Non-observability and reaction-structure interplay

Topical lecture week:
Nuclear Structure from Spectroscopy and Direct Reactions

Kai Hebeler
Darmstadt, Feb 24, 2016

SFB 1245

TECHNISCHE UNIVERSITÄT DARMSTADT

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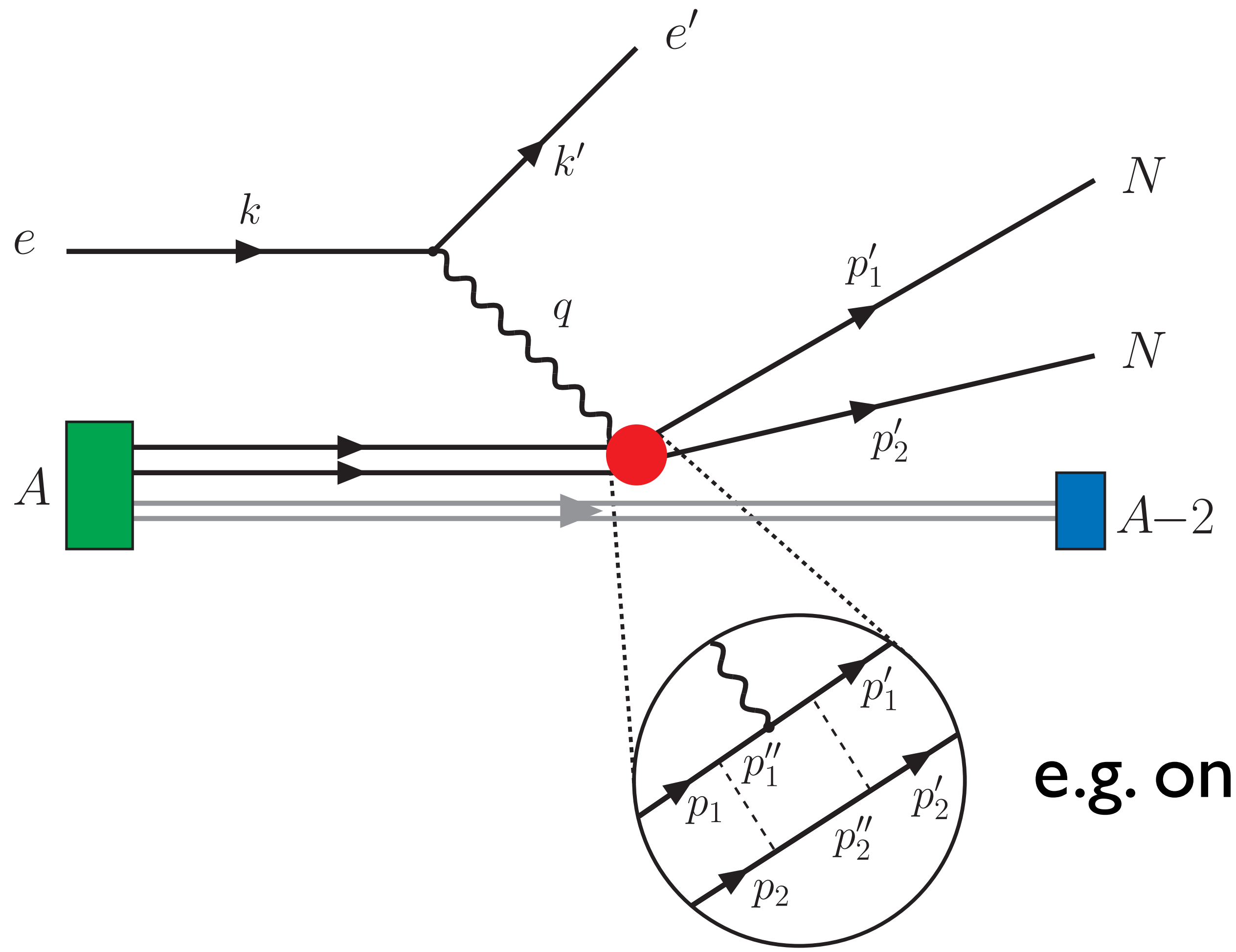
Reaction processes within ab initio frameworks

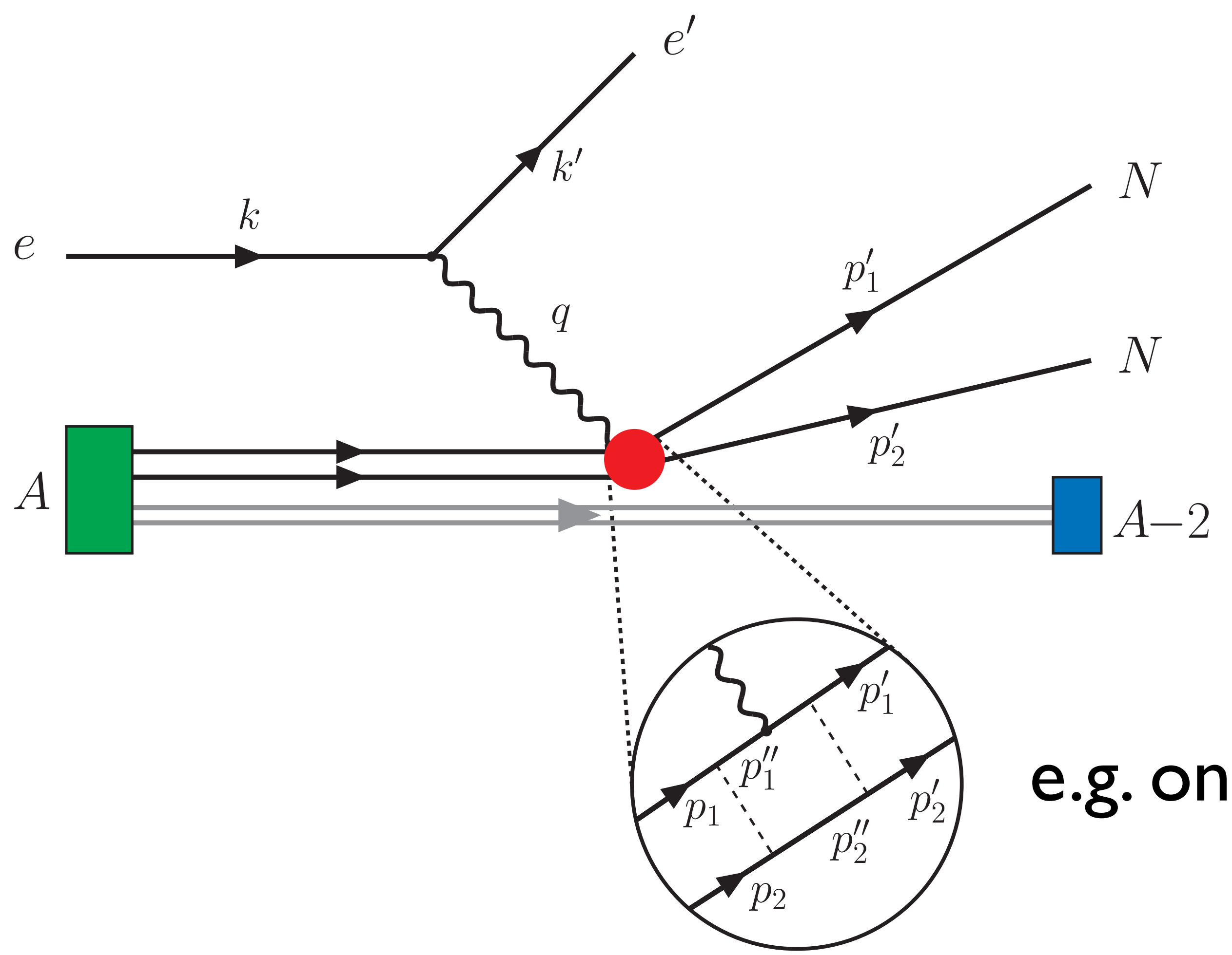


e.g. one-body interaction

Calculations generally involve:

- **nuclear structure** part for description of initial/final state wave functions
- **reaction part** describes interaction with external probes

Reaction processes within ab initio frameworks



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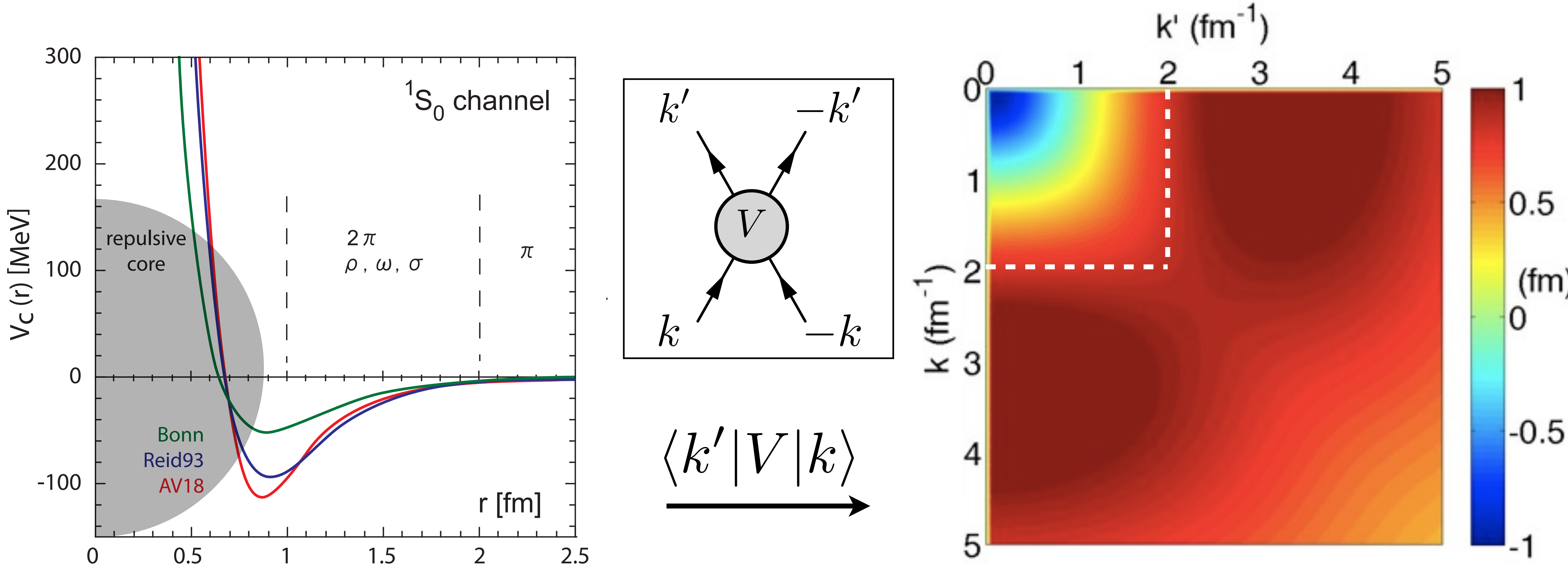
For calculations of observables a factorization of structure and reaction parts has to be assumed:

$$\sigma \sim \langle \psi_F(\lambda) | O(\lambda) | \psi_A(\lambda) \rangle$$

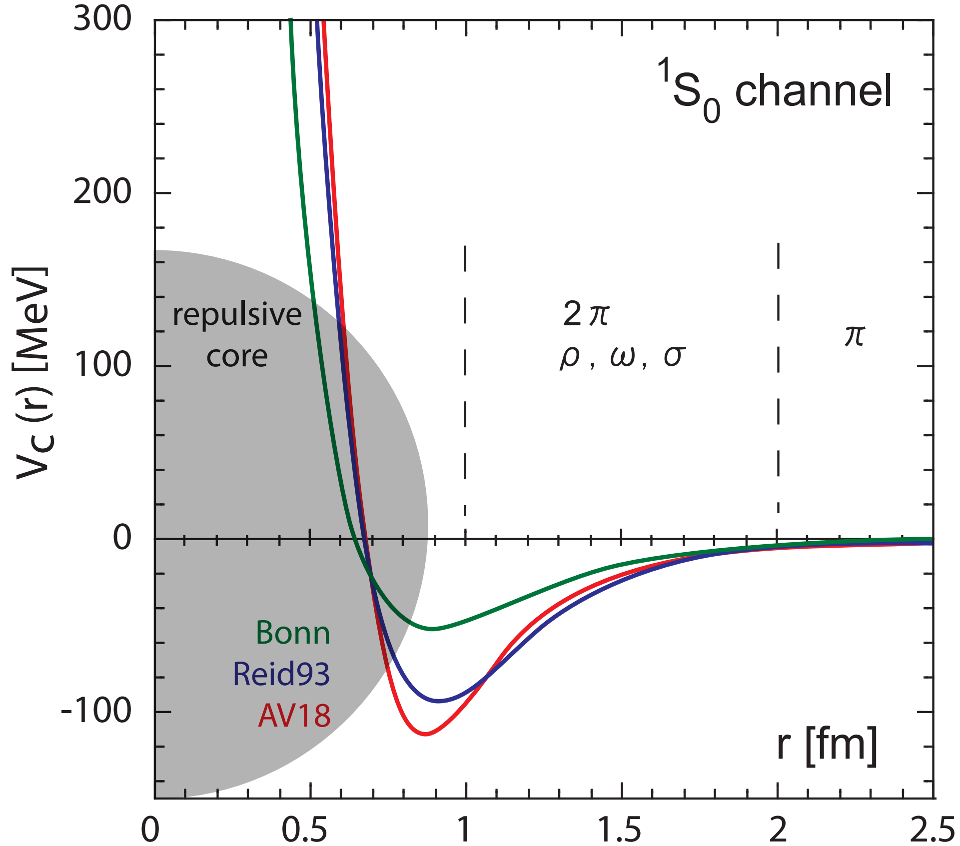
whereas λ is a chosen resolution scale

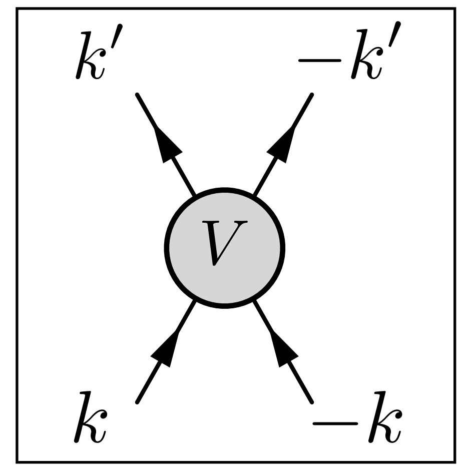
“Traditional” NN interactions



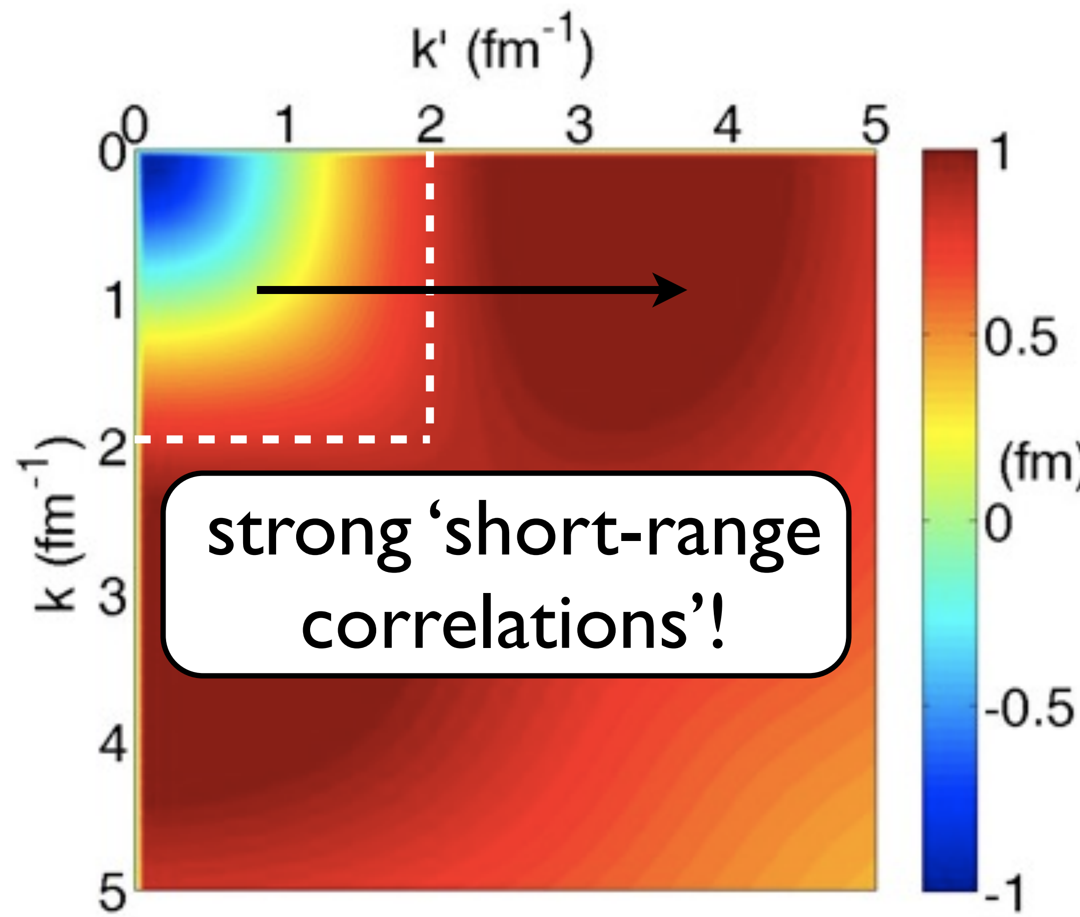
- constructed to fit scattering data (long-wavelength information)
- long-range part dominated by one pion exchange interaction
- short range part strongly model dependent!
- traditional NN interactions contain strongly repulsive core at small distance
  - strong coupling between low and high-momenta
  - many-body problem hard to solve using basis expansion!

"Traditional" NN interactions





$$\langle k' | V | k \rangle$$



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Changing the resolution scale

$$\langle\psi_F(\lambda_0)|O(\lambda_0)|\psi_A(\lambda_0)\rangle = \langle\psi_F(\lambda_0)|U^\dagger U(\lambda) O(\lambda_0) U^\dagger U(\lambda)|\psi_A(\lambda)\rangle$$
$$= \langle\psi_F(\lambda)|O(\lambda)|\psi_A(\lambda)\rangle$$

with

$$|\psi(\lambda)\rangle = U(\lambda)\,|\psi(\lambda_0)\rangle \qquad O(\lambda) = U^\dagger(\lambda) O(\lambda_0) U(\lambda)$$

$$U(\lambda)U^\dagger(\lambda) = 1$$

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Convenient to choose resolution scale λ such that

- wave functions include only momentum scales that are constrained by scattering phase shifts (reduction of scheme dependence)
- nuclear structure calculations are simplified

One solution: the Similarity Renormalization Group

- generate unitary transformation which decouples low- and high momenta:

$$H_\lambda = U_\lambda H U_\lambda^\dagger$$ with the resolution parameter λ

- change resolution systematically in small steps: $$\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$$

- generator η_λ can be chosen and tailored to different applications

- observables are preserved due to unitarity of transformation

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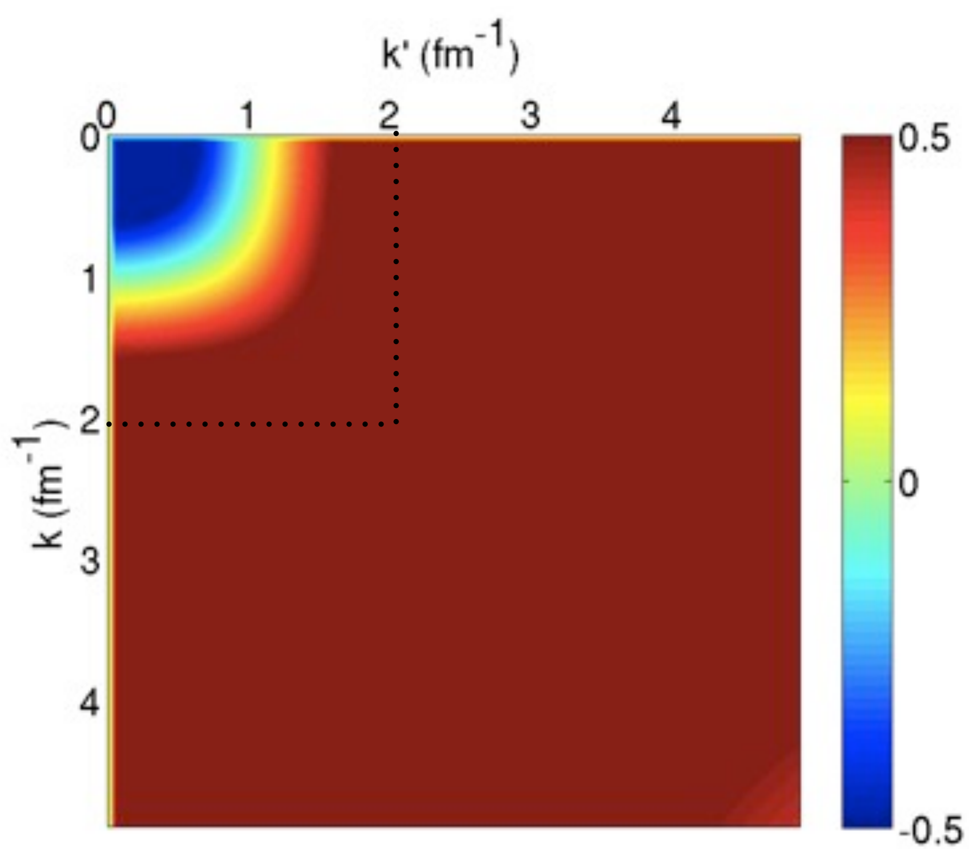

