Keele University

Darmstadt, March 18

KAVLI IPMU

Stellar Evolution Course

L5: (Other) Physical Ingredients

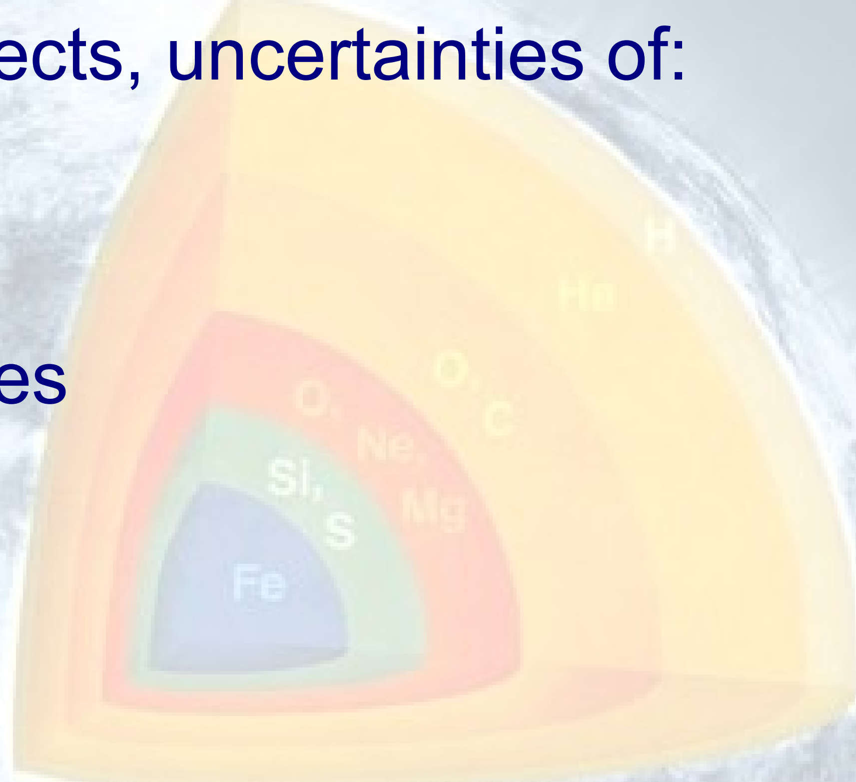
&

L6: Solving SE equations

L5: (Other) Physical Ingredients

Importance, basics, effects, uncertainties of:

- Nuclear reactions
- Neutrino energy losses
- Convection
- Mass loss
- Rotation
- Magnetic fields
- Binarity



including metallicity dependence

Changes due to nuclear reactions

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}[\sum_j r_{ji} - \sum_k r_{ik}]$$

Energy released is $\epsilon_{ij} = \frac{1}{\rho} r_{ij} e_{ij}$.

r_{ij}: number of reactions per second

e_{ij}: energy released per reaction

$q_{ij} = e_{ij}/m_i$ energy released per particle mass

For the conversion of hydrogen into helium, for example, we get

$$\frac{\partial X}{\partial t} = -\frac{\epsilon_H}{q_H} = -\frac{\partial Y}{\partial t}$$

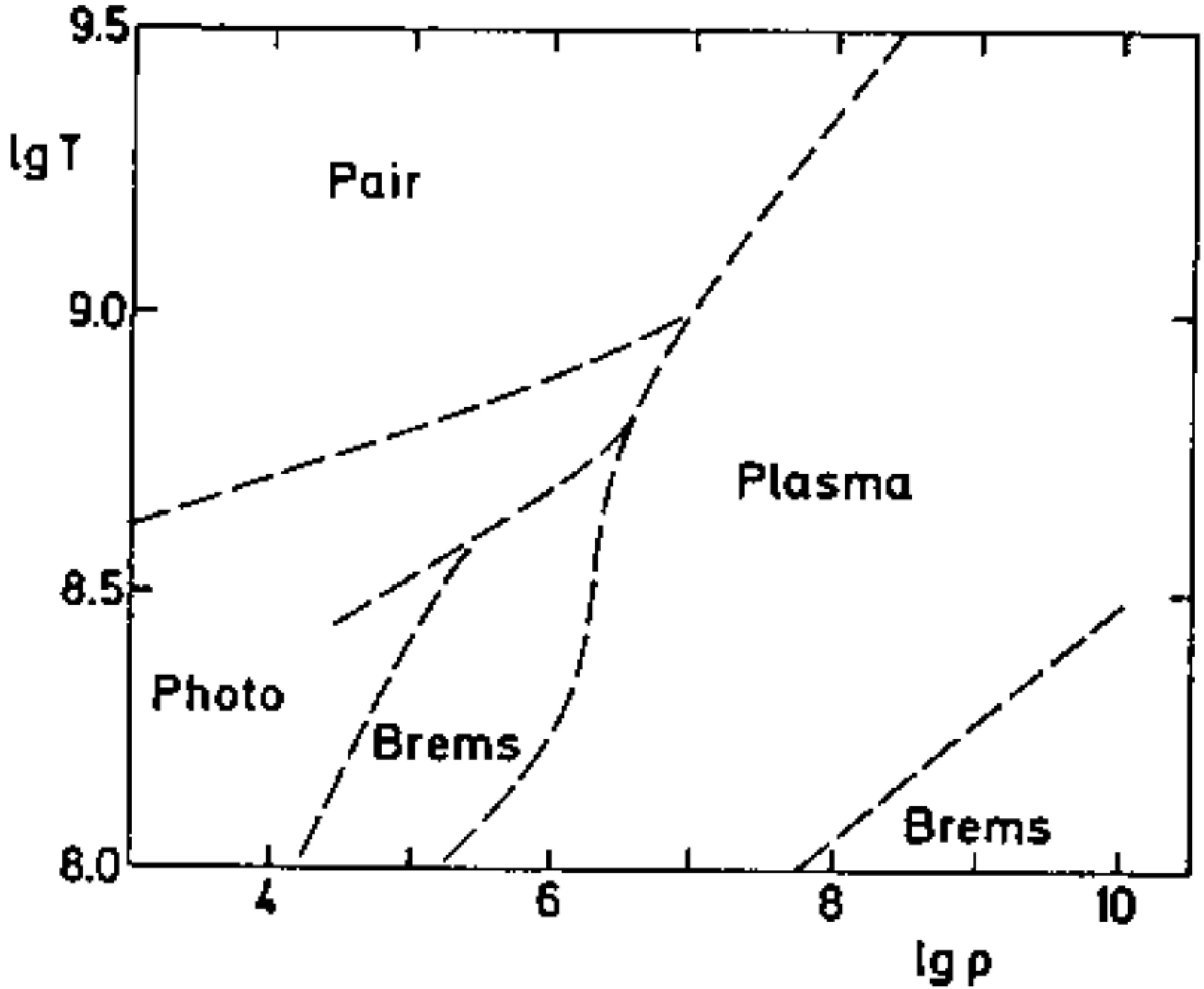
Slides taken from Achim Weiss' lectures:
http://www.mpa-garching.mpg.de/~weiss/lectures.html

Plasma neutrino emission

Stellar plasma emits neutrinos, which leave star without interaction and lead to energy loss L_ν.
Processes are:

1. Pair annihilation: $e^- + e^+ \rightarrow \nu + \bar{\nu}$ at $T > 10^9$ K.
2. Photoneutrinos: $\gamma + e^- \rightarrow e^- + \nu + \bar{\nu}$ (as Compton scattering, but with ν-pair instead of γ).
3. Plasmaneutrinos: $\gamma_{\rm pl} \rightarrow \nu + \bar{\nu}$; decay of plasma state $\gamma_{\rm pl}$
4. Bremsstrahlung: inelastic nucleus–e^- scattering, but emitted photon replaced by a ν-pair.
5. Synchroton neutrinos: as synchroton radiation, but again a photon replaced by a ν-pair.

Regions of plasma-neutrino processes



Rotation

Importance:

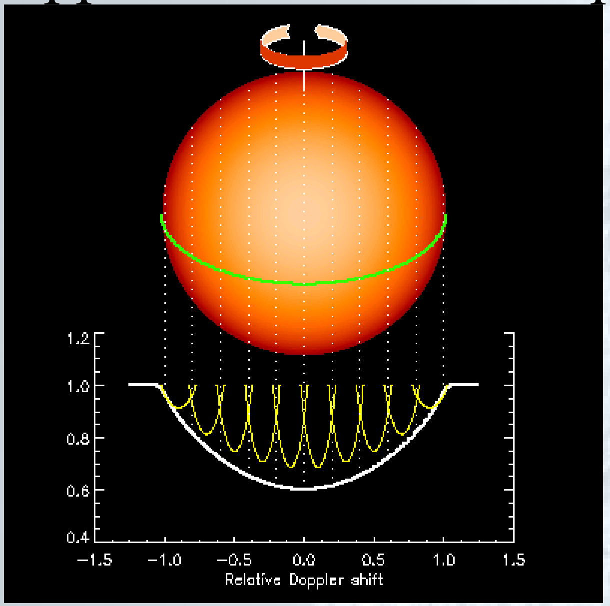
- Induces instabilities in radiative zones (none otherwise) → additional mixing of composition

- Changes properties of stars: shape, L, T_{eff}

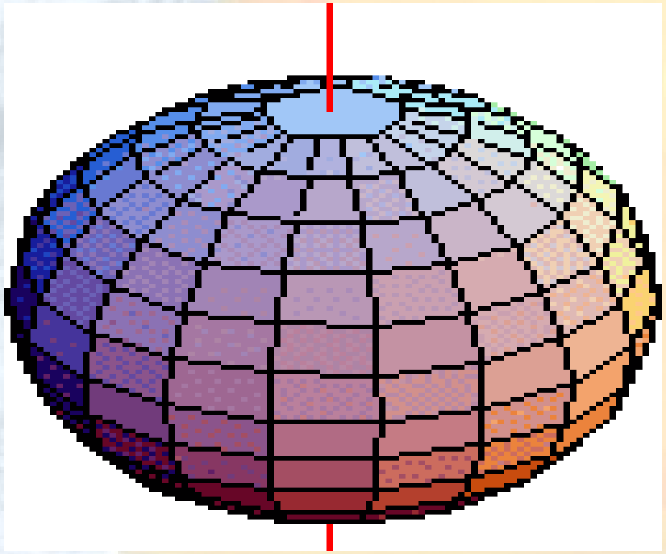
- Powers some stellar explosions (e.g. GRB, magnetars).

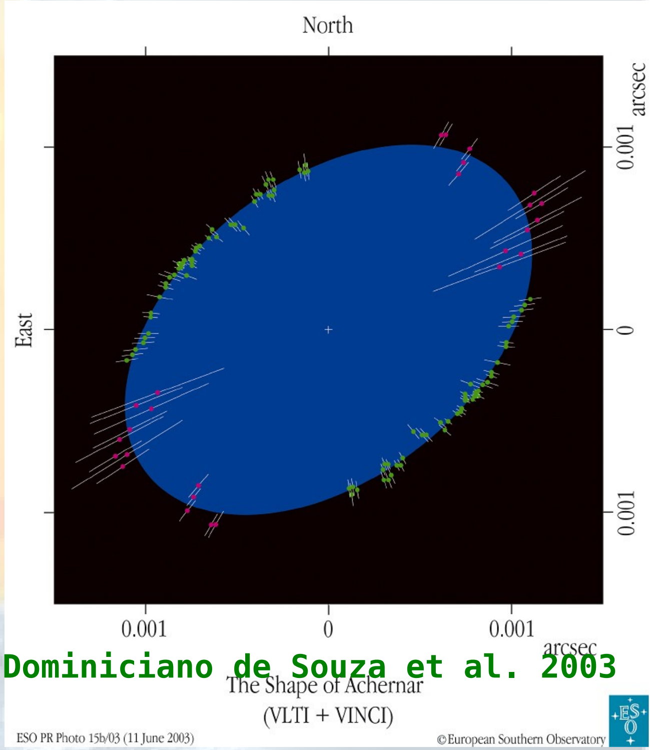
Rotational Effects on Surface

Doppler-broadened line profile



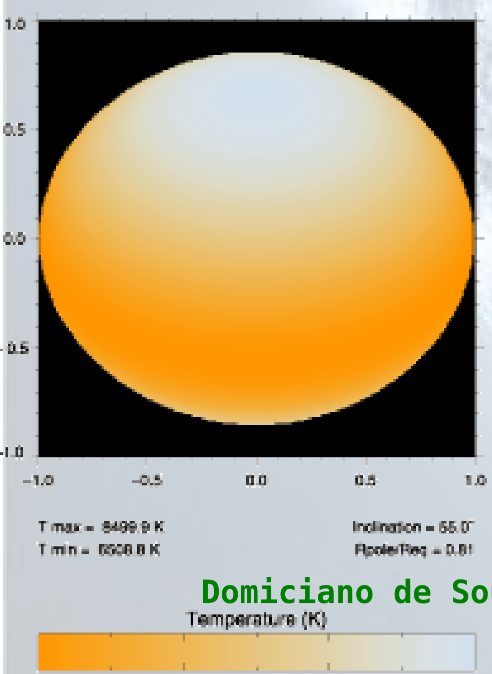
Fast rotators –> oblate shape:





Dominiciano de Souza et al. 2003

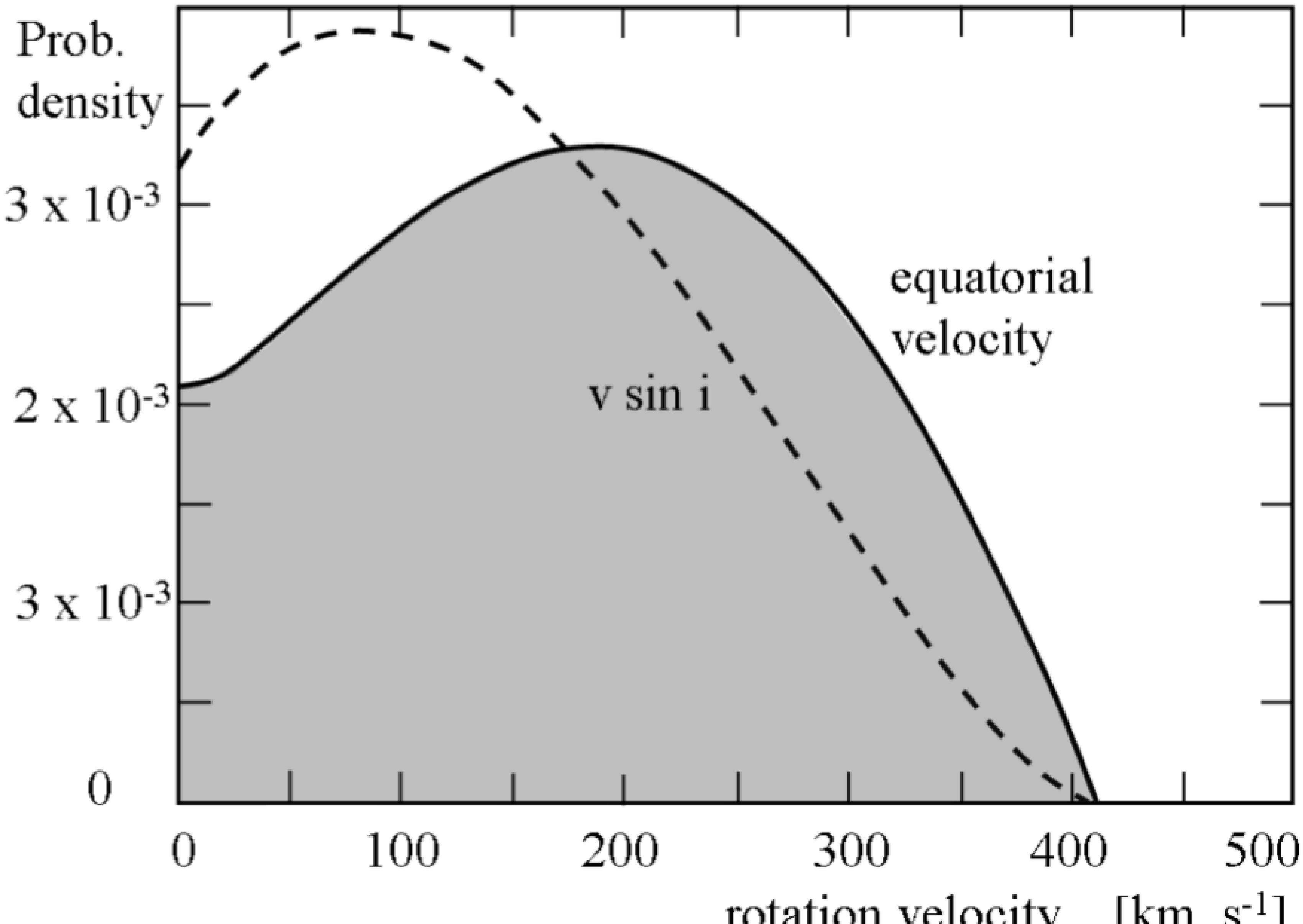
$T_{\rm eff}$ map (BMAD)



← Altair: pole brighter than equator: Effect compatible with von-Zeipel theorem (~1920s)

Domiciano de Souza et al. 2005

Rotation velocity Distribution



Data from Huang & Gies 2006, Figure from Maeder 2009

Geneva Stellar Evolution Code

1.5D hydrostatic code (Eggenberger et al 2008)

Rotation: (Maeder & Meynet 2008)

Centrifugal force: KEY FOR GRB prog.

$$\vec{g}_{\rm eff} = \vec{g}_{\rm eff}(\Omega,\theta) = \left(-\frac{GM}{r^2} + \Omega^2 r \sin^2\theta\right)\vec{e}_r + \Omega^2 r \sin\theta\cos\theta\,\vec{e}_\theta$$

Shellular rotation → still 1D: (Zahn 1992)

- Energy conservation:

$$\frac{\partial L_P}{\partial M_P} = \epsilon_{nucl} - \epsilon_\nu + \epsilon_{grav} = \epsilon_{nucl} - \epsilon_\nu - c_P\frac{\partial \overline{T}}{\partial t} + \frac{\delta}{\overline{\rho}}\frac{\partial P}{\partial t} \qquad (2.9)$$

- Momentum equation:

$$\frac{\partial P}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P \qquad (2.10)$$

- Mass conservation (or continuity equation):

$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi r_P^2 \overline{\rho}} \qquad (2.11)$$

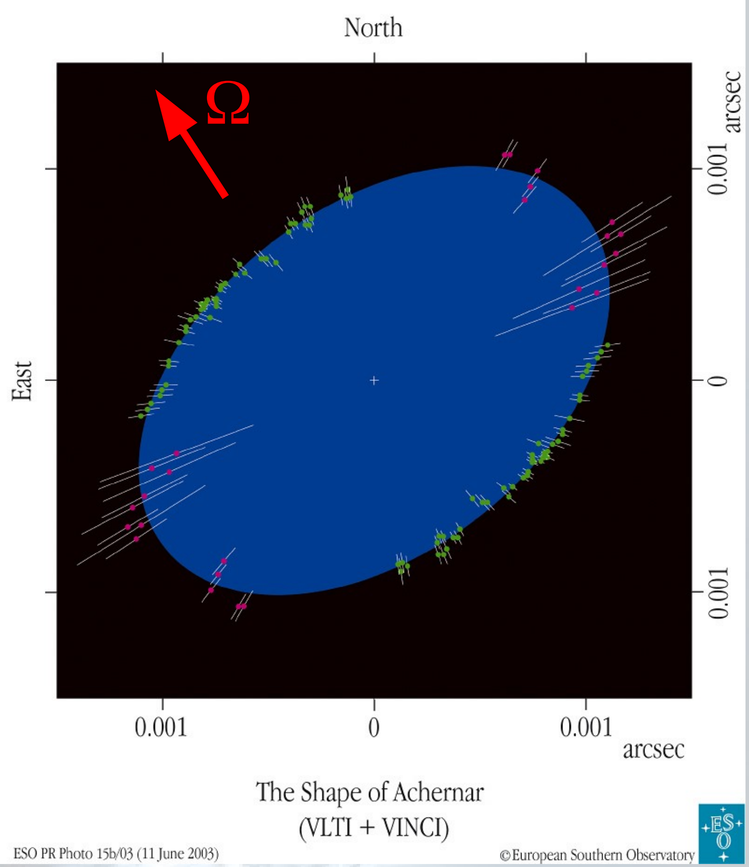
- Energy transport equation:

$$\frac{\partial \ln \overline{T}}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P \min[\nabla_{\rm ad}, \nabla_{\rm rad}\frac{f_T}{f_P}] \qquad (2.12)$$

where

$$\nabla_{\rm ad} = \frac{P\delta}{\overline{T}\overline{\rho}c_P} \quad \text{(convective zones)},$$

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa l P}{m\overline{T}^4} \quad \text{(radiative zones)},$$



$$f_P = \frac{4\pi r_P^4}{GM_P S_P}\frac{1}{<g^{-1}>},$$

$$f_T = \left(\frac{4\pi r_P^2}{S_P}\right)^2 \frac{1}{<g><g^{-1}>},$$

(Meynet and Meynet 97)

Rotation Induced Transport

Zahn 1992: strong horizontal turbulence

Transport of angular momentum:

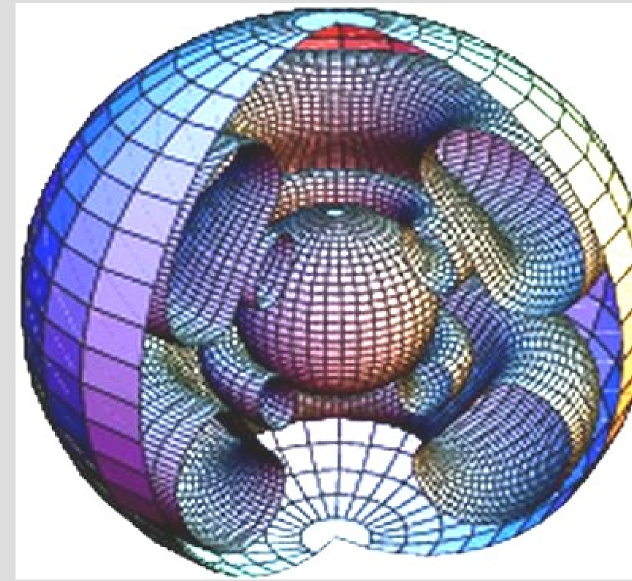
$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\bar{\Omega}\right)_{M_r} = \underbrace{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4 \bar{\Omega} U(r)\right)}_{\text{advection term}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho D r^4 \frac{\partial \bar{\Omega}}{\partial r}\right)}_{\text{diffusion term}}$$

Transport of chemical elements:

$$\rho \frac{\mathrm{d}X_i}{\mathrm{d}t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2 \left[D + D_{eff}\right]\frac{\partial X_i}{\partial r}\right) + \left(\frac{\mathrm{d}X_i}{\mathrm{d}t}\right)_{\text{nucl}}$$

