Beta decay probing weak interaction properties

Part 1

TU Darmstadt, Sept. 3-5, 2018

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Outline- 1

1. Introduction / 2 lectures

- role of beta decay in weak interaction physics
- beta decay Hamiltonian
- beta decay angular distribution

2. ft-values / 3 lectures

- definition
- corrected ft-values
- test of CKM matrix unitarity
- role of mirror beta transitions and neutron decay

3. Correlation measurements / 5 lectures

- correlation formula
- physics content and opportunities
- testing parity violation
- searching for time reversal violation
- probing the structure of the weak interaction (scalar and tensor currents)

Outline - 2

4. Status of new physics searches / 1 lecture

- overview
- prospects and comparison to LHC
- weak magnetism

5. Beta spectrum shape / 1 lecture

- description
- ongoing and planned experiments

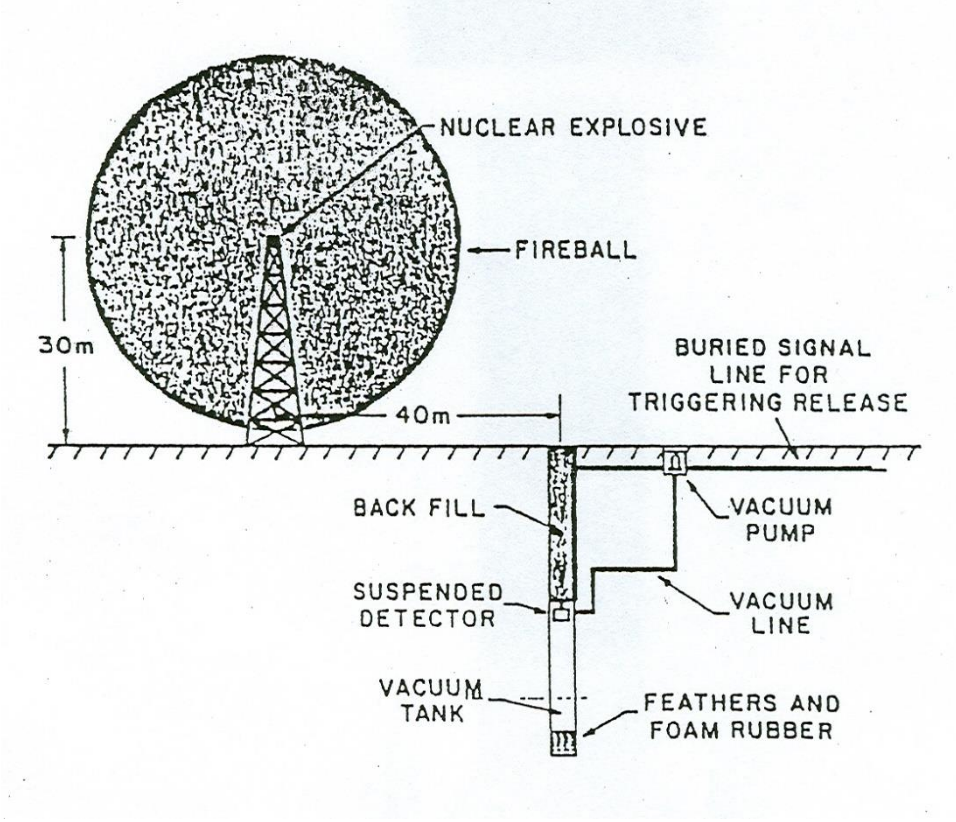
6. Reactor neutrino anomaly / 1 lecture

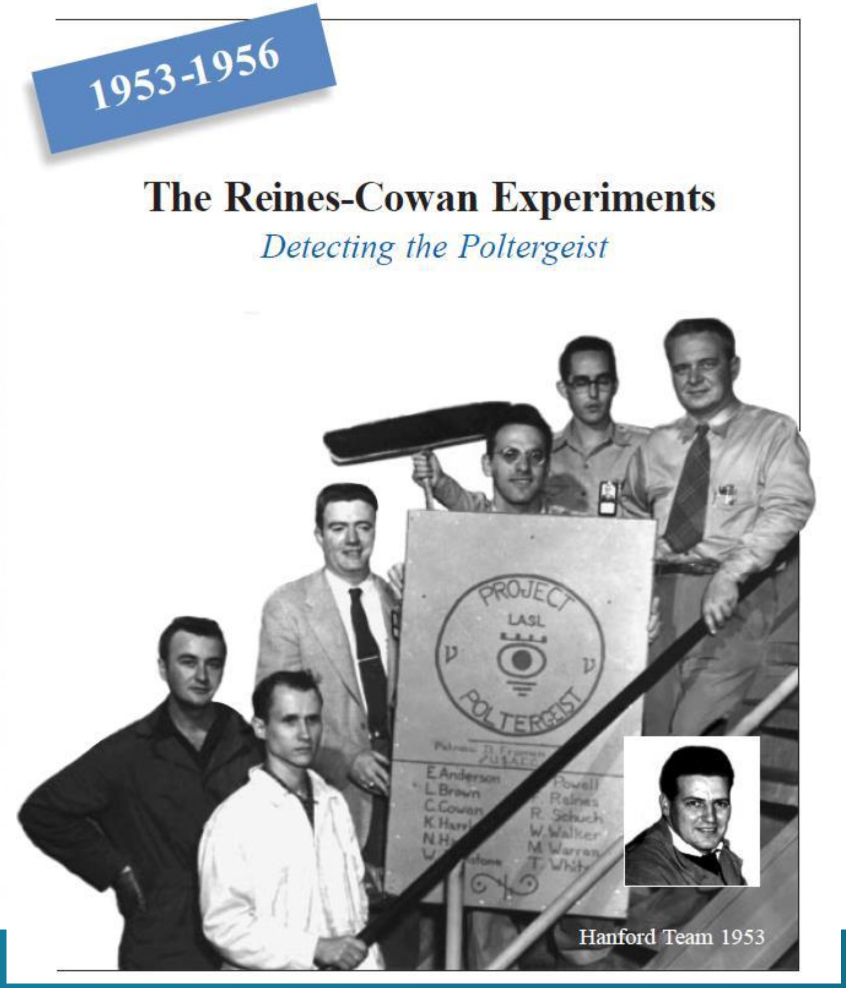
- the problem (rate and bump)
- critical analysis
- searches for a fourth, sterile neutrino
- role of first-forbidden beta transitions

Contribution of beta decay to weak interaction

Beta decay has played and is still playing a visible role in the determination of properties of the weak interaction, e.g.

1. discovery of the neutrino



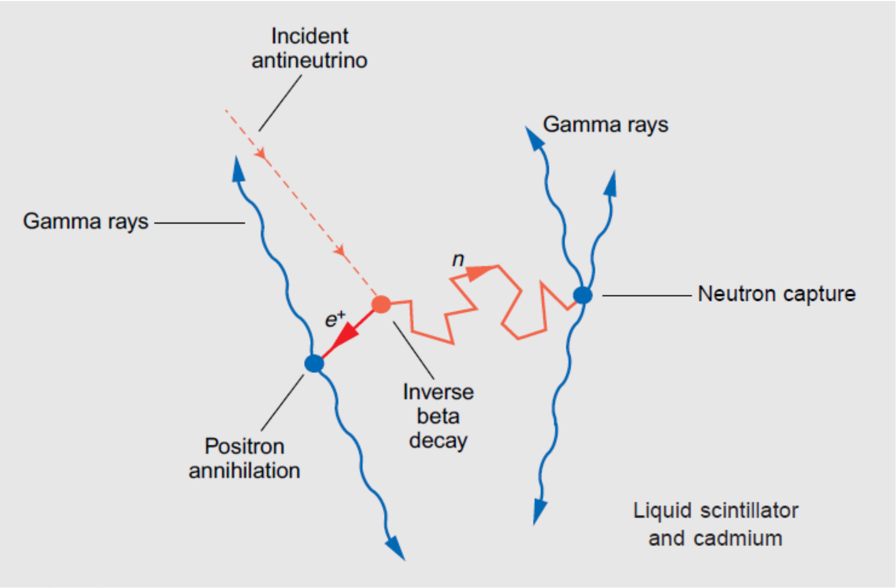

1953-1956

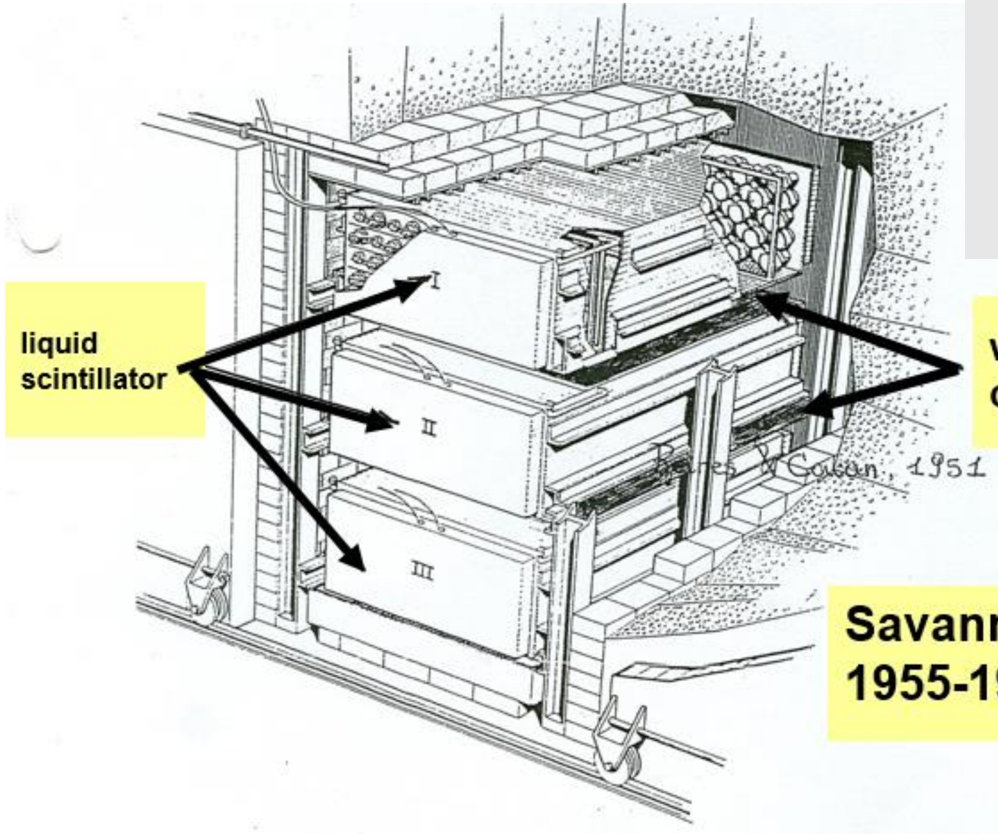
The Reines-Cowan Experiments

Detecting the Poltergeist

Hanford Team 1953

searching for $\bar{\nu} + p \rightarrow n + e^{+}$.