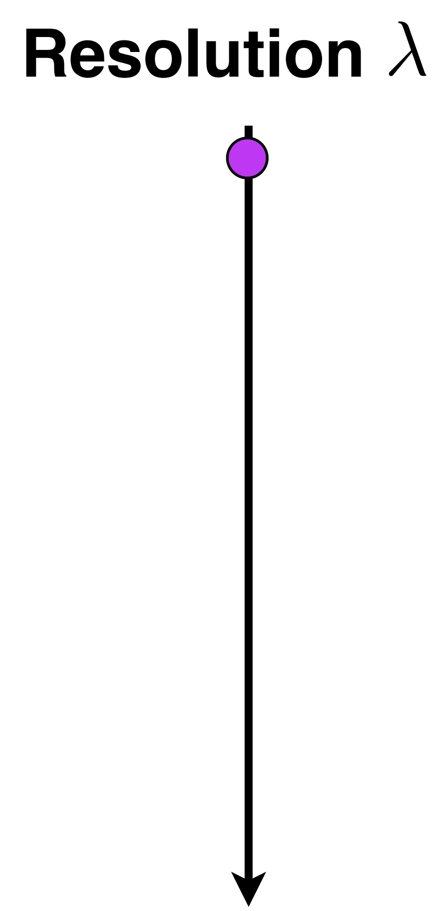

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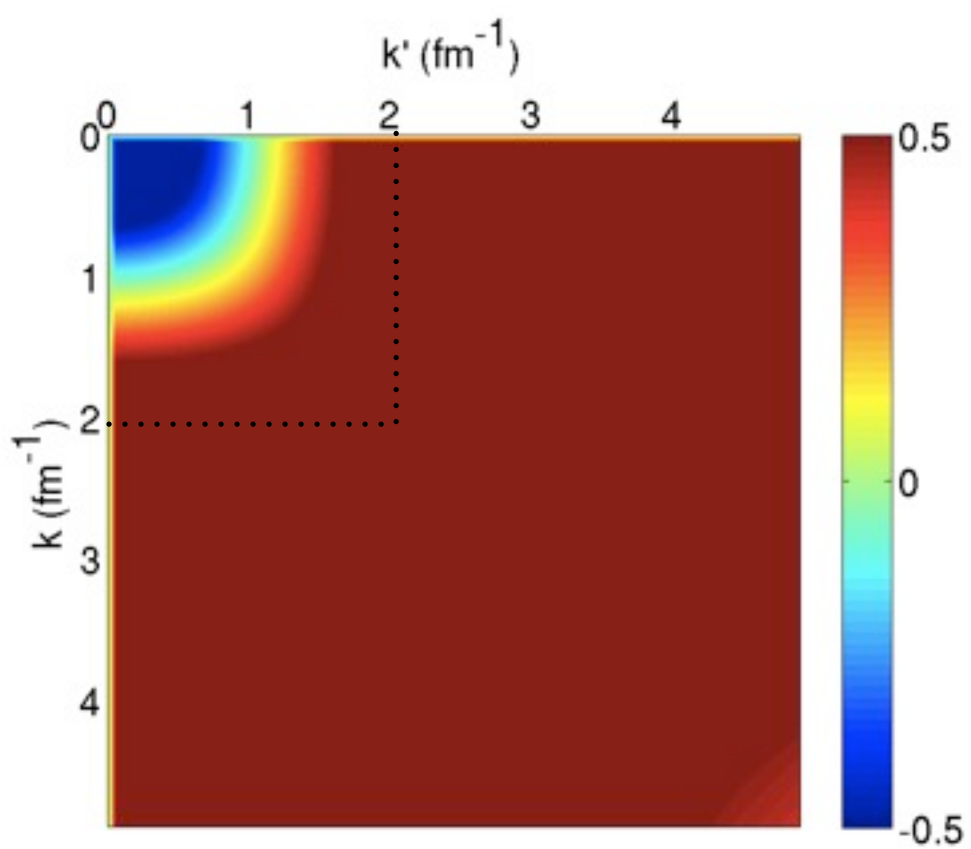
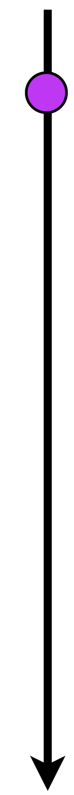


Resolution λ



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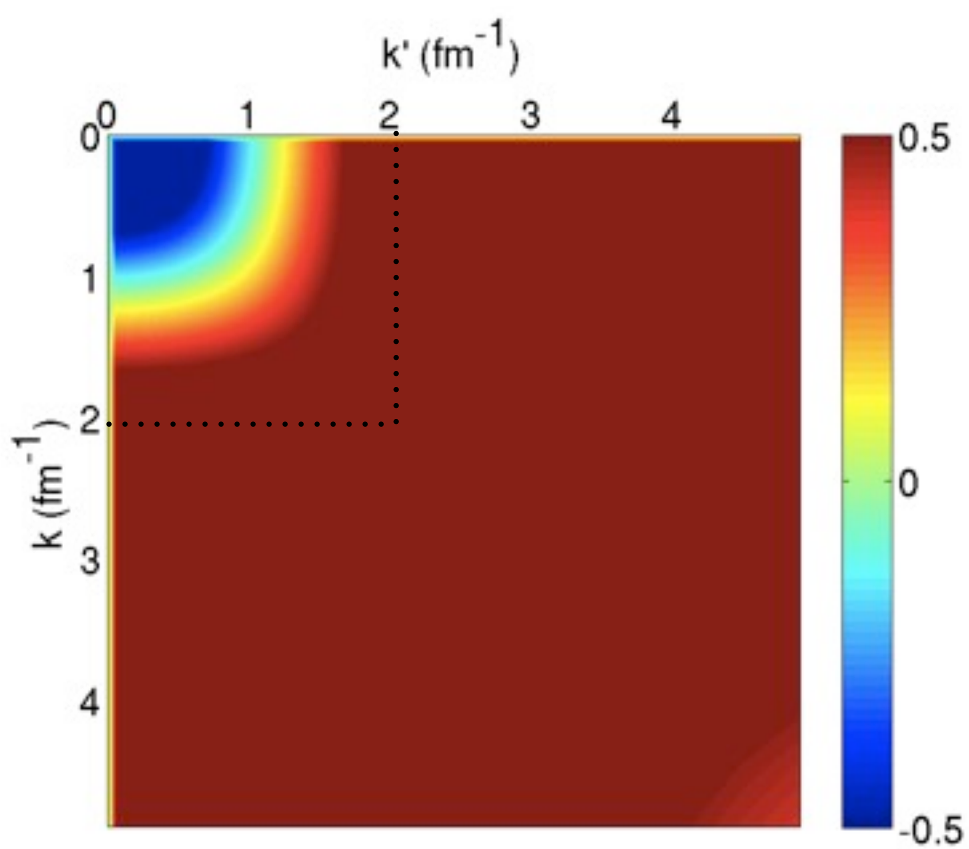

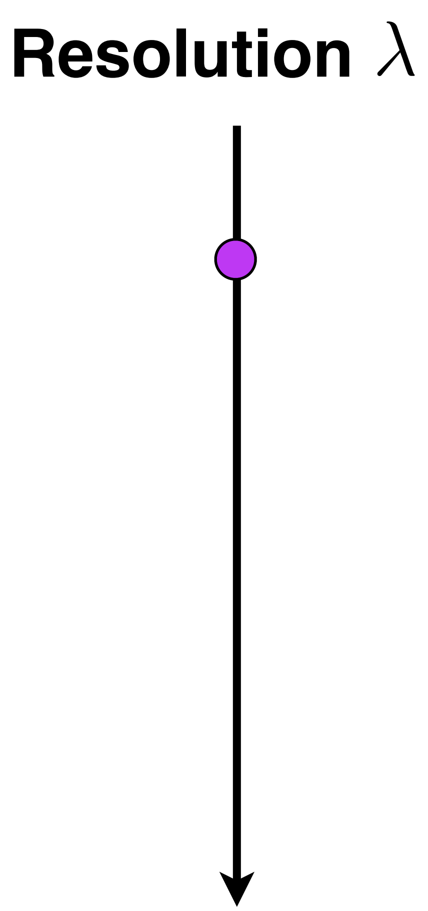

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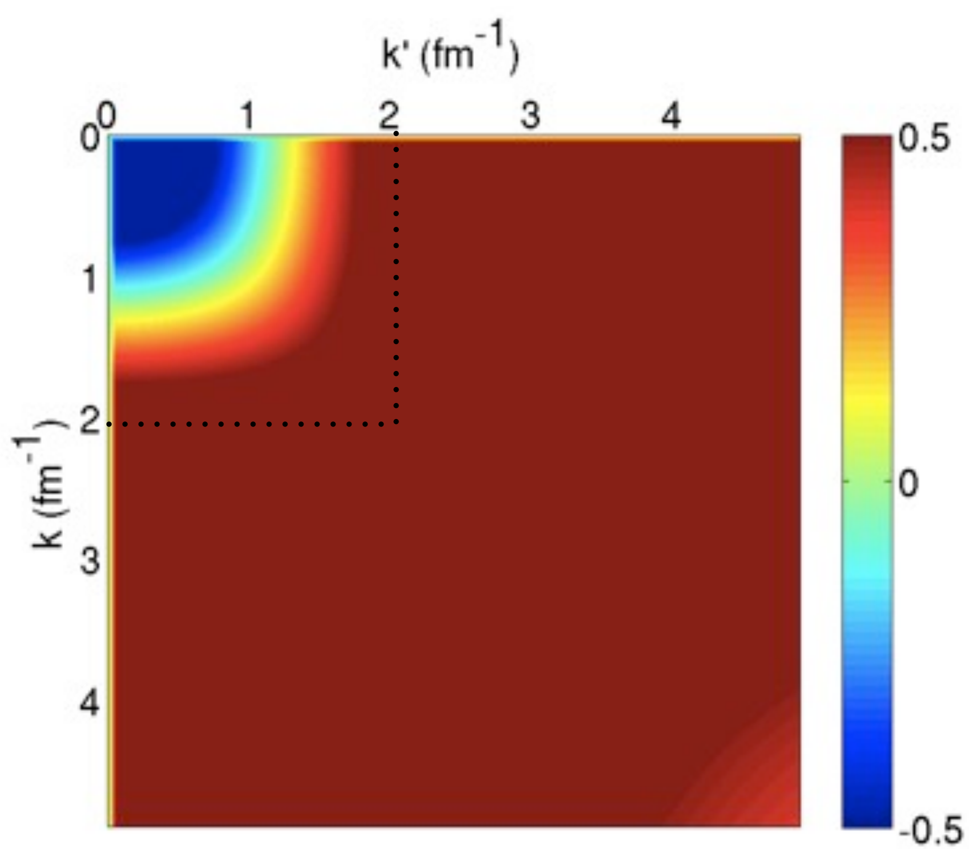

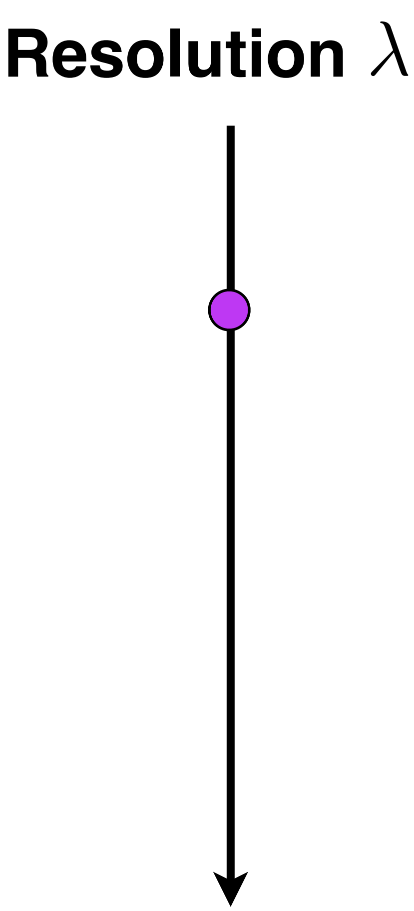

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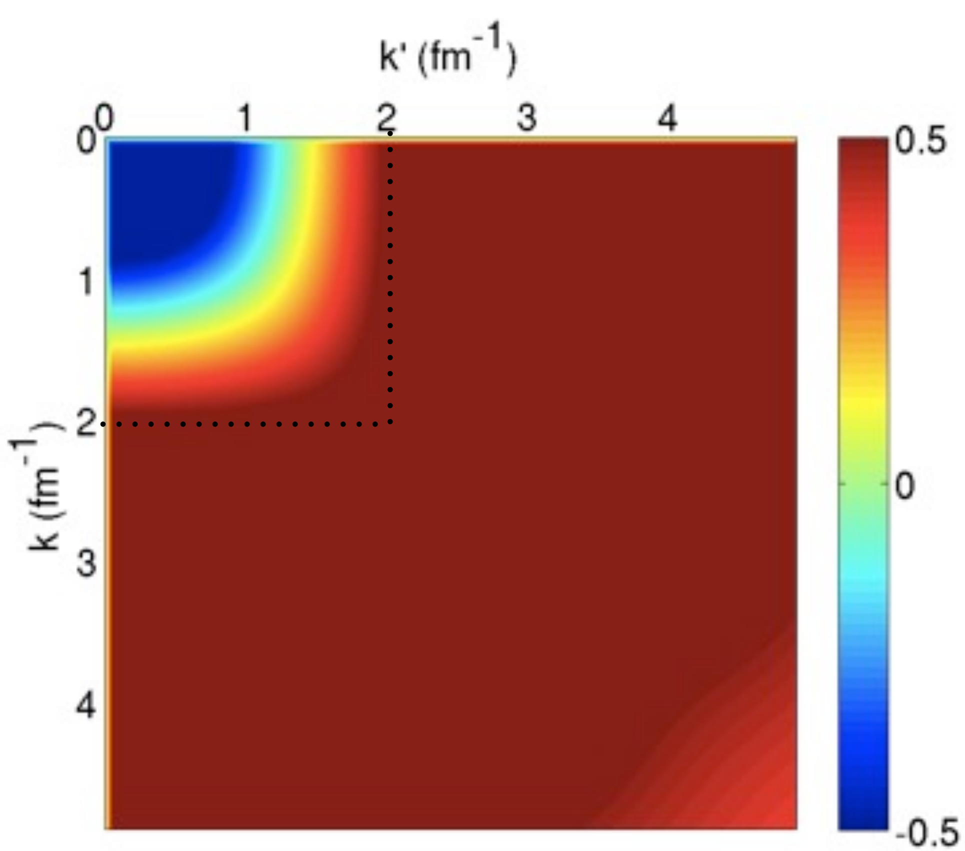

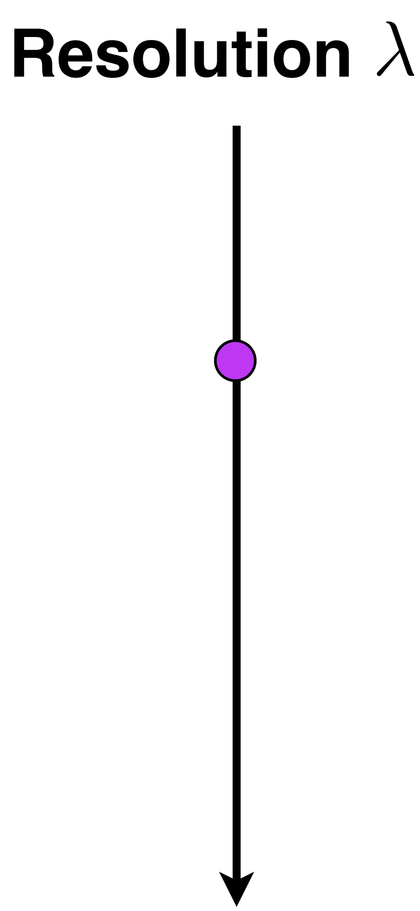

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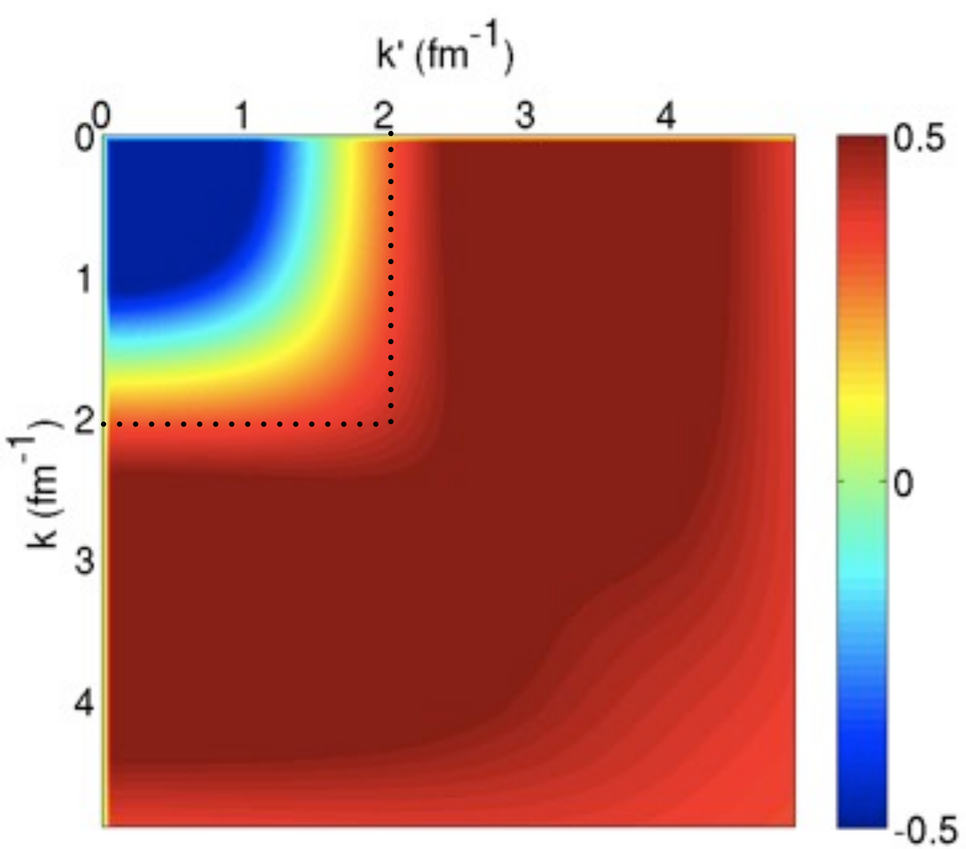

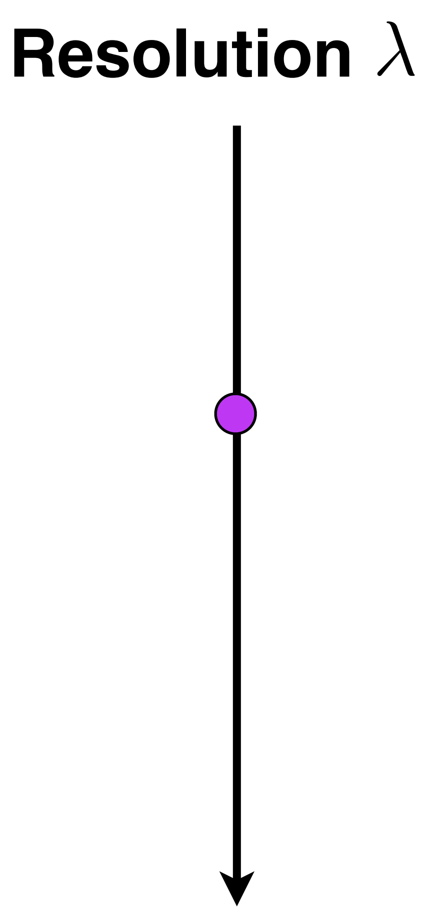

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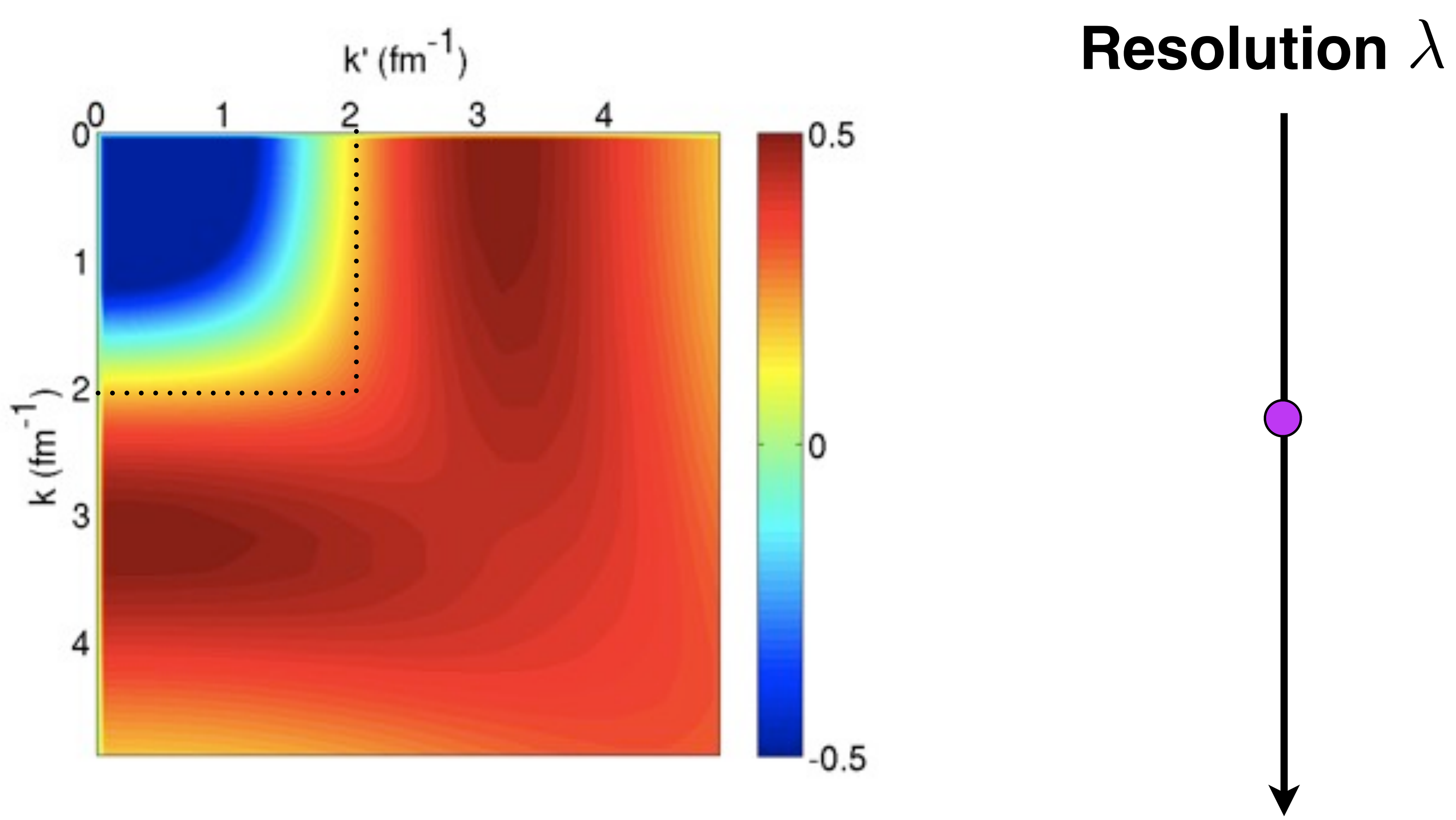