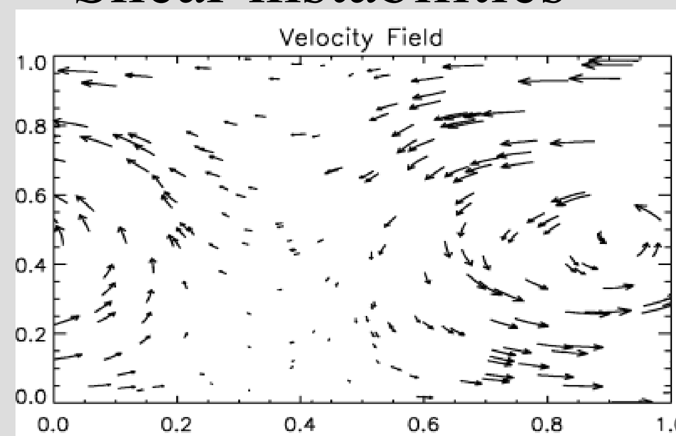
D: diffusion coeff. due to various transport mechanisms (convection, shear)

D_{eff}: diffusion coeff. due to meridional circulation + horizontal turbulence



Meynet & Maeder 2000

Shear instabilities



Brueggen & Hillebrandt 2000

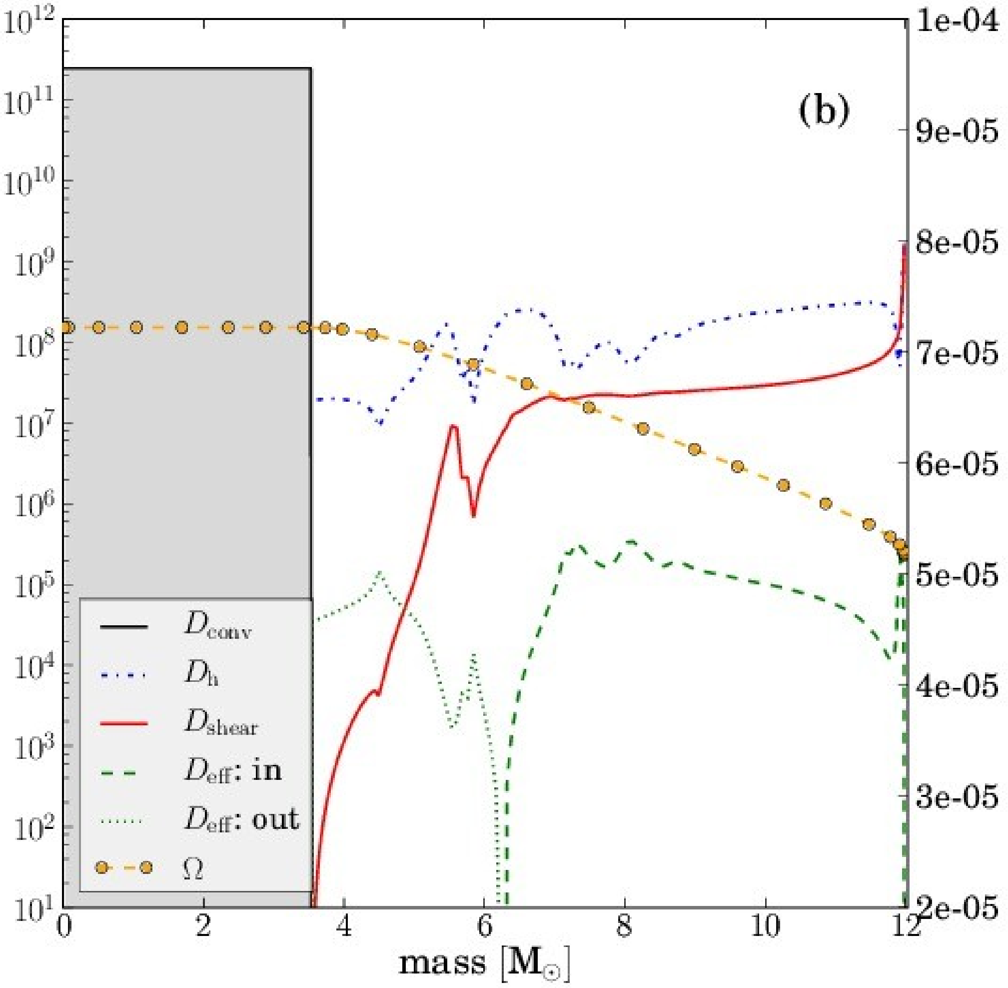
Rotation Induced Transport: Prescriptions

D_h: three prescriptions
Zahn (1992); Maeder (2003); Mathis et al. (2004)

D_{shear}: two prescriptions
Talon & Zahn (1997); Maeder (1997)

Different zones concerned inside the star

mixing ± strong (4%,12%)



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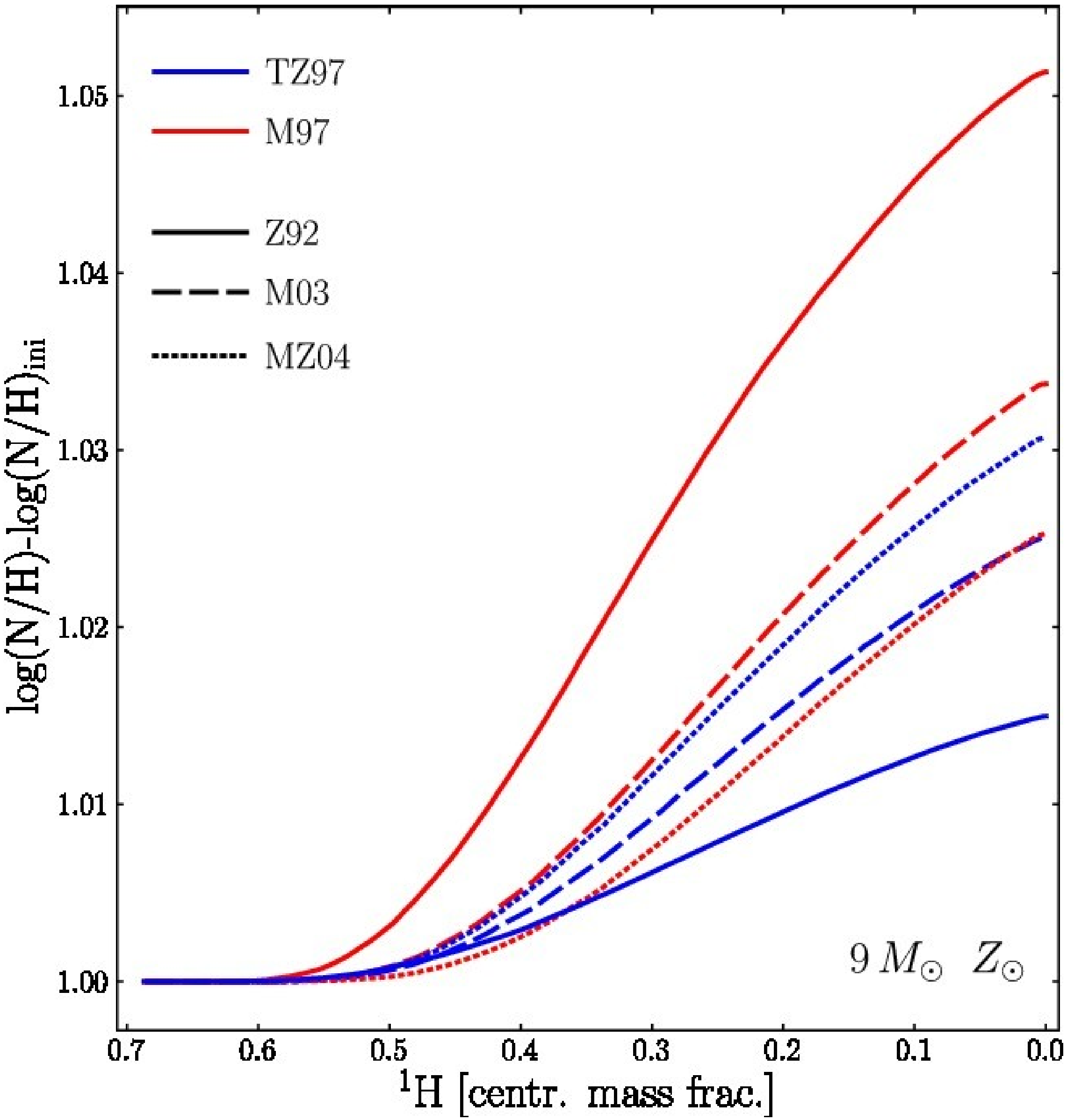


See also Chieffi & Limongi 13

Mass Loss

Mass loss prescription without rotation:

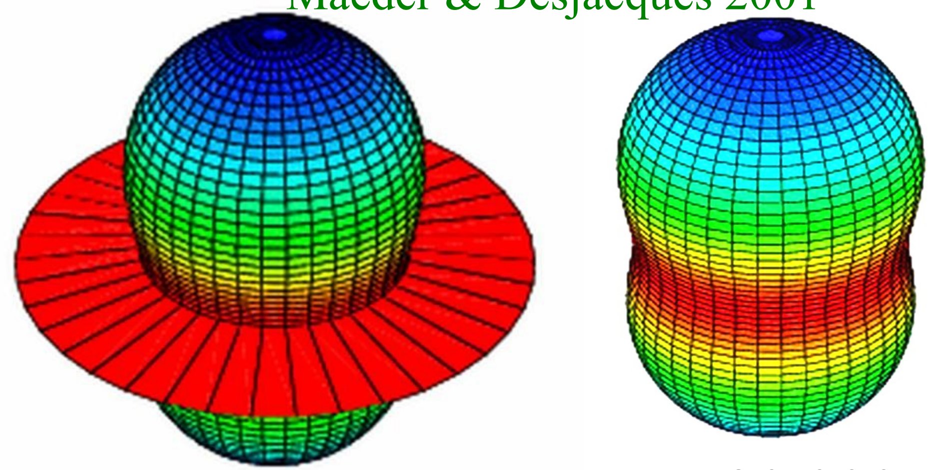
- O-type & LBV stars (bi-stab.): Vink et al 2000, 2001 and RSG de Jager et al 1988
- WR stars (clumping effect): N

Effects of rotation:

- Enhancement: Maeder & Meynet 2000

$$\frac{\dot{M}(\Omega)}{\dot{M}(0)} \approx \frac{(1-\Gamma)^{\frac{1}{\alpha}-1}}{\left[1-\frac{4}{9}\frac{v^2}{v^2_{crit,1}}-\Gamma\right]^{\frac{1}{\alpha}-1}}$$

Maeder & Desjacques 2001



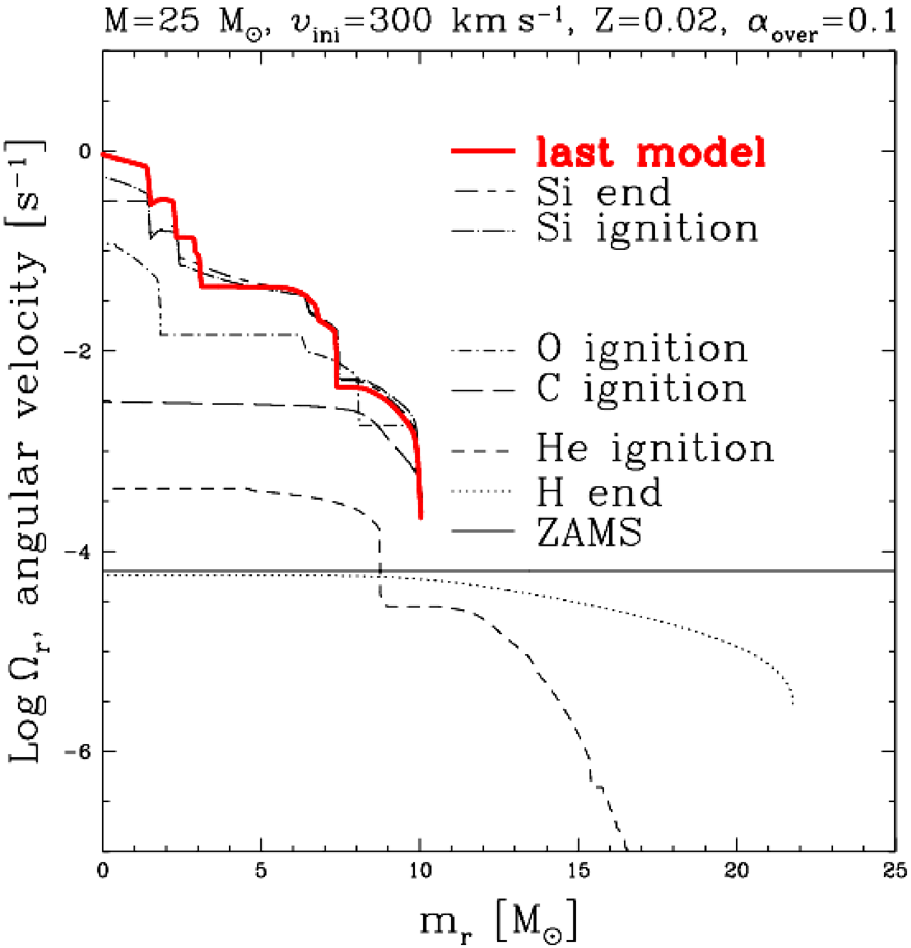
T_{eff} =25 000 K T_{eff} =30 000 K

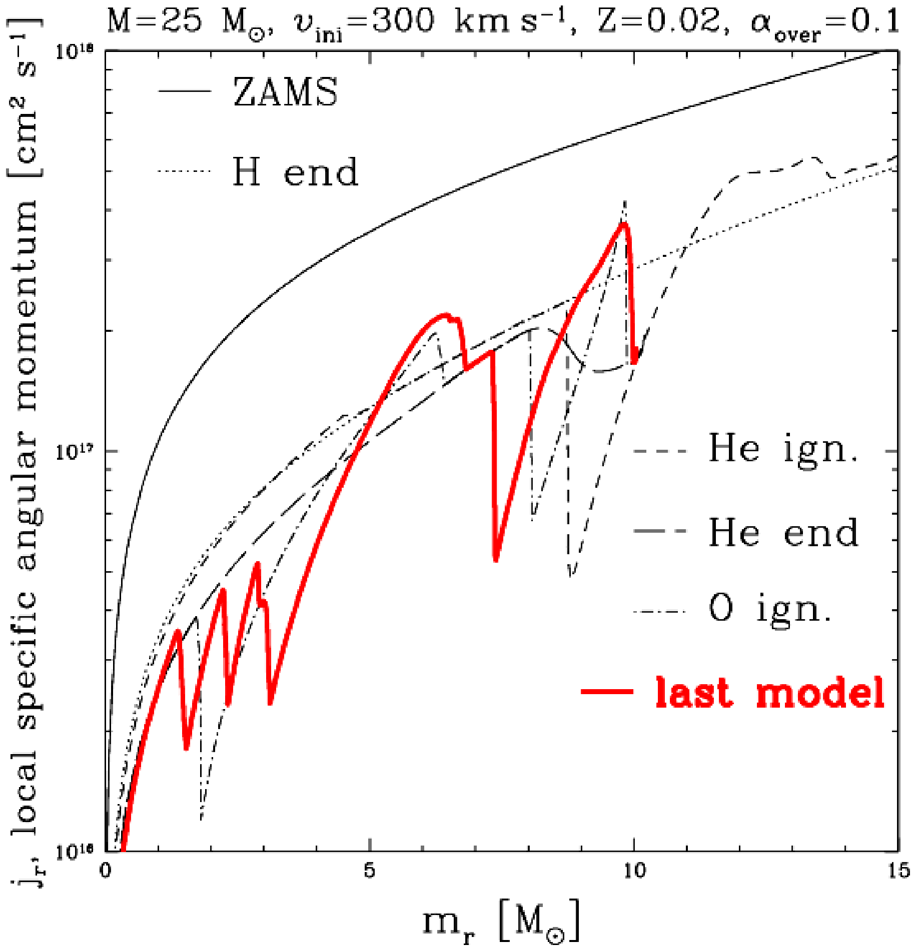
- & anisotropy:

$F_{rad} \sim g_{eff}$: Von Zeipel, 1924 → affects angular momentum loss

Angular velocity: Ω↑
end Si: Ω~ 1 s^{-1}

Angular momentum, j↓
j(end Si) ~ j(end He)

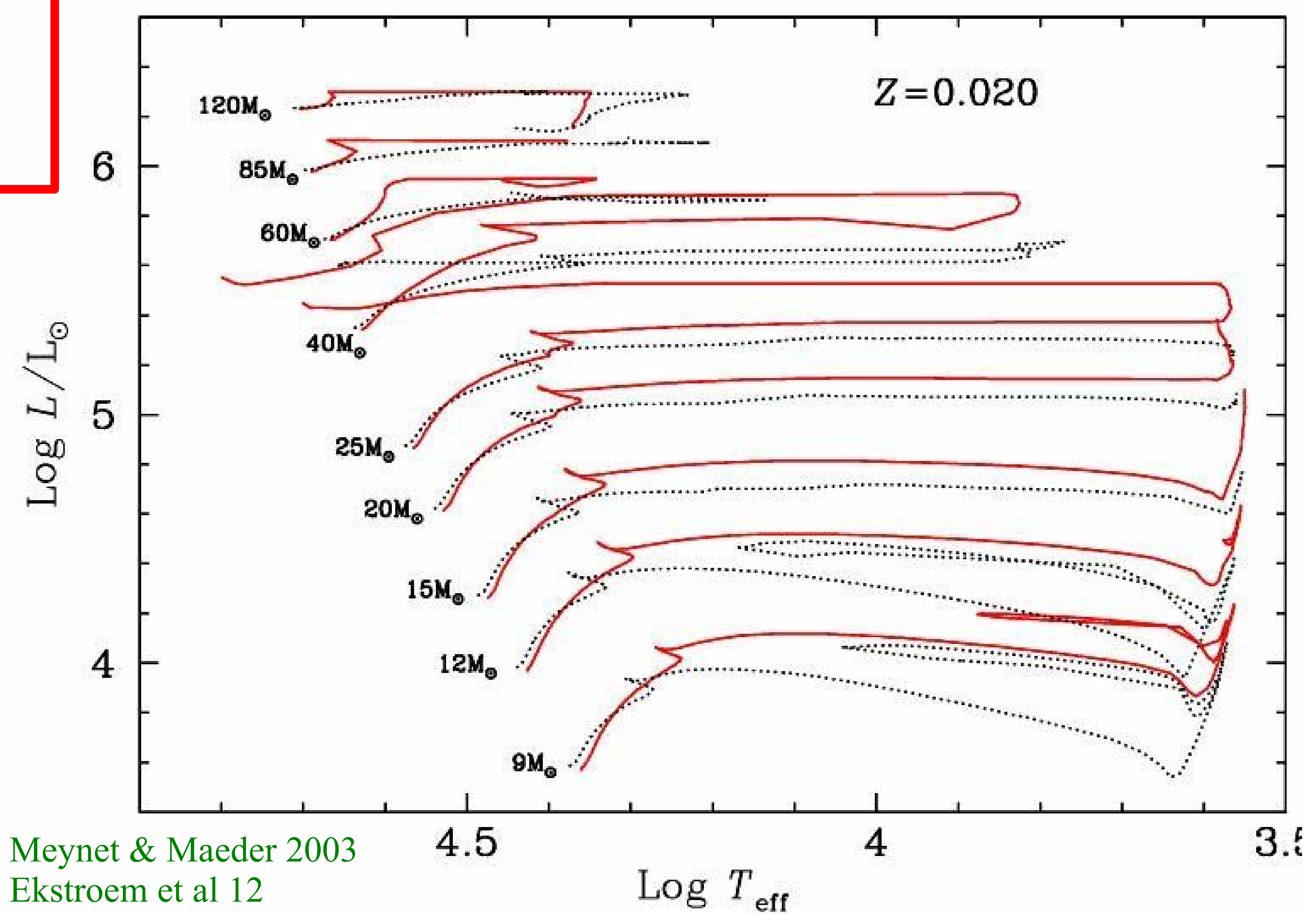




Hirschi et al 2004, A&A

Impact of Rotation (@ solar Z)

υ_{ini} =
300 km/s
0 km/s



Meynet & Maeder 2003
Ekstroem et al 12

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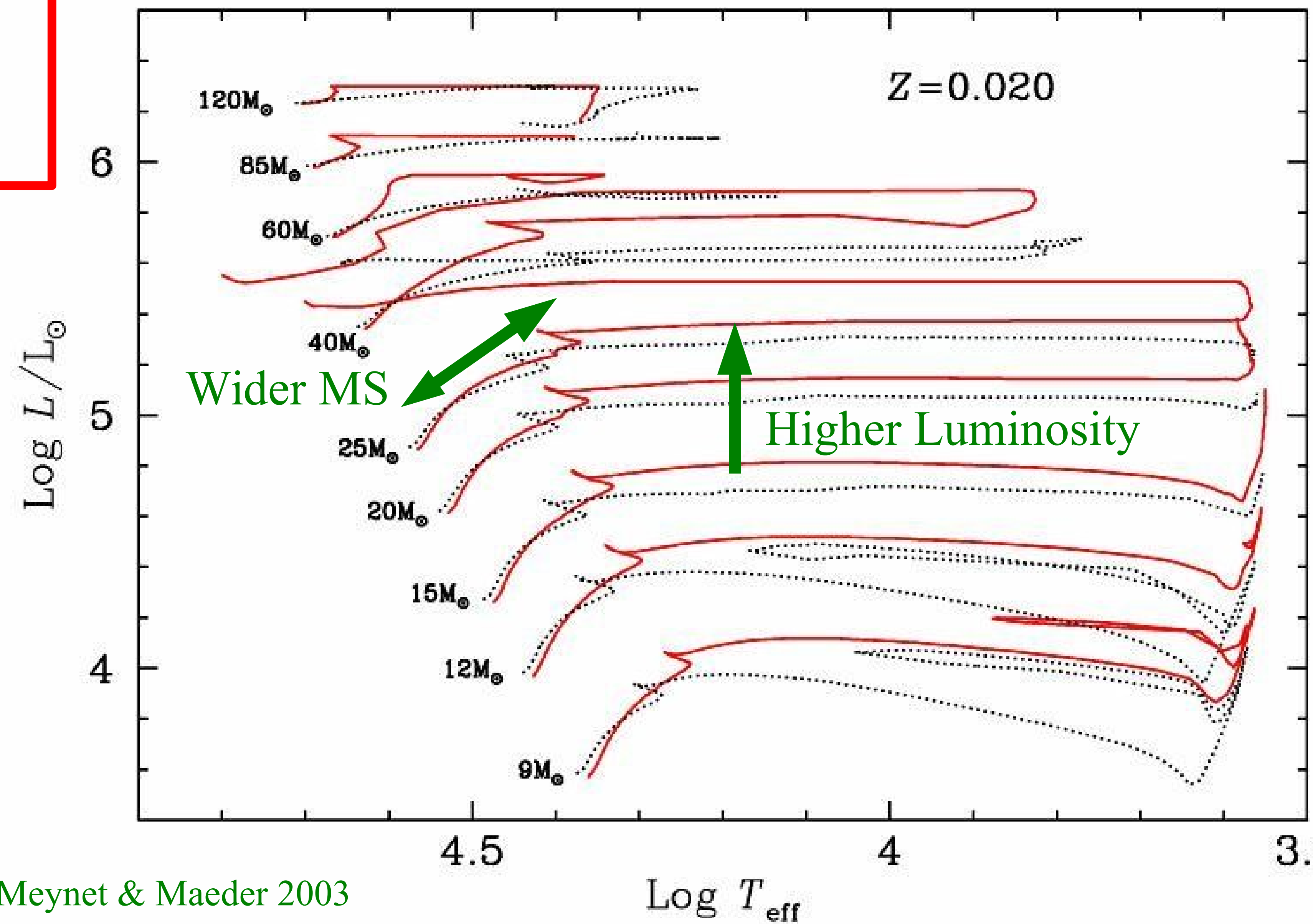