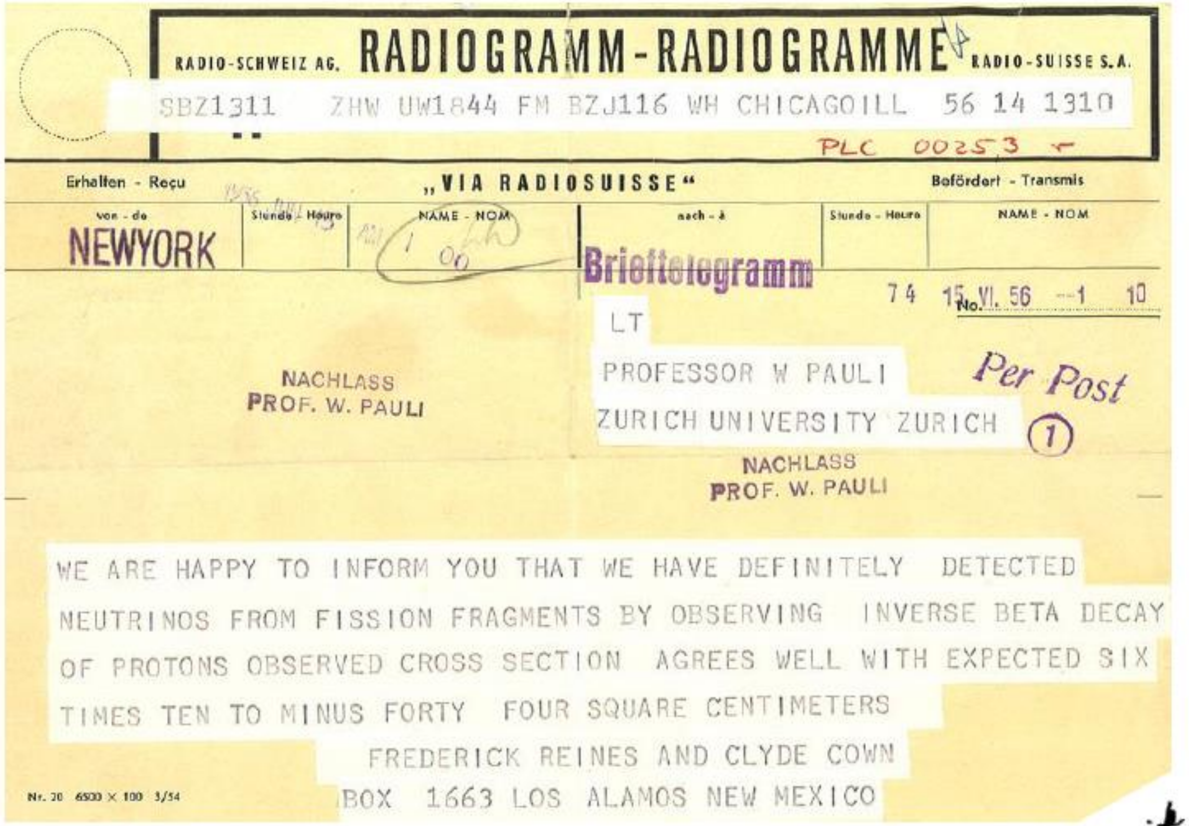
liquid scintillator

water and cadmium-chloride

Savannah-River reactor 1955-1956

F. Reines and C.L. Cowan, Phys. Rev. 113 (1959) 273

The Savannah River Experiment



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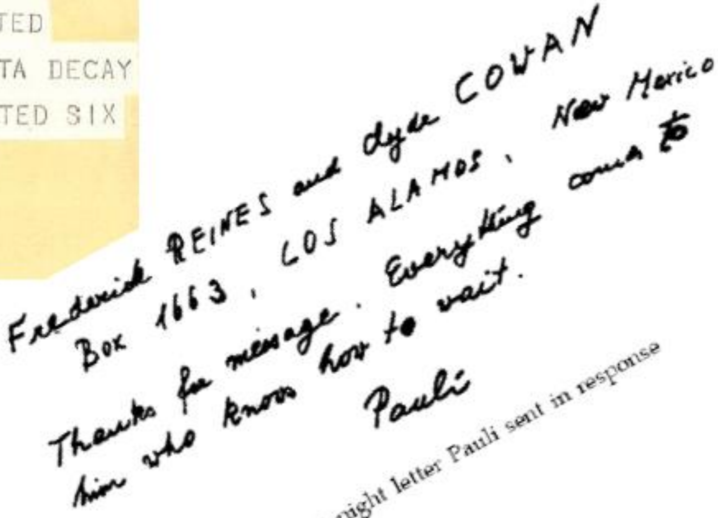
WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

FREDERICK REINES AND CLYDE COWN
BOX 1663 LOS ALAMOS NEW MEXICO

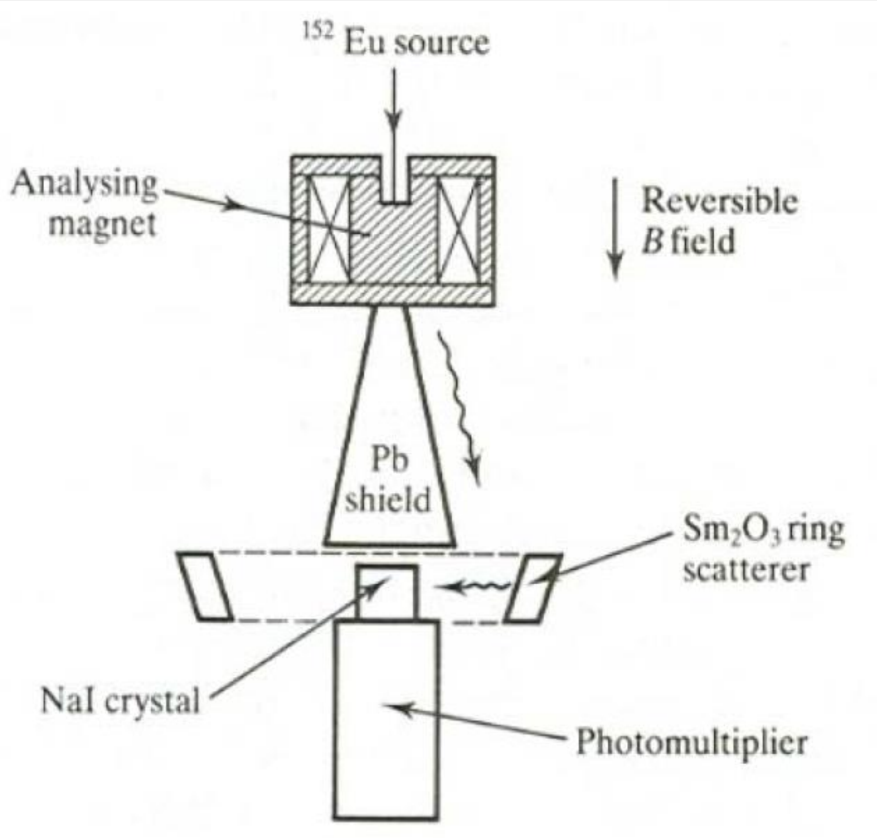
Nr. 20 6500 × 100 3/54

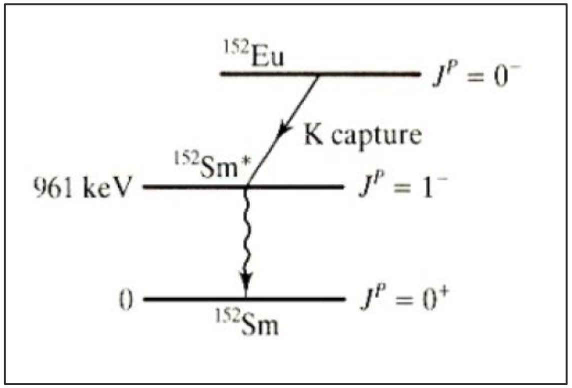


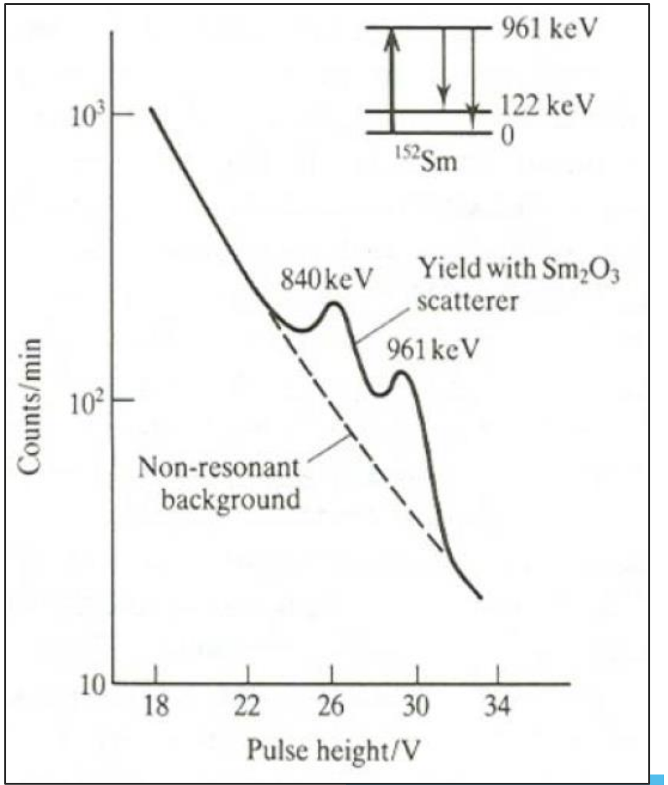
Frederick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico
Thanks for message. Everything comes to
him who knows how to wait.
Pauli