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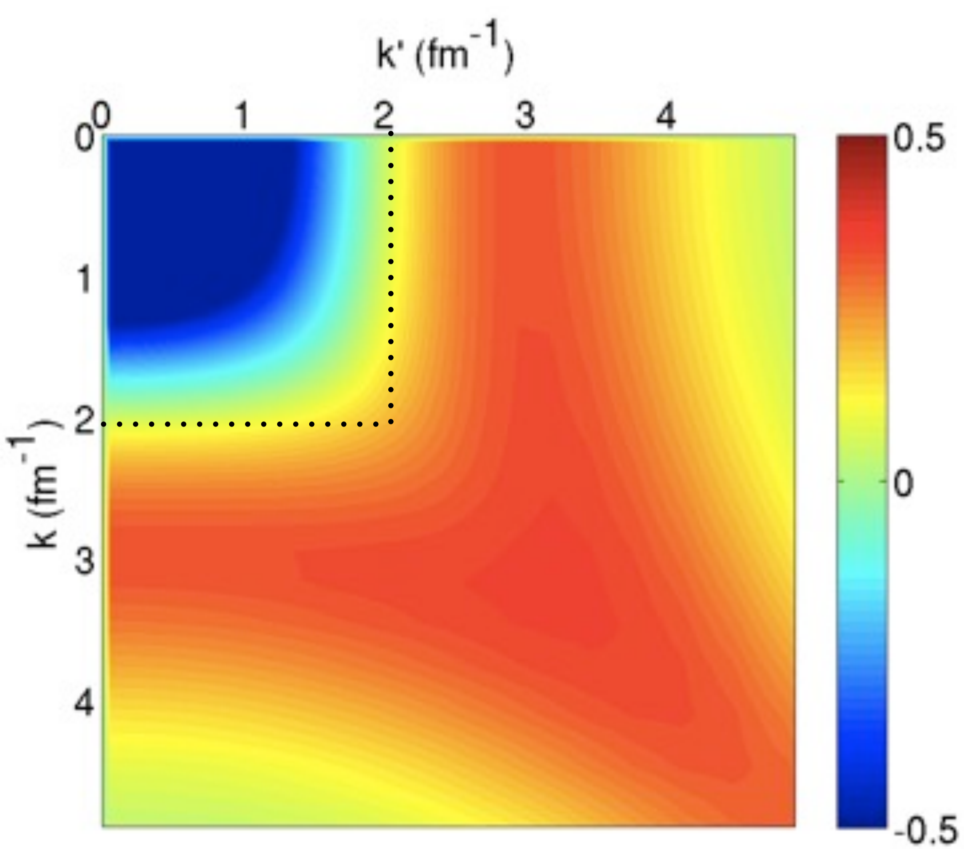

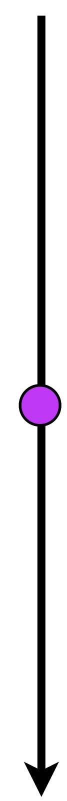

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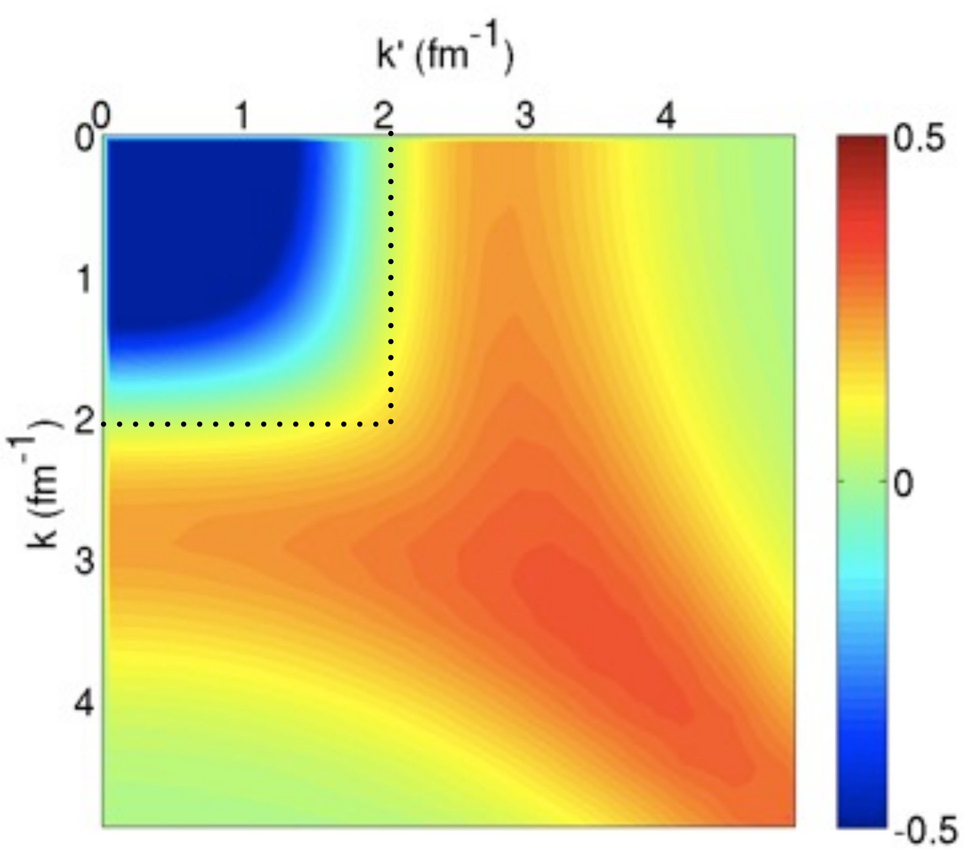

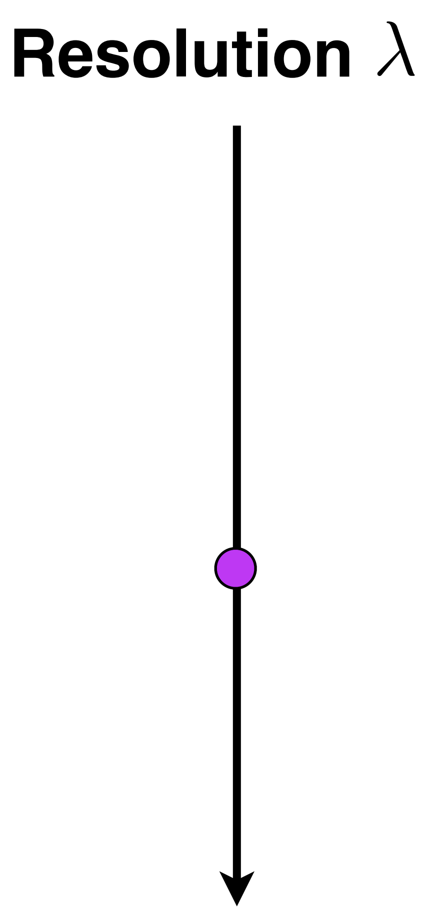

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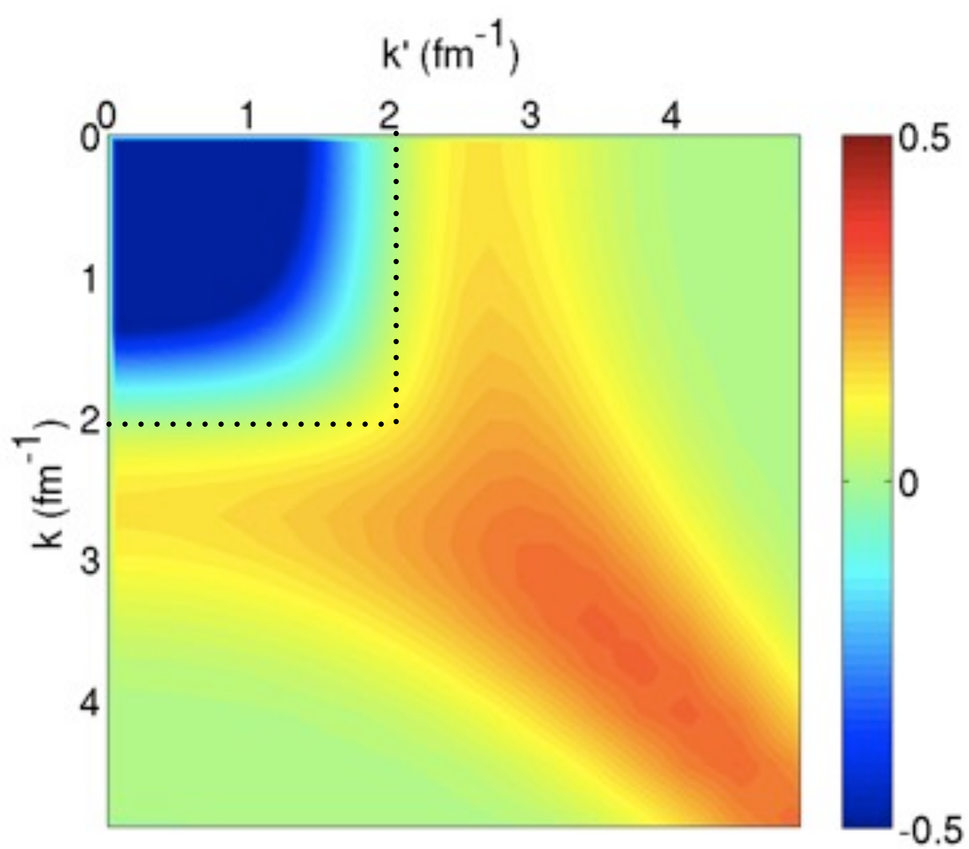

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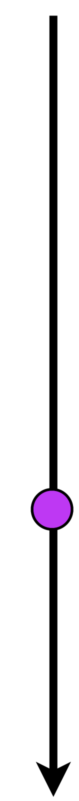

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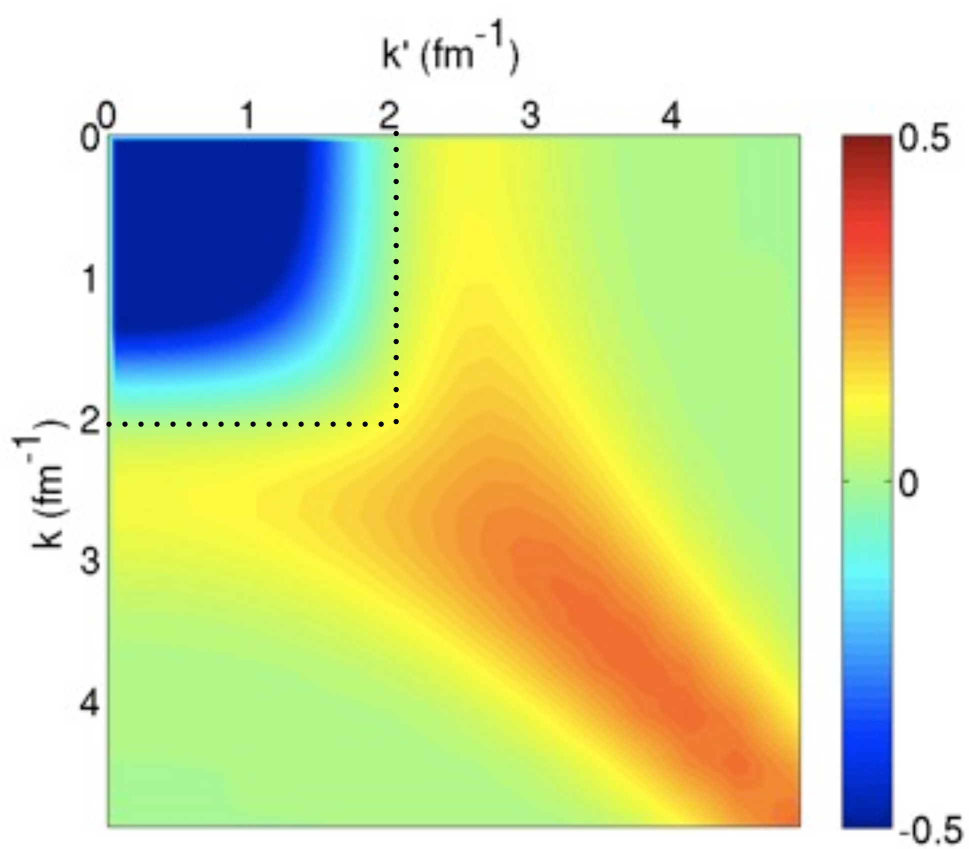

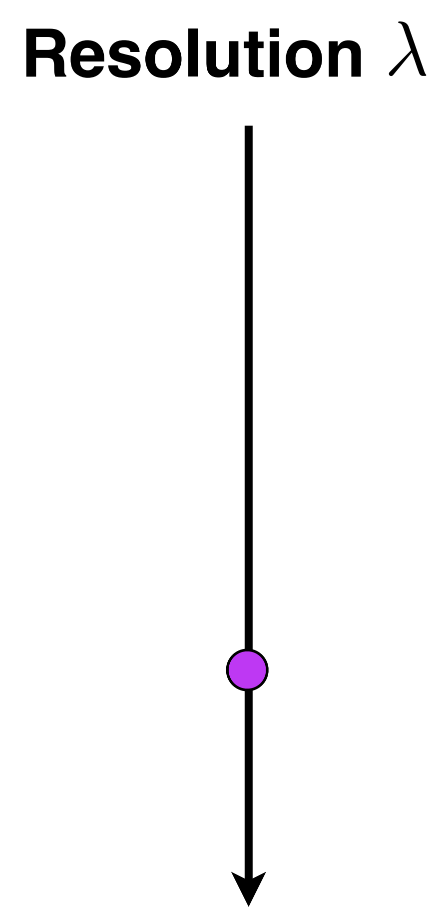

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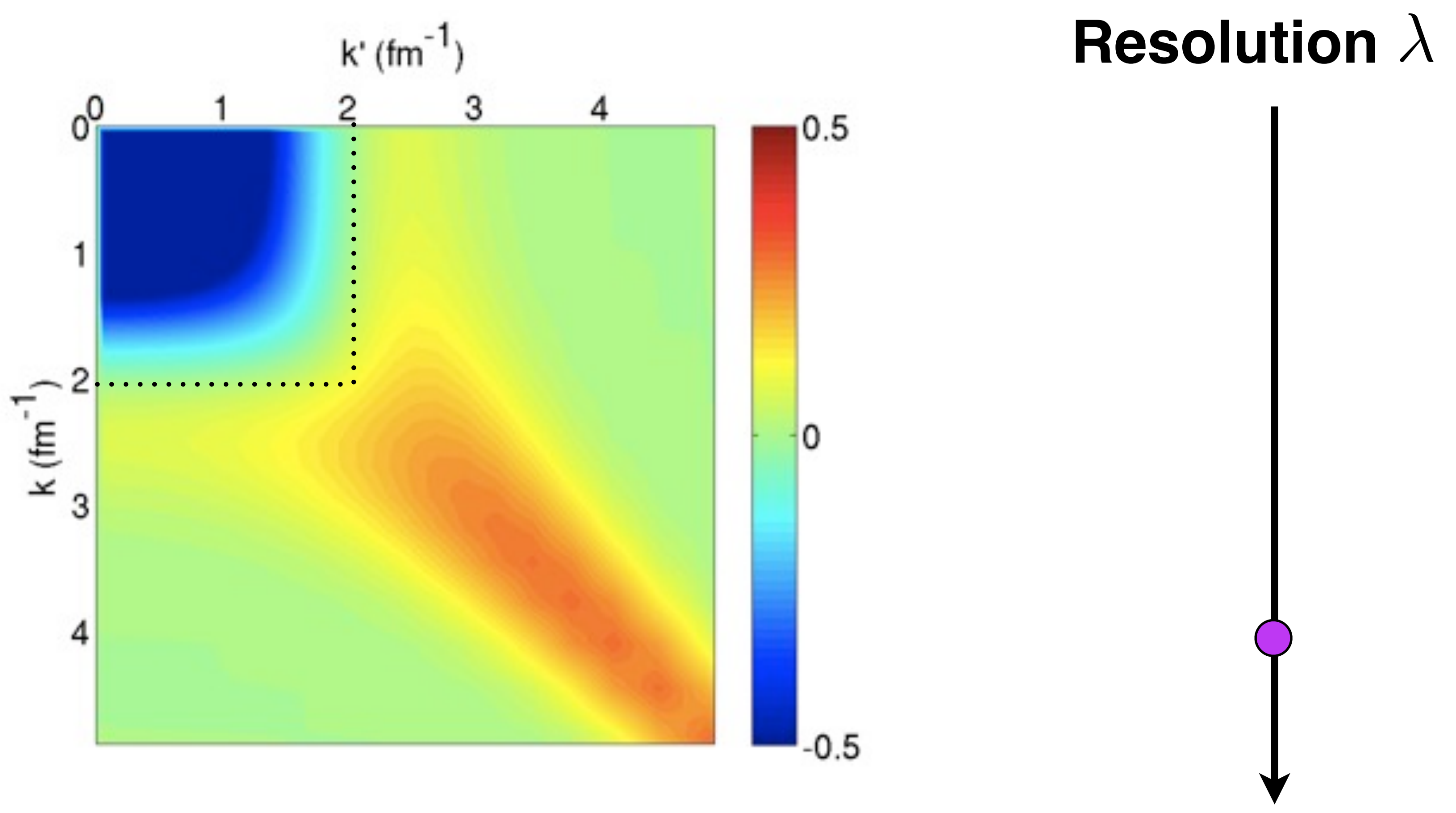

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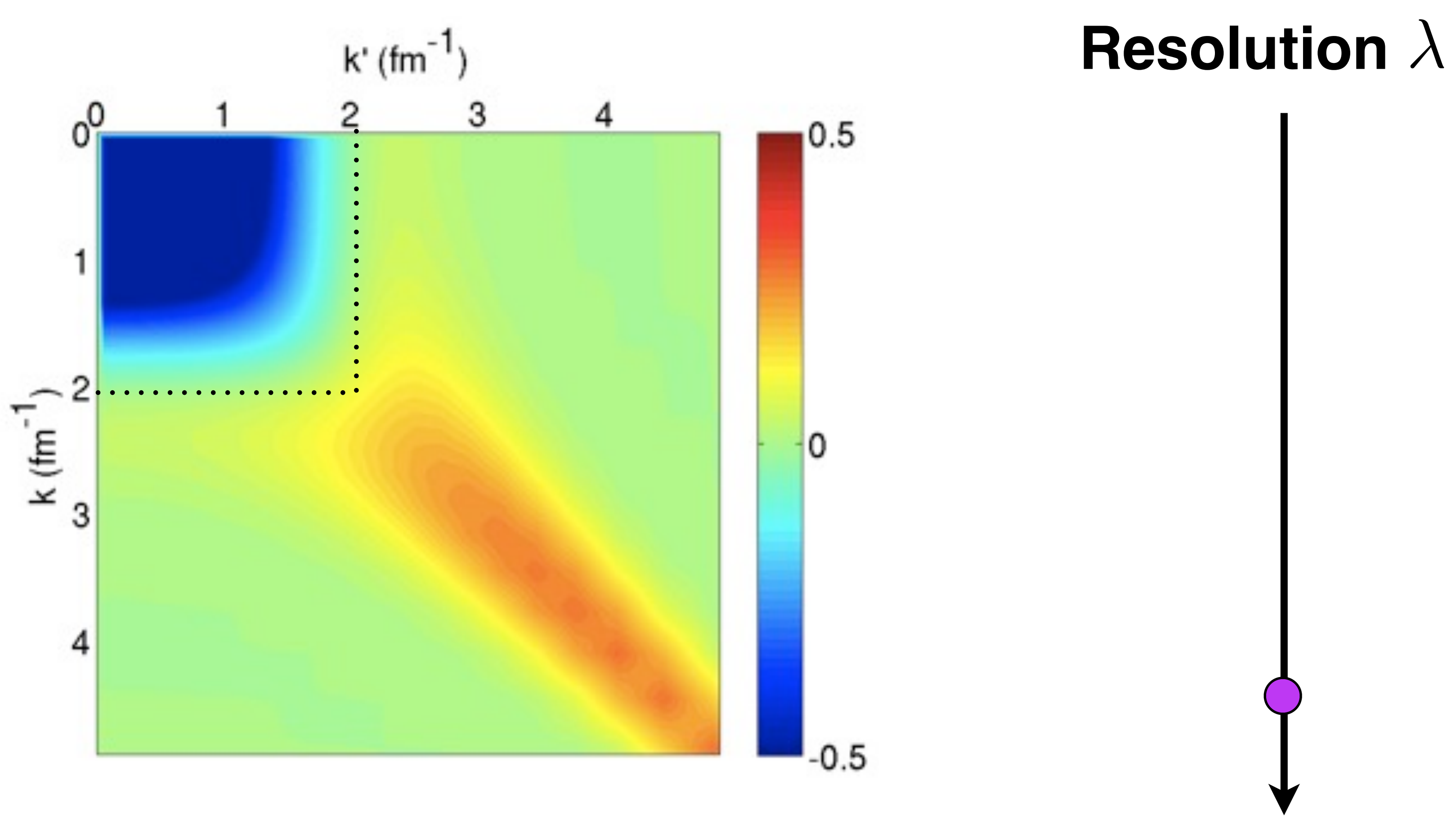