Meynet & Maeder 2003

Impact of Rotation (@ solar Z): T_{eff}

Roche model: $R_{\mathrm{eq,crit}} = \frac{3}{2} R_{\mathrm{pol,crit}}$

Modification of the gravity:

$$\vec{g}_{\mathrm{eff}} = \vec{g}_{\mathrm{eff}}(\Omega, \theta) =$$

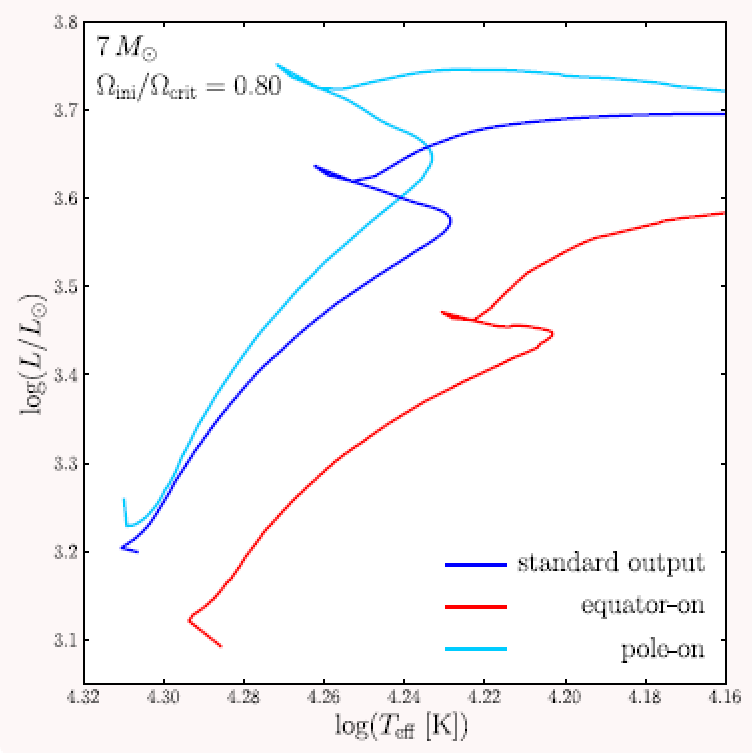
$$\left(-\frac{G\,M}{r^2} + \Omega^2 r\ \sin^2\theta\right)\ \vec{e}_r + \left(\Omega^2 r\ \sin\theta\ \cos\theta\ \right)\vec{e}_\theta$$

and thus of the T_{eff}:

$$T_{\mathrm{eff}} = T_{\mathrm{eff}}(\Omega, \theta) = \left[\frac{L}{4\pi\sigma\,G\,M^\star}\ g_{\mathrm{eff}}(\Omega, \theta)\right]^{1/4}$$

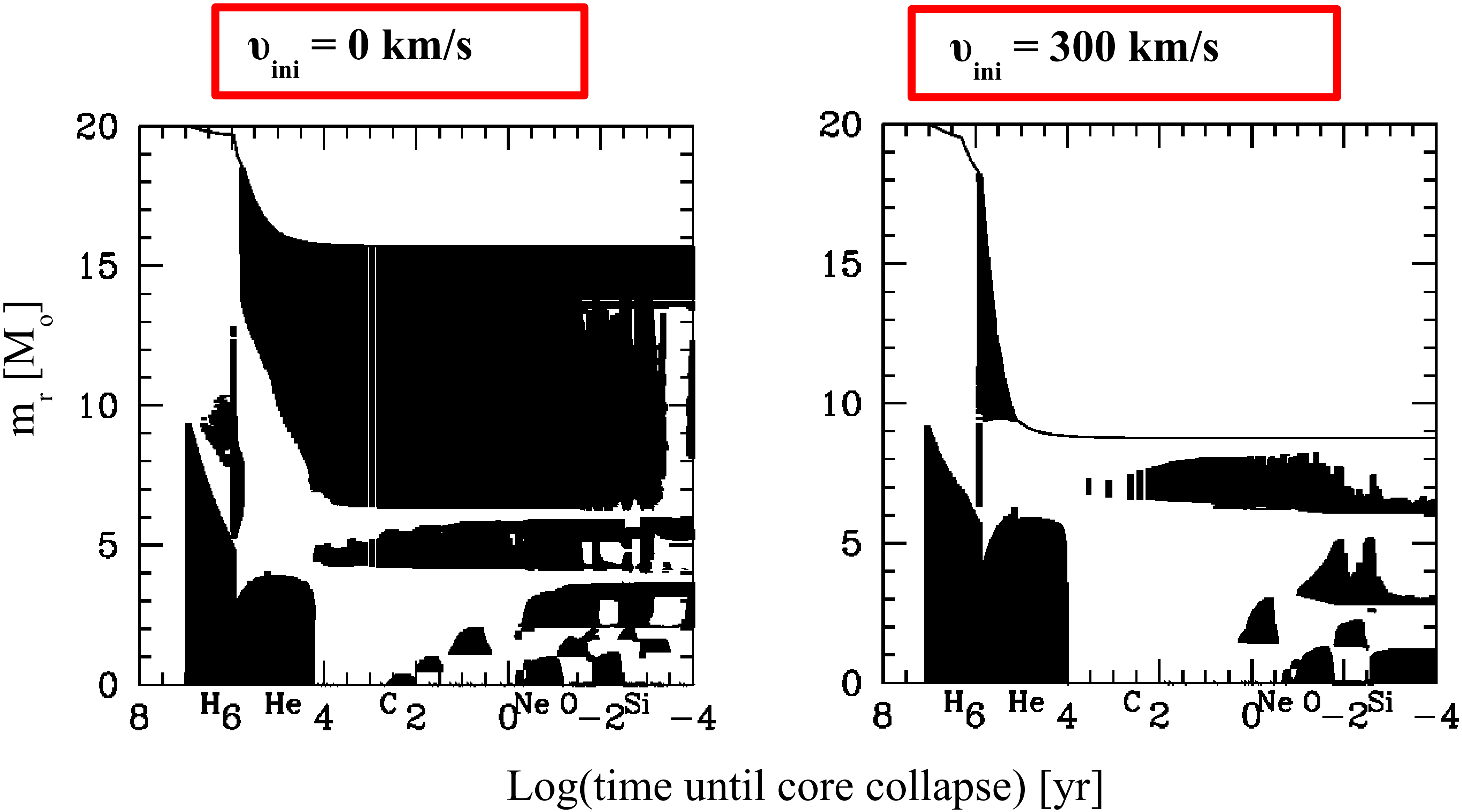
Standard outputs of the models: $= (L/\sigma S_P)^{1/4}$
S_P: true deformed surface

model from Georgy et al. (2013)



Correction for limb darkening according to Claret 2000

Impact of Rotation, $\mathcal{M}_{ini} = 20\ \mathcal{M}_\odot$



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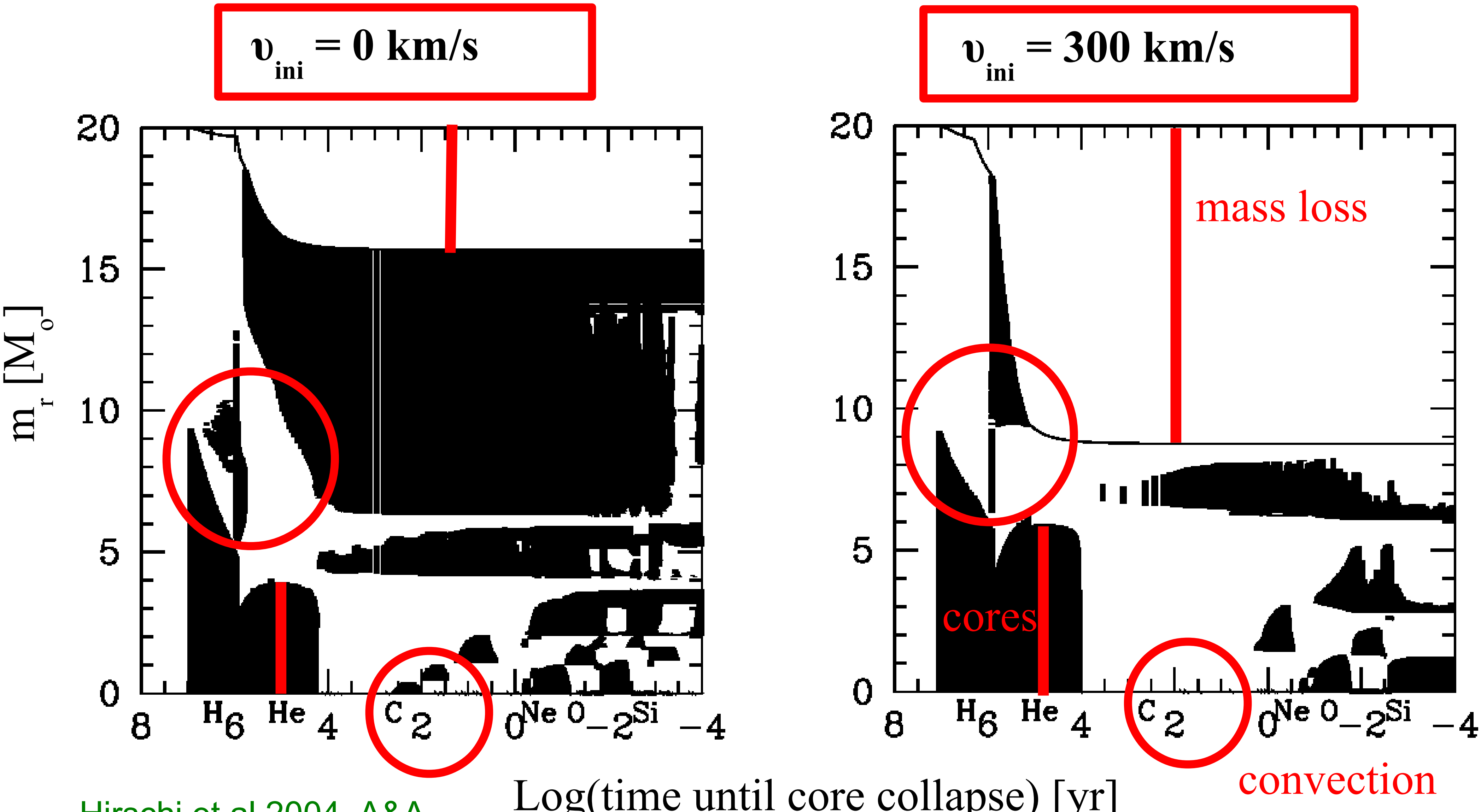
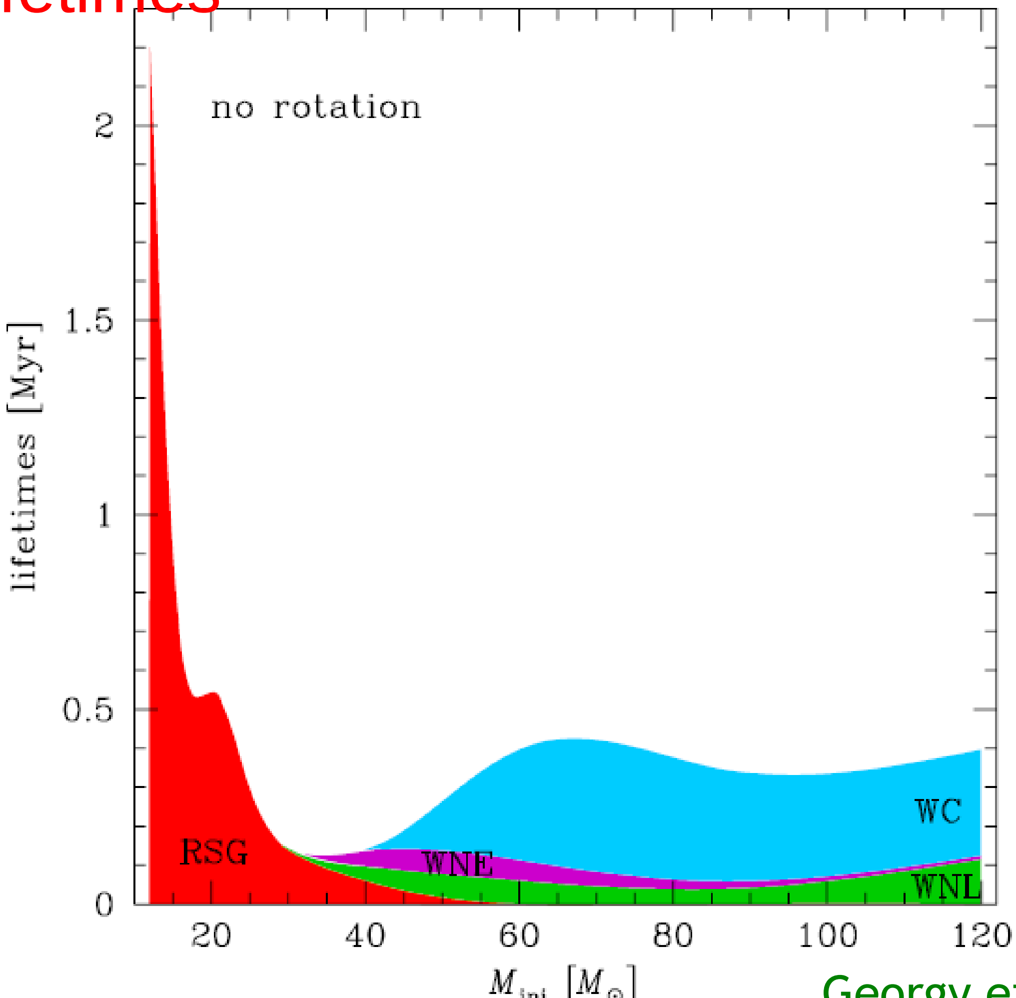


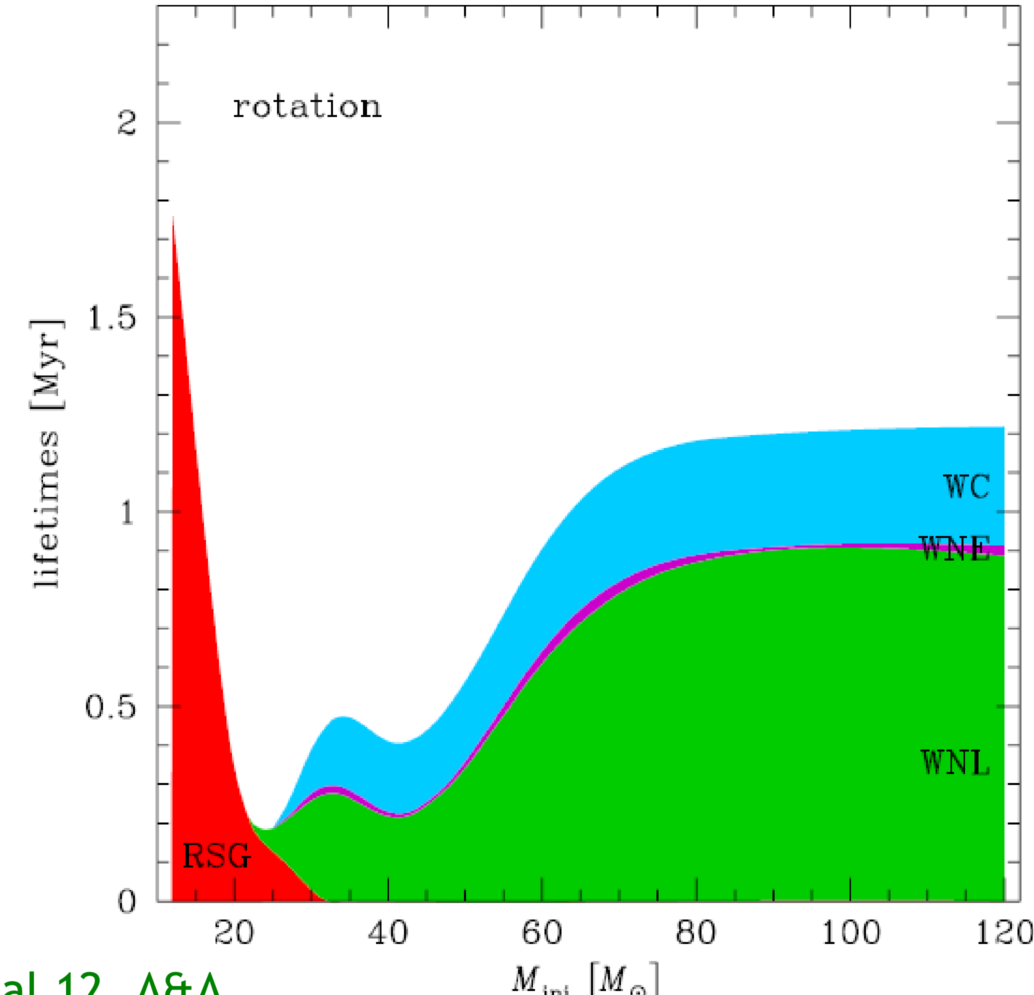
Hirschi et al 2004, A&A

WR Lifetimes @ solar Z

Rotation: decrease of M_{min} for WR formation & increase in WR lifetimes

Meynet & Maeder 03





Georgy et al 12, A&A

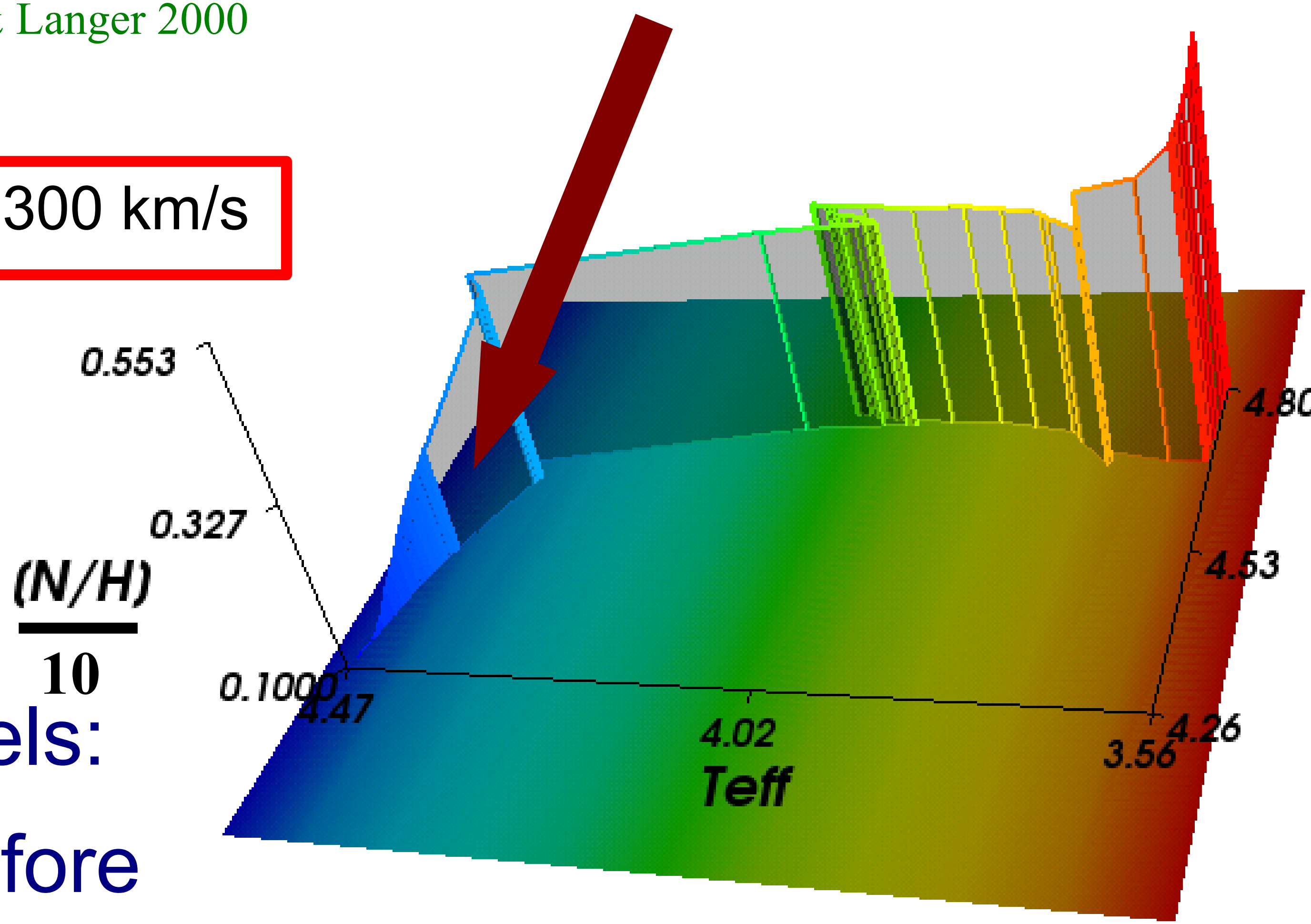
NO ROT: $M_{min} \approx$ 25-30 M_o

ROT: $M_{min} \approx$ 20 M_o

Rotating models: enrichment starts during MS

Meynet & Maeder 2000, Heger & Langer 2000

15 M_o model with u_{ini}=300 km/s



Non-rotating models:

no enrichment before

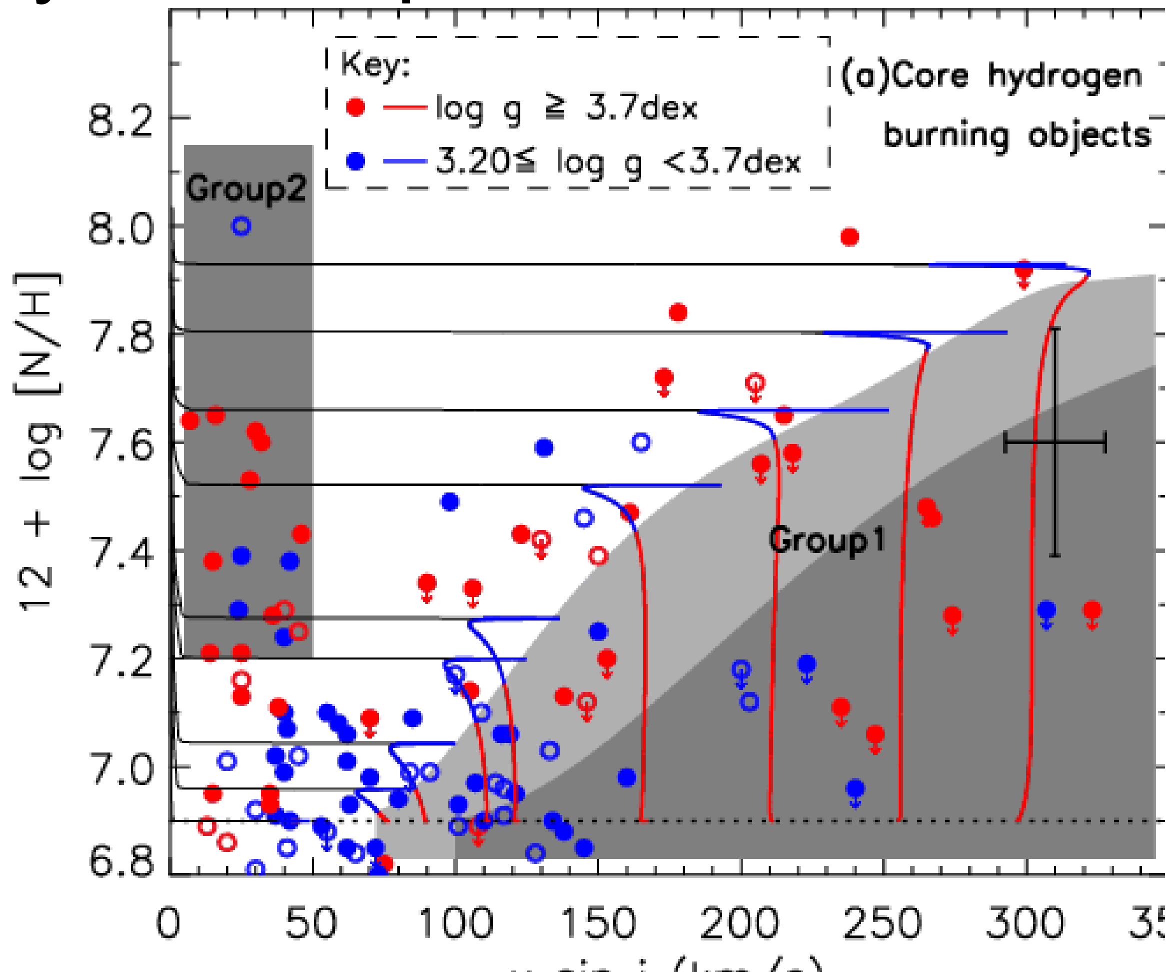
1st dredge up (RSG)

Mixing questioned by FLAMES survey (Hunter et al 08,09)

Nitrogen Surface Enrichment

Flames survey: many stars explained BUT

Explanations:



Single stars:

G1: less evolved/

lower mass

G2: pole-on / B-f?