The night letter Pauli sent in response

2. determination of the helicity of the neutrino

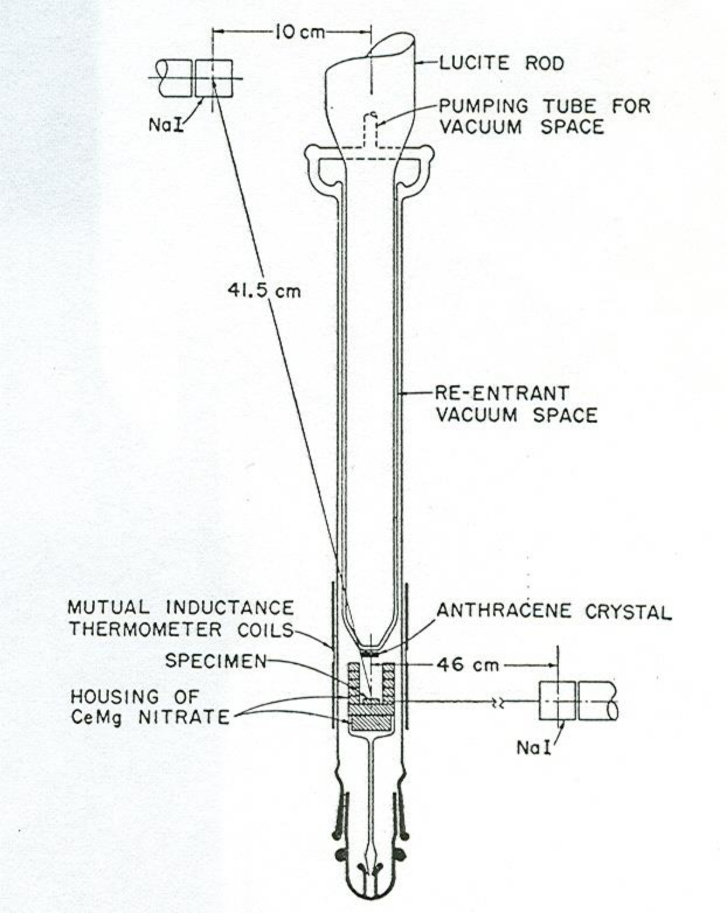


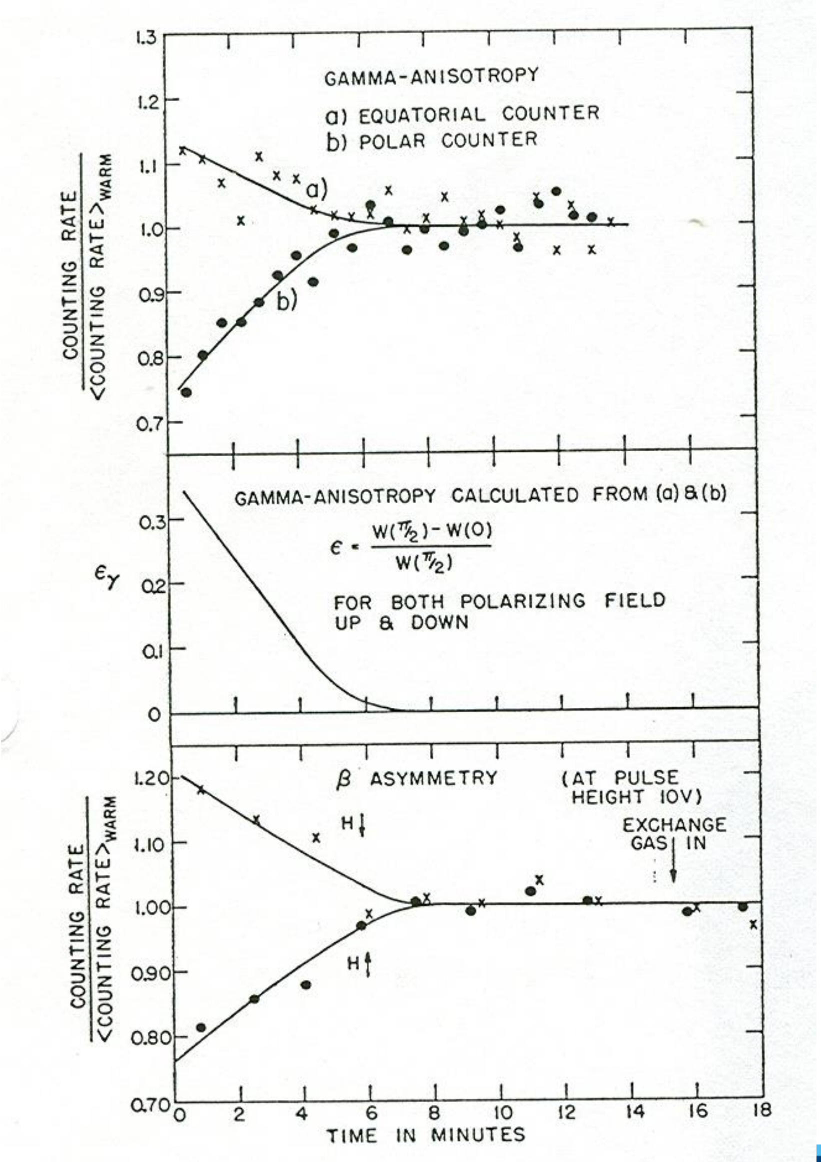




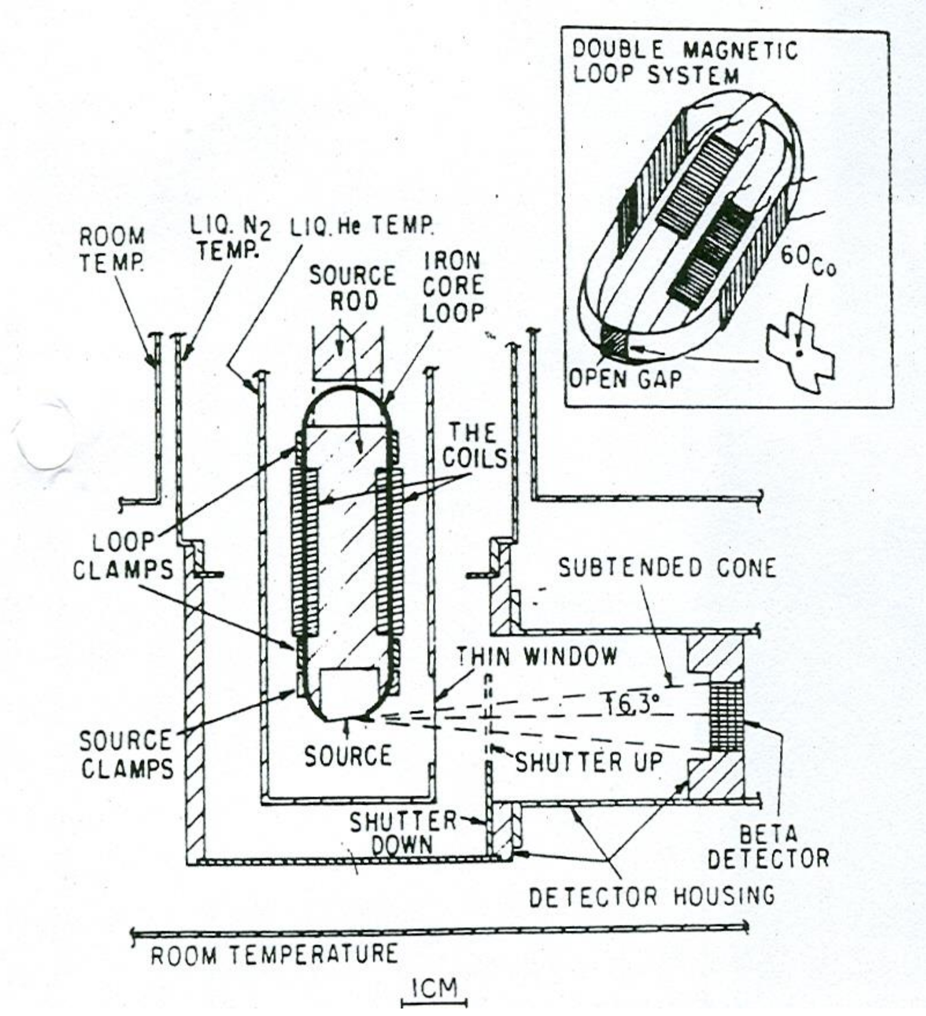
M. Goldhaber et al., Phys. Rev. 109 (1958) 1015