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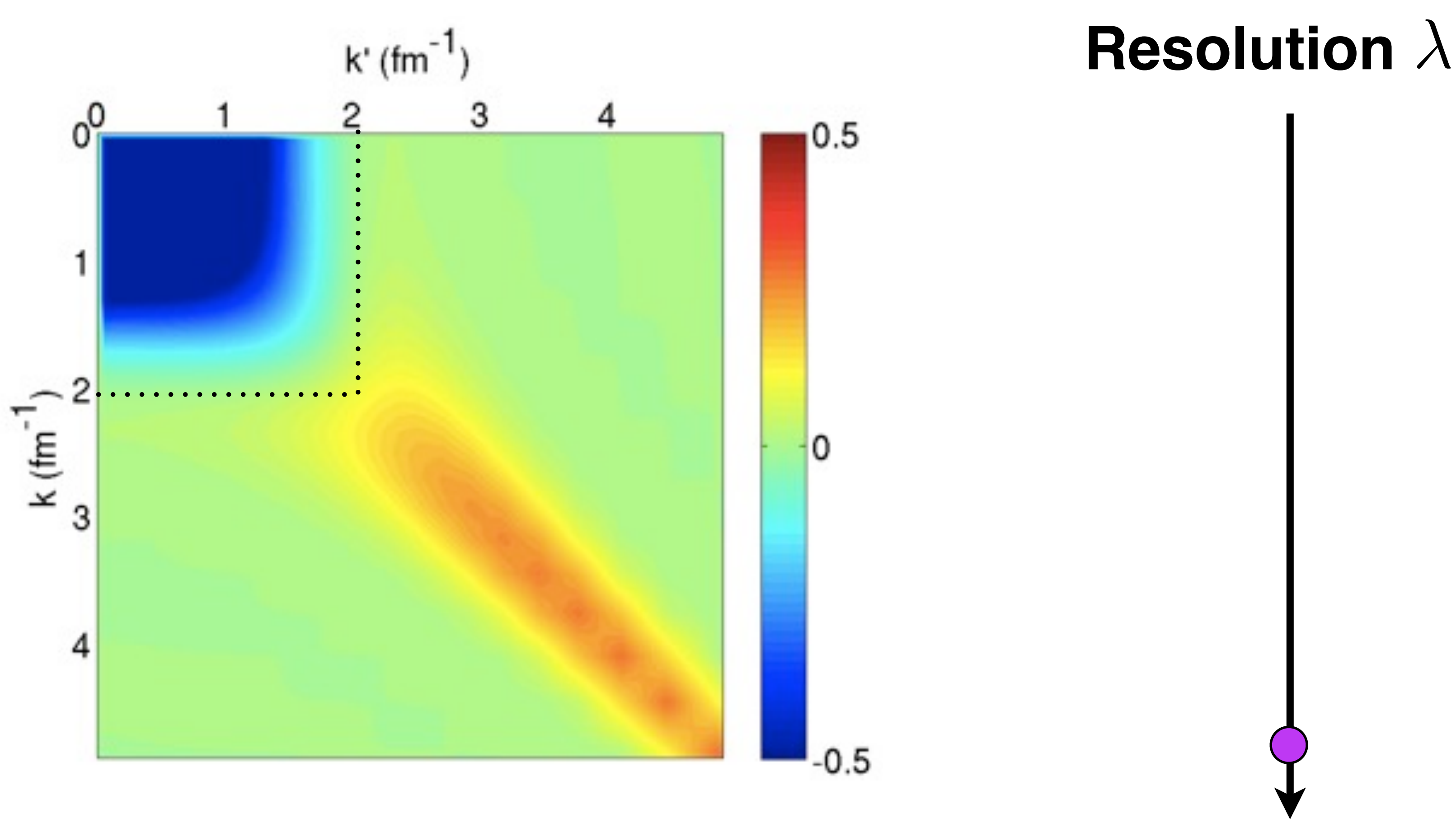
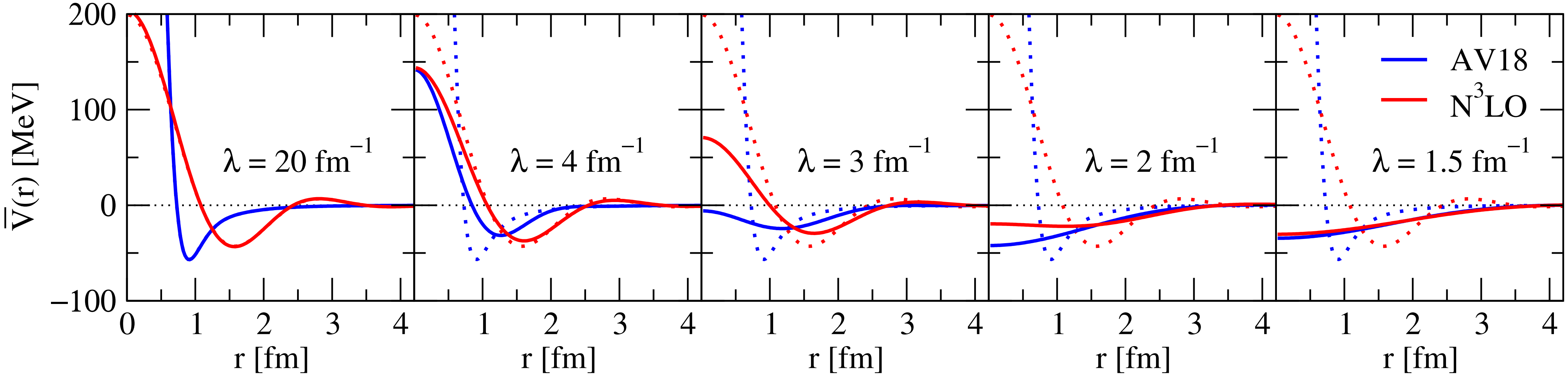
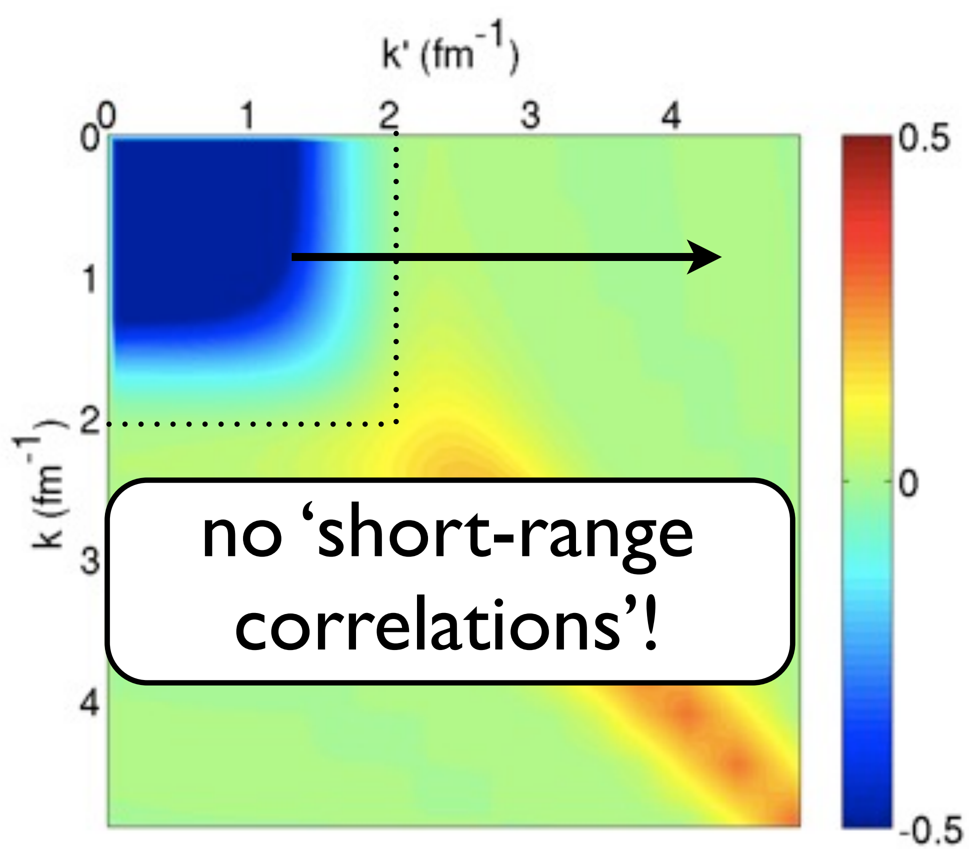


One solution: the Similarity Renormalization Group

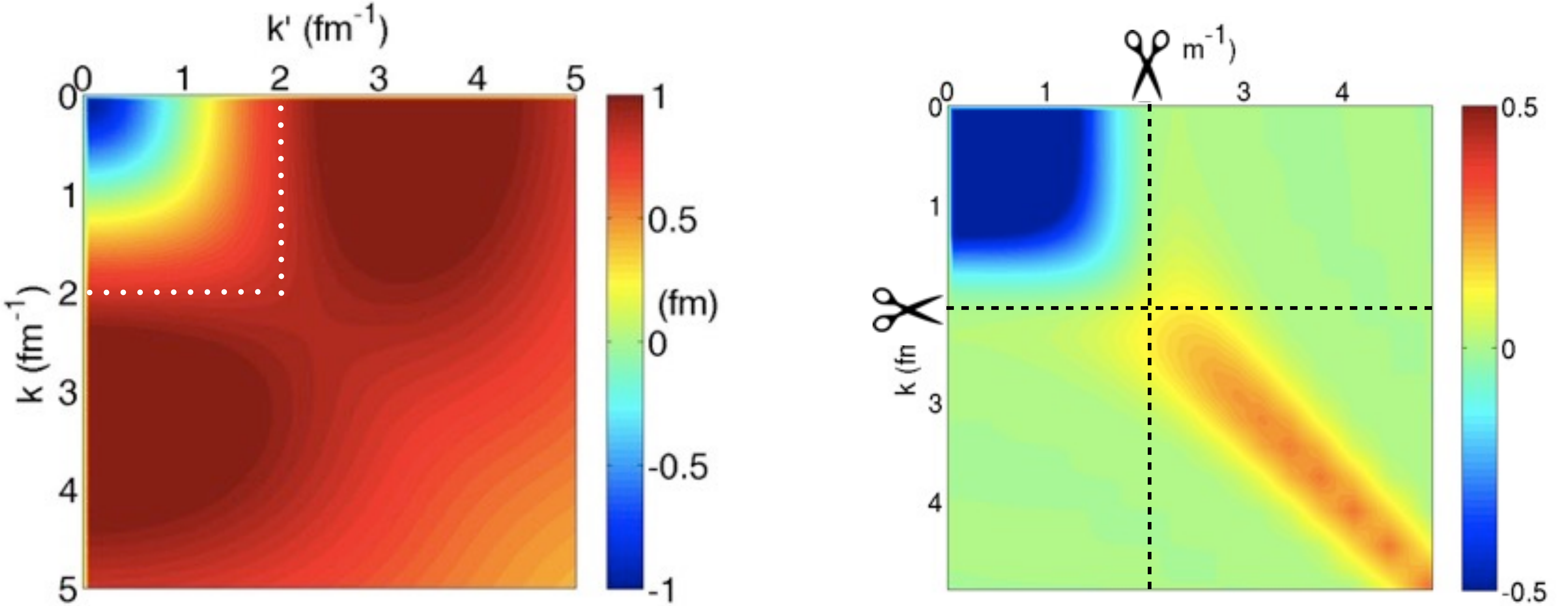
$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r, r')$$





Resolution λ

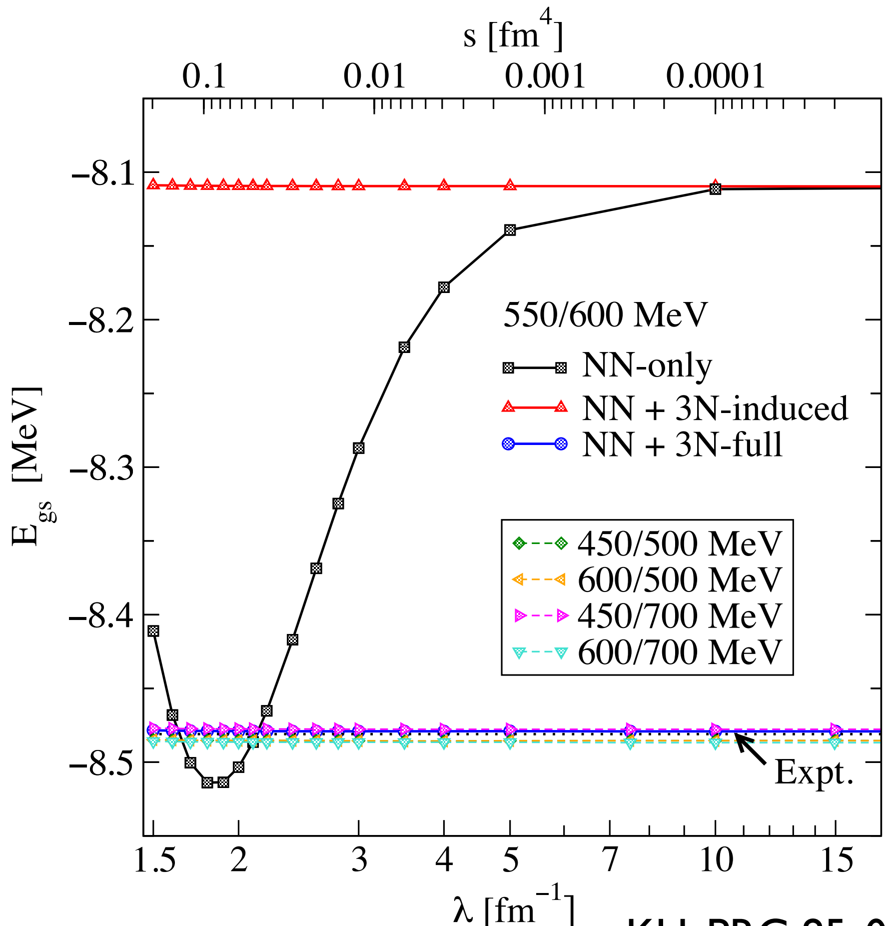
Systematic decoupling of high-momentum physics: the Similarity Renormalization Group



- elimination of coupling between low- and high momentum components,
  ⟶ simplified many-body calculations!
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformations also change three-body (and higher-body) interactions...

Application of SRG-evolved NN+3N forces to nuclei



KH, PRC 85, 021002 (2012)

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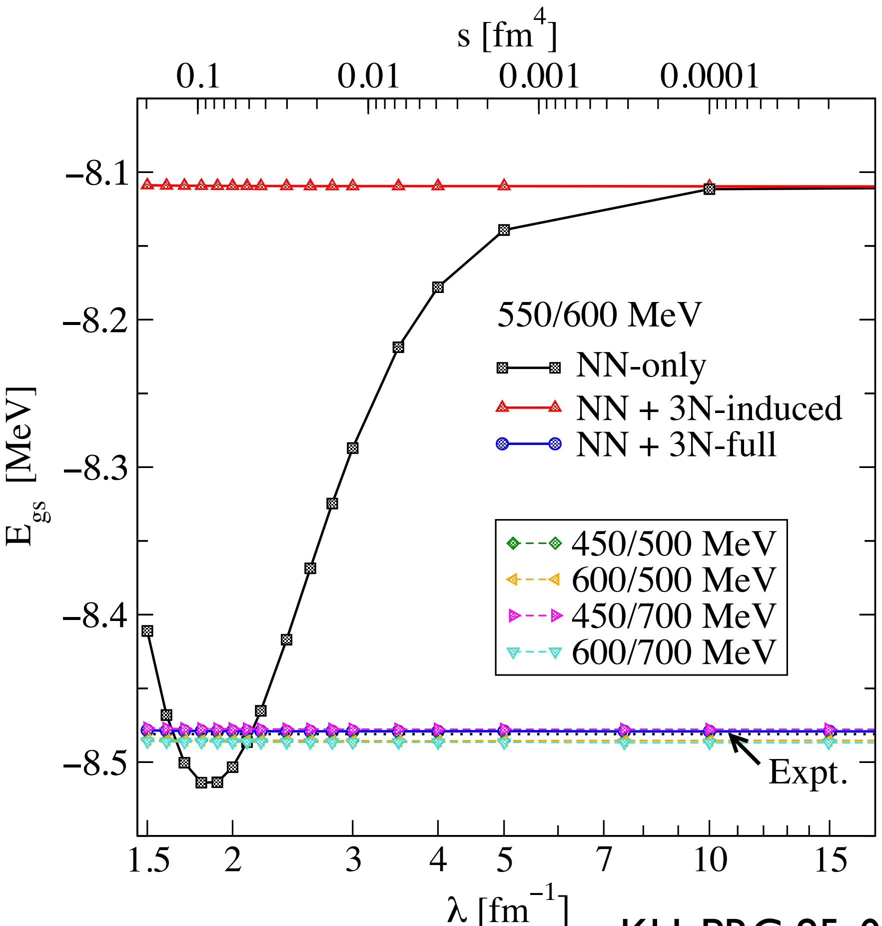

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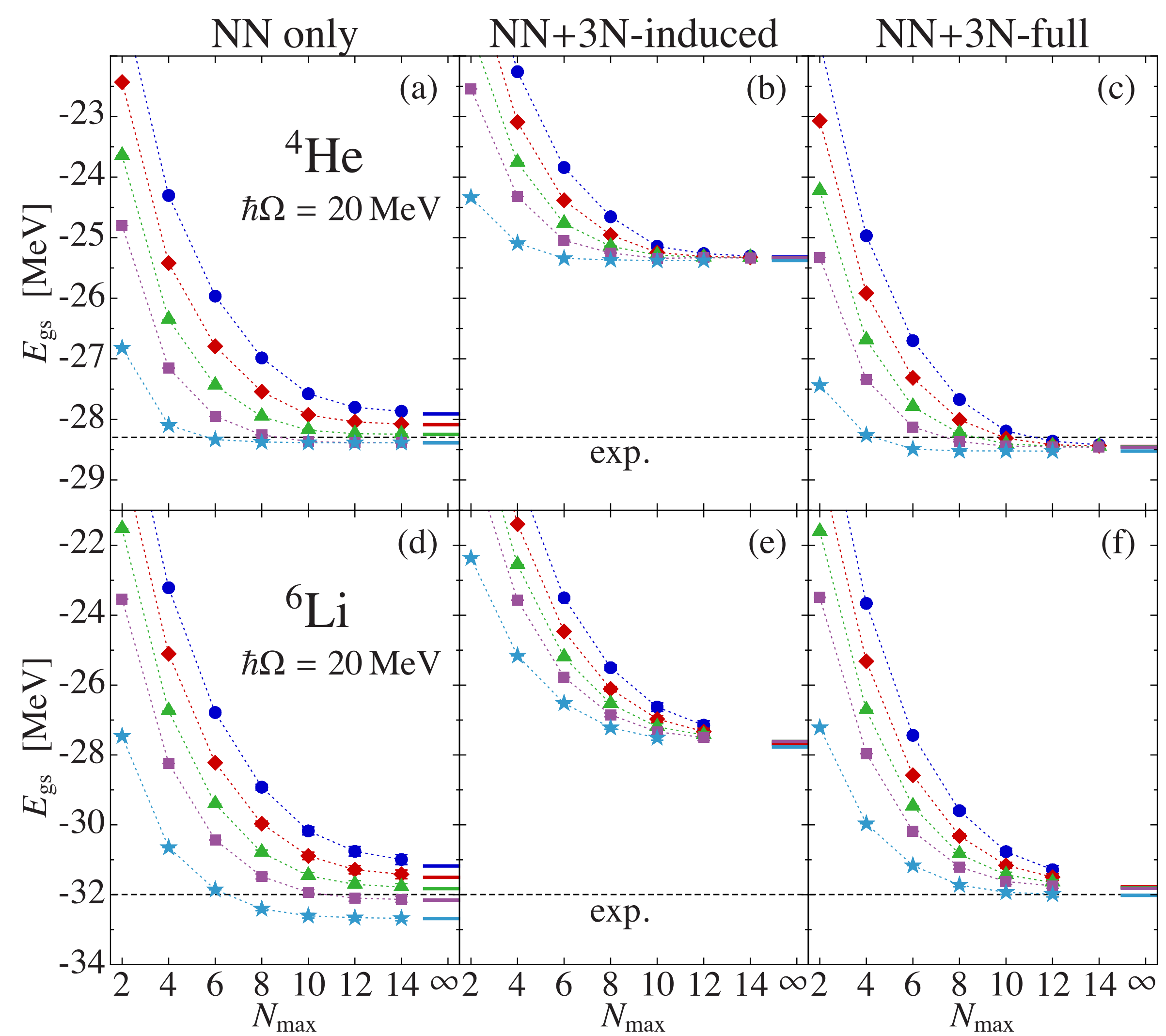

Roth et al. PRL 107, 072501 (2011)

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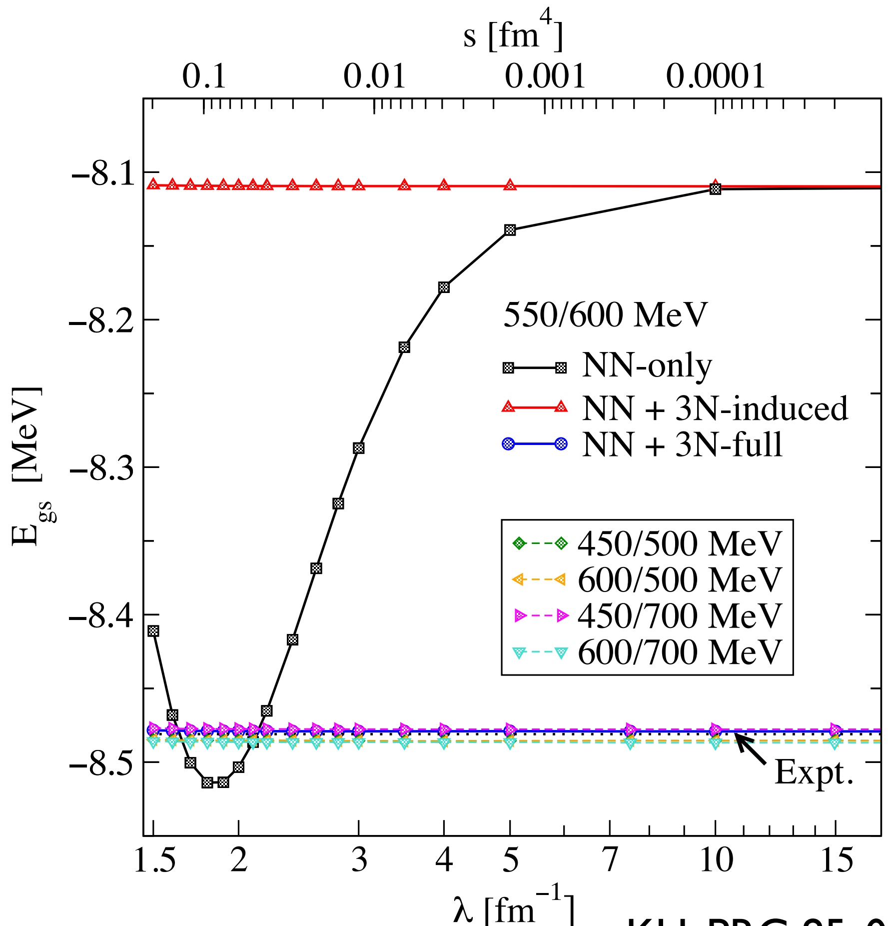

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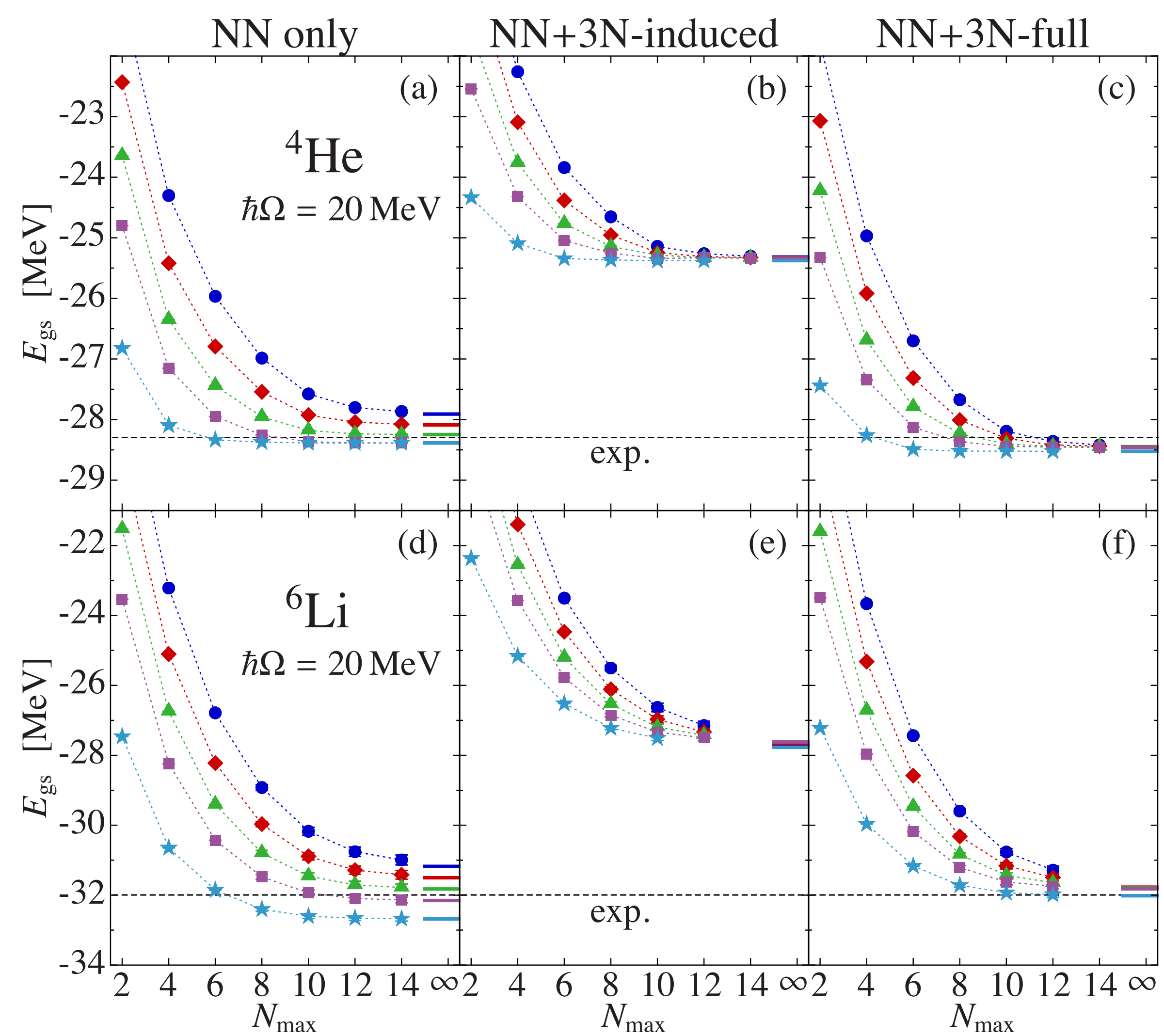