Binary stars: (Langer etal 08)

G1: N-poor matter accr.

G2: * slowed down / B-f?

Other issues: Initial composition, overshooting, enriched blue supergiants

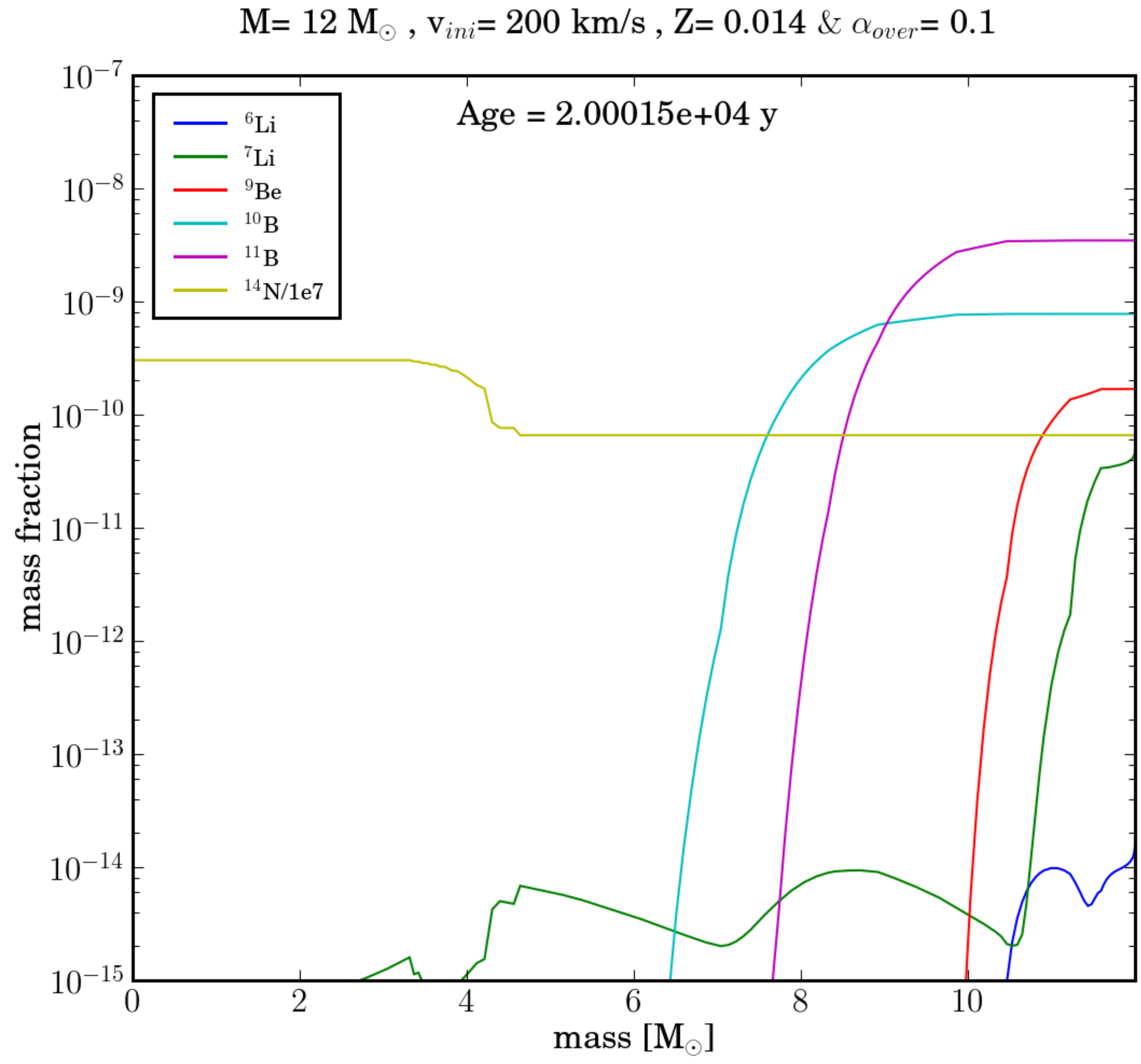
Boron can help distinguish between rotation and binarity

VLT FLAMES survey of massive stars: Hunter etal 07,08,09, ...

(Brott etal 08, 11a,b, Langer etal)

Boron Surface Depletion: Models

- Boron is depleted in the stellar interior.

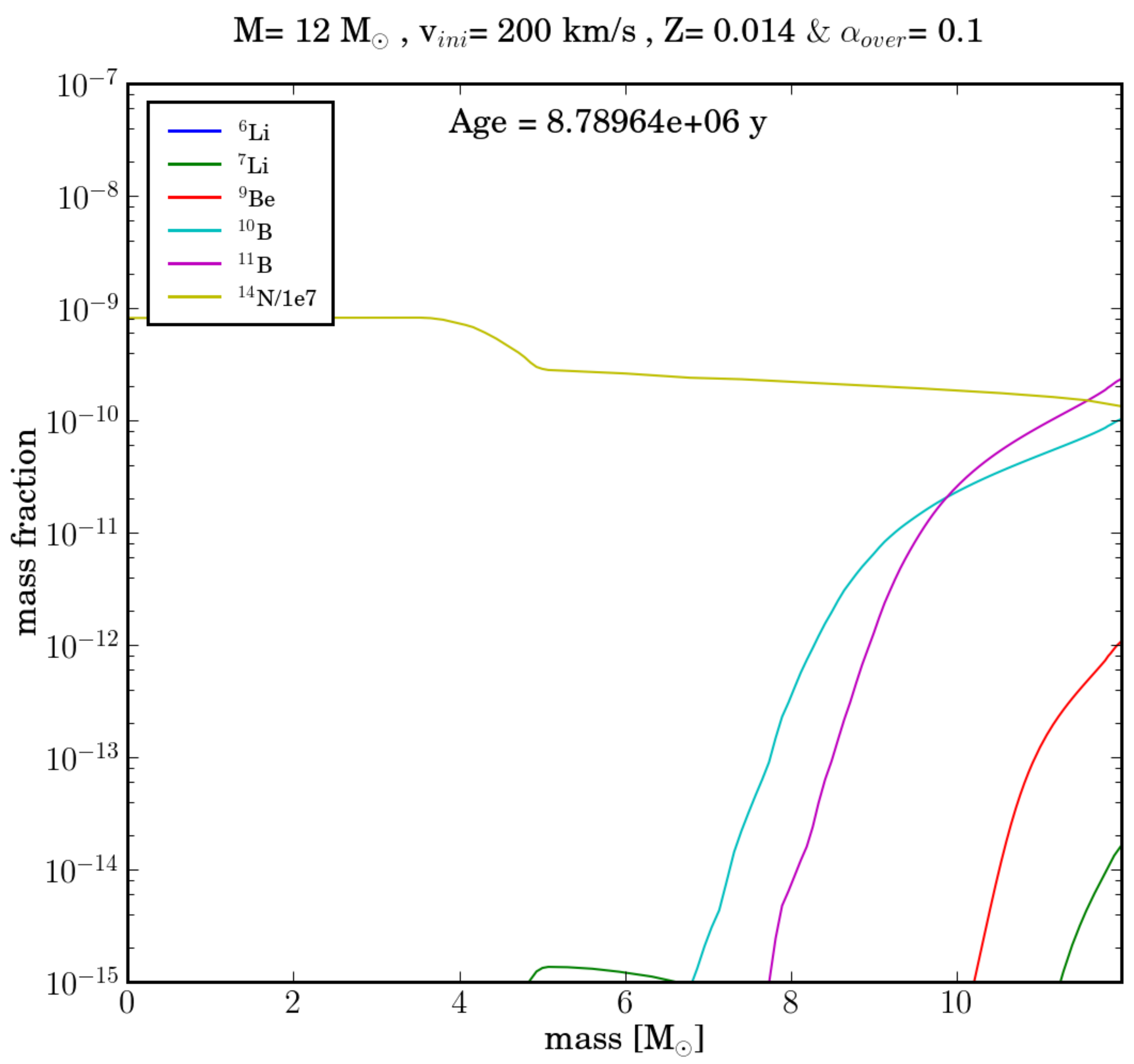


Boron Surface Depletion: Models

Rotational mixing

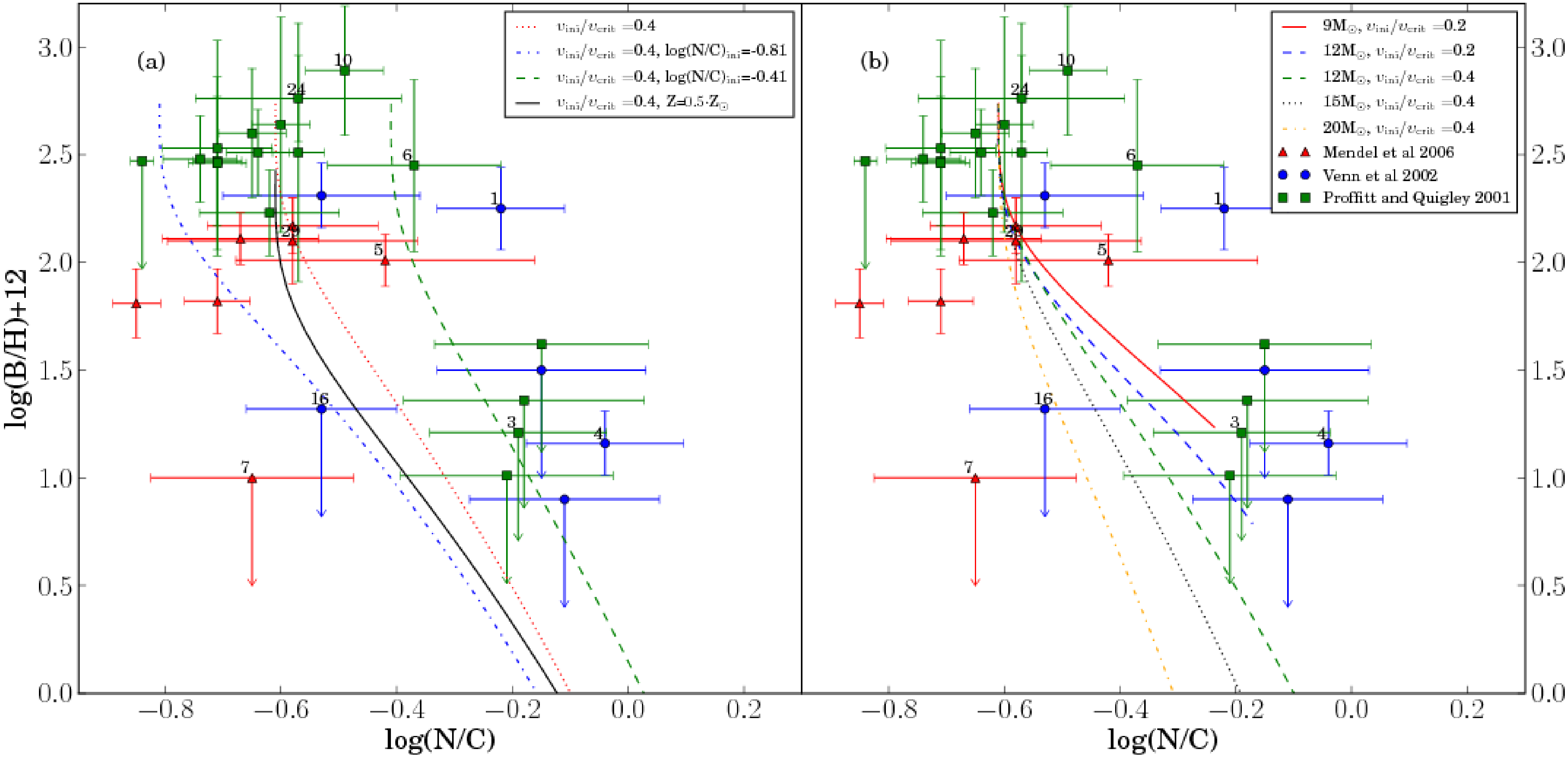
-> surface

boron depletion



Boron Surface Depletion

Rotational mixing -> surface boron depletion



Frischknecht et al A&A 2010

Binaries cannot explain B depletion without N enrichment (Langer et al

Magnetic Fields

Importance:

- Guides charged-particle

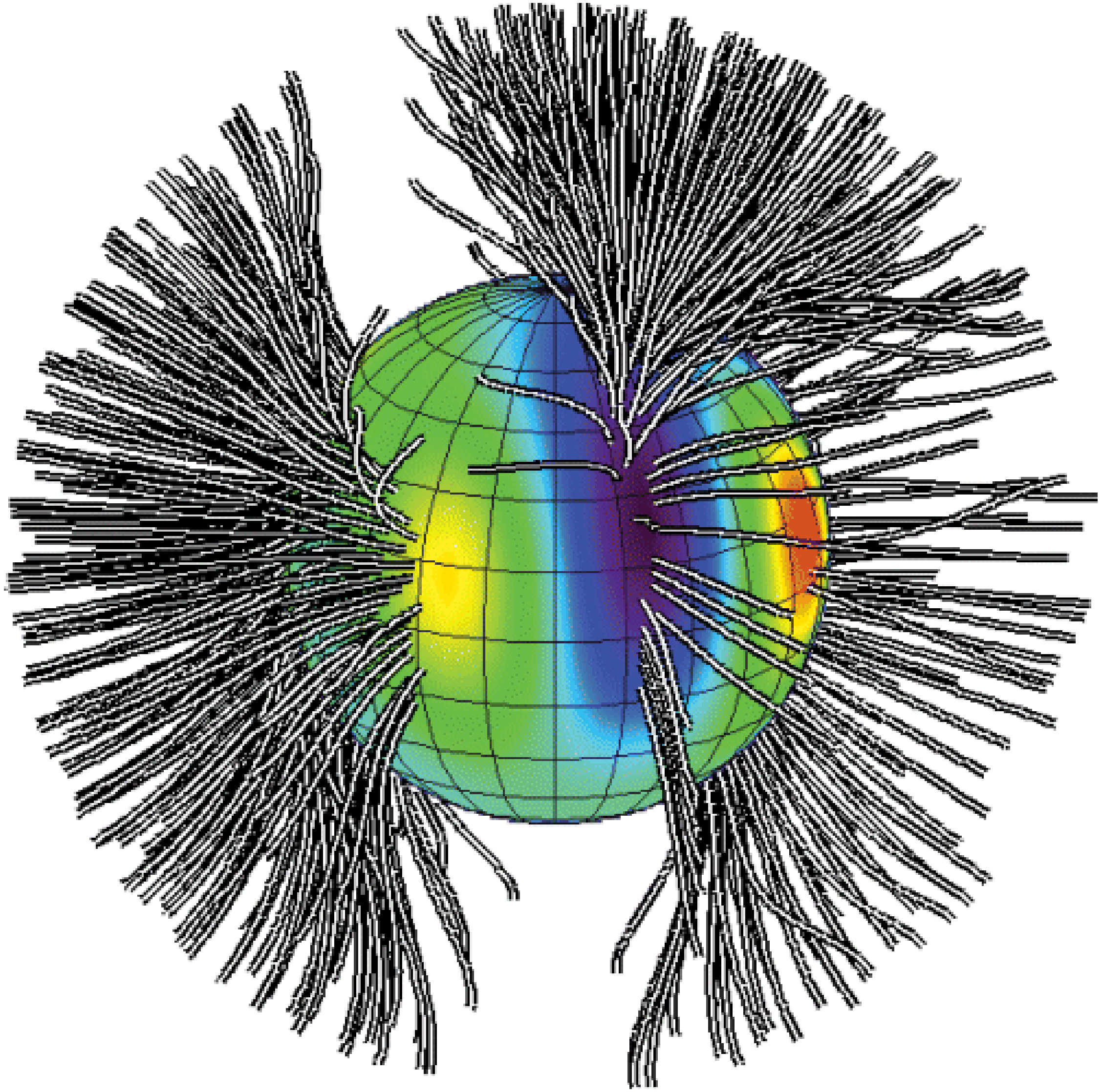
- Shapes stellar winds

- Couples rotation of different parts of the star

Importance debated

Surface Magnetic Fields

τ Sco



Donati et al. 2006

Surface Magnetic Fields

A few dozen He-peculiar stars

7 magnetic OB stars

	Ref	Sp. T.	Vsini Km/s	Prot days	M Msol	Incl. Deg.	β Deg.	Bpol G
HD191612	(6)			538			45	~1500
Θ Ori C	(1)	O4-6V	20	15.4	45	45	42+-6	1100+-100
βCep	(2)	B1IVe	27	12.00	12	60+-10	85+-10	360+-40
τ Sco	(7)	B0.2V		41				~500
V2052 Oph	(3)	B1V	63	3.64	10	71+-10	35+-17	250+-190
ζCas	(4)	B2IV	17	5.37	9	18+-4	80+-4	340+-90
ωOri	(5)	B2IVe	172	1.29	8	42+-7	50+-25	530+-200
He-peculiar		B1-B8p		0.9-22	<10			1000-10000

Only 2 magnetic O star known

(1) Donati et al. 2003 (2) Henrichs et al. 2000 (3,4,5) Neiner et al. 2003abc, (6,7) Donati et al. 2006ab

Angle between the magnetic axis and the rotation axis

Large ongoing surveys: e.g. MiMes
Most magnetic stars show abundance anomalies: Bp, Ap stars

Magnetic Fields

Question: are these values compatible with magnetic fields observed in pulsars?

