwenk, PRC **95**, 024302



local and semilocal NN forces:

- up to N^2LO and N^4LO
- R_0 = 0.9, 1.0, 1.1 and, 1.2 fm

new uncertainties (Epelbaum *et al.*)

order-by-order analysis in the chiral expansion (LO neglected)

findings:

- large uncertainties: breakdown of the chiral expansion **?**

Gezerlis *et al.*, PRC **90**, 054323
Epelbaum *et al.*, Eur. Phys. J. A **51**, 53

Nuclear Equation of State from *ab-initio* point-of-view

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Summary | Outlook

Improved Normal-Ordering Method

- applicable to all 3N forces (incl. N^3LO)
- **asymmetric matter**: results for EOS, symmetry energy, …

More Applications

- studied **many-body convergence** in neutron matter: N^3LO 3N forces beyond Hartree-Fock and in SCGF method
- **BCS pairing gaps** in $^1S_0$ and $^3P_2$–$^3F_2$:
  - N^3LO 3N contributions to previous NN potentials
  - recent (semi-)local NN potentials, new uncertainties

Extensions – a selection

- finite temperatures, consistently-evolved forces, ...
- **constrain next-gen. potentials: saturation**, … Carlsson *et al.*, PRX **6**, 011019

Nuclear Equation of State from *ab-initio* point-of-view

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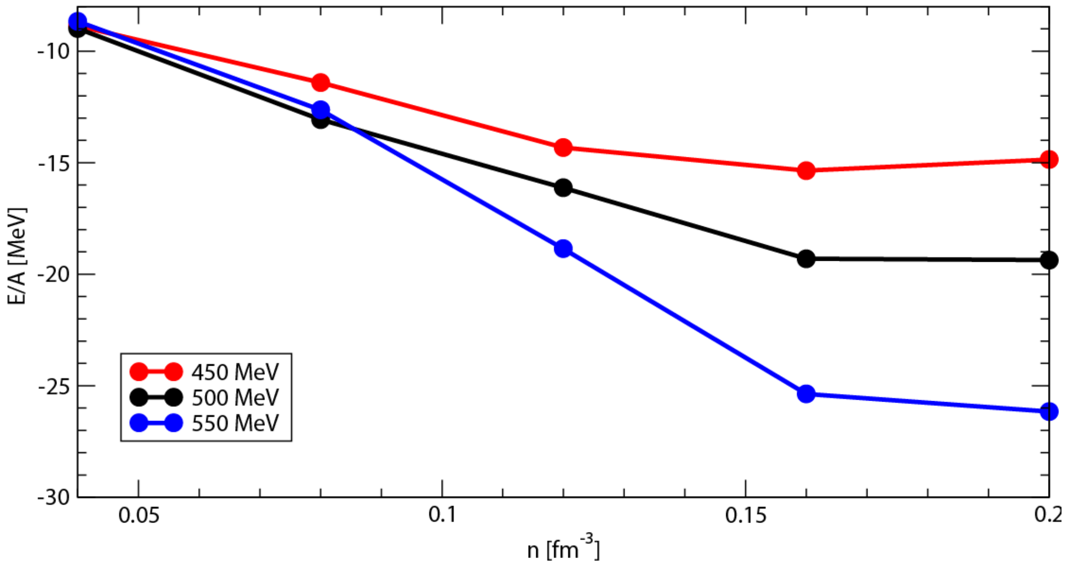
First Results: Isospin-Symmetric Matter

Drischler *et al.*, work in progress

preliminary



less perturbative

N^2LOsim	$E_{NN}^{(3)}(n_0)$ [MeV]	$E_{NN}^{(4)}(n_0)$ [MeV]	$E_{tot}^{(IV)}(n_0)$ [MeV]
450 MeV	**+1.1**	**+0.3**	**–15.4**
500 MeV	+1.6	–1.5	–19.3
550 MeV	+3.3	–4.1	–25.4