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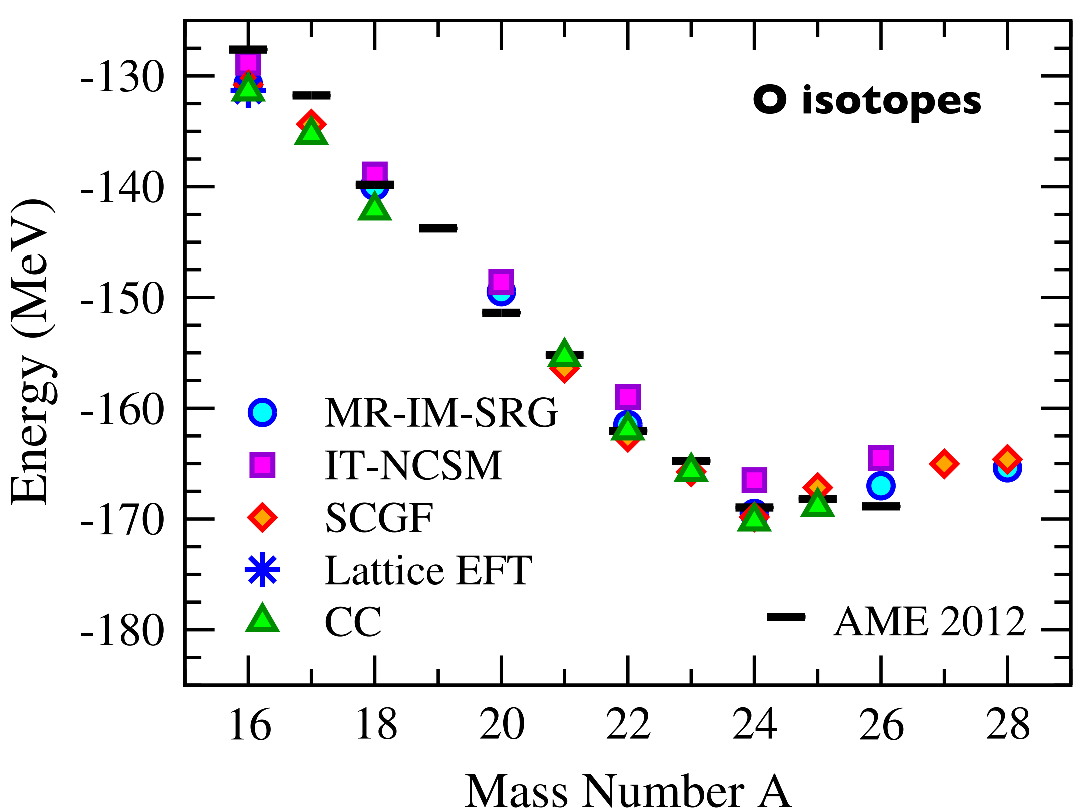

KH et al. , Ann. Rev. Nucl. Part. Sci.165, 457 (2015)

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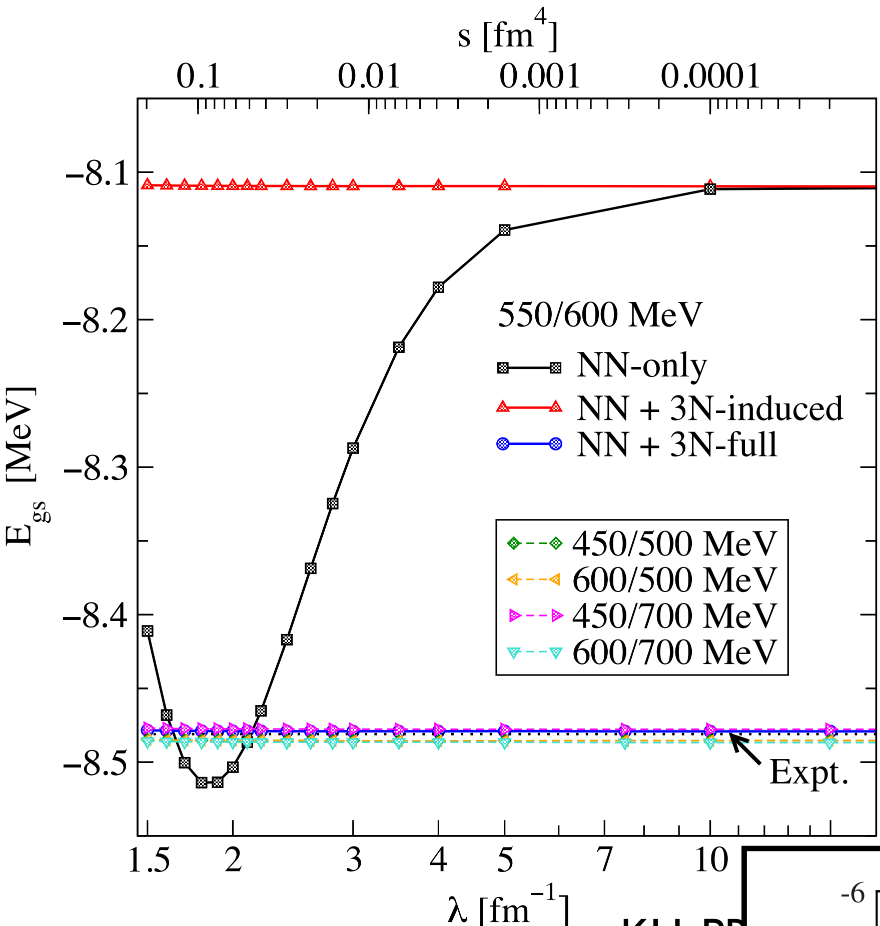
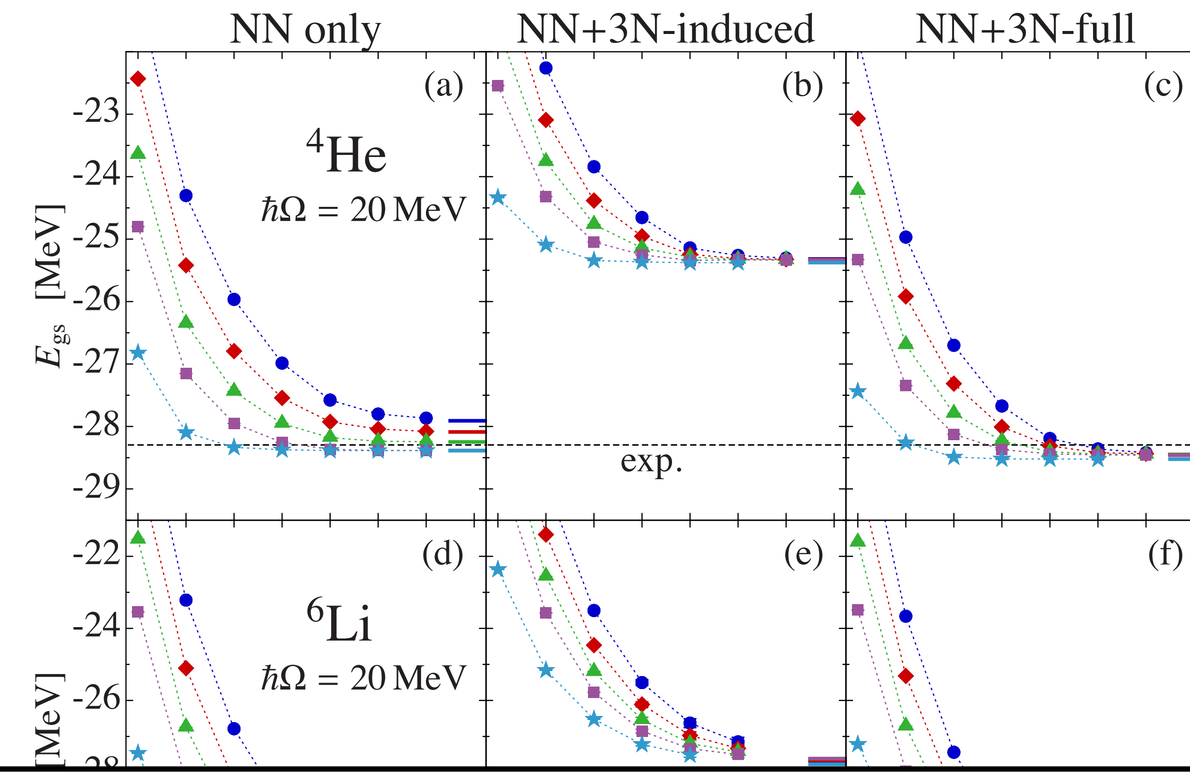
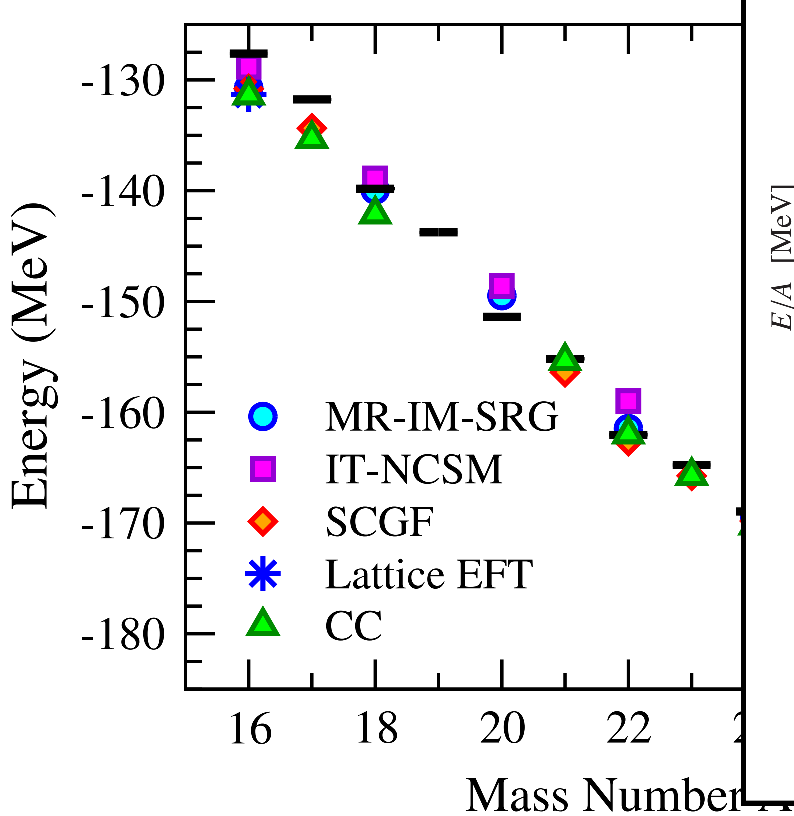
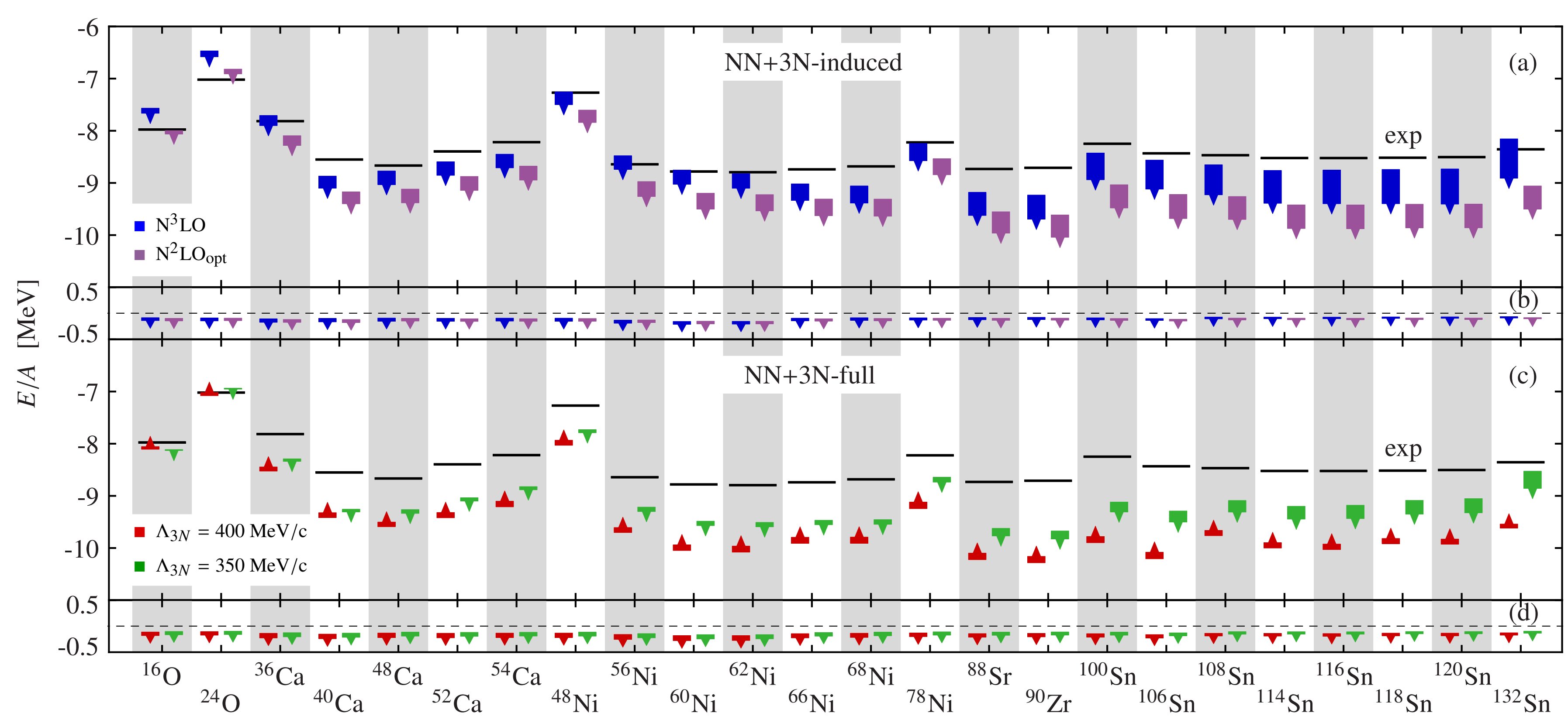


KH, PR

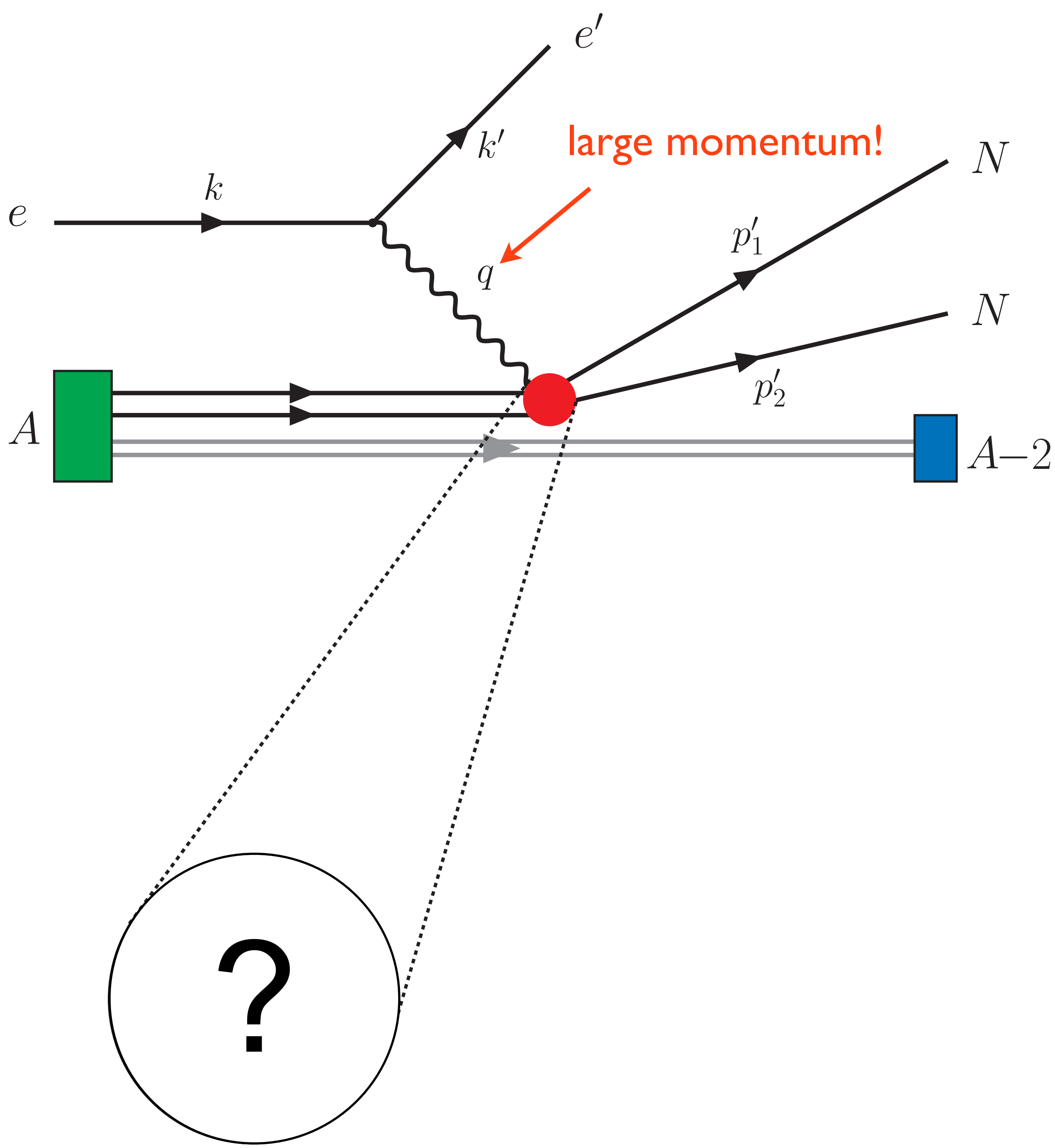


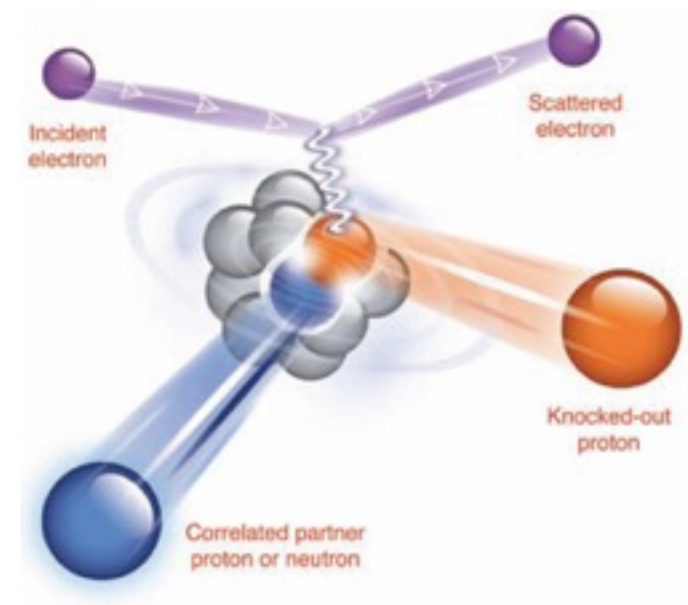




Binder et al. , Phys. Lett B 736, 119 (2014)

Application: deep-inelastic knock-out reactions





- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

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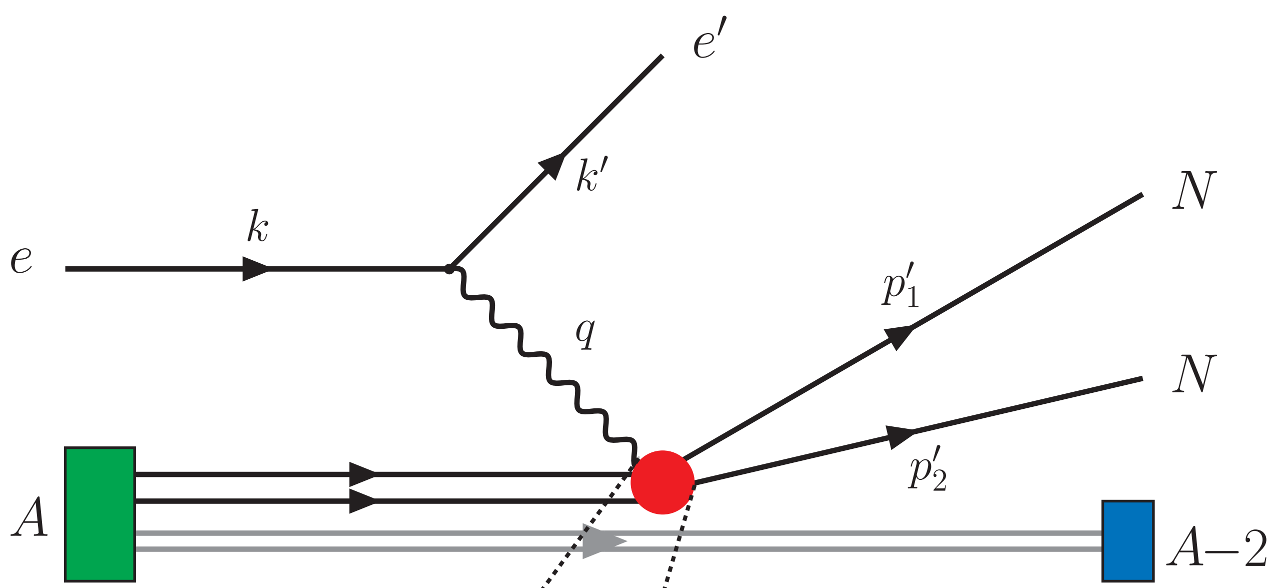