Pulsars→ 10^{12} G

$$Br^2 = const.$$

$(10\ \text{km}/5\ R_{sol})^2 \times 10^{12}$ G ~ 10 G.

$$B_+ / B_- = (r_- / r_+)^2$$

Answer: observed magnetic are one-two orders of magnitude higher→More compatible with progenitors of magnetars 10^{15} G

Question: may the observed values have an impact on the wind?

$$\eta(r) \equiv \frac{B^2/8\pi}{\rho v^2/2}$$

if $\eta > 1$ → wind behavior

ud-Doula & Owocki (2002)

Answer: YES. For early-type stars, $\eta > 1$ for B~ 50-100 G

Magnetic Fields: Theory

Taylor Instability (1973)

Small initial horizontal field:
instability of the field lines
→Small vertical component
→Differential rotation winds up
→New horizontal field lines closer and denser:
DYNAMO (Spruit 2002)

More general expressions
(Maeder 04,
Maeder & Meynet 2005)

Criteria for field amplification:

$$\Omega > \omega_A = \frac{B}{r\sqrt{4\pi\,\bar{\varrho}}}$$

$$q = -\frac{\partial \ln \Omega}{\partial \ln r} > q_{min} = \left(\frac{N}{\Omega}\right)^{7/4} \left(\frac{\eta}{N\,r^2}\right)^{1/4}$$

$$\text{where } N^2 = \frac{\eta/K}{\eta/K+2} N_T^2 + N_\mu^2$$

Transport coefficients:

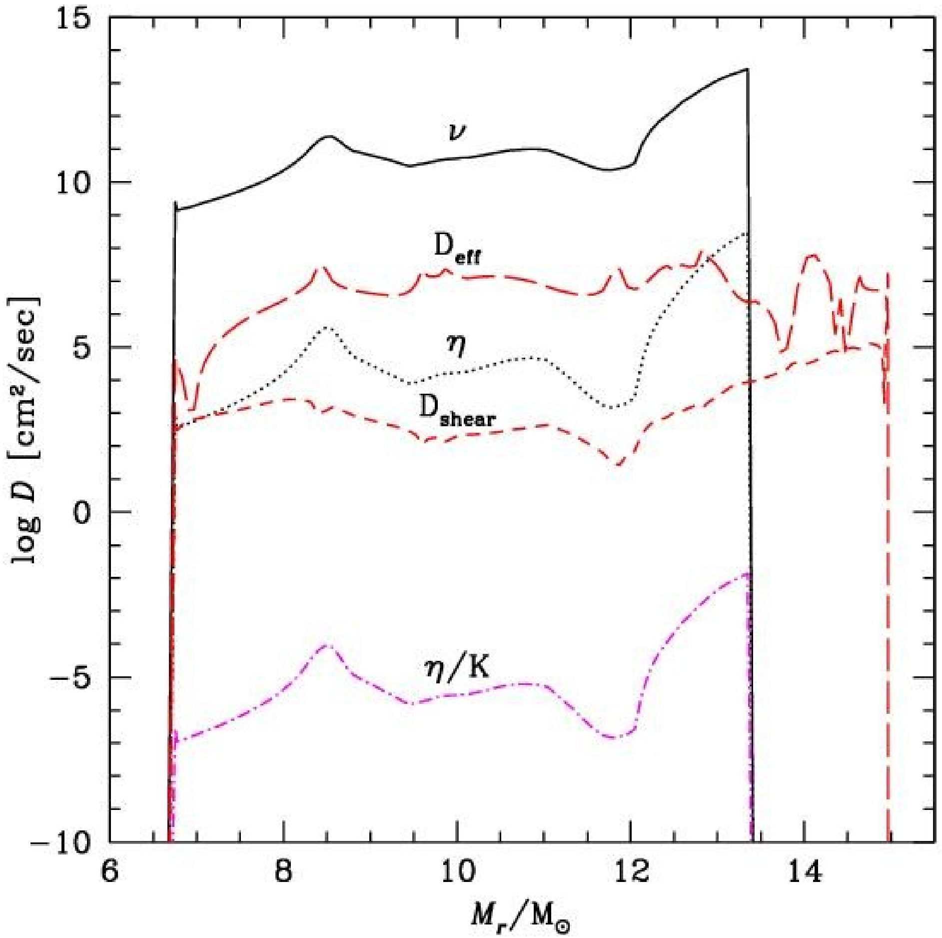
$$D_{\rm chem} = \frac{r^2\,\Omega}{q^2} \left(\frac{\omega_A}{\Omega}\right)^6$$

$$D_\Omega = \frac{r^2\,\Omega}{q^2} \left(\frac{\omega_A}{\Omega}\right)^3 \left(\frac{\Omega}{N}\right)$$

but Taylor-Spruit dynamo debated (e.g. Zahn et al 07)

Magnetic Fields: Models

15 M_o, Z=0.02 & v_{ini} =300 km/s



(Maeder & Meynet 2005)

Transport of Ω (ν):

dominated by B-fields (v)

Flatter Ω profiles

Transport of X_i (η):

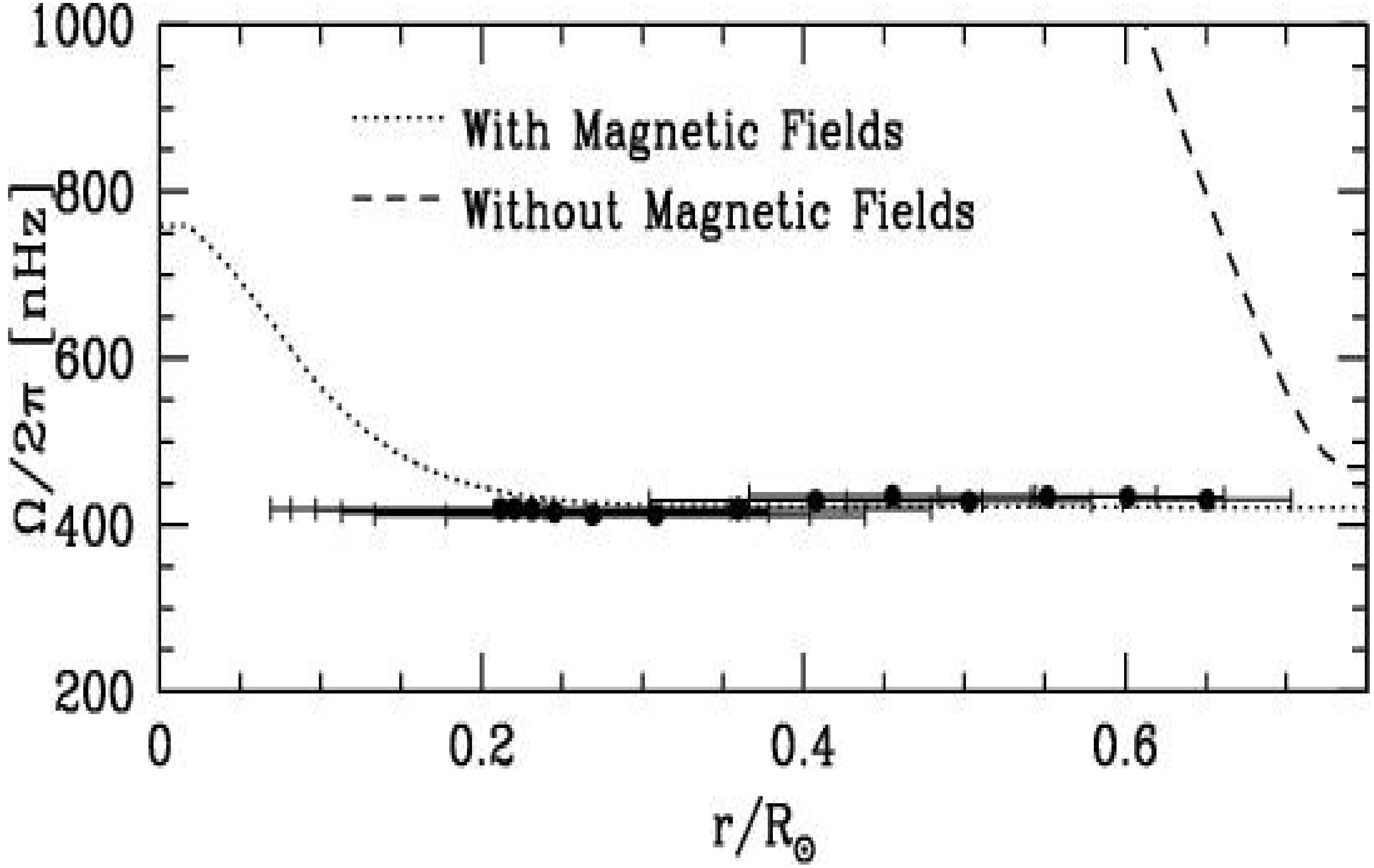
Dominated by meridional circulation (D_{eff})

Stronger mixing

Magnetic Fields: Rotation of the Sun

Sun rotation profile compatible with helioseismology

(Eggenberger et al 2005)



Taylor-Spruit dynamo debated

Brun & Zahn 2009

Gravity waves can also help

(Charbonnel & Talon 2005, Arnett & Meakin 2006)

Magnetic Fields: Massive Stars

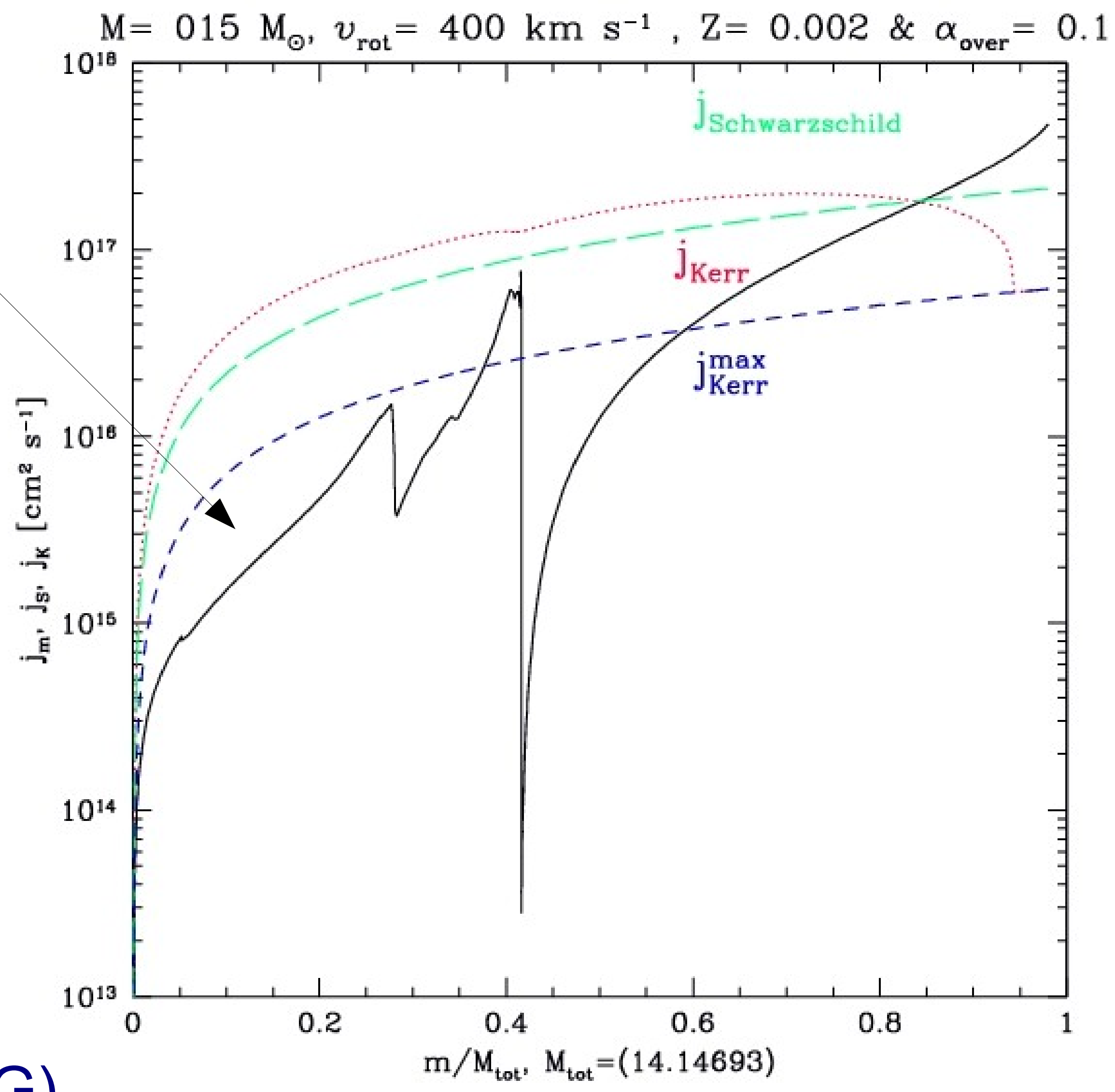
Taylor-Spruit dynamo (Spruit 02, Maeder & Meynet 05) :

Better for pulsar periods

(see also Heger et al 2005)

Not enough for WDs

(Suijs et al 08)



Other mechanisms?

- Dynamo in conv. env.
- During/after explosions

(see discussion in Meynet et al 11,13)

GRBs/MHD explosions?

← Quasi chemically-homog. evol. of fast rot. stars (avoid RSG)

(Yoon et al 06,07, Woosley & Heger 2006)

Magnetic Fields: Low & Massive Stars

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Better for pulsar periods

(see also Heger et al 2005)

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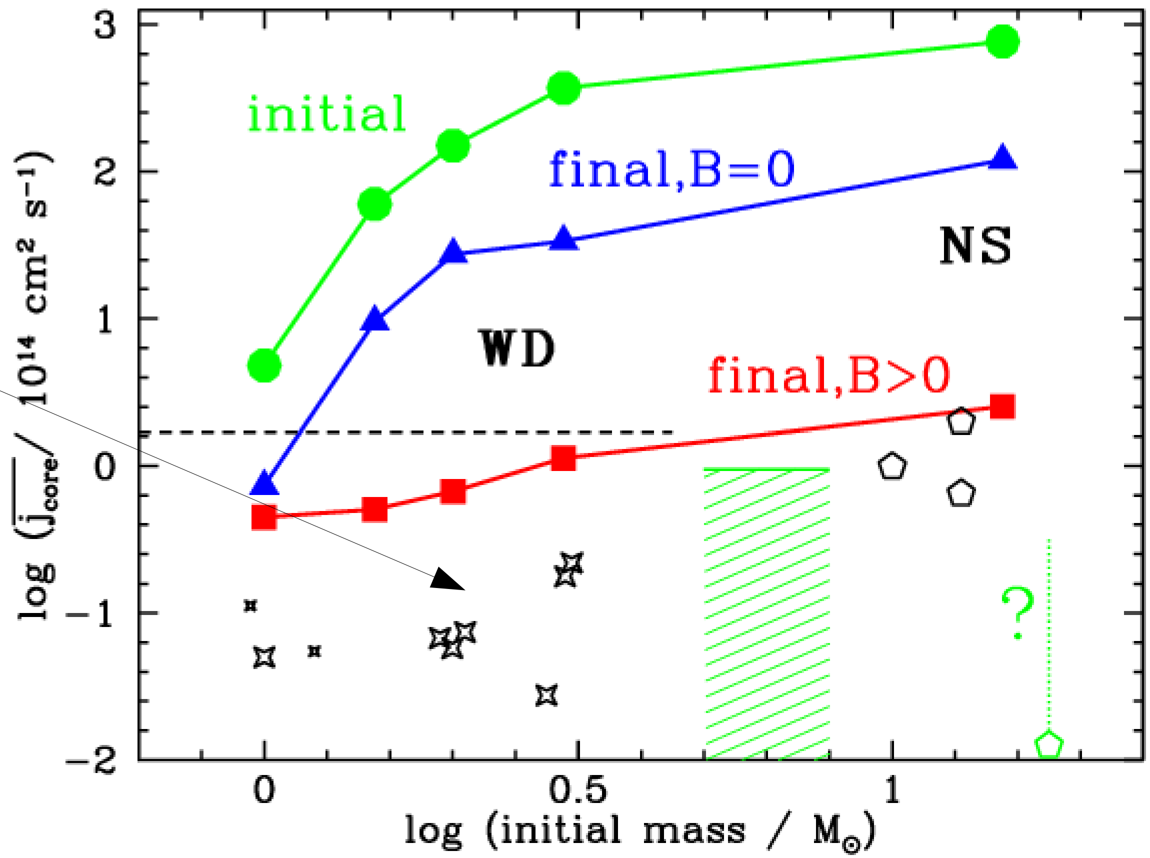
(Suijs et al 08)



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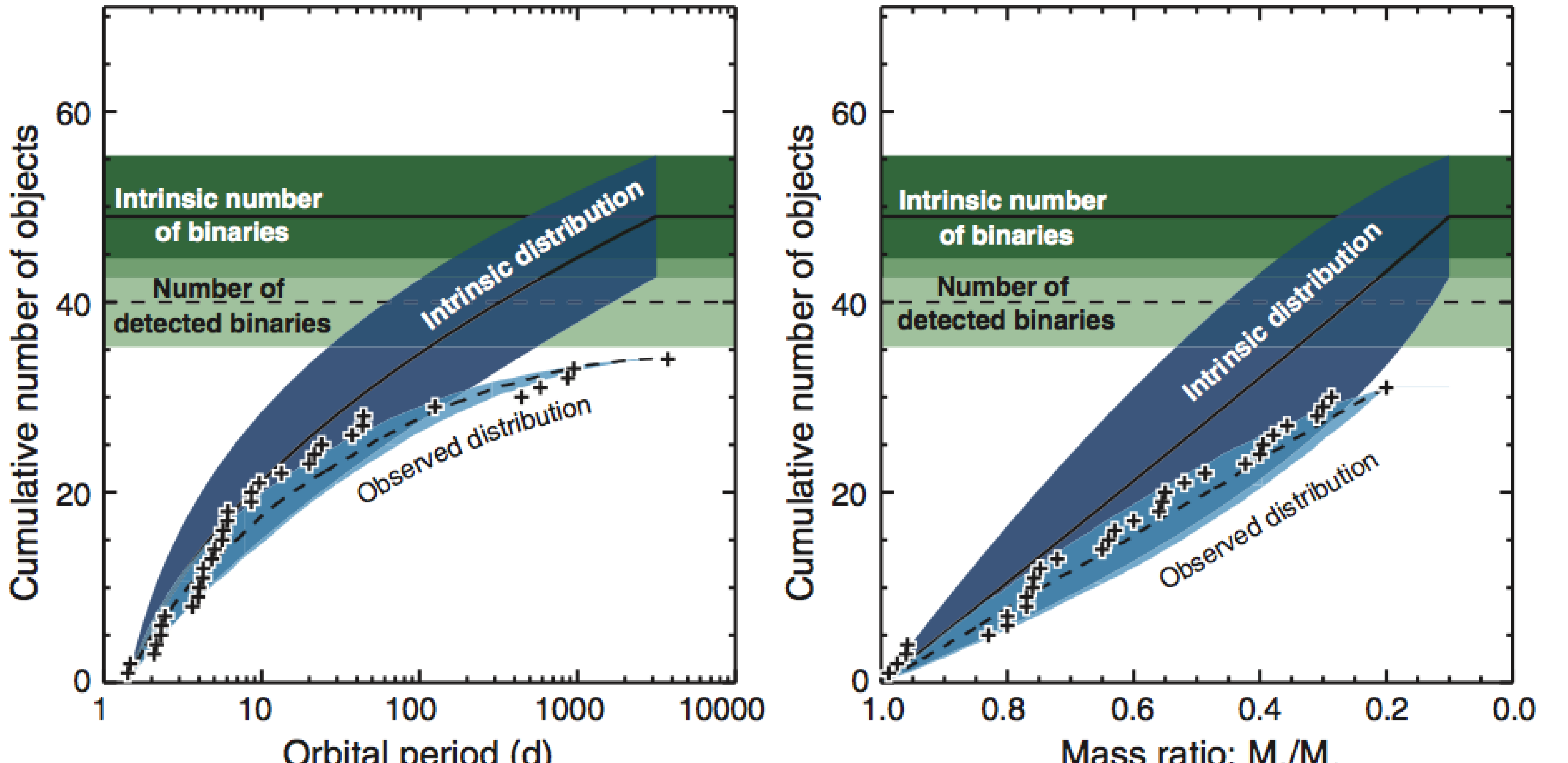
evol. of fast rot. stars (avoid RSG)

(Yoon et al 06,07, Woosley & Heger 2006)

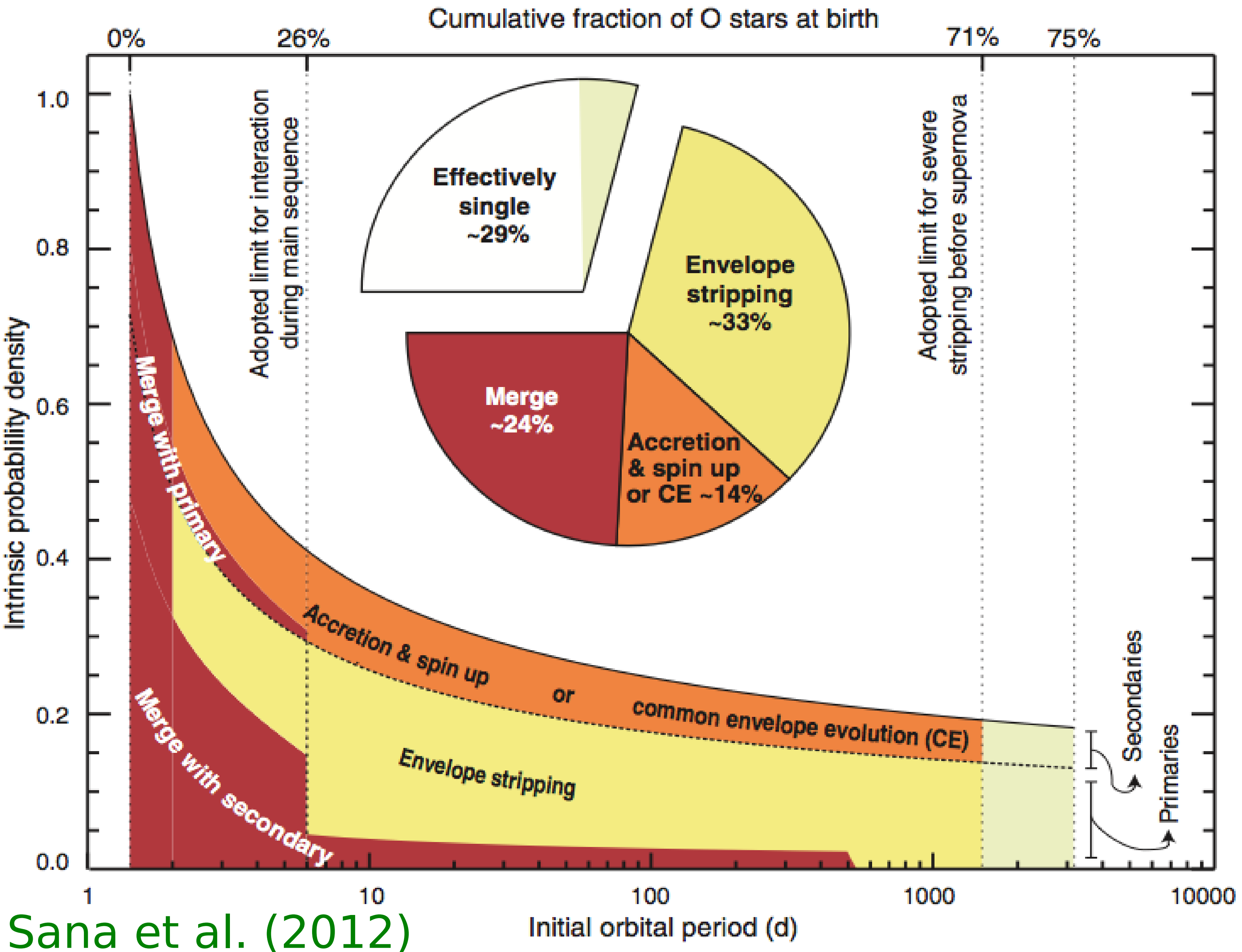
Binarity

- Stars in six nearby **galactic** open clusters→
- 71 single and multiple O-type objects
- 40 detected binaries



Sana et al. (2012)

Binarity



Sana et al. (2012)

Binarity

The VLT-FLAMES Tarantula Survey★ (LMC)

VIII. Multiplicity properties of the O-type star population

H. Sana[1], A. de Koter[1,2], S.E. de Mink[3,4★★], P.R. Dunstall[5], C.J. Evans[6], V. Hénault-Brunet[7], J. Maíz Apellániz[8], O.H. Ramírez-Agudelo[1], W.D. Taylor[7], N.R. Walborn[3], J.S. Clark[9], P.A. Crowther[10], A. Herrero[11,12], M. Gieles[13], N. Langer[14], D.J. Lennon[15,3], and J.S. Vink[16]

(Affiliations can be found after the references)