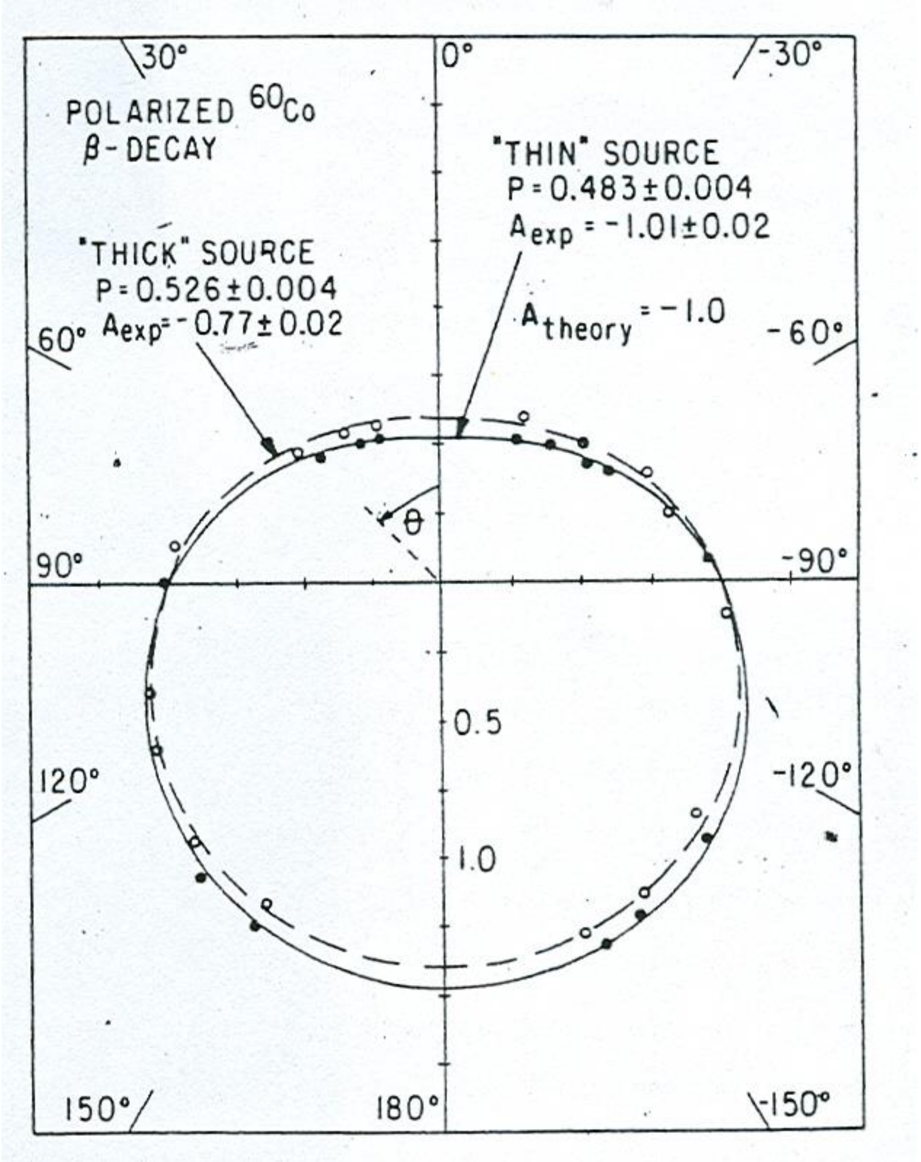
3. discovery of parity violation





C.S. Wu et al., Phys. Rev. 105 (1957) 1413



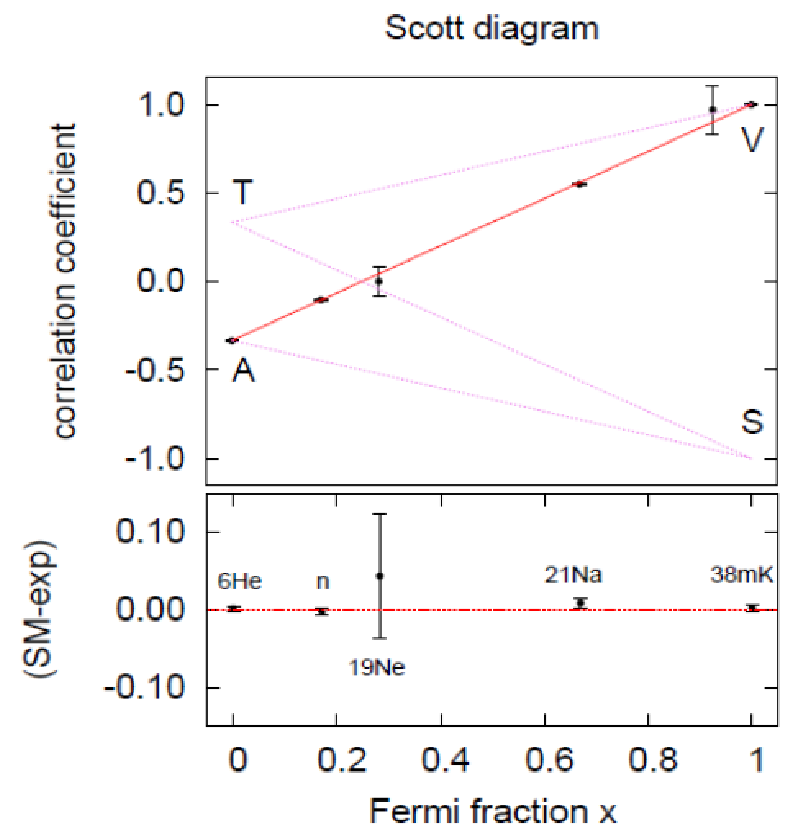


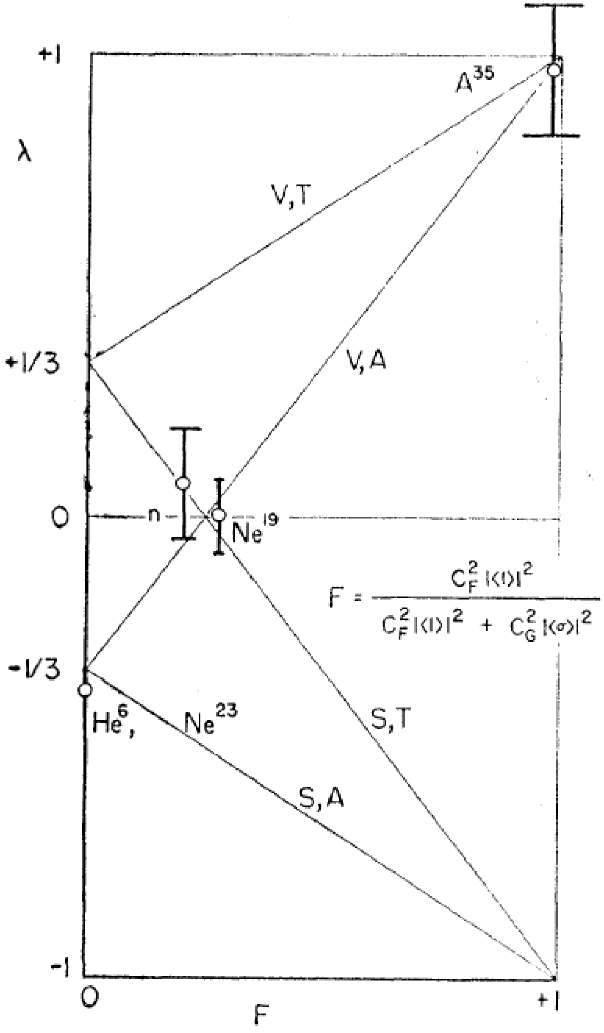
L.M. Chirovsky, C.S. Wu et al., Nucl. Instr. Meth. 219 (1984) 103

4. determination of the structure of the weak interaction

measured beta-neutrino correlation
for several beta decays
with different Fermi fraction;

extracted values for
the correlation coefficients
corresponded to line for V, A currents



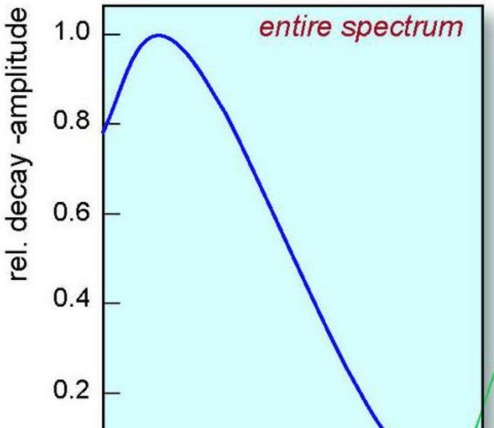


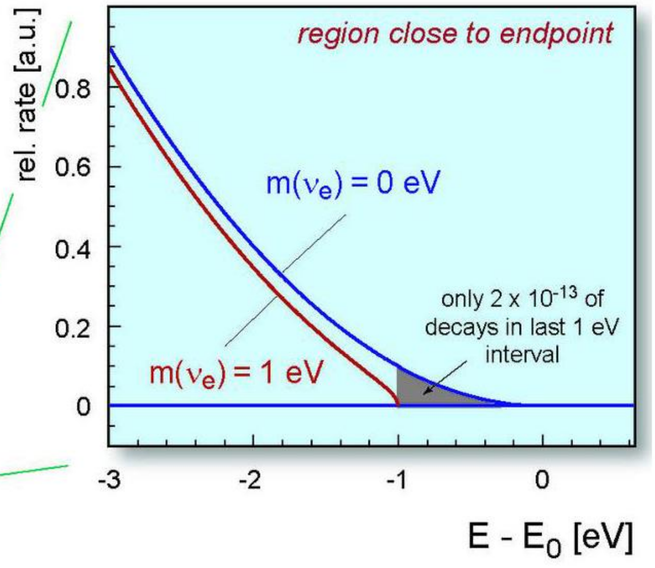
J.S. Allen et al., Phys. Rev. 116 (1959) 134

5. determination of the absolute neutrino mass

Tritium:

- ✓ very low endpoint energy
- ✓ simple nuclear structure
- ✓ suitable lifetime





$$m(\nu_e) < 2 \text{ eV} \quad (95\% \text{ CL})$$

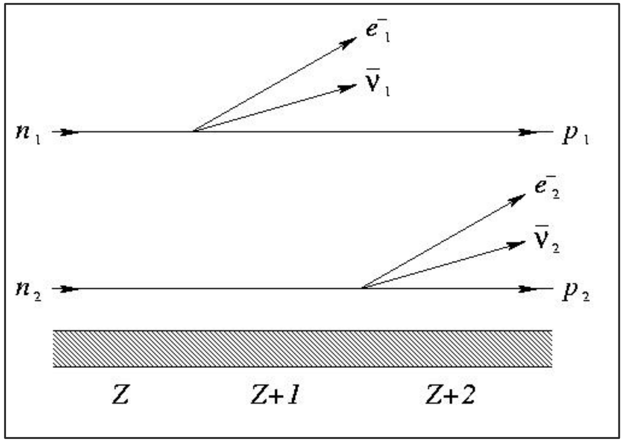
E.W. Otten and C. Weinheimer, Rep. Prog. Phys. 71 (2008) 086201