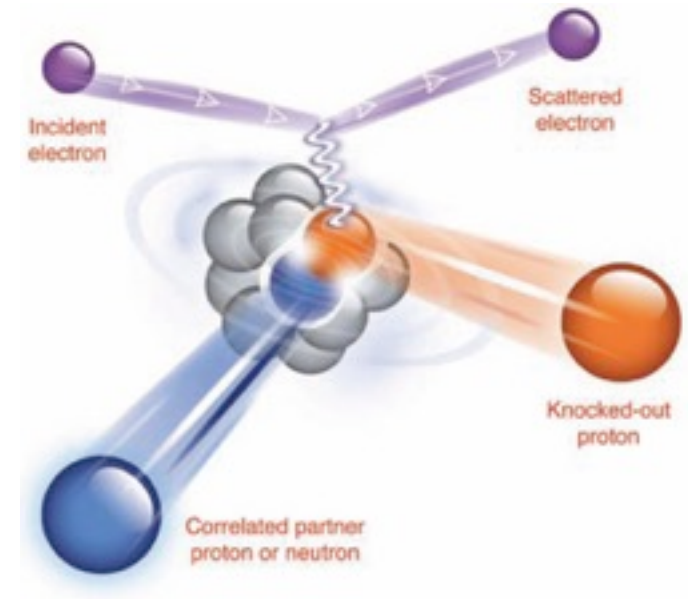

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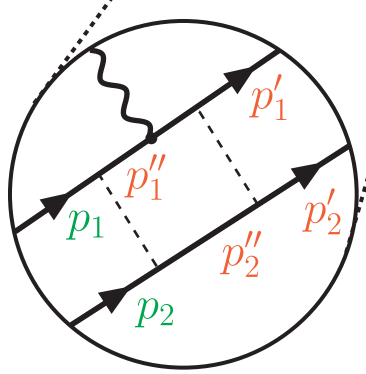
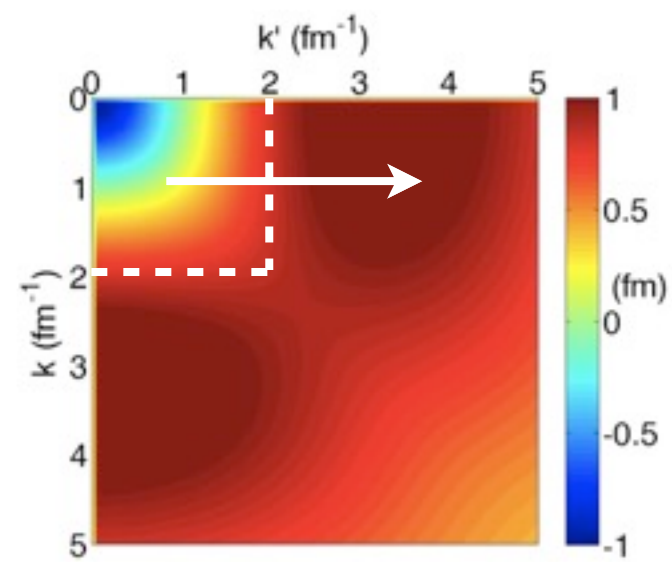
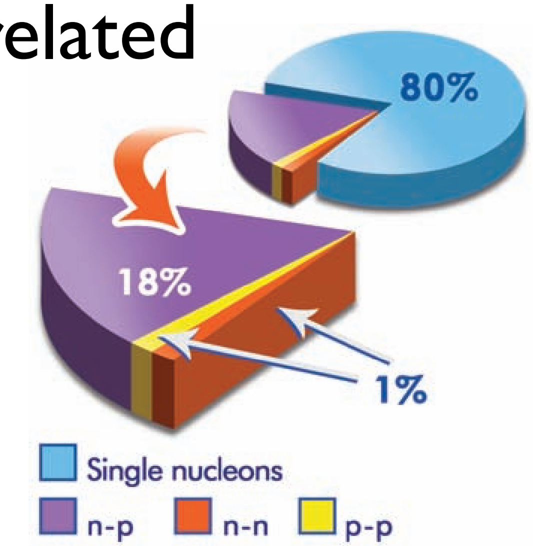
Short-range-correlation interpretation (SRC):

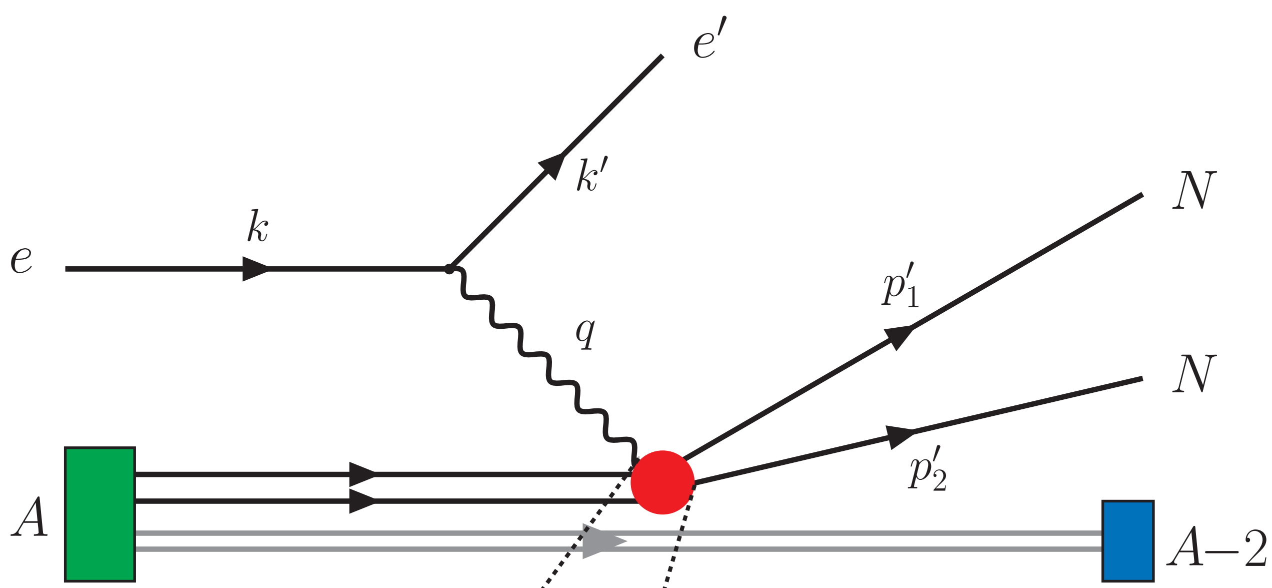


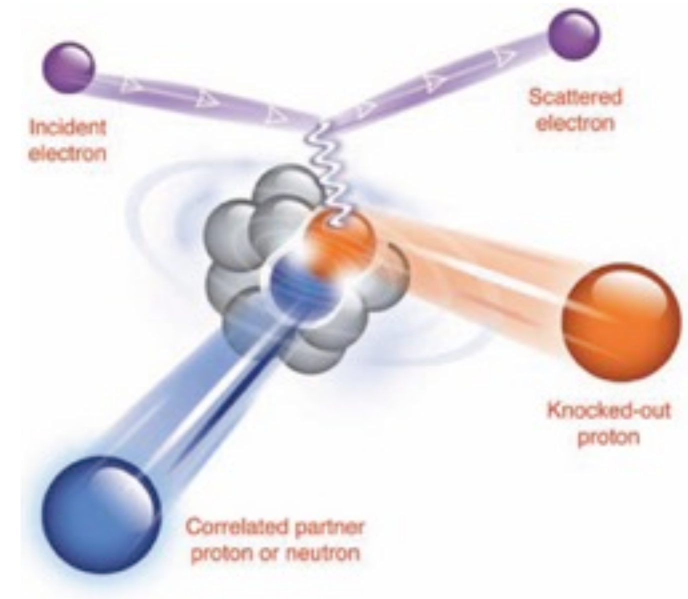


'measurement' of number of short range-correlated pairs in ^{12}C



Application: deep-inelastic knock-out reactions

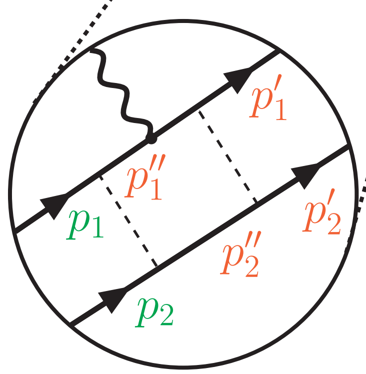


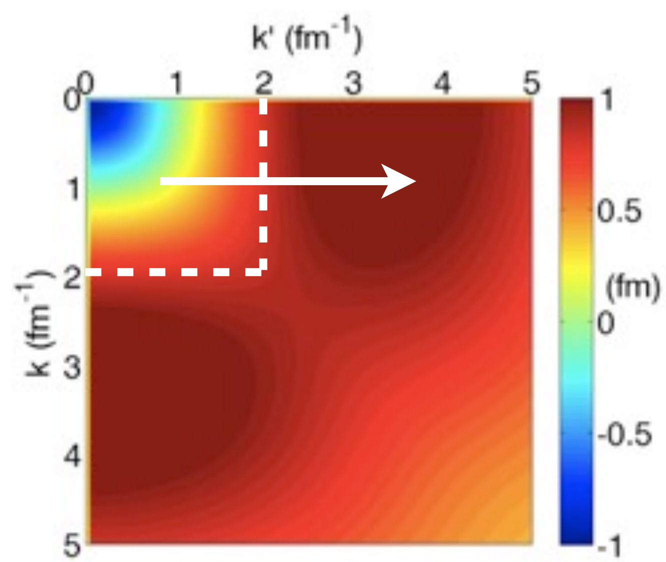


• detection of knocked out pairs with large relative momenta
• excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

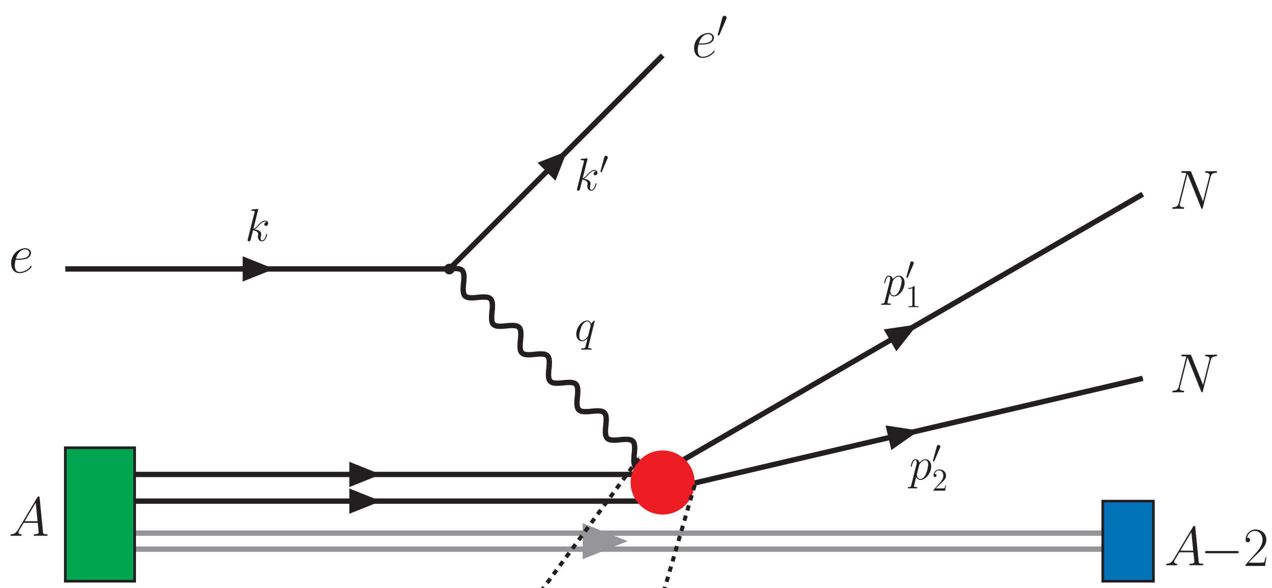
Short-range-correlation interpretation (SRC):

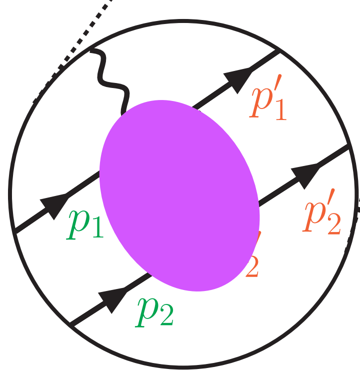




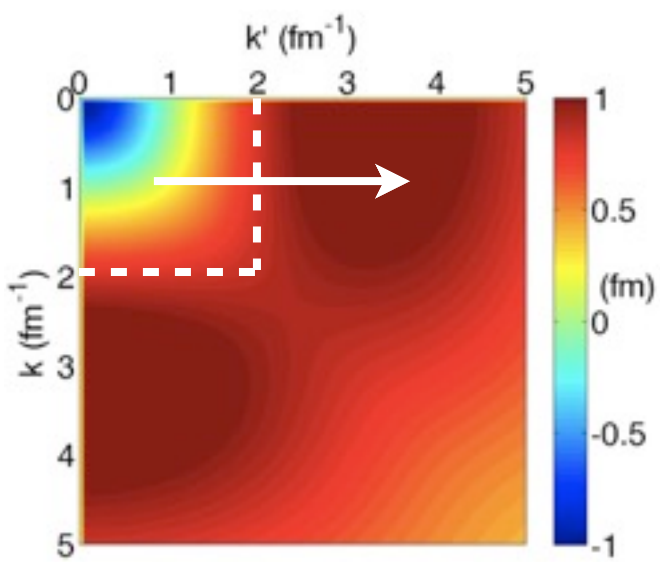
Explanation in terms of low-momentum interactions?

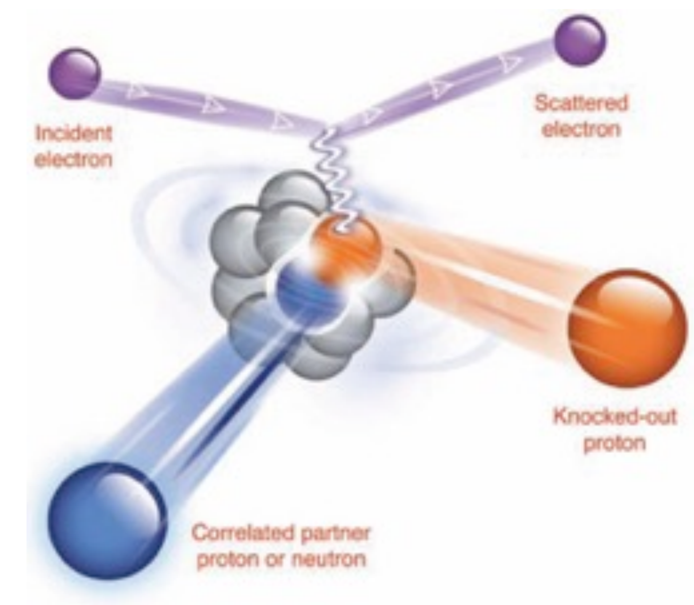
Application: deep-inelastic knock-out reactions





Short-range-correlation interpretation (SRC):





- detection of knocked out pairs with large relative momenta
- excess of np pairs over pp pairs

Subedi et al., Science 320, 1476 (2008)

Explanation in terms of low-momentum interactions?

Vertex depends on the resolution!
One-body current and SRC changes to two-body current and simple wave function.

Furnstahl, KH, Rep. Prog. Phys. 76, 126301 (2013)

RG evolution of operators

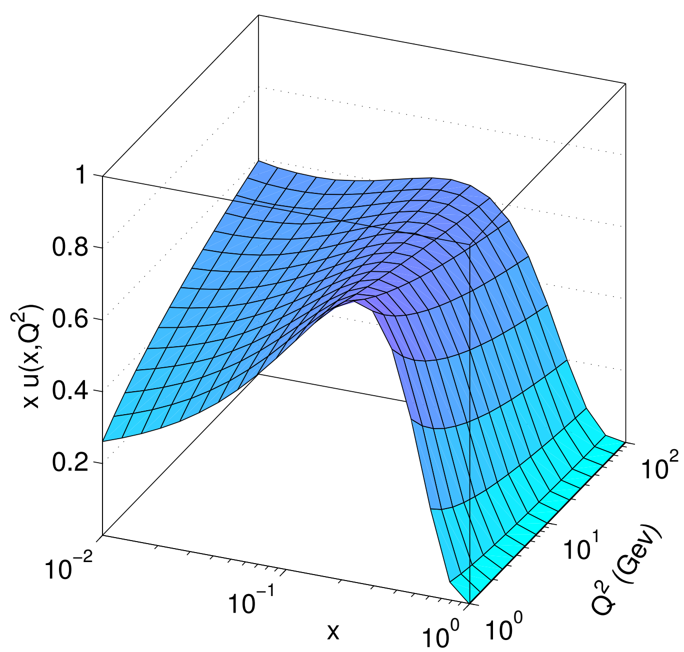
one unitary SRG transformation renormalizes all operators

$$O_\lambda = U_\lambda O U_\lambda^\dagger, \quad \frac{dO_\lambda}{d\lambda} = [\eta_\lambda, O_\lambda]$$

$\Rightarrow$ quantities like momentum distributions are generally scale dependent:

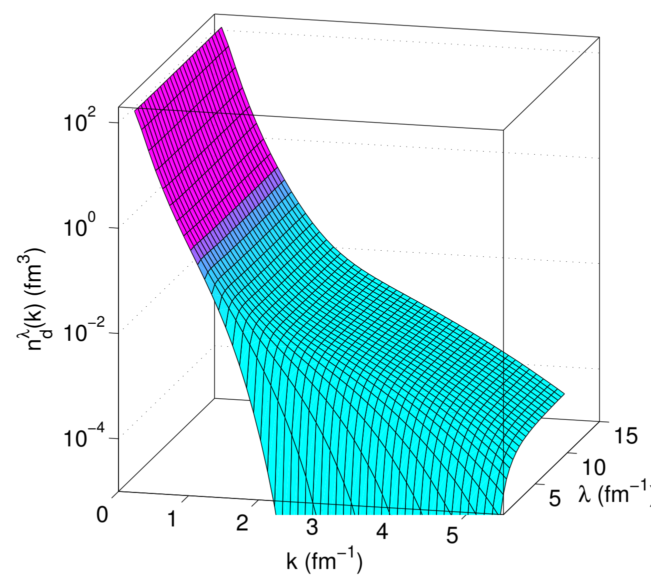
quark parton distribution

$xq(x, Q^2)$



deuteron momentum distribution

$n(k, \lambda)$

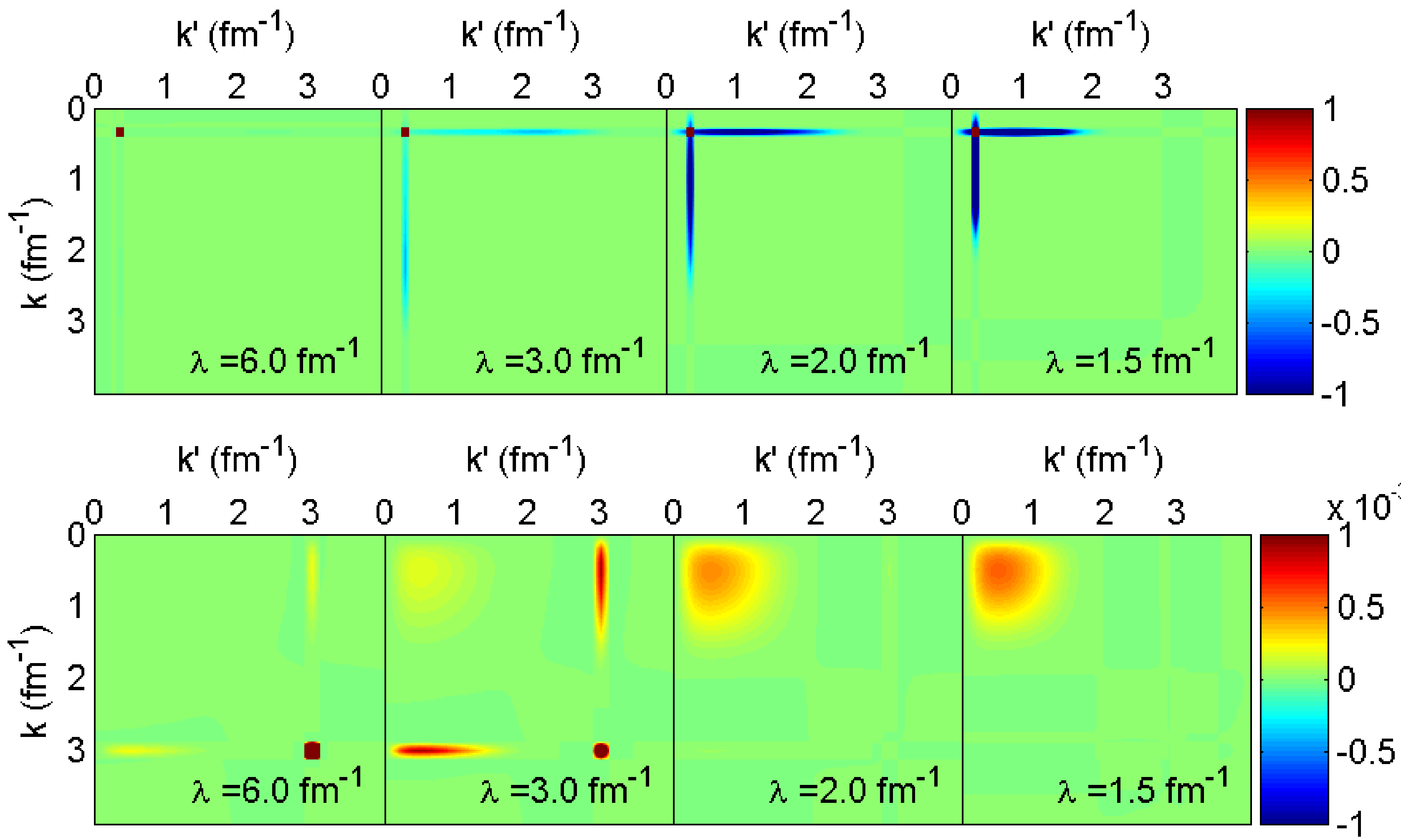


Furnstahl, KH, Rep. Prog. Phys. 76, 126301 (2013)

- applies generally to all quantities like spectroscopic factors, short-range corr.,...
- consistency requires consistent RG evolution of reaction and structure parts
- key for all momenta involving high-momentum components

Evolved density operator in the deuteron

investigate $\langle \psi_D(\lambda) | U(\lambda) a_q^\dagger a_q U^\dagger(\lambda) | \psi_D(\lambda) \rangle$