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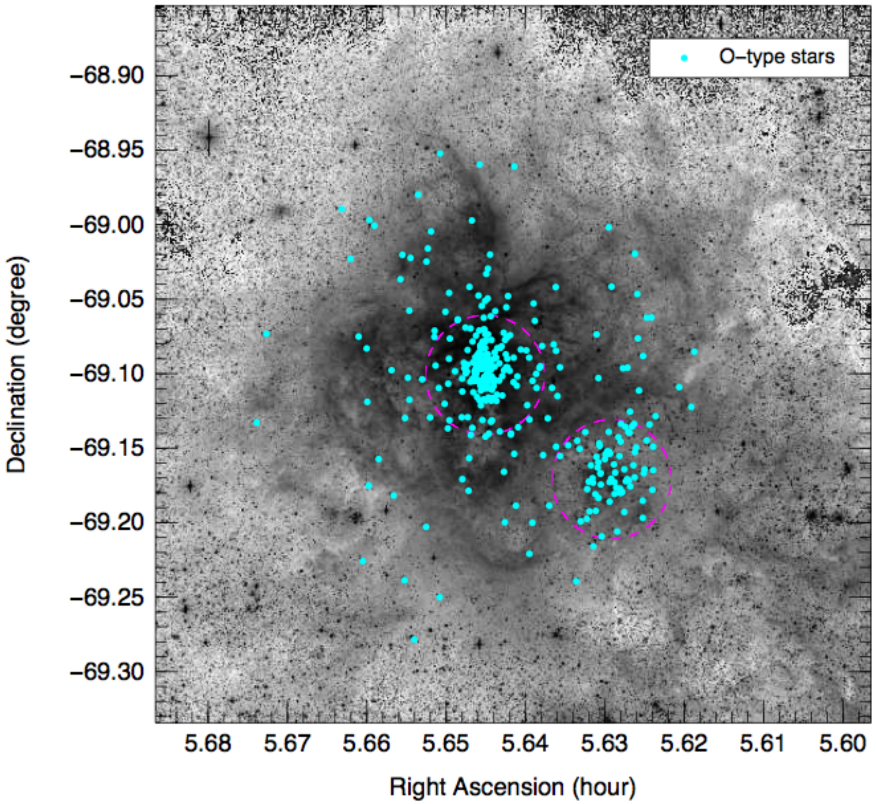
360 O-type stars
Intrinsic binary fraction 51%



Sana et al. (2012)

The overall problem

- $m = M_r$: Lagrangian coordinate
- r, P, T, L_r: independent variables
- X_i: composition variables
- ρ, κ, ϵ, $\ldots$: dependent variables
- initial value problem in time: $r(m, t = 0)$, $P(m, t = 0)$, $T(m, t = 0)$, $L_r(m, t = 0)$, $\vec{X}(m, t = 0) = \vec{X}(t = 0) \rightarrow$ integration with time
- boundary value problem in space: $r(m = 0, t) = 0$, $L_r(m = 0, t) = 0$ and $L = 4\pi\sigma R^2 T_{\mathrm{eff}}^4$, $P(m = M) = P(\tau = 2/3)$

The equations

The four structure equations to be solved are:

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho}$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2}$$

$$\frac{\partial L_r}{\partial m} = \epsilon_n - \epsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t}$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla$$

The equations

For energy transport, we have to find the appropriate ∇.
In case of radiative transport, this is:

$$\nabla_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{\kappa L_r P}{mT^4}$$

Transport of angular momentum:

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\bar{\Omega}\right)_{M_r} = \underbrace{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4\bar{\Omega}U(r)\right)}_{\text{advection term}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho D r^4\frac{\partial\bar{\Omega}}{\partial r}\right)}_{\text{diffusion term}}$$

Transport of chemical elements:

$$\rho\frac{\mathrm{d}X_i}{\mathrm{d}t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2\left[D + D_{eff}\right]\frac{\partial X_i}{\partial r}\right) + \left(\frac{\mathrm{d}X_i}{\mathrm{d}t}\right)_{\mathrm{nucl}}$$

Solving
Equations
on Computers

Numerical Procedure

In addition: $\rho(P,T,X_i)$, $\kappa(P,T,X_i)$, $r_{jk}(P,T,X_i)$, $\epsilon_n(P,T,X_i)$, $\epsilon_\nu(P,T,X_i)$, $\ldots$

In space (mass $0 \le m \le M$): boundary value problem with boundary conditions:

- at center: $r(0) = 0$, $L_r(0) = 0$
- at $r = R$: P and T either from $P = 0$, $T = 0$ ("zero b.c.") or from atmospheric lower boundary and:
- Stefan-Boltzmann-law $L = 4\pi\sigma R^2 T_{\rm eff}^4$

Central conditions

Series expansion in m around center:

$$r = \left(\frac{3}{4\pi\rho_c}\right)^{1/3} m^{1/3}$$

$$P = P_c - \frac{3G}{8\pi}\left(\frac{4\pi}{3}\rho_c\right)^{4/3} m^{2/3}$$

$$L_r = (\epsilon_g + \epsilon_n - \epsilon_\nu)_c m$$

Stellar atmospheres (hydrostatic, grey):

- optical depth: $\tau := \int_R^\infty \kappa\rho \mathrm{d}r = \bar{\kappa}\int_R^\infty \rho \mathrm{d}r$;
- pressure $P(R) := \int_R^\infty g\rho \mathrm{d}r \approx g_R \int_R^\infty \rho \mathrm{d}r \Rightarrow P = \frac{GM}{R^2}\frac{2}{3}\frac{1}{\kappa}$

In time, we have an initial value problem (zero-age model).

Numerical methods

Spatial problem

1. **Direct integration** : e.g. Runge-Kutta
2. **Difference method** : difference equations replace differential equations
3. **Hybrid methods** : direct integration between fixed mesh-points; multiple-fitting method, but the variation of the guesses at the fixed points is done via a Newton-method

The Henyey-method

Write the equations in a general form:

$$A_i^j := \frac{y_i^{j+1} - y_i^j}{m_i^{j+1} - m_i^j} - f_i(y_1^{j+1/2}, \dots, y_4^{j+1/2})$$

upper index: grid-point ($j + 1/2$ mean value); lower index: i-th variable
Outer and inner boundary conditions:

$$B_i = 0 \quad i = 1, 2 \qquad C_i = 0 \quad i = 1, \dots, 4$$

where the inner ones are to be taken at grid-point $N - 1$ and the expansions around $m = 0$ have been used already.

Henyey-method (contd.)

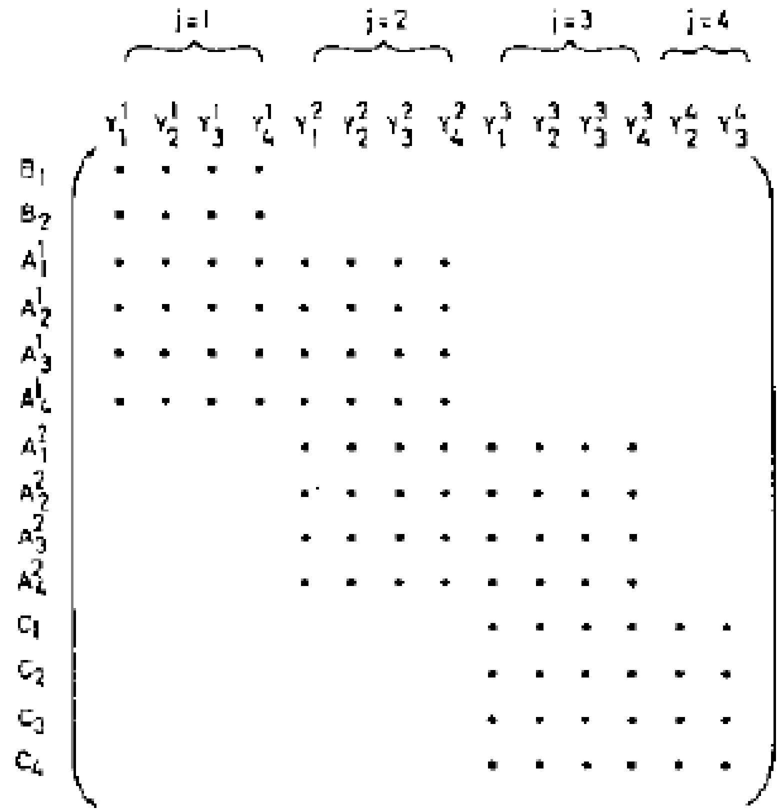
$\rightarrow 2 + 4 + (N-2)\cdot 4 = 4N-2$ equs.
for $4\times N$ unknowns -2 b.c.
Newton-approach for corrections δy_i

$$A_i^j + \sum_i \frac{\partial A_i^j}{\partial y_i}\delta y_i = 0$$

$$H \begin{pmatrix} \delta y_1^1 \\ \delta y_2^1 \\ \vdots \\ \delta y_3^N \\ \delta y_4^N \end{pmatrix} = \begin{pmatrix} B_1 \\ \vdots \\ A_i^j \\ \vdots \\ C_4 \end{pmatrix}$$

Henyey-scheme

Matrix H contains all derivatives and is called Henyey-matrix. It contains non-vanishing elements only in blocks. This leads to a particular method for solving it (Henyey-method).



Expresse some of corrections in terms of others, e.g.:
$\delta y_1^1 = U_1 \delta y_3^2 + V_1 \delta y_4^2 + W_1 \Rightarrow$ matrix equations for U_i, V_i, W_i.

Henyey-scheme

First block-matrix $j = 1, 2$:

$$
\begin{bmatrix}
\frac{\partial B_1}{\partial y_1^1} & \frac{\partial B_1}{\partial y_2^1} & \cdots & 0 \\
\frac{\partial B_2}{\partial y_1^1} & \frac{\partial B_2}{\partial y_2^1} & \cdots & 0 \\
\frac{\partial A_1^1}{\partial y_1^1} & \frac{\partial A_1^1}{\partial y_2^1} & \cdots & \frac{\partial A_1^1}{\partial y_2^2} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial A_1^4}{\partial y_1^1} & \frac{\partial A_1^4}{\partial y_2^1} & \cdots & \frac{\partial A_1^4}{\partial y_2^2}
\end{bmatrix}
\begin{bmatrix}
U_1 & V_1 & W_1 \\
U_2 & V_2 & W_2 \\
U_3 & V_3 & W_3 \\
\vdots & \vdots & \vdots \\
U_6 & V_6 & W_6
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & -B_1 \\
0 & 0 & -B_2 \\
-\frac{\partial A_1^1}{\partial y_3^2} & -\frac{\partial A_1^1}{\partial y_4^2} & -A_1^1 \\
\vdots & \vdots & \vdots \\
-\frac{\partial A_4^1}{\partial y_3^2} & -\frac{\partial A_4^1}{\partial y_4^2} & -A_4^1
\end{bmatrix}
$$

Integration in time

$$X_i(t + \triangle t) = X_i(t) + \frac{\partial X_i}{\partial t}(T(t), P(t), \ldots)\triangle t$$

Improvement:
backward differencing (e.g. nuclear network)

$$X_i(t + \triangle t) = X_i(t) + \triangle t \sum_i r_{ij}(t) X_i(t + \triangle t) X_j(t + \triangle t)$$

done for chemical evolution, mixing, diffusion, etc.

Nowadays matrices can be inverted without splitting them into small sections and without decoupling of space and time, see e.g. MESA code: Paxton et al 2011

Homology Relations
& Stability
if time permits

Mass Loss: General Dependence on Stellar Mass

More massive stars have stronger winds because they are much more luminous:

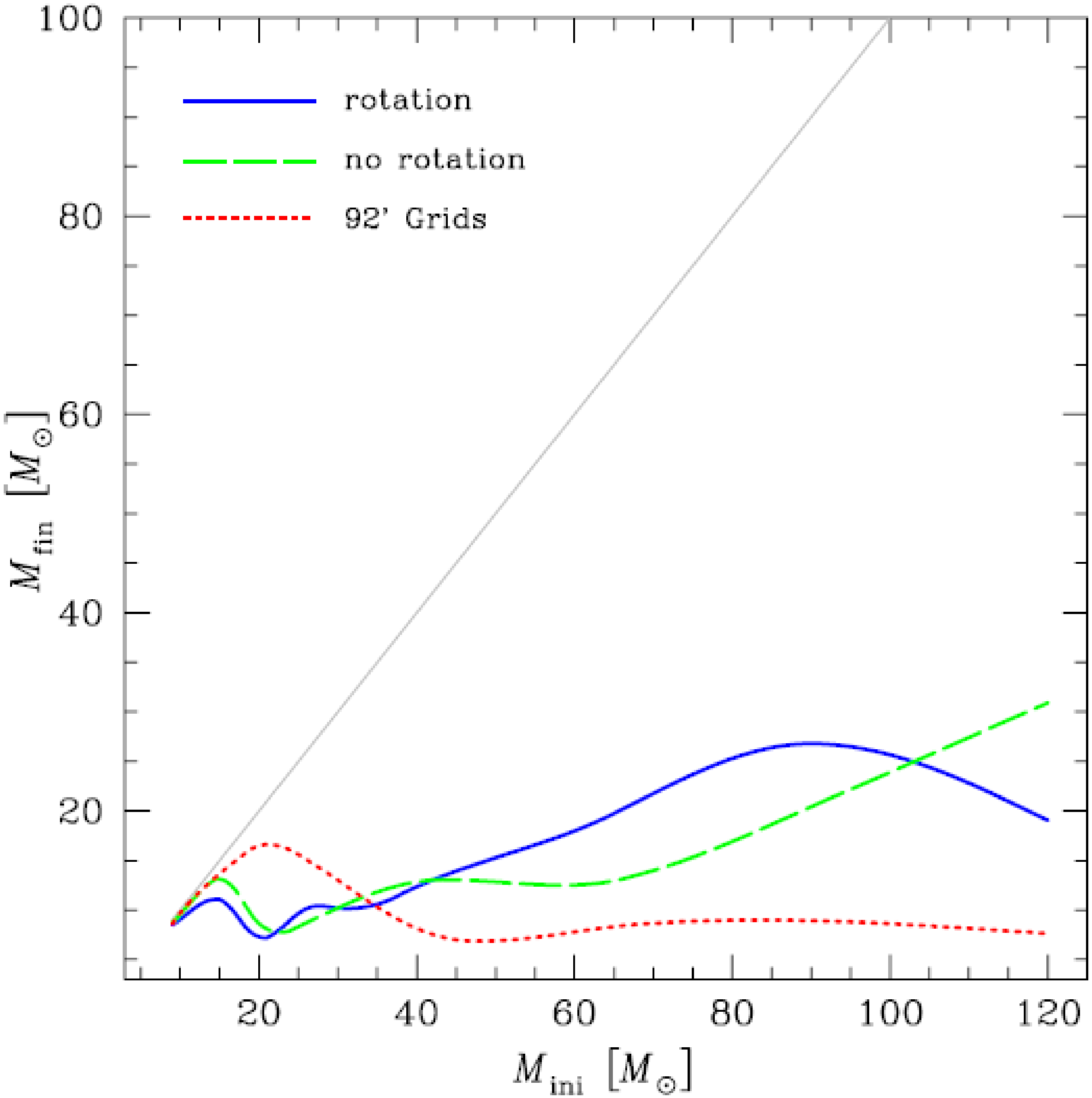
For low-mass stars:
$$L \propto \frac{\beta^4 \mu^4 M^3}{\kappa}$$

For high-mass stars:
$$L \propto \frac{\mu^4 M}{\kappa}$$

Where β is the ratio of gas to total pressure: 1 → 0 from low to high-mass stars

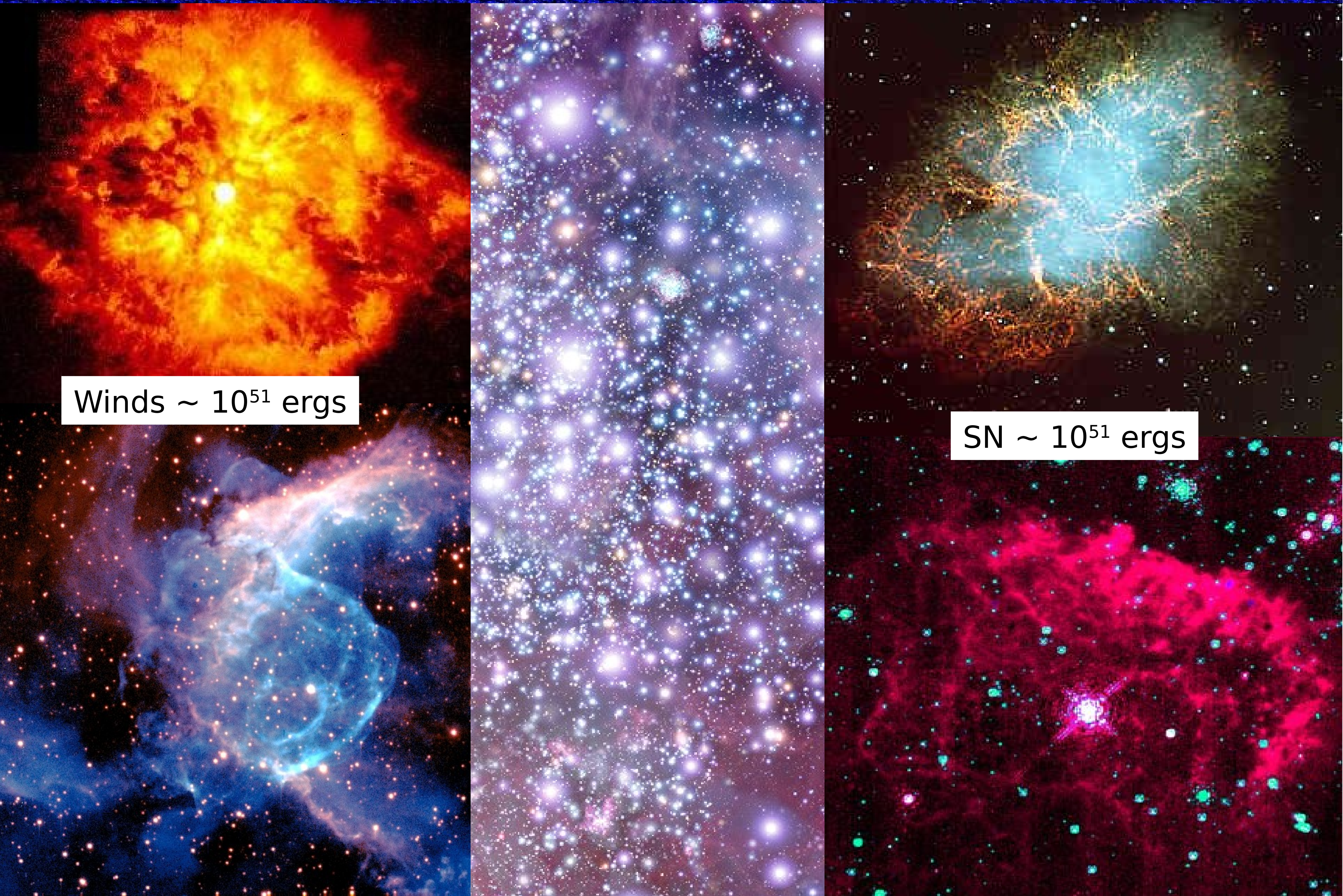
Spare slides (mass loss, GRBs SNII,Ib,Ic)

Importance of Mass Loss



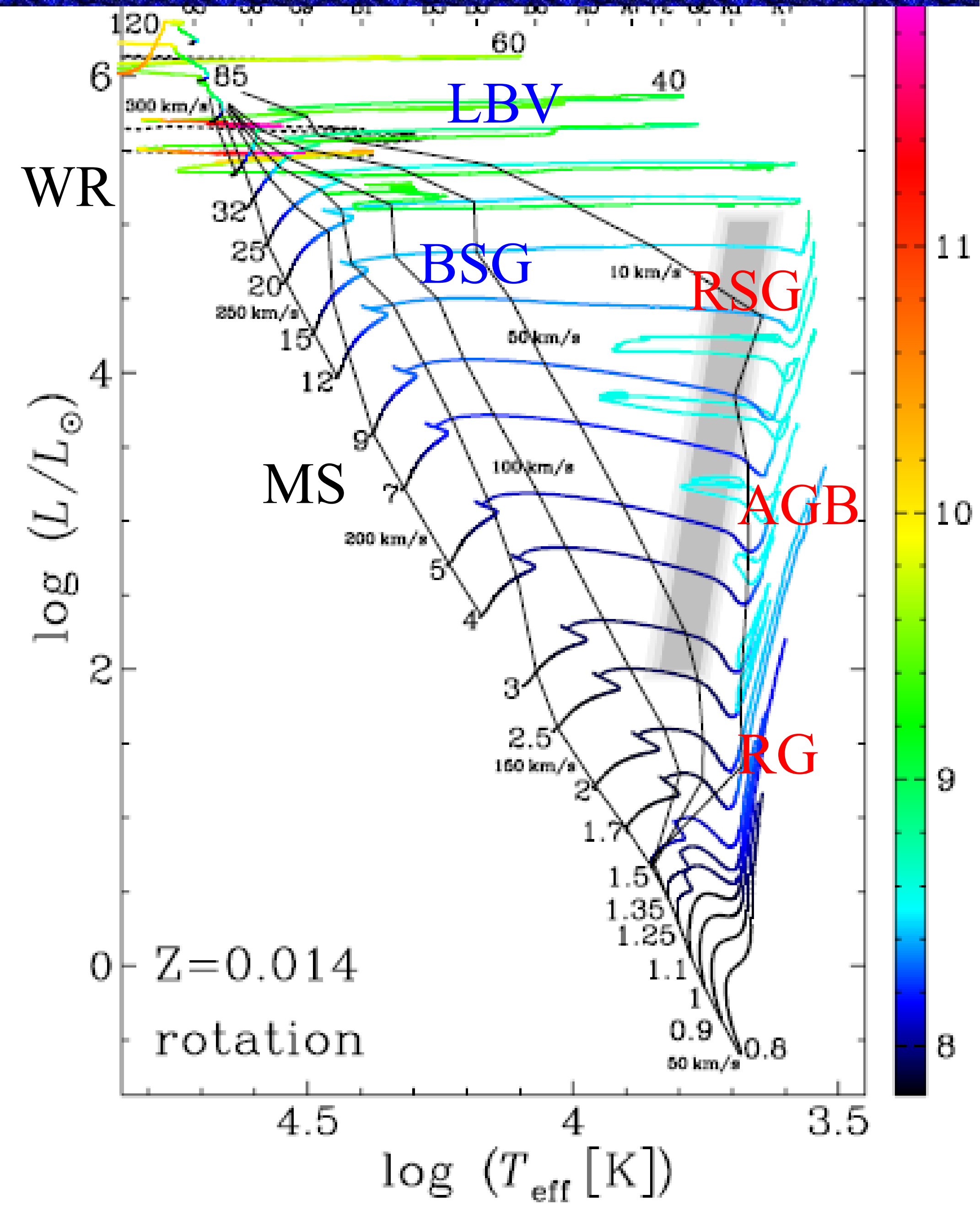
Ekström et al 12, see also Chieffi & Limongi 13

Injection of Mechanical Energy



Main Phases of Stellar Evolution

Mass loss driving mechanism and prescriptions are very different for different evolutionary stages