KATRIN experiment at KIT,
towards sensitivity of 0.2 eV

6. double beta decay

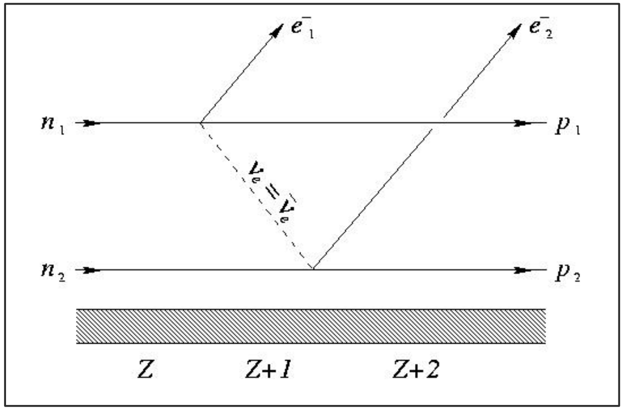
2νββ

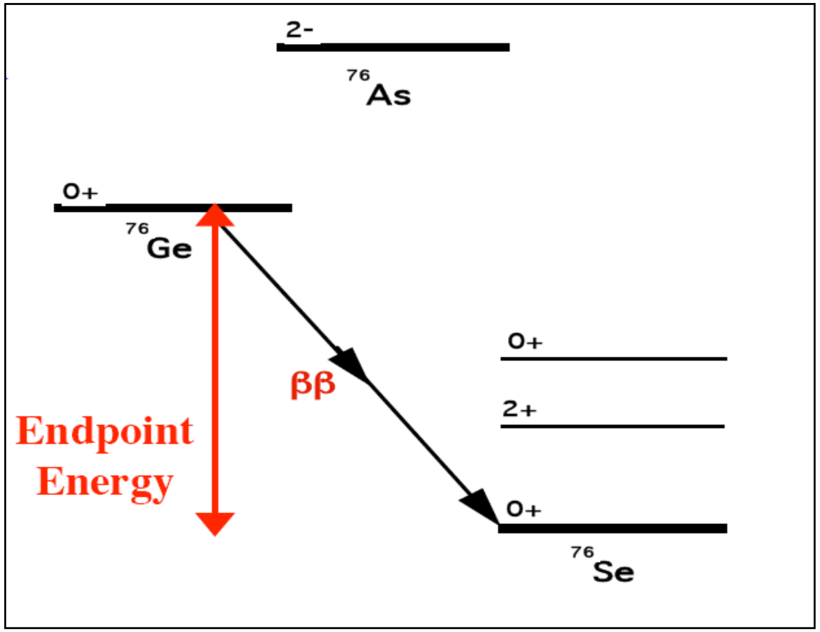
$T_{1/2} \cong 10^{18} - 10^{21}$ y

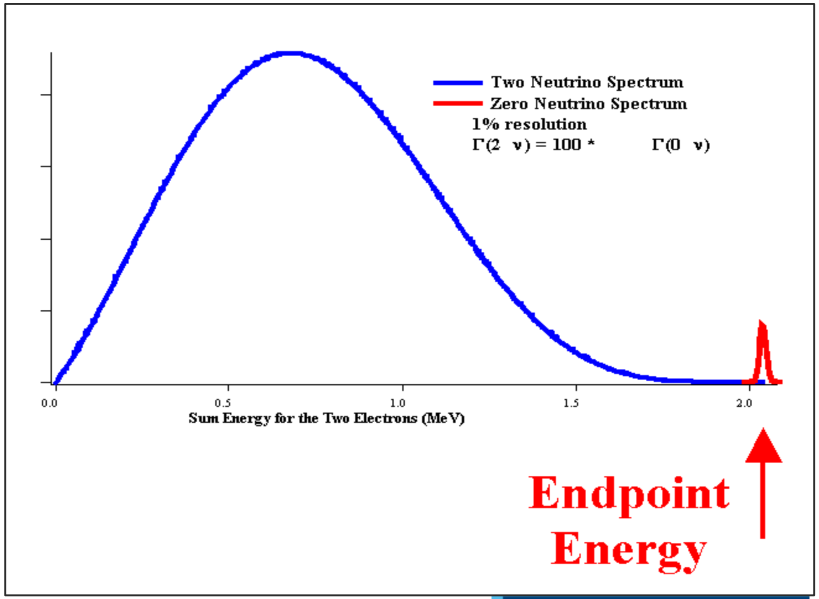


0νββ

$T_{1/2} > 10^{22} - 10^{26}$ y



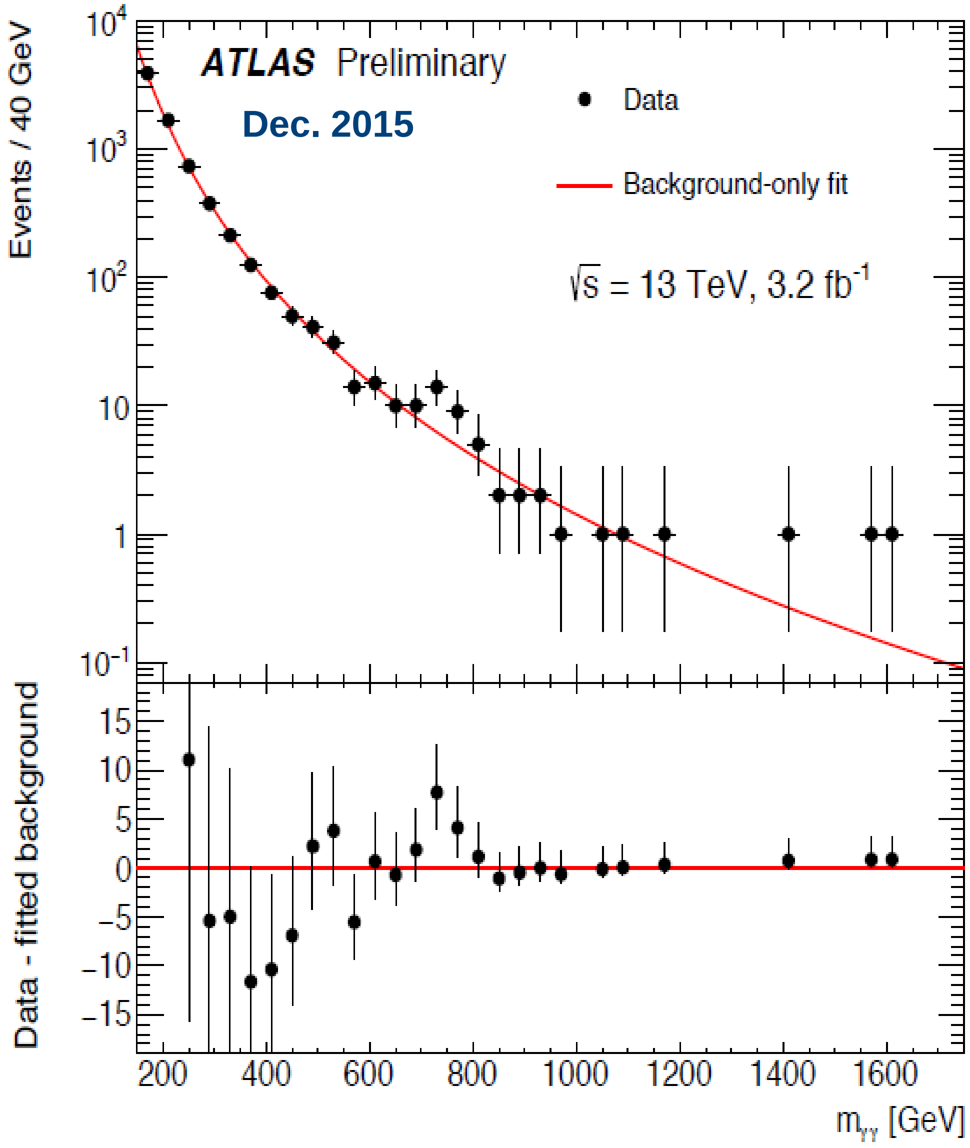




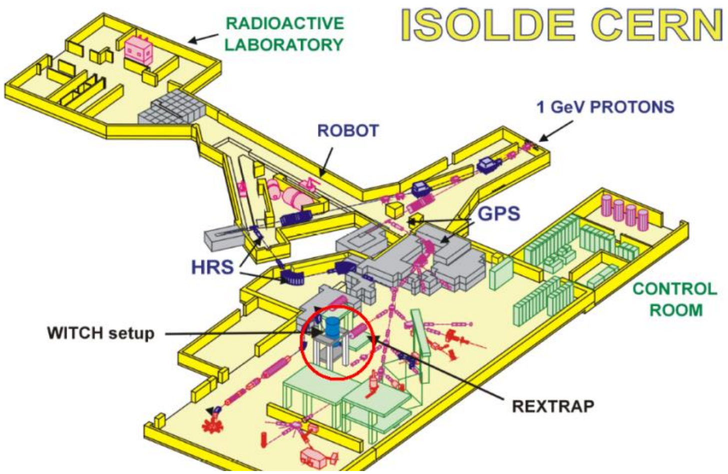
e.g. B. Pritychenko, Nuclear Data Sheets 120 (2014) 102–105

Experiments at the frontiers of standard theory

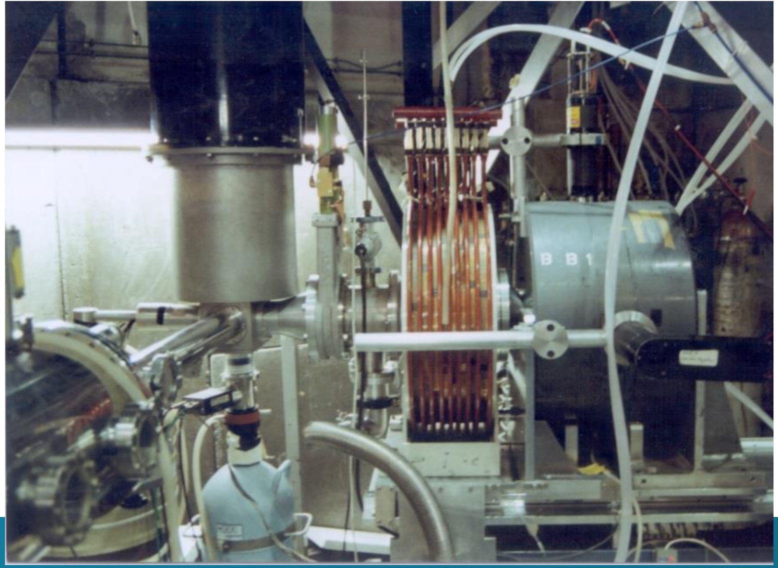
High energy frontier



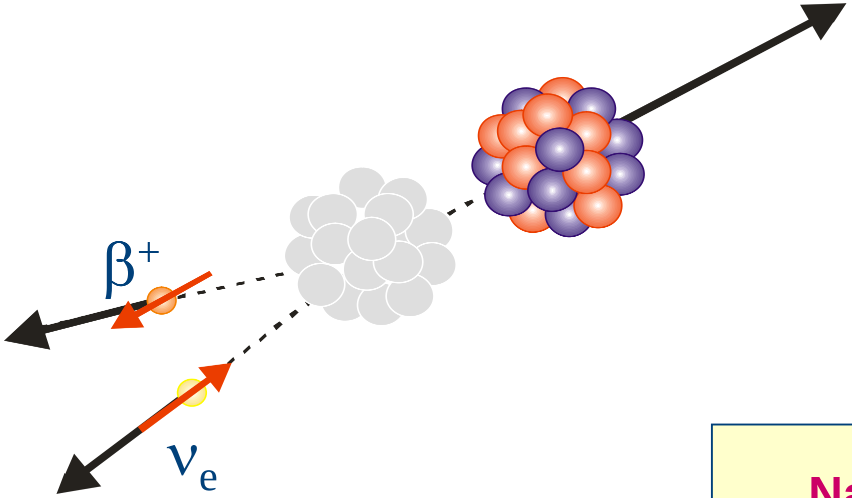
Precision frontier



$$\tilde{A} = A_{SM}\left[1-k\left(C_T + C_T^{'}\right)\right]$$



1. Introduction -
β decay hamiltonian
and angular distribution



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the Standard Model and beyond:

- $C_V \equiv 1$; $C_A = -1.27$ (g_A/g_V from n-decay)
- $C_V' = C_V$ & $C_A' = C_A$ (maximal P-violation)
- $C_S = C_S' = C_T = C_T' = C_P = C_P' \equiv 0$
  (only V- and A-currents)
- no time reversal violation (except for the CP-violation described by the phase in the CKM matrix)

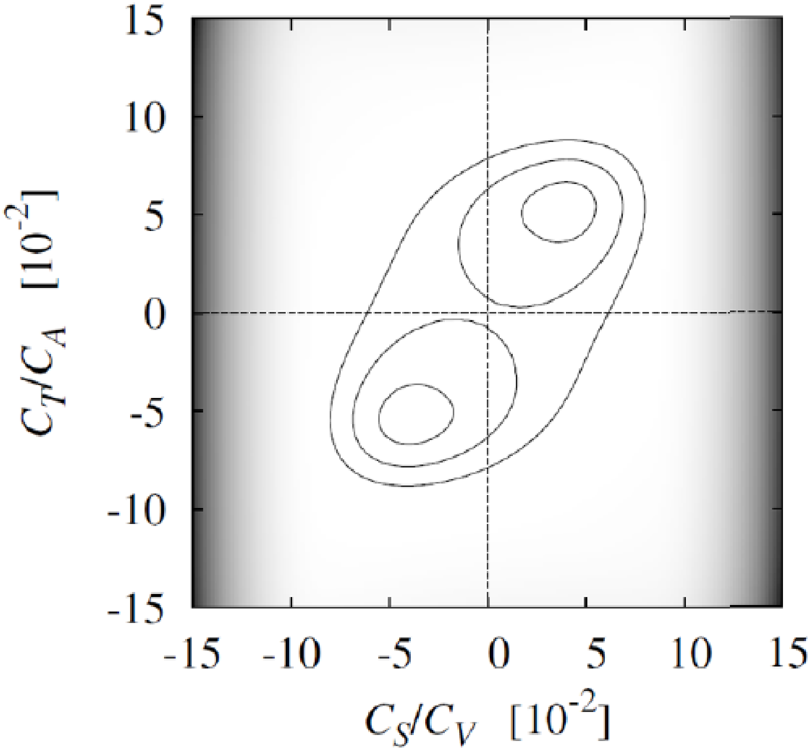
5% level → ~ 350 GeV
per mille level → ~ 2.5 TeV

$$C_i \propto \frac{M_W^2}{M_{new}^2}$$

experimental upper limits on scalar and tensor couplings involving right-handed neutrinos are at the 5 to 10 % level

(neutron and nuclear β-decay)

M. Gonzalez-Alonso, O. Naviliat-Cuncic, NS, Prog. Part. Nucl. Phys. (arXiv:1803.08732)



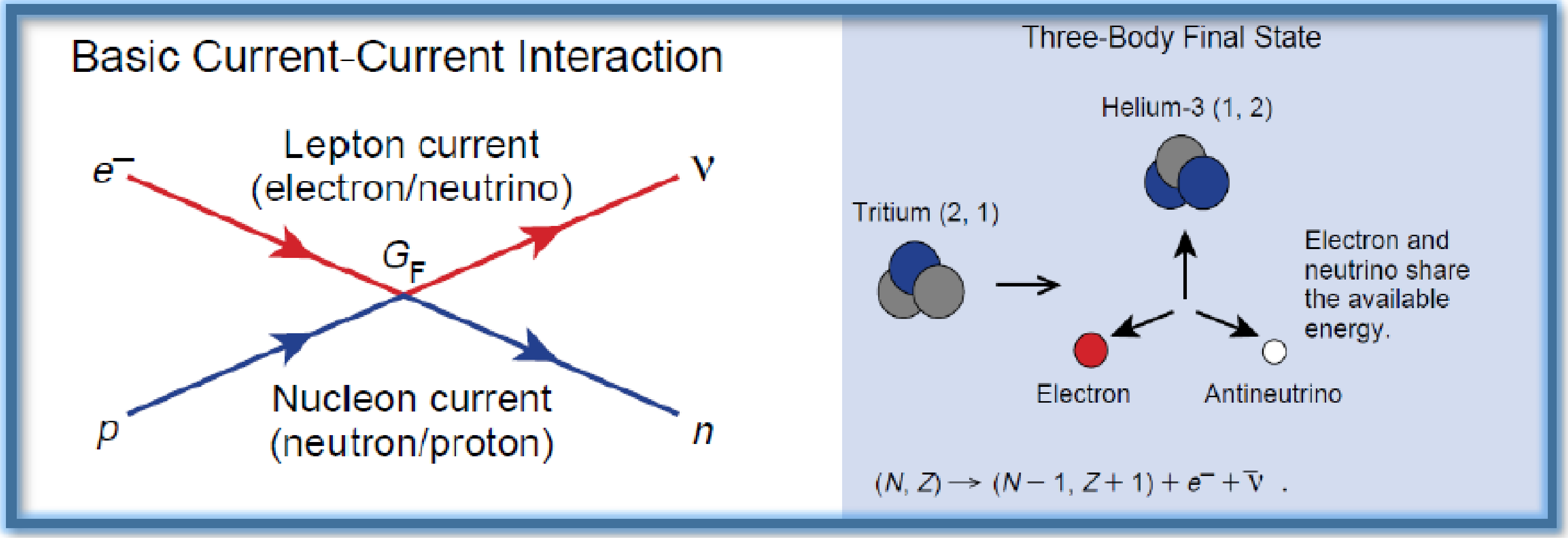
β decay Hamiltonian

Fermi, 1934
Analogous to electromagnetic interaction Fermi chooses a vector interaction

$$\mathcal{H} = g\left(\overline{\Psi}_p\gamma_\mu\Psi_n\right)\left(\overline{\Psi}_e\gamma^\mu\Psi_\nu\right) + h.c.$$

(γ_μ are Dirac matrices)

Nucleon potential Lepton potential



Intermezzo: the Pauli matrices and **the Dirac** γ matrices

The Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

sometimes also labeled as σ_1 , σ_2 and σ_3 , and with

$$\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}I \qquad \text{(anticommutation)}$$

$$\sigma_i\sigma_j = i\sigma_k \qquad \text{(cyclic permutation of indices)}$$

$$(\sigma_i)^2 = I$$

There exist at most three 2 x 2 matrices that anti-commute,

e.g. the Pauli matrices σ_1 , σ_2 and σ_3 .

But, one can construct an anti-commuting set of 4 x 4 matrices out of the Pauli matrices and the 2 x 2 unit matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

namely :

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix} \quad \text{with} \quad k = 1,2,3 \quad \text{and} \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

These matrices are hermitian and anti-commuting, e.g.

$$\alpha_k \beta = -\beta \alpha_k$$

They are also unitary, i.e.

$$\alpha_k^\dagger \alpha_k = \alpha_k^2 = I$$

The Dirac γ matrices:

$$\gamma_k = -i\beta\alpha_k \qquad k = 1,2,3$$
$$\gamma_4 = \beta$$

Thus :

$$\gamma_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma_2 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma_3 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix}$$

$$\beta = \qquad \gamma_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma_5 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

The Dirac γ-matrices obey the following rules :

$$\gamma_\mu\gamma_\nu = -\gamma_\nu\gamma_\mu \qquad \mu \neq \nu$$
$$\gamma_\mu^2 = I$$
$$\gamma_5 \equiv \gamma_1\ \gamma_2\ \gamma_3\ \gamma_4 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$
$$\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5 \qquad \mu = 1,2,3,4$$
$$\gamma_5^2 = I$$
$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} \qquad \mu,\nu = 1,2,3,4,5$$

Wave functions

The Dirac γ-matrices act on the wave functions

$$\psi(\overline{r},t) \;=\; \psi \;\exp\left[\frac{i}{\hbar}(\overline{p}\cdot\overline{r} - Et)\right]$$

where ψ ('spinor') describes the spin part of the wave function. This spinor ψ is a (1-column, 4-row) matrix :

$$\psi = \begin{pmatrix} \phi^1 \\ \phi^2 \\ \xi^1 \\ \xi^2 \end{pmatrix}$$

with ϕ and ξ two-component spinors, and describes the spin state of a particle (e.g. ϕ) and its anti-particle (e.g. ξ). Thus

$$\phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

describe a spin up and a spin down state, respectively.

Free particle solutions of the Dirac equation

Consider the plane wave equation for a free particle :

$$\psi(\overline{r},t) \;=\; \psi\, \exp\left[\frac{i}{\hbar}\left(\overline{p}\cdot\overline{r}-Et\right)\right] \tag{2.14}$$

with ψ the wave function at the origin, a 4-component spinor describing the spin of the particle. This ψ satisfies the Dirac equation (2.9)

$$E\ \psi = \left(c\ \overline{\alpha}\cdot\overline{p}\ + m_0 c^2 \beta\right)\psi \tag{2.15}$$

with the 4-component spinor

$$\psi = \begin{pmatrix} \phi \\ \xi \end{pmatrix} \tag{2.16}$$

made up of two 2-component spinors ϕ and ξ.

Inserting this form for ψ in eq. (2.15) and writing $\bar{\alpha} = -\gamma_5 \bar{\sigma}$ with $\bar{\sigma}$ the 4 x 4 Pauli matrices

$$\sigma_k = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} \qquad (\; k = 1,2,3 \;) \tag{2.17}$$

(the σ_k at the right-hand side are the 2 x 2 Pauli matrices; whether σ_k is a 2 x 2 or a 4 x 4 matrix should be clear from the context), this becomes :

$$E \begin{pmatrix} \phi \\ \xi \end{pmatrix} = \left[\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} c \; \bar{\sigma} \cdot \bar{p} \; + \; \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} m_0 c^2 \right] \begin{pmatrix} \phi \\ \xi \end{pmatrix} \tag{2.18}$$

or also :

$$\begin{cases} c \; (\bar{\sigma} \cdot \bar{p}) \; \xi = (E - m_0 c^2) \; \phi \\ c \; (\bar{\sigma} \cdot \bar{p}) \; \phi = (E + m_0 c^2) \; \xi \end{cases} \tag{2.19}$$

For a state of positive energy, i.e. with $E_+ = +(c^2 p^2 + m_0{}^2 c^4)^{1/2}$ one has :

$$\xi = \frac{c\,(\overline{\sigma}\cdot\overline{p})}{E_+ + m_0 c^2}\,\phi \qquad (2.20)$$

In the non-relativistic limit, i.e. $E_+ \rightarrow m_0 c^2$,

$$\xi \;\rightarrow\; \frac{c\,(\overline{\sigma}\cdot\overline{p})}{2m_0 c^2}\,\phi \;=\; \frac{\overline{\sigma}\cdot\overline{v}}{2c}\,\phi \;\xrightarrow{\text{if }\overline{v}\rightarrow 0}\; 0 \qquad (2.21)$$

Therefore, ϕ is associated with the positive-energy states. It is therefore also sometimes called the “large” component of ψ. Similarly, ξ (“small” component) is associated with the negative-energy states.