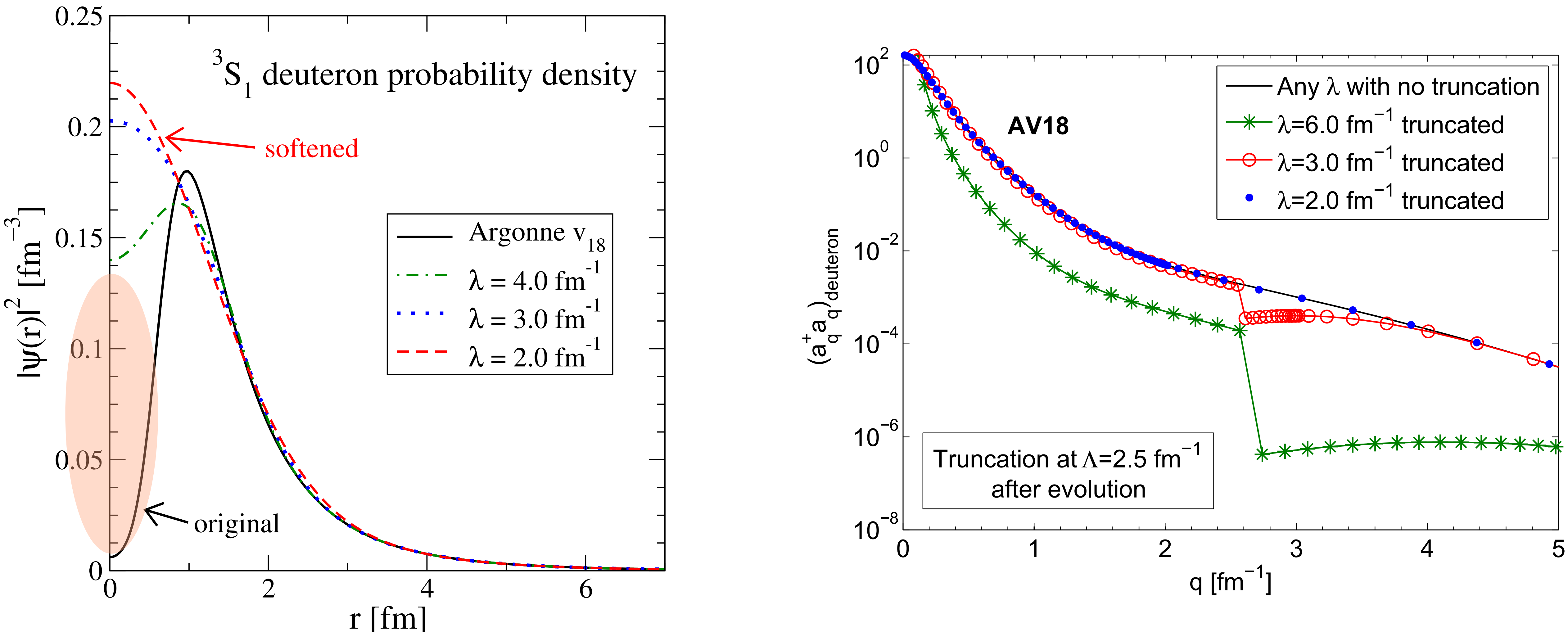
$q = 0.32\ \mathrm{fm}^{-1}$

$q = 3.0\ \mathrm{fm}^{-1}$

Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 (2010)

- for low-momentum operators RG evolution provides only small corrections
- for high-momentum operators induced two-body contributions at small momenta completely dominate contribution at small resolution scales

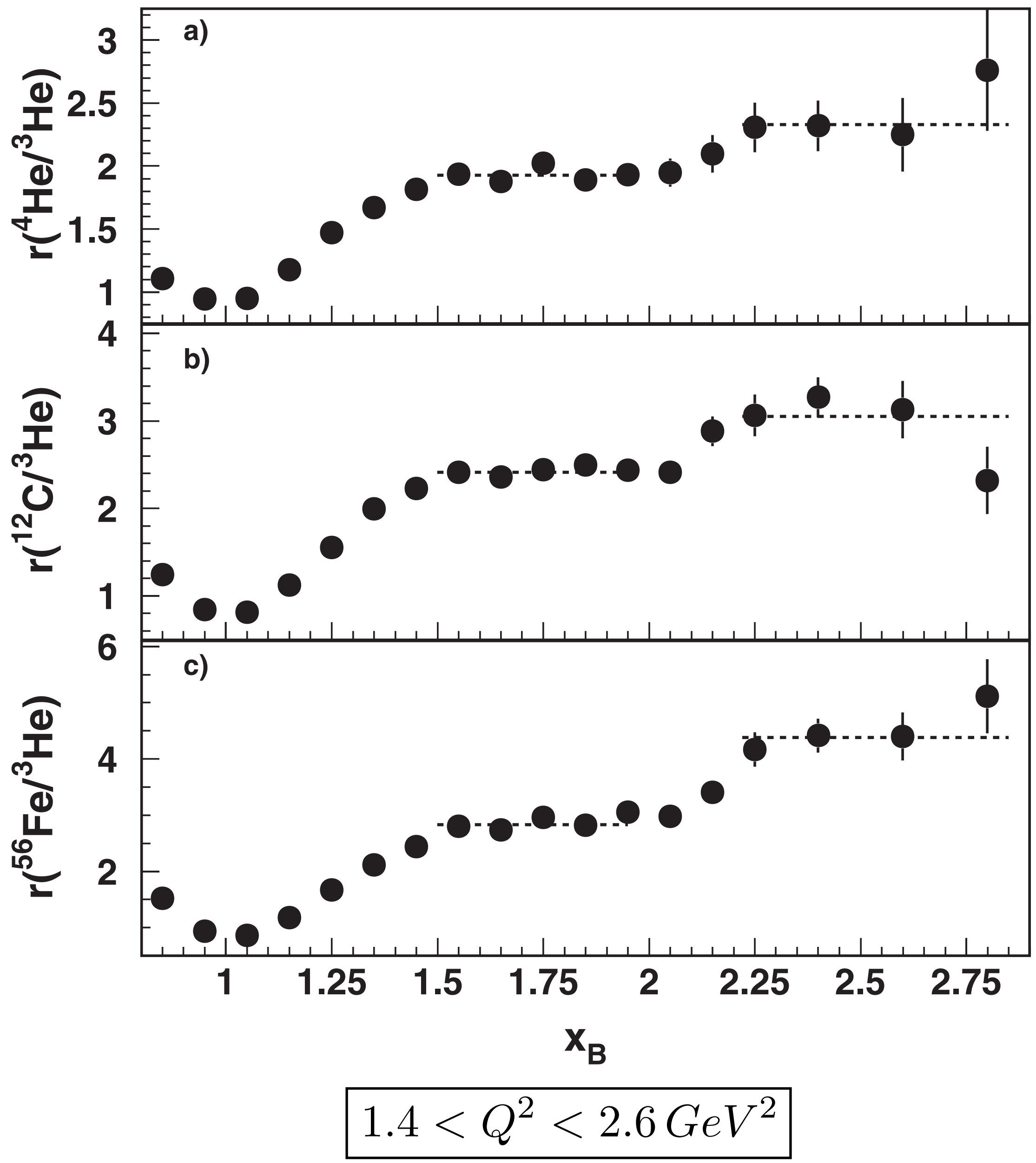
Evolved density operator in the deuteron



Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 (2010)

- short-distance correlations in wave function at very resolution dependent
- perfect invariance of momentum distribution function with evolved density operator
- $U_\lambda(k,q)$ factorizes for $k < \lambda$ and $q \gg \lambda$: $U_\lambda(k,q) \approx K_\lambda(k) Q_\lambda(q)$

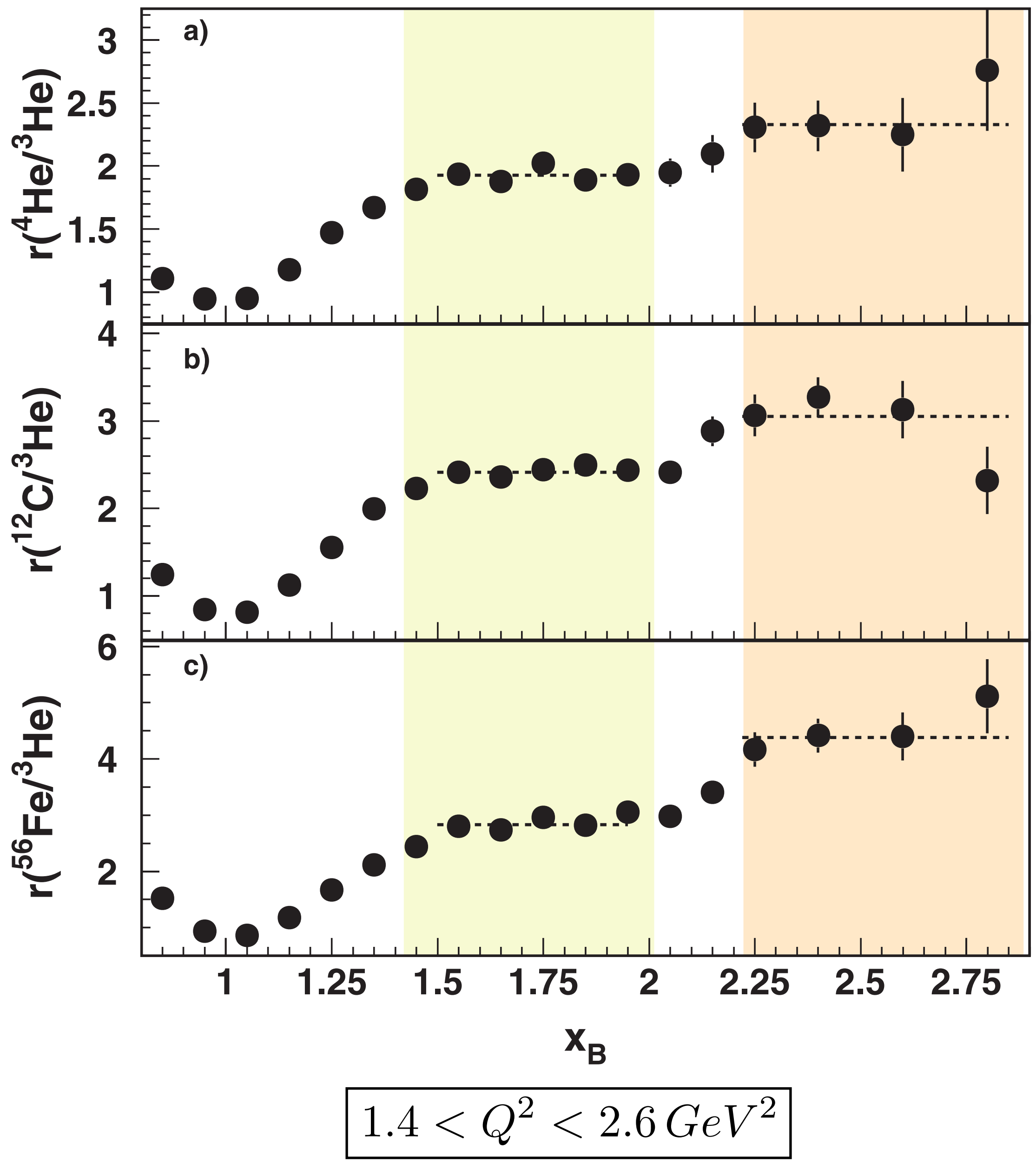
Electron inclusive cross sections at high momentum: Scaling in nuclear systems



Egiyan et al. PRL 96, 1082501 (2006)

- idea: high-energy electrons probe the high-momentum part/small distance part of the nucleon wave function
- the form of the high-momentum tail should be universal since it is dominated by short-range correlations
- universality shows up as a plateaus in ratios of cross sections

Electron inclusive cross sections at high momentum: Scaling in nuclear systems



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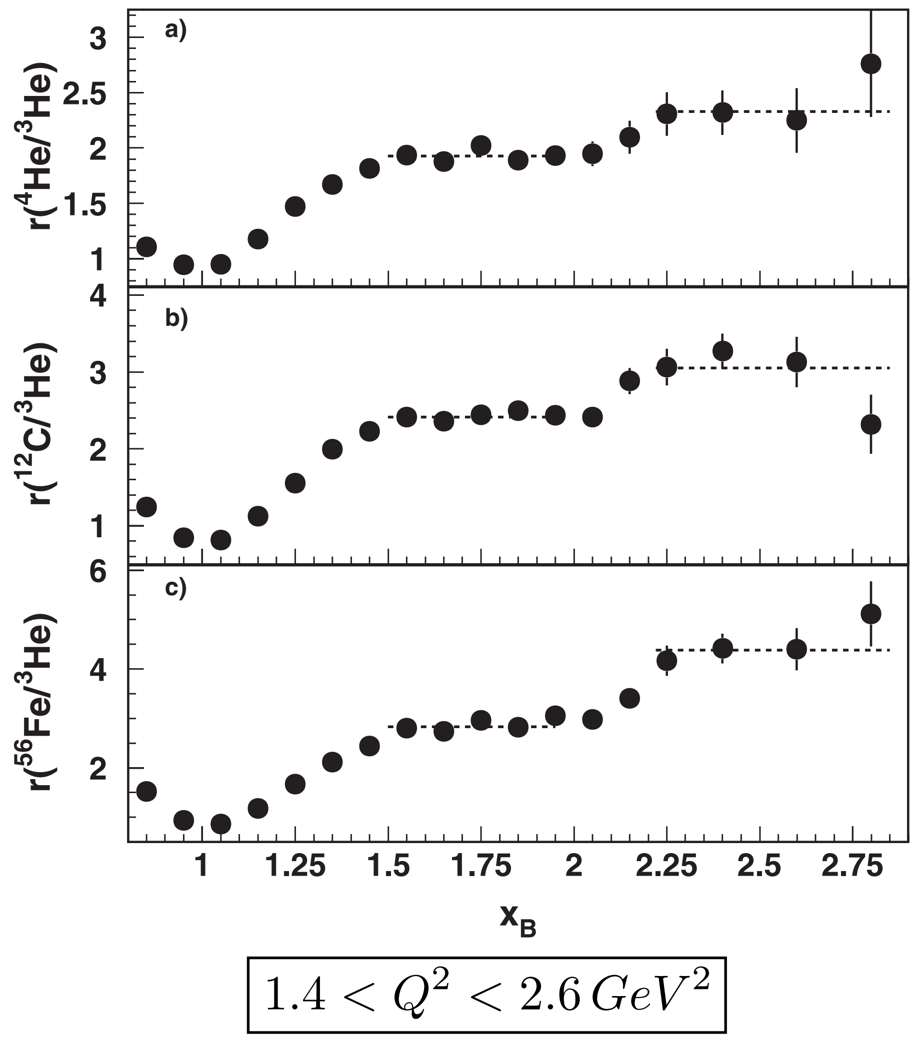
- idea: high-energy electrons probe the high-momentum part/small distance part of the nucleon wave function
- the form of the high-momentum tail should be universal since it is dominated by short-range correlations
- universality shows up as a plateaus in ratios of cross sections
- kinematically, 2-body correlations are limited to $x_B \sim 1$ to 2 and 3-body correlations to $x_B \sim 2$ to 3

Scaling in nuclear systems



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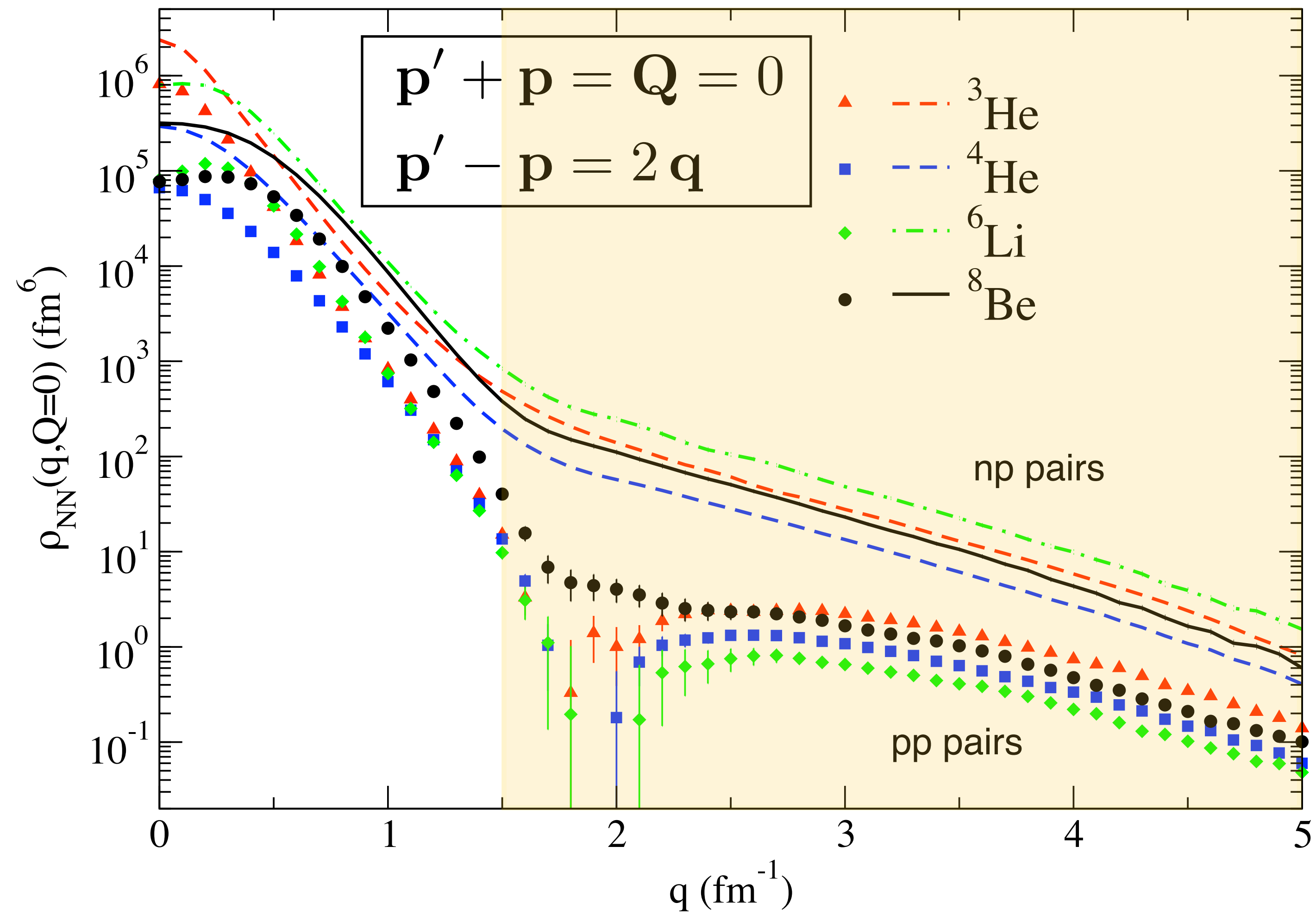


Schiavilla et al., PRL 98, 132501 (2007)

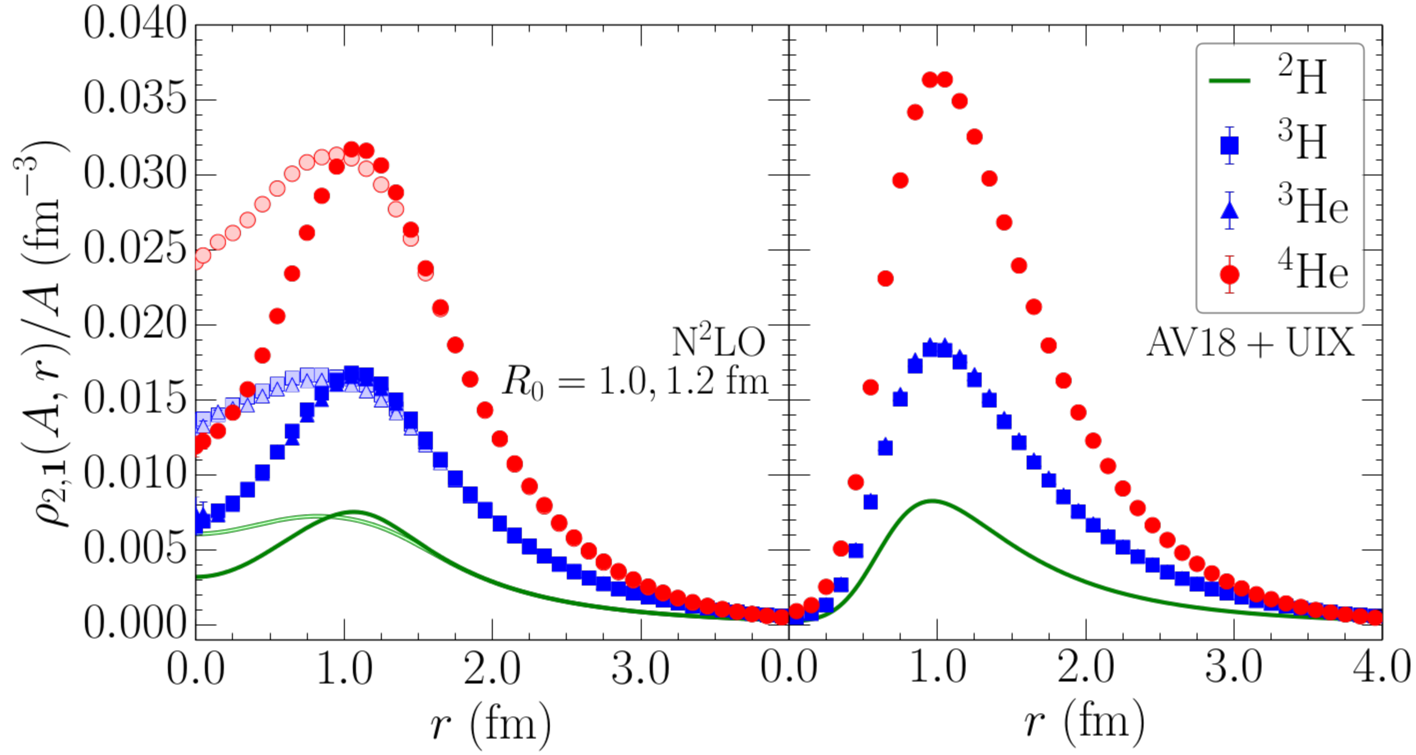
- scaling behavior of momentum distribution function:

  $\rho_{\mathrm{NN}}(q, Q=0) \approx C_A \times \rho_{\mathrm{NN,Deuteron}}(q, Q=0)$ at large q

- dominance of np pairs over pp pairs
- “hard” (high resolution) interaction used
- dominance explained by short-range tensor forces

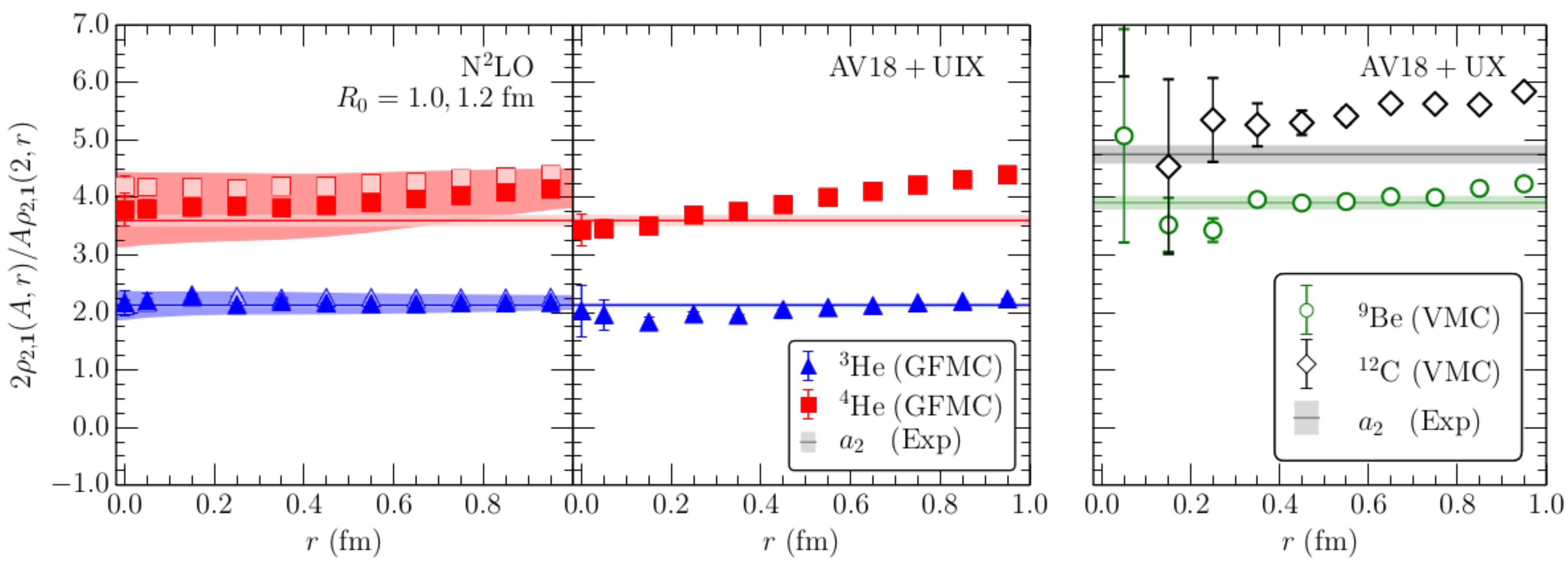
Nuclear scaling within chiral EFT

$$P \gg \Lambda_b \gg m_\pi \Rightarrow F_2^A(x, Q^2) \sim g_2(A, \Lambda) f_2(x, Q^2, \Lambda)$$



two-body distribution functions scheme and scale dependent!