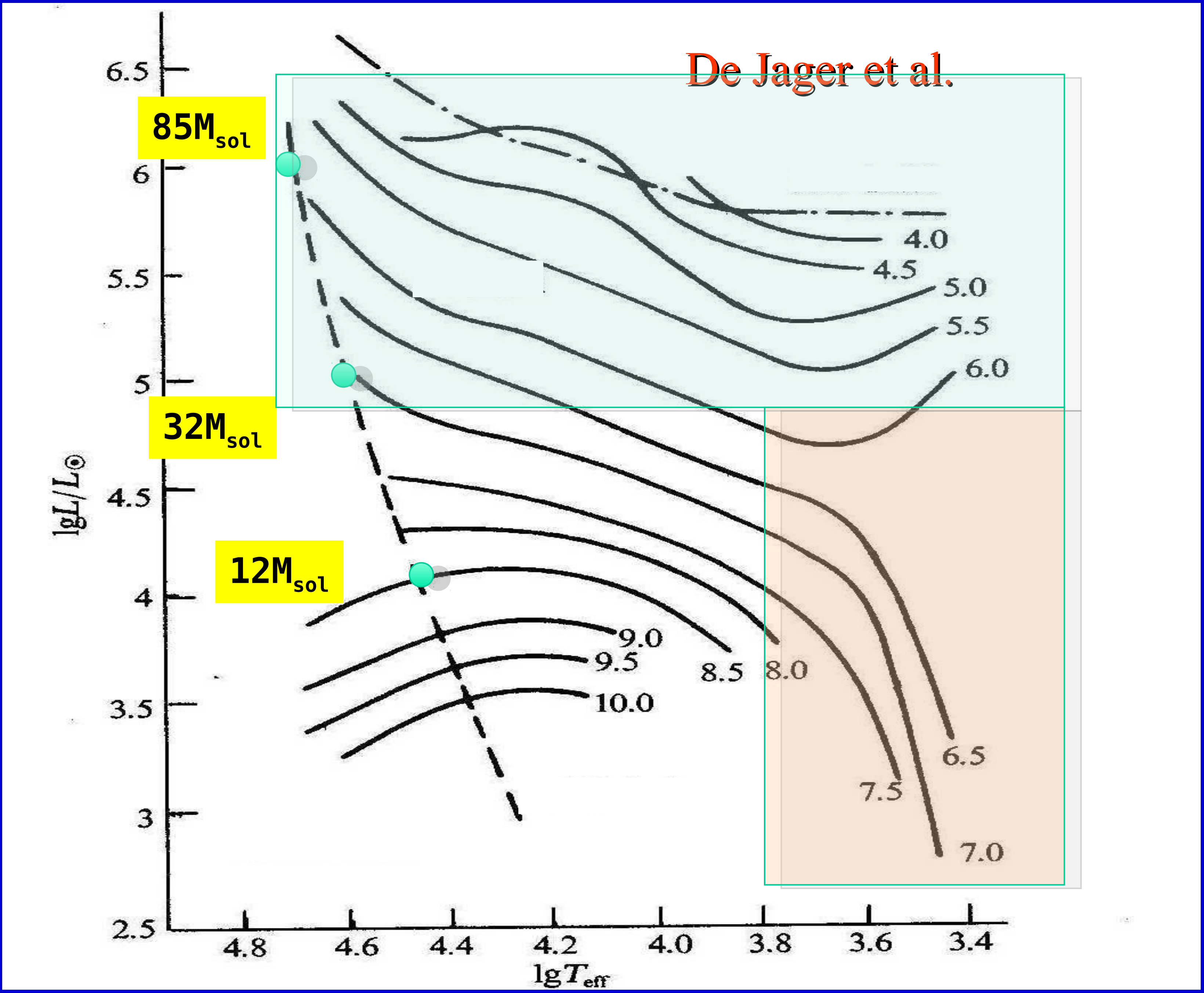


Ekstroem et al 12

Mass Loss: Types, Driving & Recipes

Mass loss driving mechanism and prescriptions at different stages:

- O-type & “LBV” stars (bi-stab.): line-driven Vink et al 2000, 2001
- WR stars (clumping effect): line-driven Nugis & Lamers 2000, Gräfener & Hamann (2008)
- RSG: Pulsation/dust? de Jager et al 1988
- RG: Pulsation/dust? Reimers 1975,78, with η=~0.5
- AGB: Super winds? Dust Bloecker et al 1995, with η=~0.05
- LBV eruptions: continuous driven winds? Owocki et al
- ...



De Jager et al.
85M_sol
32M_sol
12M_sol
lgL/L☉
lgT_eff
6.5
6
5.5
5
4.5
4
3.5
3
2.5
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10.0

What changes at low Z?

- Stars are more compact: $R \sim R(Z_o)/4$ (lower opacities) at $Z=10^{-8}$
- Mass loss weaker at low Z: → faster rotation

$$\dot{M}(Z)=\dot{M}(Z_0)(Z/Z_0)^{\alpha}$$

- α = 0.5-0.6 (Kudritzki & Puls 00, Ku02) (Nugis & Lamers, Evans et al 05)

- α = 0.7-0.86 (Vink et al 00,01,05)

$Z(LMC) \sim Z_o/2.3$ => Mdot/1.5 – Mdot/2

$Z(SMC) \sim Z_o/7$ => Mdot/2.6 - Mdot/5

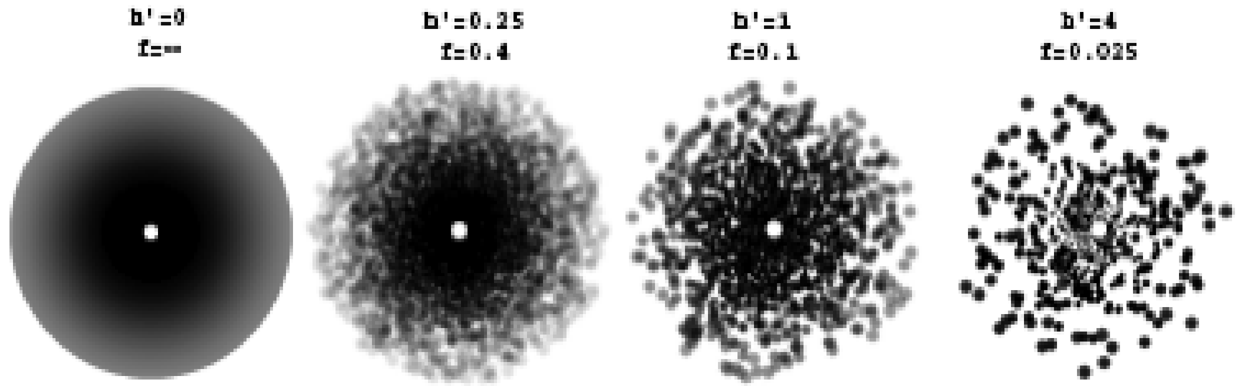
Mass loss at low Z still possible?

RSG (and LBV?): no Z-dep.; CNO? (Van Loon 05, Owocky et al)

Mechanical mass loss ← critical rotation

(e.g. Hirschi 2007, Ekstroem et al 2008, Yoon et al 2012)

CLUMPING



ρ^2

diagnostics

If wind clumped in reality but supposed to be homogeneous

Excess emission from inhomogeneities →
incorrectly interpreted as
arising from a smooth but denser medium

MASS LOSS OVERESTIMATED

Fullerton et al 05: Mdot/10

Bouret et al 05: Mdot/3 or smaller

Surlan et al 13: problem resolved?

RSG/YSG/WR – SN II, IIb, Ibc

Observational constraints:

- RSG Upper Luminosity: Log (L/L_{SUN}) ~ 5.2-5.3 *(median value of the most 5 L_{SUN} stars)* (Levesque et al 05)

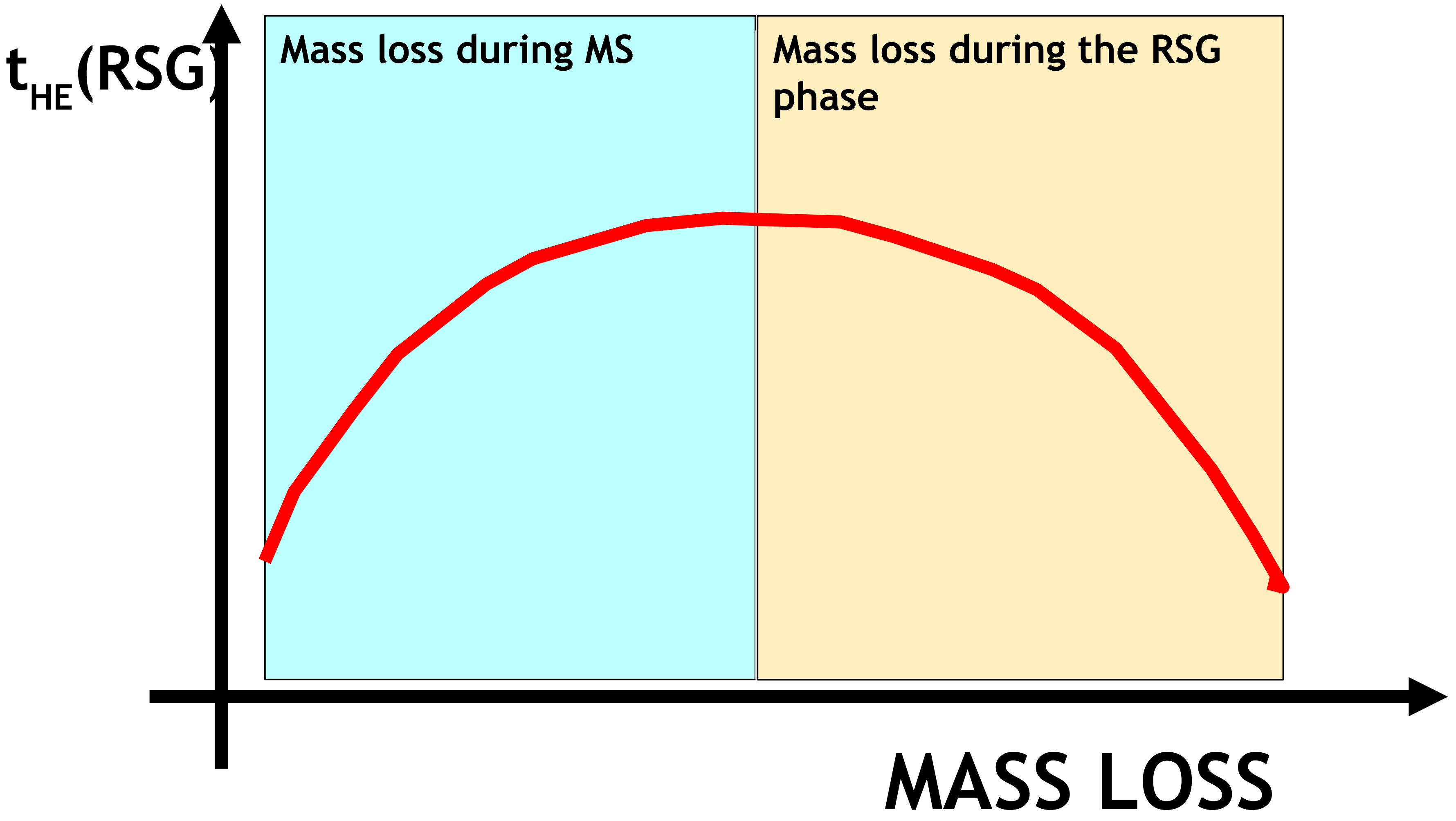
- SNII-P Log (L/L_{SUN}) <~5.1 (Smartt et al. 2009)

- No clear dependence on Z for these upper limit

- WR/O, RSG/BSG ratios vary with Z

CHANGE OF MASS LOSS

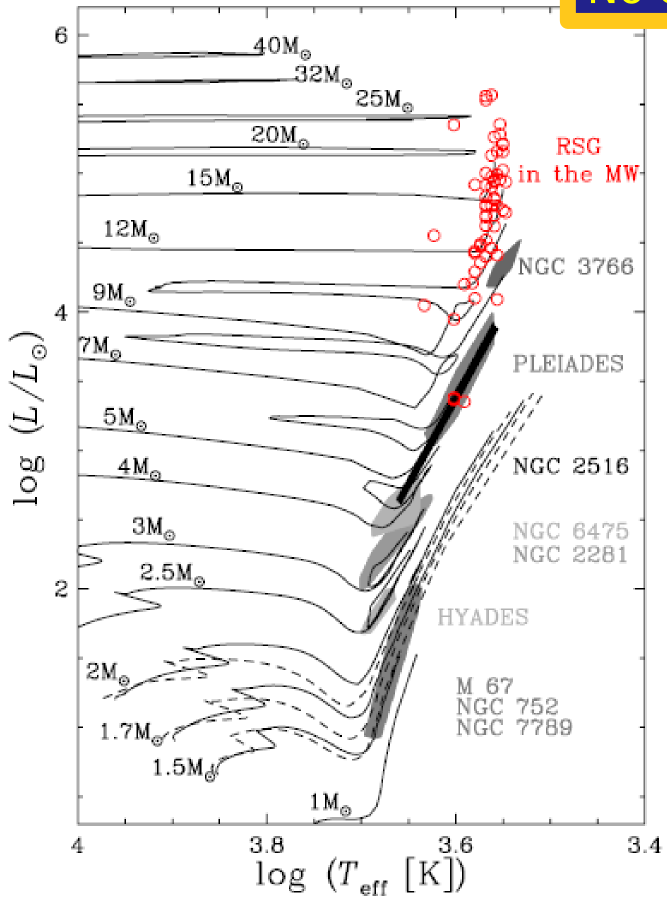
For a given initial mass

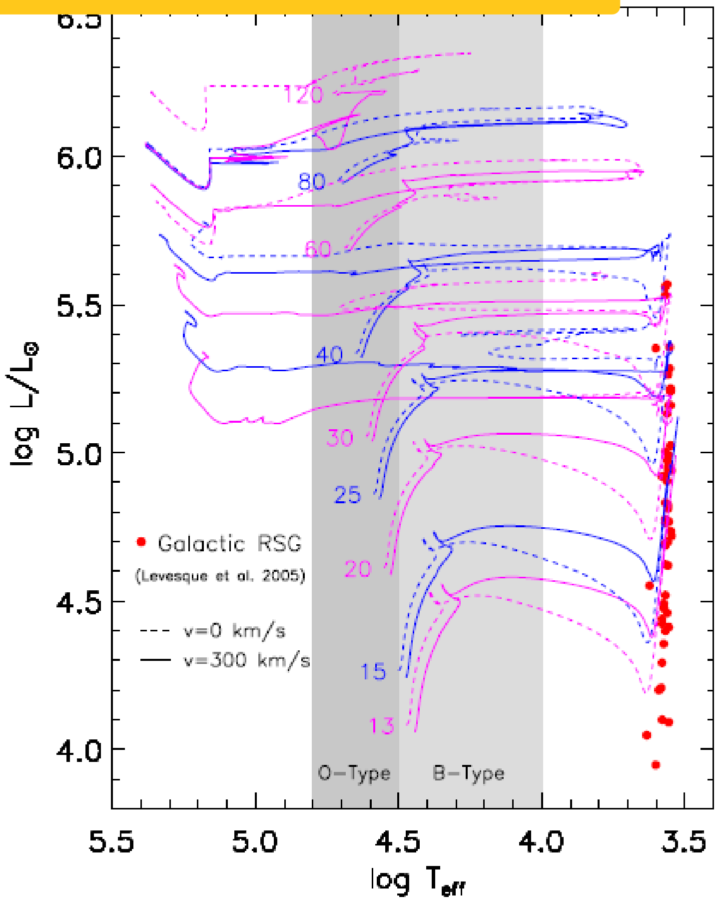


RSG/YSG/WR – SN II, IIb, Ib, Ic

RSG Upper Luminosity ~ 5.2-5.3
(median value of the most 5 L_{SUN} stars)
SNII-P ~ 5.1 (Smartt et al. 2009)

No clear dependence on Z





- **Tracks: Ekstroem et al 12**
- **grey areas: obs. See MM89**
- **red circles: Levesque et al 05**

- **Tracks: Chieffi & Limongi 13 (CL13)**

Pressure and energy

Pressure as momentum transfer $\perp$ area= $n(\epsilon) \cdot \vec{p}(\epsilon) \cdot \vec{v}(\epsilon)$
Mean over incident angle ($1/3 \cdot p \cdot v$) and particle energy ϵ:

$$P = \frac{1}{3} \int_0^\infty n(\epsilon) p(\epsilon) v(\epsilon) \mathrm{d}\epsilon$$

relativistic case:

$\gamma := \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$; $p = \gamma m v$; $\epsilon = (\gamma - 1) m c^2 \Rightarrow$

$$P = \frac{1}{3} \int_0^\infty n\epsilon \left(1 + \frac{2mc^2}{\epsilon}\right) \left(1 + \frac{mc^2}{\epsilon}\right)^{-1} \mathrm{d}\epsilon$$

Pressure and energy

limiting cases

- non-relativistic: $mc^2 \gg \epsilon \rightarrow$
  $P_{\mathrm{NR}} = \frac{2}{3}\int n\epsilon \mathrm{d}\epsilon = \frac{2}{3}\langle n\epsilon\rangle = \frac{2}{3}U_{\mathrm{NR}}$
- extrem-relativistic: $mc^2 \ll \epsilon \rightarrow P_{\mathrm{ER}} = \frac{1}{3}U_{\mathrm{ER}}$

$\Rightarrow$ general relation $P \sim U$ (energy density)

Radiation pressure:

$$P_{\mathrm{rad}} = \frac{1}{3}U = \frac{a}{3}T^4 \ \left(a = 7.56 \cdot 10^{-15}\,\frac{\mathrm{erg}}{\mathrm{cm}^3\mathrm{K}^4}\right)$$

Typical mass-loss rates for galactic O-type stars on the MS

0.5-20 x 10^{-6} M_{sol} year^{-1}

$$\dot{M} \propto L^{1.7}$$

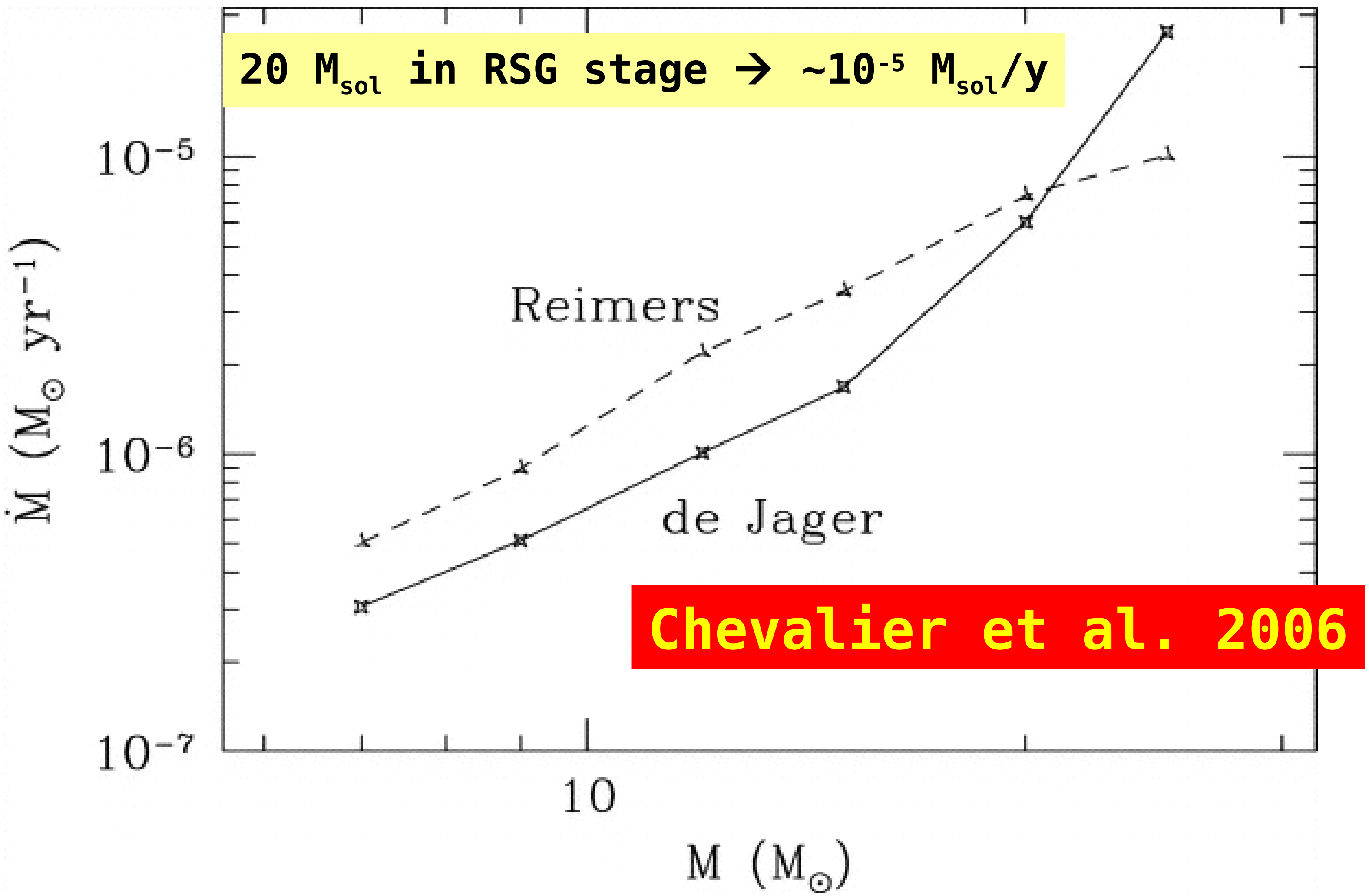
$$\Rightarrow \quad \dot{M} \propto M^{3.4}$$

$$L \propto M^2$$

$$\tau_{MS} \propto M^{-0.6}$$

$$\Rightarrow \quad \Delta M \propto M^{2.8}$$

$$\Delta M / M \propto M^{1.8}$$

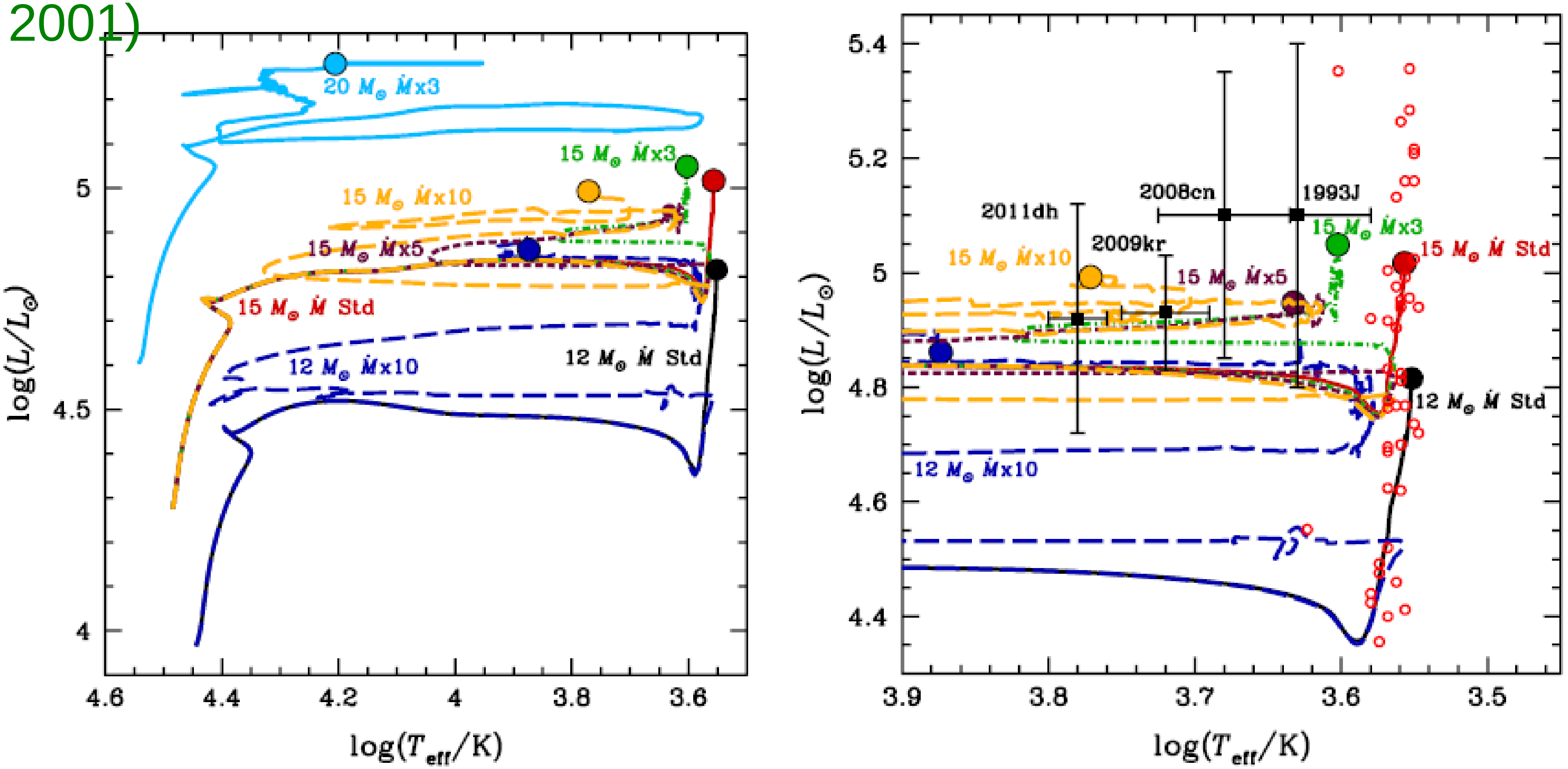


Dust enshrouded red supergiant may have higher mass loss (factor between 3 and 50) van Loon et al. (2005).