The 4-component spinor

$$\psi = \begin{pmatrix} \phi \\ \xi \end{pmatrix}$$

was interpreted by Dirac as describing a spin-1/2 particle-antiparticle combination, e.g. an electron (ϕ) and a positron (ξ), with the two components of ϕ and ξ describing the two possible spin-orientations of the spin-1/2 particles.

Writing this more explicitly one has :

$$\psi_\sigma^+ = \begin{pmatrix} \phi \\ \xi \end{pmatrix} = \begin{pmatrix} \chi_\sigma \\ \dfrac{c\,(\overline{\sigma}\cdot\overline{p})}{E_+ + m_0c^2}\,\chi_\sigma \end{pmatrix} \quad \text{with} \quad \chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.22)$$

or also

$$\psi_\sigma^+ = \begin{pmatrix} \phi \\ \xi \end{pmatrix} = \begin{pmatrix} \chi_\sigma \\ (\eta\ \overline{\sigma}\cdot\hat{p})\ \chi_\sigma \end{pmatrix} \quad \text{with} \quad \eta = \frac{cp}{|\,E\,| + m_0c^2} \qquad (2.23)$$

A few specific cases:

1. for a spin-1/2 particle with positive energy $E_+ > 0$, and with the spin either

in the positive z-direction, i.e. $\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

or

in the negative z-direction, i.e. $\chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

eq. (2.23) becomes (note that $\bar{\sigma} \cdot \hat{p} = \sigma_x \hat{p}_x + \sigma_y \hat{p}_y + \sigma_z \hat{p}_z$):

$$\psi_\uparrow^+ = \frac{1}{\sqrt{1+\eta^2}} \begin{pmatrix} 1 \\ 0 \\ \eta\, p_z / p \\ \eta\, (p_x + ip_y) / p \end{pmatrix} \text{, resp. } \psi_\downarrow^+ = \frac{1}{\sqrt{1+\eta^2}} \begin{pmatrix} 0 \\ 1 \\ \eta\, (p_x - ip_y) / p \\ -\eta\, p_z / p \end{pmatrix} \quad (2.24)$$

with $\sqrt{1+\eta^2}$ a normalization factor.

2. for a spin-1/2 particle with negative energy $E_- < 0$, and with the spin either in the positive or in the negative z-direction one has

$$\psi_\sigma^- = \begin{pmatrix} \phi \\ \xi \end{pmatrix} = \begin{pmatrix} -(\eta\ \bar{\sigma} \cdot \hat{p})\ \chi_\sigma \\ \chi_\sigma \end{pmatrix} \tag{2.25}$$

and eq. (2.23) becomes

$$\psi_\uparrow^- = \frac{1}{\sqrt{1+\eta^2}} \begin{pmatrix} -\eta\ p_z / p \\ -\eta\ (p_x + ip_y) / p \\ 1 \\ 0 \end{pmatrix} \text{, resp. } \psi_\downarrow^- = \frac{1}{\sqrt{1+\eta^2}} \begin{pmatrix} -\eta\ (p_x - ip_y) / p \\ \eta\ p_z / p \\ 0 \\ 1 \end{pmatrix} \tag{2.26}$$

Solutions of the Dirac equation for a massless spin-1/2 particle

For massless particles (i.e. the neutrino), $m_0 = 0$ such that $\eta = 1$. If we further assume that the particle:

- is in a positive energy state E_+,
- has its spin along the z-axis, and
- its momentum in the (x,z)-plane,

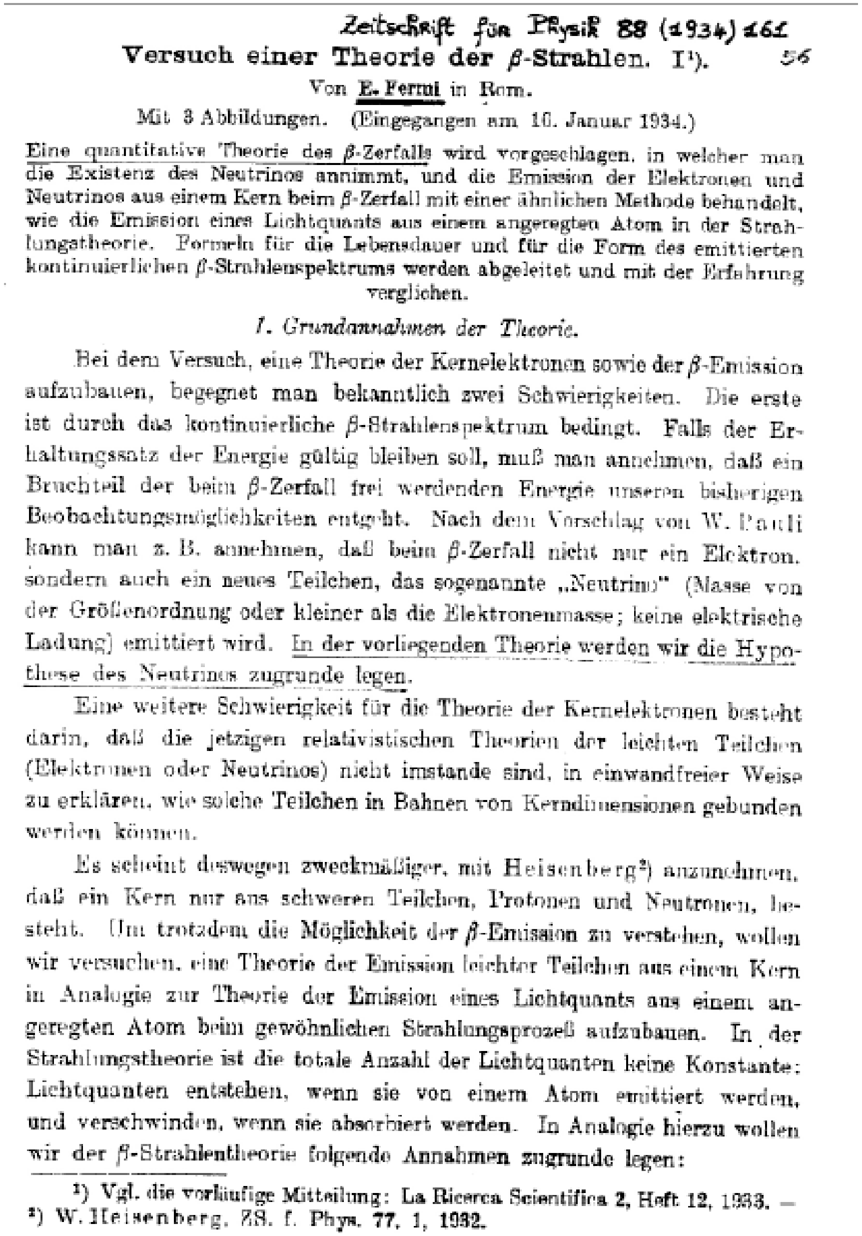
then

$$\psi_\uparrow^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ \cos\theta \\ \sin\theta \end{pmatrix} \quad \text{and} \quad \psi_\downarrow^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \sin\theta \\ -c\,os\,\theta \end{pmatrix} \qquad (2.27)$$

If one supposes, in addition, that the spin is along the positive z-derection, i.e. $\overline{p} = +p\,\hat{p}_z$, then :

$$\psi_\uparrow^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \psi_\downarrow^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \qquad (2.28)$$

β decay Hamiltonian



Zeitschrift für Physik 88 (1934) 161

Versuch einer Theorie der β-Strahlen. I[1]). 56

Von **E. Fermi** in Rom.

Mit 3 Abbildungen. (Eingegangen am 10. Januar 1934.)

Eine quantitative Theorie des β-Zerfalls wird vorgeschlagen, in welcher man die Existenz des Neutrinos annimmt, und die Emission der Elektronen und Neutrinos aus einem Kern beim β-Zerfall mit einer ähnlichen Methode behandelt, wie die Emission eines Lichtquants aus einem angeregten Atom in der Strahlungstheorie. Formeln für die Lebensdauer und für die Form des emittierten kontinuierlichen β-Strahlenspektrums werden abgeleitet und mit der Erfahrung verglichen.

1. Grundannahmen der Theorie.

Bei dem Versuch, eine Theorie der Kernelektronen sowie der β-Emission aufzubauen, begegnet man bekanntlich zwei Schwierigkeiten. Die erste ist durch das kontinuierliche β-Strahlenspektrum bedingt. Falls der Erhaltungssatz der Energie gültig bleiben soll, muß man annehmen, daß ein Bruchteil der beim β-Zerfall frei werdenden Energie unseren bisherigen Beobachtungsmöglichkeiten entgeht. Nach dem Vorschlag von W. Pauli kann man z. B. annehmen, daß beim β-Zerfall nicht nur ein Elektron, sondern auch ein neues Teilchen, das sogenannte „Neutrino" (Masse von der Größenordnung oder kleiner als die Elektronenmasse; keine elektrische Ladung) emittiert wird. In der vorliegenden Theorie werden wir die Hypothese des Neutrinos zugrunde legen.

Eine weitere Schwierigkeit für die Theorie der Kernelektronen besteht darin, daß die jetzigen relativistischen Theorien der leichten Teilchen (Elektronen oder Neutrinos) nicht imstande sind, in einwandfreier Weise zu erklären, wie solche Teilchen in Bahnen von Kerndimensionen gebunden werden können.