Chen, Detmold, Lynn, Schwenk, arXiv: 1607.03065 (2016)



...ratios are not!

SRC Correlation Factors

Detailed comparison of experiment and theory

	N^2LO ($R_0 = 1.0 - 1.2$ fm)	AV18+UIX	Exp
^3H	2.1(2) – 2.3(3)	2.0(4)	
^3He	2.1(2) – 2.1(3)	2.0(4)	2.13(4)
^4He	3.8(7) – 4.2(8)	3.4(3)	3.60(10)

Nuclear scaling at low resolution

$\langle\psi_\lambda|O_\lambda|\psi_\lambda\rangle$ **factorizes** into a low-momentum structure and a **universal** high momentum part if the initial operator only weakly couples low and high momenta ⟶ explains scaling!

key: $U_\lambda(k,q) \approx K(k)Q(q)$ for $k < \lambda$ and $q \gg \lambda$

factorization!

That leads to:

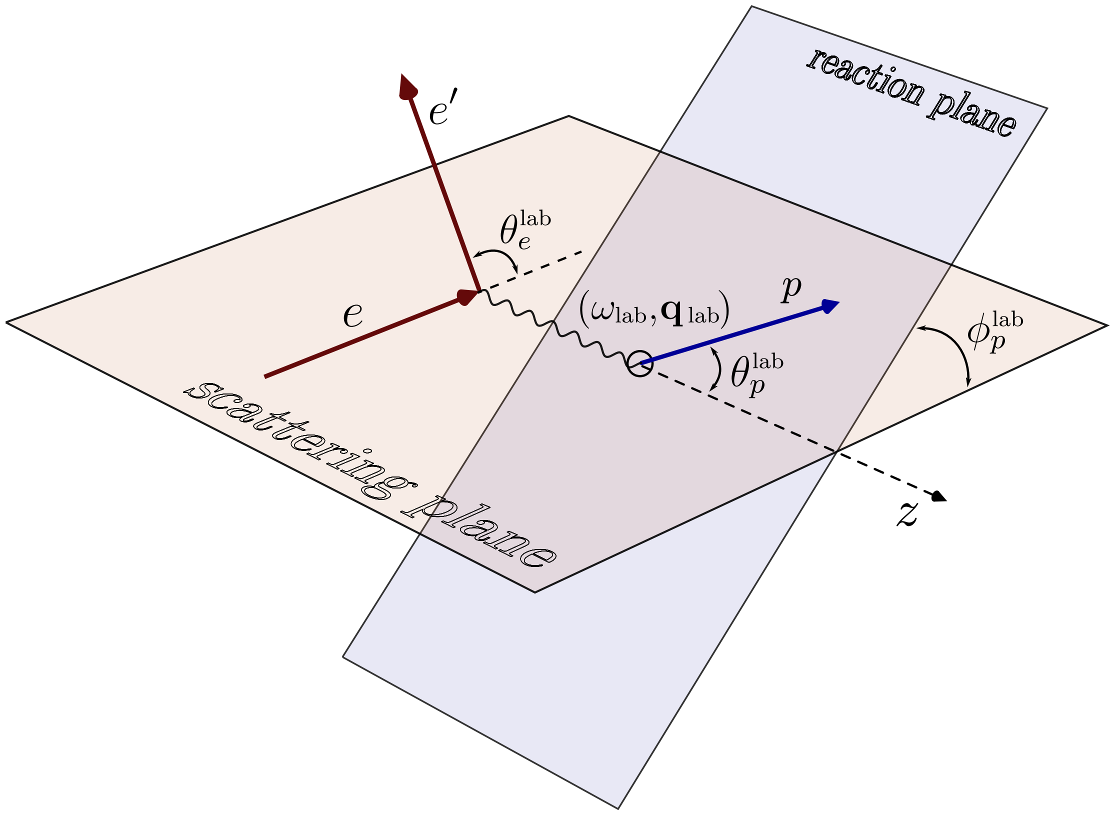
$$\begin{aligned}\langle\psi_\lambda|\,O_\lambda\,|\psi_\lambda\rangle &= \int_0^\lambda dk\,dk' \int_0^\infty dq\,dq' \psi^\dagger(k) U_\lambda(k,q) O(q,q') U_\lambda(q',k') \psi_\lambda(k') \\ &\approx \int_0^\lambda dk\,dk'\,\psi_\lambda^\dagger(k) \left[\int_0^\lambda dq\,dq' K(k)K(q)O(q,q')K(q')K(k') + I_{QOQ}K(k)K(k')\right]\psi_\lambda(k')\end{aligned}$$

with the **universal** quantity:

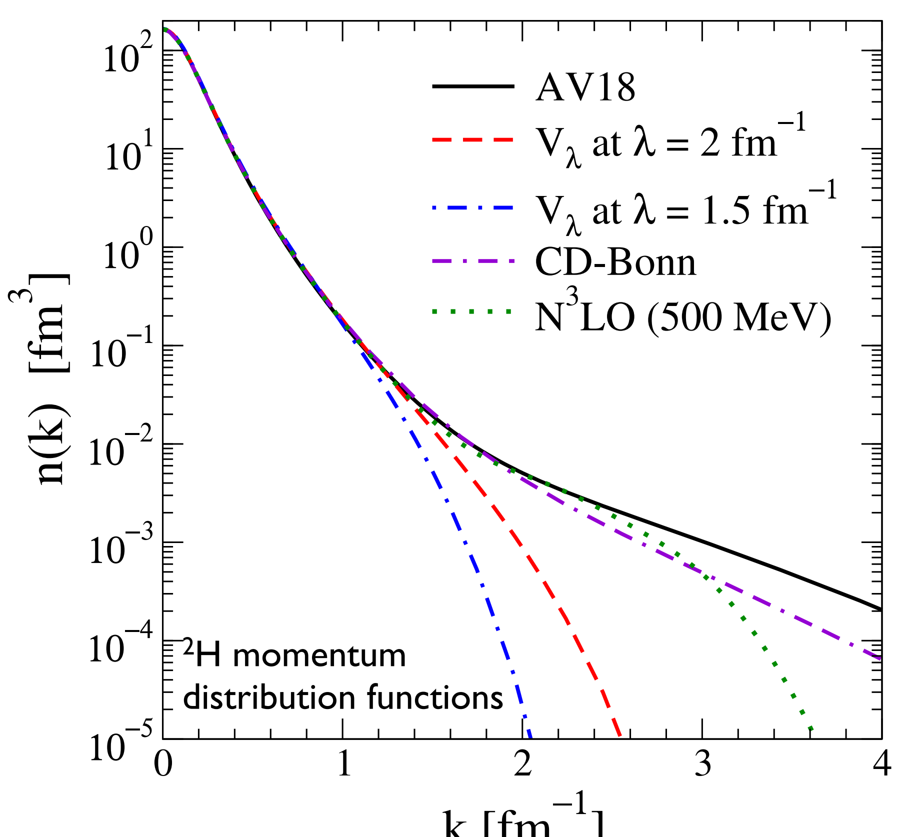
$$I_{QOQ} = \int_\lambda^\infty dq\,dq' Q(q)O(q,q')Q(q')$$

valid if initial operator weakly couples low and high momenta

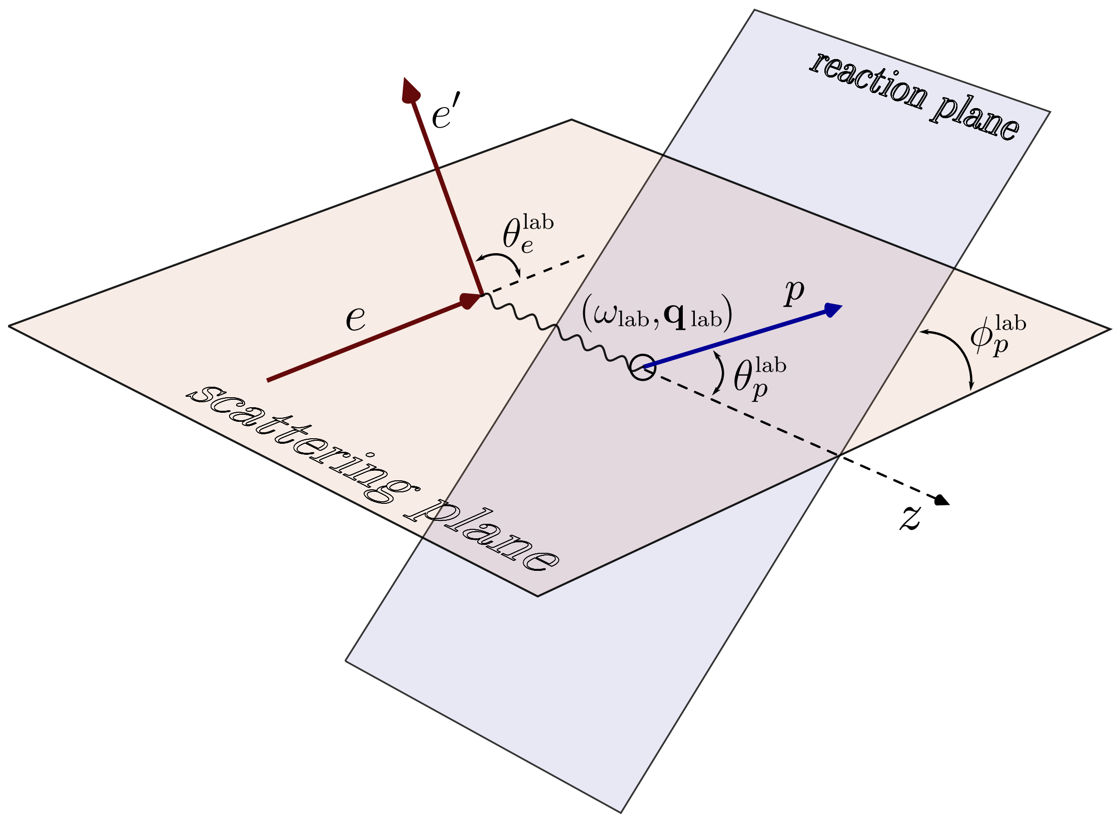
Deuteron disintegration at low resolution scales



More, König, Furnstahl, KH,
PRC 92, 064002 (2015)



Deuteron disintegration at low resolution scales



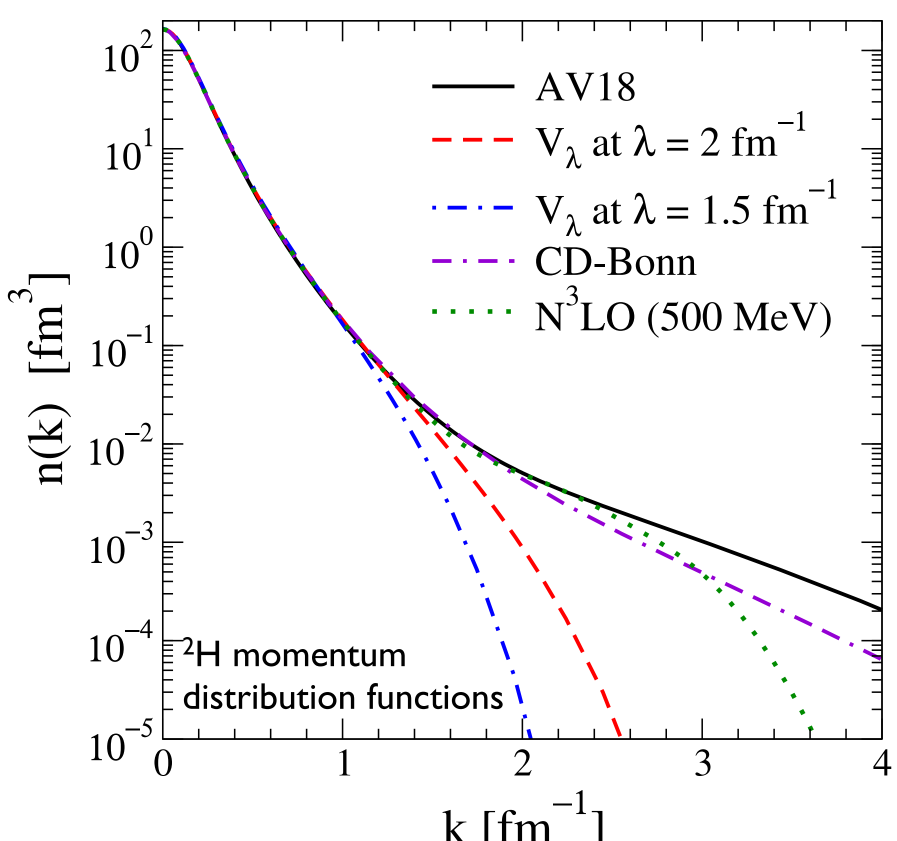
More, König, Furnstahl, KH, PRC 92, 064002 (2015)

consider initial one-body current:

$$\langle \mathbf{k}_1\, T_1 |\, J_0(\mathbf{q}) \,|\, \mathbf{k}_2\, T = 0 \rangle = \frac{1}{2}\left(G_E^p + (-1)^{T_1} G_E^n\right) \delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{q}/2) + \frac{1}{2}\left((-1)^{T_1} G_E^p + G_E^n\right) \delta(\mathbf{k}_1 - \mathbf{k}_2 + \mathbf{q}/2)$$

$$\langle \psi_f |\, J_0 \,| \psi_i \rangle = \underbrace{\langle \phi |\, J_0 \,| \psi_i \rangle}_{\text{IA}} + \underbrace{\langle \phi | t^\dagger\, G_0^\dagger\, J_0 \,| \psi_i \rangle}_{\text{FSI}}$$
$$= \left\langle \psi_f | U^\dagger U J_0 U^\dagger U | \psi_i \right\rangle$$
$$= \left\langle \psi_f^\lambda | J_0^\lambda | \psi_i^\lambda \right\rangle$$

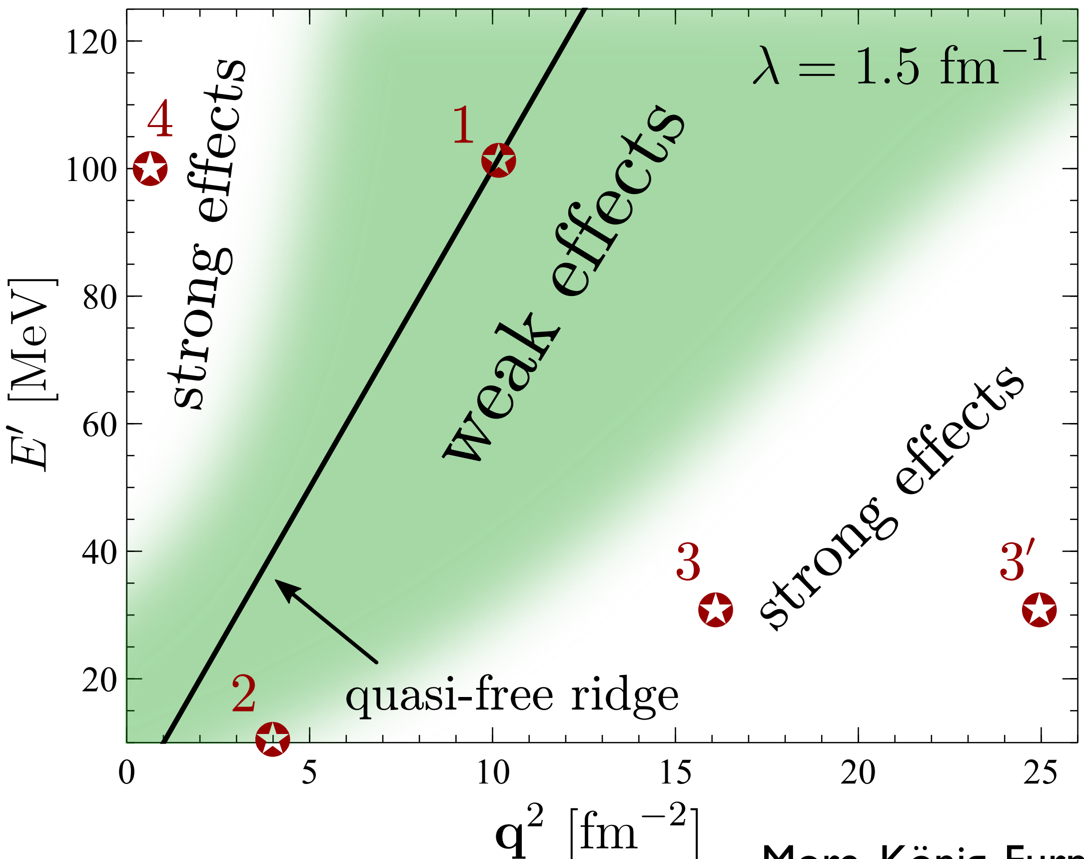
$$\mathcal{T}_{S, m_{s_f}, \mu, m_{J_d}} = -\pi \sqrt{2\alpha |\mathbf{p}'| E_p E_d / M_d}\, \langle \psi_f \,|\, J_\mu(\mathbf{q}) \,|\psi_i \rangle$$



study longitudinal structure function

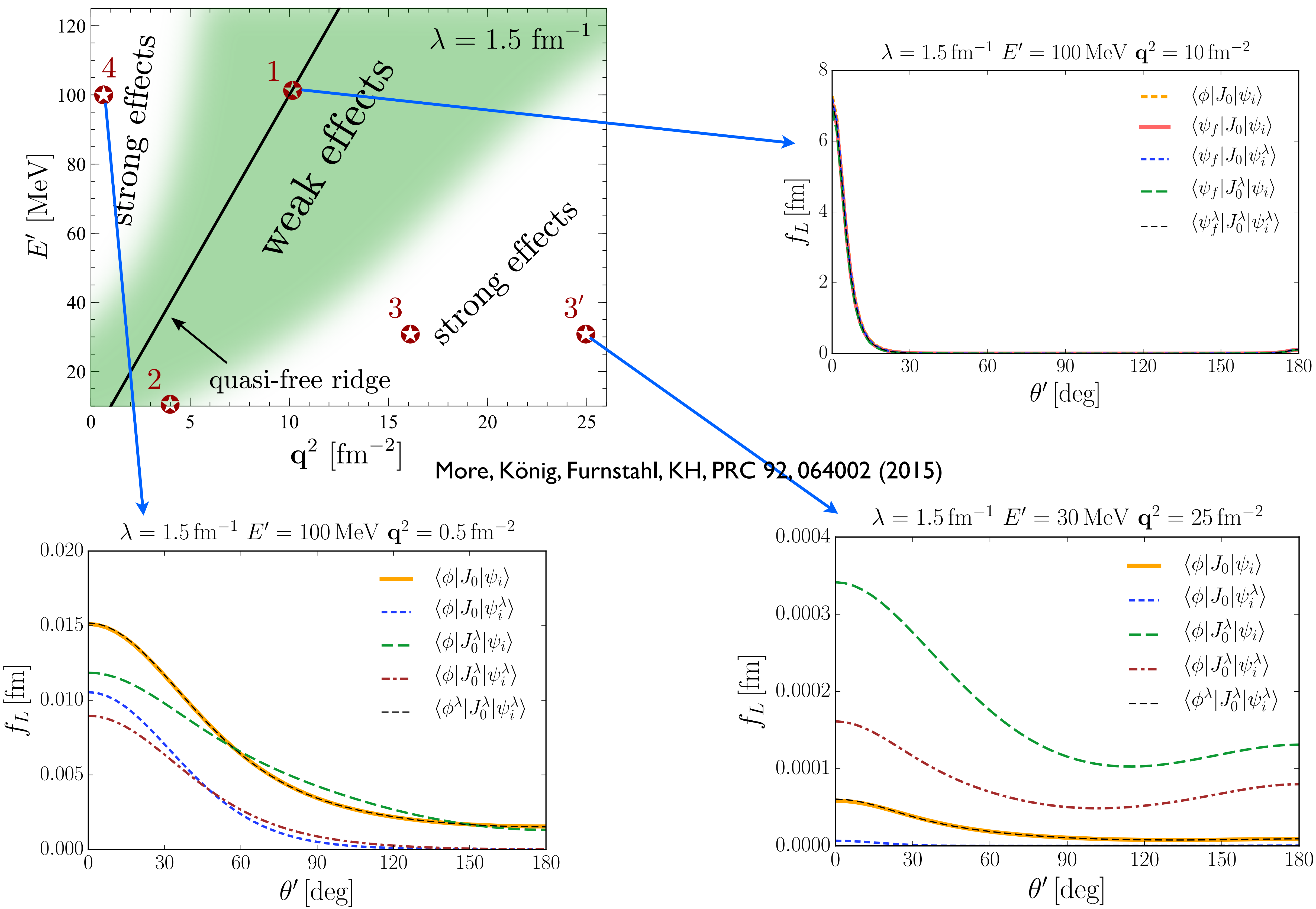
$$f_L = \sum_{\substack{S_f, m_{s_f} \\ m_{J_d}}} \mathcal{T}_{S_f, m_{s_f}, \mu=0, m_{J_d}}(\theta', \varphi')\, \mathcal{T}^*_{S_f, m_{s_f}, \mu=0, m_{J_d}}(\theta', \varphi')$$

Deuteron disintegration at low resolution scales



More, König, Furnstahl, KH, PRC 92, 064002 (2015)

Deuteron disintegration at low resolution scales



More, König, Furnstahl, KH, PRC 92, 064002 (2015)

Summary

- for ab-initio studies of reactions it is crucial to treat structure and reaction part consistently and simultaneously
- theoretical interpretation sensitively depends on the resolution scale
- resolution scale change will shift contributions between structure and reaction parts
- deep inelastic cross sections usually explained in terms of short-range correlations, scheme dependent, all observables can also be explained by separation of scales and factorization
- studied deuteron disintegration based on RG evolved interactions and currents
  - ✦ found perfect RG invariance of longitudinal structure function
  - ✦ impact of RG evolution strongly depends on kinematics