RSG/YSG/WR – SN II, IIb, Ib, Ic

RSG Mdot: - log(Teff/K) > 3.7: de Jager et al. (1988)
- log(Teff/K) < 3.7: linear fit from the data of Sylvester et al. (1998) and van Loon et al. (1999) (see Crowther 2001)

$$\dot{M} = -1.479 \times 10^{-14} \times \left(\frac{L}{L_\odot}\right)^{1.7}$$

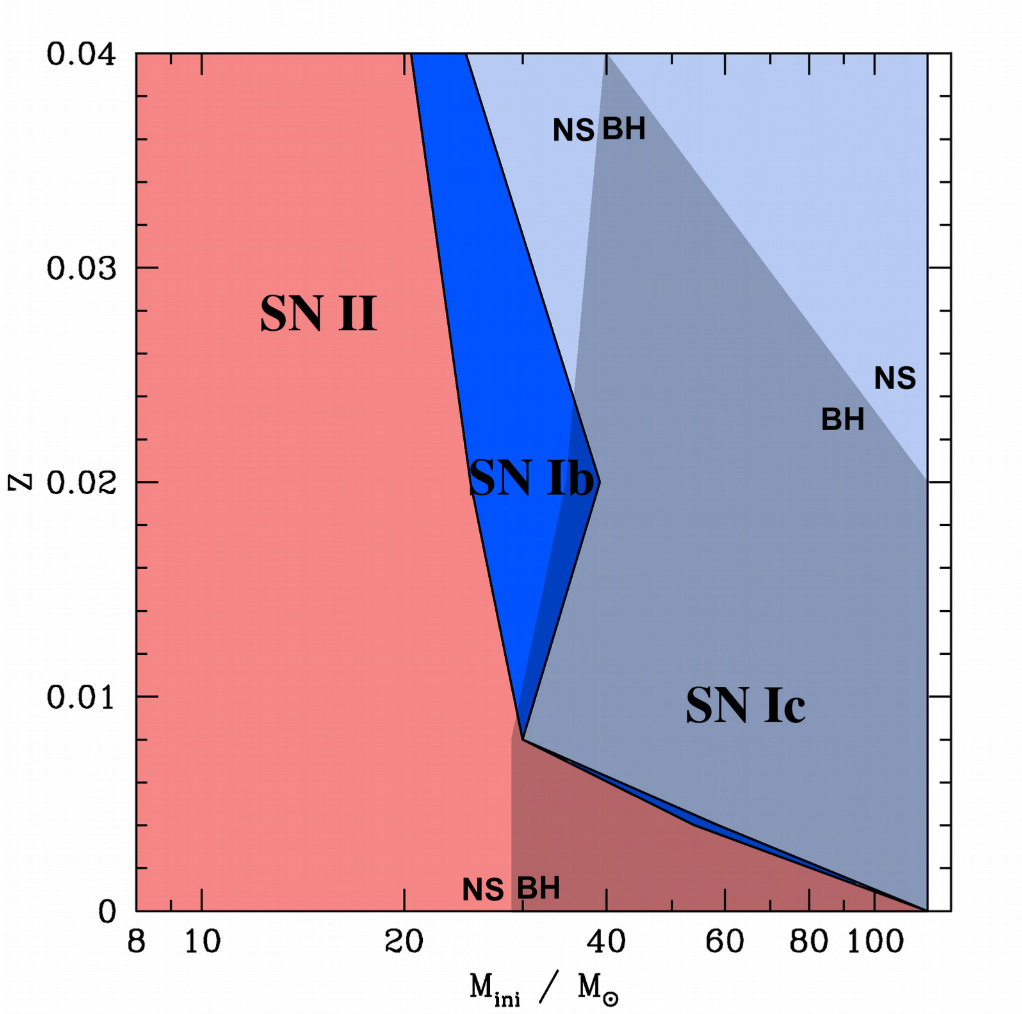


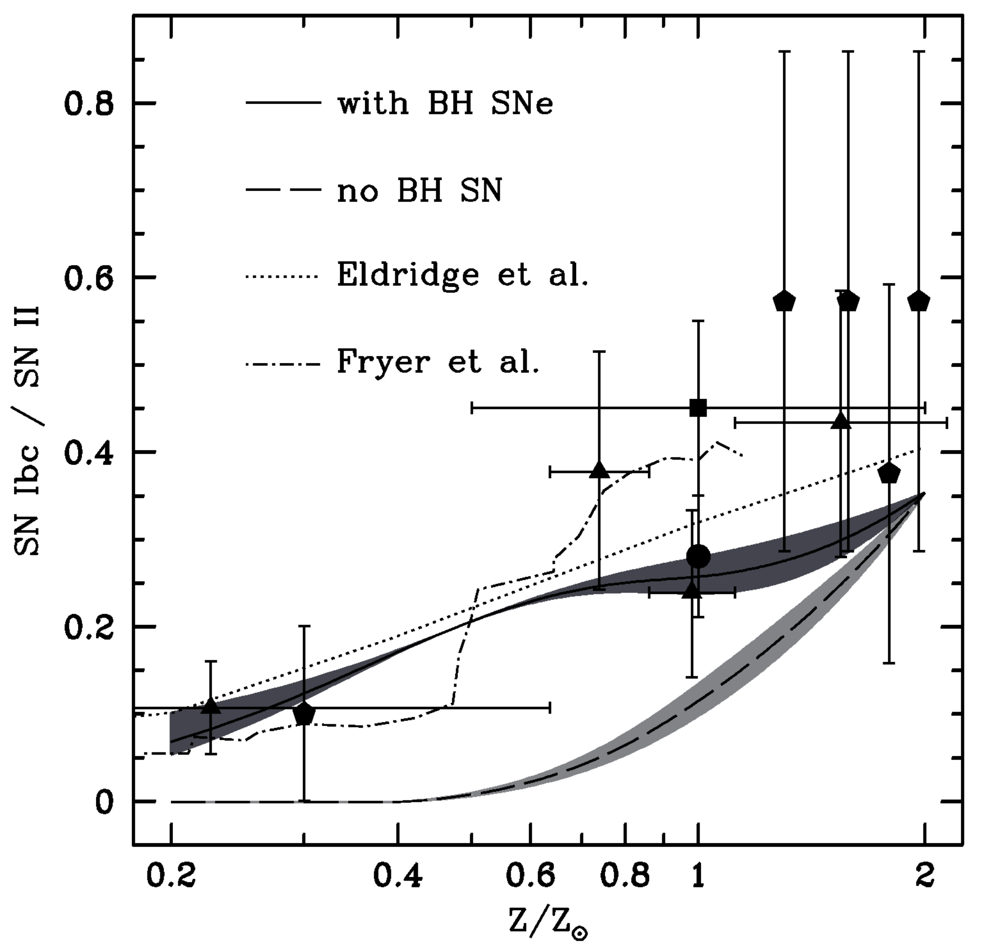
Models: Georgy 12 (see also Eldridge et al 13)

Super-Eddington layers → increased Mdot (see Ekstroem et al 13)

Final stages & SN type

Ratio SNIbc/SNII: tests final type





Georgy et al 09

- THEORY: Georgy et al 09 (solid line)
  binaries: Eldridge etal 08 (dotted)
- OBS: Prantzos & Boissier 03
  (triangles)

Long soft GRB-SN Ic connection: GRB060218/SN2006aj

Cusumano et al 2006, ...

GRB 031203-SN 2003lw / GRB 030329-SN 2003dh / GRB 980425-SN 1998bw, ...

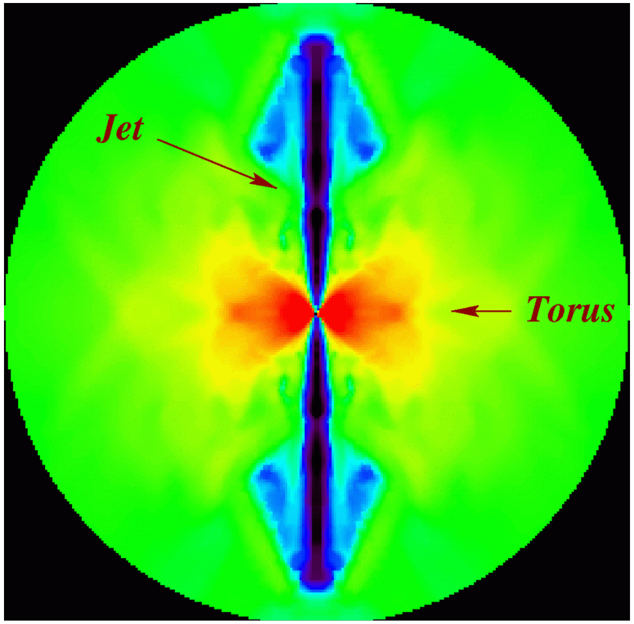
Tagliaferri, G et al 2004 / Matheson 2003, ... / Iwamoto, K. 1999, ...

Collapsar progenitors must: (Woosley 1993, A. Mc Fadyen)

Form a BH

Lose their H-rich envelope → WR star

Core w. enough angular momentum



(simulation by Mc Fadyen)

Observational info:

Z of close-by GRBs is lower than solar

~ Z (Magellanic clouds)

(Stanek et al 06, Le Floc'h er al 2003, Fruchter et al 2006)

Theoretical GRB rates (without B-fields)

Obs: R(GRB)=3x10^{-6} to 6x10^{-4} & R(SNII,Ib,c)~7x10^{-3}

[yr^{-1}]

GRB from all WR types:

Podsiadlowski et al 04

Too many

GRB from WO (SN Ic):

OK with obs.

Hirschi et al A&A, 443, 581, 2005

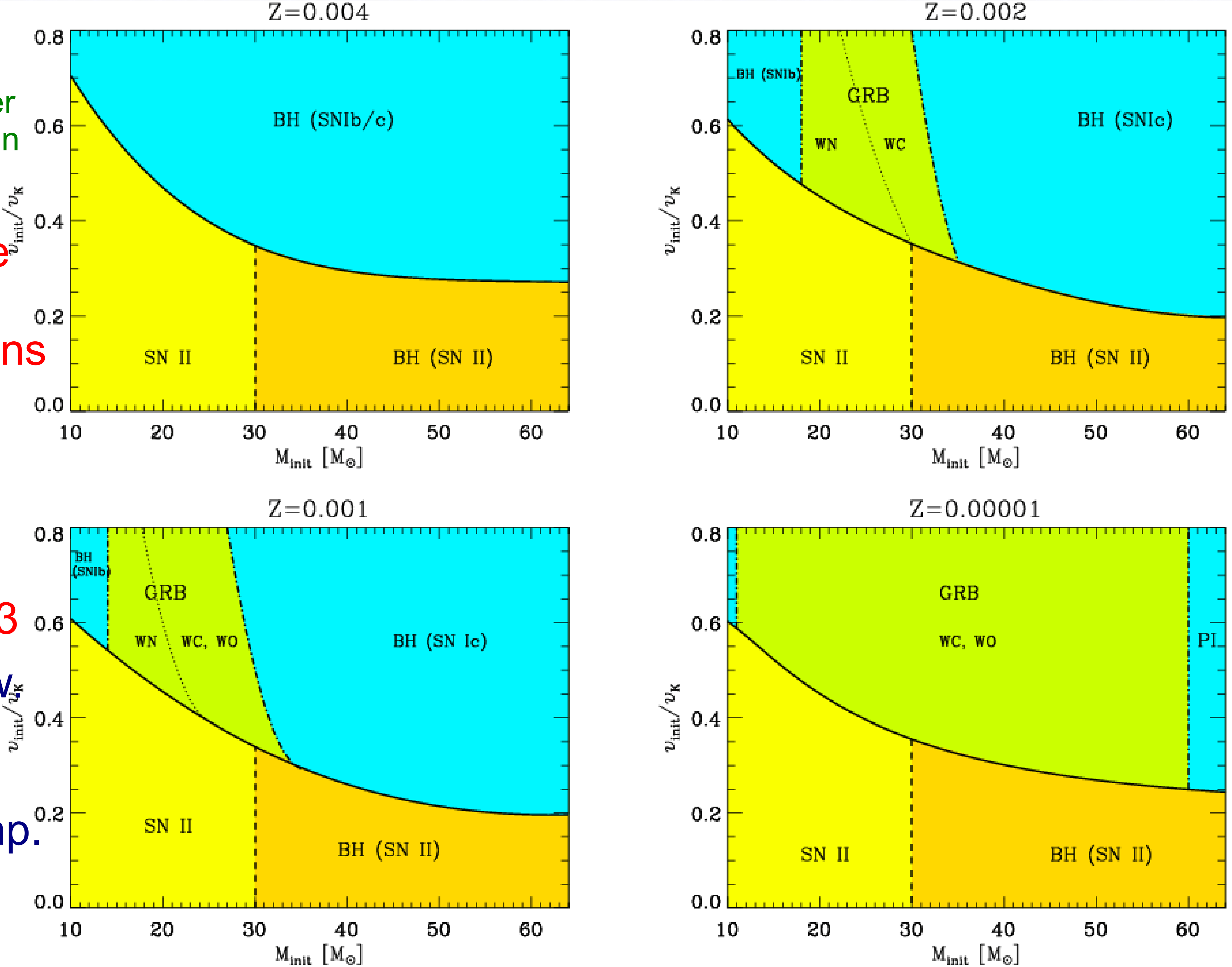
	Z_{SMC}	Z_{LMC}	Z_{o}	Z_{GC}
M_{GRB}^{min}(WR)	32	25	22	21
M_{GRB}^{max}(WR)	95	95	75	55
R_{GRB}^{max}(WR)	1.15E-03	1.74E-03	2.01E-03	1.92E-03
M_{GRB}^{min}(WO)	50	45	-	-
M_{GRB}^{max}(WO)	95	95	-	-
R_{GRB}^{max}(WO)	4.74E-04	5.99E-04	-	-

GRB favoured at low Z, maybe also very low Z (85 Mo)

GRB progenitors with B-Fields

Yoon, Langer & Norman 06

Rates compatible with observations



Z_{max}~0.003 is a bit low.

Dep. on Mdot & Solar comp.

GRB progenitors with B-Fields

Taylor-Spruit dynamo (Spruit 2002) : better for NS (Heger et al 2005, Suijs etal 08)

No A_{BH}>1 in Fe-core @ pre-SN stage with B-fields (Petrovic et al 2005, ...)

- A_{BH}~1 ← Quasi chemically-homog. evol. of fast rot. stars (avoid RSG)
  (Yoon & Langer 06, Woosley & Heger 2006)

40 M_o models

V_{ini} [km/s]	Z_o	Z(SMC)	$Z=10^{-3}$	$Z=10^{-5}$	$Z=10^{-8}$
~230	-	-	-	No	-
~300	-	WR	-	-	-
400-500	WR	WR	WR	WR	No
700	-	-	-	-	WR

This study

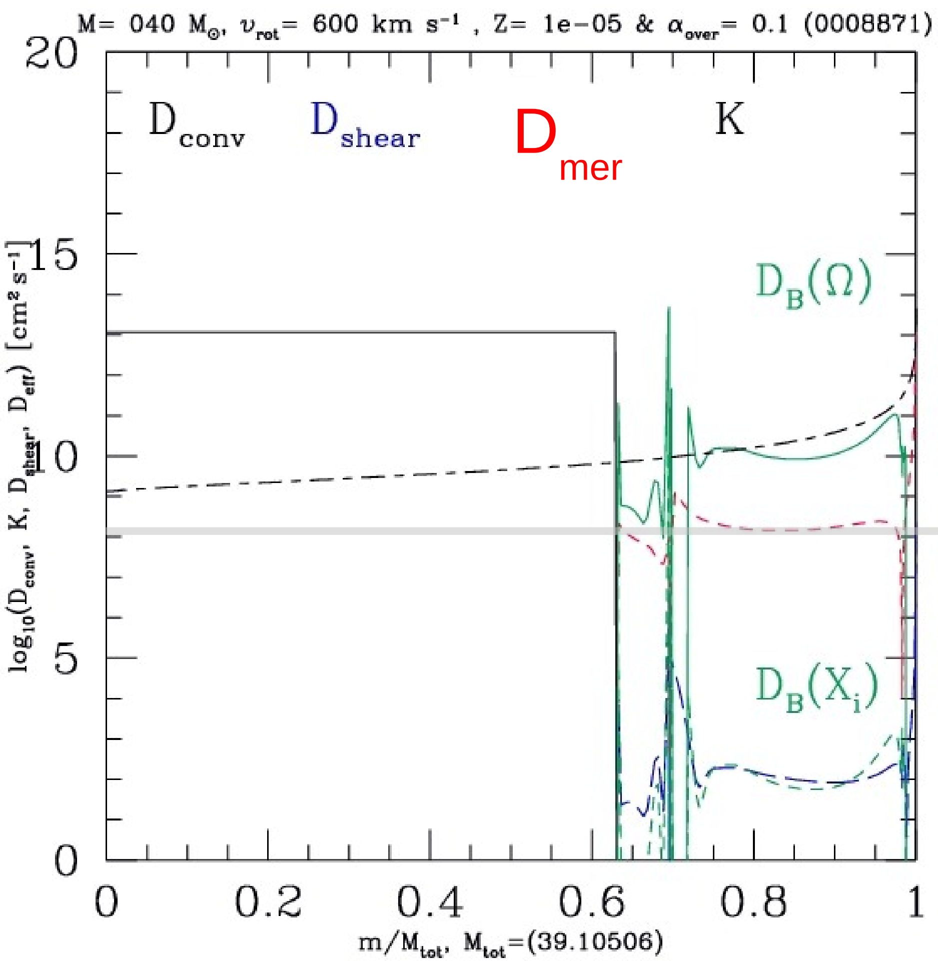
- WR (SNIb,c) & GRBs predicted down to Z=~0 (Yoon et al 06)

Question:

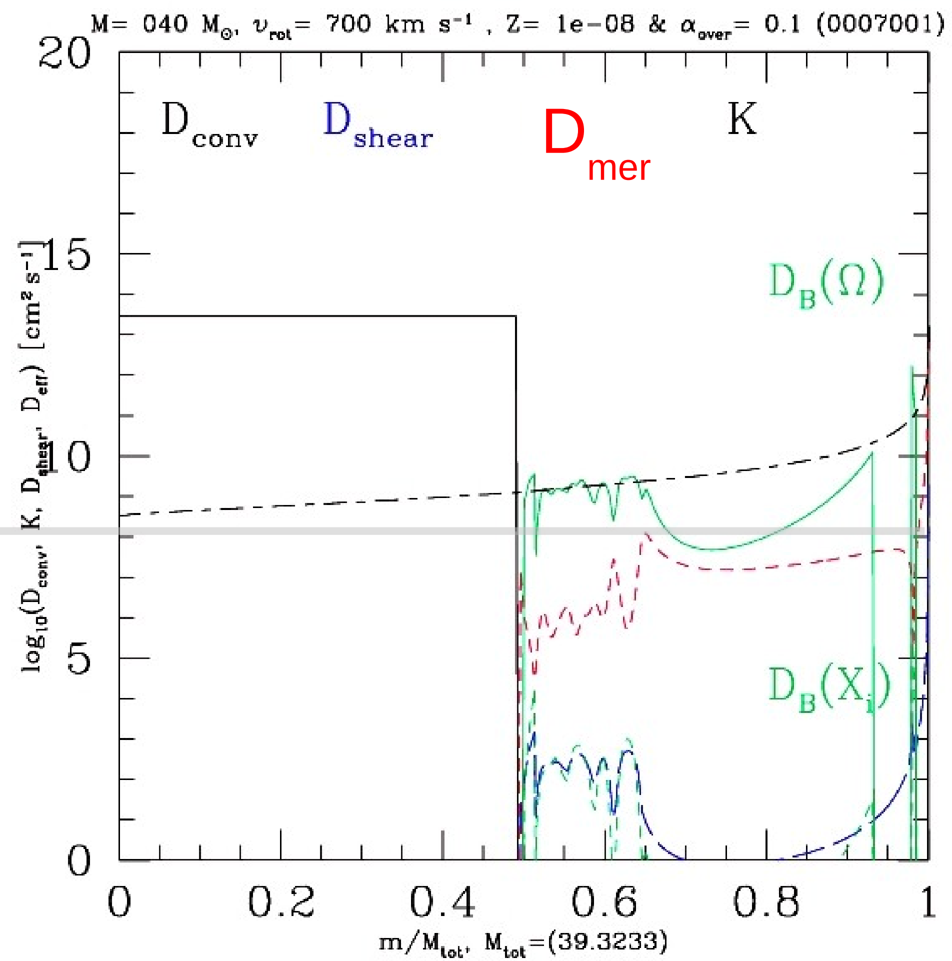
- GRBs around Z(LMC) & Z(SMC)? Dep. On mass loss / NO GRB @ Z_o
  (Meynet & Maeder 2007)

Quasi-Chem. Evol. @ very low Z? 40M_o models

Z=1e-5, v_{ini}=600 km/s (v_{ini}/v_{crit} =0.59)



Z=1e-8, v_{ini}=700 km/s (v_{ini}/v_{crit} =0.55)



Diff. Coeff. Smaller --> Quasi-Chem. Evol. harder for the first stellar generations