Es scheint deswegen zweckmäßiger, mit Heisenberg[2]) anzunehmen, daß ein Kern nur aus schweren Teilchen, Protonen und Neutronen, besteht. Um trotzdem die Möglichkeit der β-Emission zu verstehen, wollen wir versuchen, eine Theorie der Emission leichter Teilchen aus einem Kern in Analogie zur Theorie der Emission eines Lichtquants aus einem angeregten Atom beim gewöhnlichen Strahlungsprozeß aufzubauen. In der Strahlungstheorie ist die totale Anzahl der Lichtquanten keine Konstante: Lichtquanten entstehen, wenn sie von einem Atom emittiert werden, und verschwinden, wenn sie absorbiert werden. In Analogie hierzu wollen wir der β-Strahlentheorie folgende Annahmen zugrunde legen:

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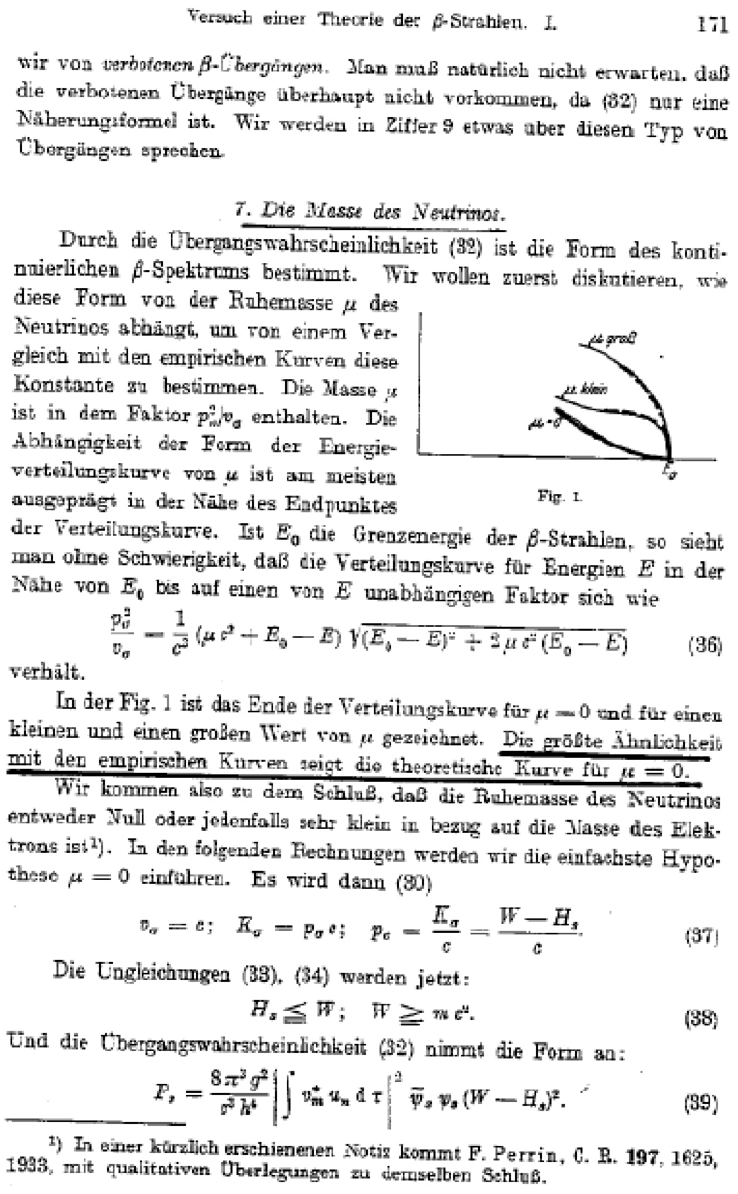
[1]) Vgl. die vorläufige Mitteilung: La Ricerca Scientifica **2**, Heft 12, 1933. — [2]) W. Heisenberg, ZS. f. Phys. **77**, 1, 1932.



wir von *verbotenen β-Übergängen*. Man muß natürlich nicht erwarten, daß die verbotenen Übergänge überhaupt nicht vorkommen, da (32) nur eine Näherungsformel ist. Wir werden in Ziffer 9 etwas über diesen Typ von Übergängen sprechen.

7. Die Masse des Neutrinos.

Durch die Übergangswahrscheinlichkeit (32) ist die Form des kontinuierlichen β-Spektrums bestimmt. Wir wollen zuerst diskutieren, wie diese Form von der Ruhemasse μ des Neutrinos abhängt, um von einem Vergleich mit den empirischen Kurven diese Konstante zu bestimmen. Die Masse μ ist in dem Faktor p_σ^2/v_σ enthalten. Die Abhängigkeit der Form der Energieverteilungskurve von μ ist am meisten ausgeprägt in der Nähe des Endpunktes der Verteilungskurve. Ist E_0 die Grenzenergie der β-Strahlen, so sieht man ohne Schwierigkeit, daß die Verteilungskurve für Energien E in der Nähe von E_0 bis auf einen von E unabhängigen Faktor sich wie

$$\frac{p_\sigma^2}{v_\sigma} = \frac{1}{c^3}(\mu c^2 + E_0 - E)\sqrt{(E_0 - E)^2 + 2\mu c^2 (E_0 - E)} \qquad (36)$$

verhält.

Fig. 1.

In der Fig. 1 ist das Ende der Verteilungskurve für $\mu = 0$ und für einen kleinen und einen großen Wert von μ gezeichnet. Die größte Ähnlichkeit mit den empirischen Kurven zeigt die theoretische Kurve für $\mu = 0$.

Wir kommen also zu dem Schluß, daß die Ruhemasse des Neutrinos entweder Null oder jedenfalls sehr klein in bezug auf die Masse des Elektrons ist[1]). In den folgenden Rechnungen werden wir die einfachste Hypothese $\mu = 0$ einführen. Es wird dann (30)

$$v_\sigma = c; \quad K_\sigma = p_\sigma c; \quad p_\sigma = \frac{K_\sigma}{c} = \frac{W - H_s}{c}. \qquad (37)$$

Die Ungleichungen (33), (34) werden jetzt:

$$H_s \leqq W; \quad W \geqq m c^2. \qquad (38)$$

Und die Übergangswahrscheinlichkeit (32) nimmt die Form an:

$$F_s = \frac{8\pi^3 g^2}{c^3 h^4}\left|\int v_m^* u_n \, d\tau\right|^2 \overline{\psi}_s \psi_s (W - H_s)^2. \qquad (39)$$

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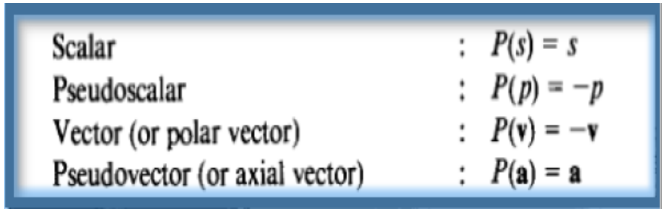
[1]) In einer kürzlich erschienenen Notiz kommt F. Perrin, C. R. **197**, 1625, 1933, mit qualitativen Überlegungen zu demselben Schluß.

Gamow and Teller (1936) include spin

$$\mathcal{H} = \sum_{i\in\{S,V,T,A,P\}} g\ C_i\ \underbrace{\left(\overline{\Psi}_p \mathcal{O}_i \Psi_n\right)}_{\text{Nucleon potential}}\ \underbrace{\left(\overline{\Psi}_e \mathcal{O}_i \Psi_\nu\right)}_{\text{Lepton potential}} + h.c.$$

coupling constants – to be determined by experiment

Name	Symbol	Current	Number of components
Scalar	S	$\overline{\psi}\psi$	1
Vector	V	$\overline{\psi}\gamma^\mu\psi$	4
Tensor	T	$\overline{\psi}\sigma^{\mu\nu}\psi$	6
Axial Vector	A	$\overline{\psi}\gamma^\mu\gamma^5\psi$	4
Pseudo-Scalar	P	$\overline{\psi}\gamma^5\psi$	1



Scalar : $P(s) = s$

Pseudoscalar : $P(p) = -p$

Vector (or polar vector) : $P(\mathbf{v}) = -\mathbf{v}$

Pseudovector (or axial vector) : $P(\mathbf{a}) = \mathbf{a}$

Extra selection rules

S and V: $\Delta I = 0$ Fermi transition: S = 0

T and A: $\Delta I = \pm 1$ and 0 Gamow-Teller transition: S = 1

$$\mathcal{H} = \sum_{i\in\{S,V,T,A,P\}} g \; C_i \underbrace{\left(\overline{\Psi}_p \mathcal{O}_i \Psi_n\right)}_{\text{Nucleon potential}} \underbrace{\left(\overline{\Psi}_e \mathcal{O}_i \Psi_\nu\right)}_{\text{Lepton potential}} + h.c.$$

and the operators O_i expressed as Dirac γ matrices

Operator O_i	Number of independent matrices	Relativistic transformation properties of $\overline{\psi}_a O_i \psi_b$
1	1	Scalar
γ_μ	4	Vector
$\gamma_\mu\gamma_\lambda$	6	antisymmetric Tensor of rank 2
$\gamma_\mu\gamma_5$ $(\gamma_\nu\gamma_\lambda\gamma_\sigma)$	4	Axial vector
γ_5 $(= \gamma_1\gamma_2\gamma_3\gamma_4)$	1	Pseudoscalar

J_μ	L_μ
S	S or P
V	V or A
T	T
A	A or V
P	P or S

Lee & Yang, 1956 – Wu et al., 1957
(maximal) violation of parity

$$\mathcal{H} = \sum_{i\in\{S,V,T,A,P\}} g\left[C_i\left(\overline{\Psi}_p\mathcal{O}_i\Psi_n\right)\left(\overline{\Psi}_e\mathcal{O}_i\Psi_\nu\right) + C_i'\left(\overline{\Psi}_p\mathcal{O}_i\Psi_n\right)\left(\overline{\Psi}_e\mathcal{O}_i\gamma_5\Psi_\nu\right)\right]$$

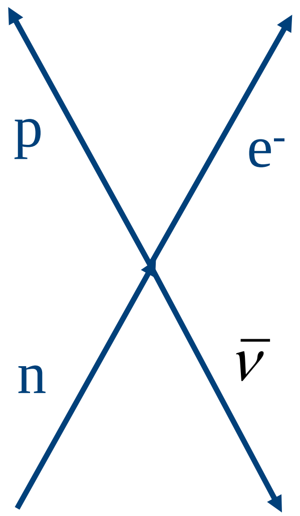
or:

$$\mathcal{H} = g_F \sum_{i=\mathrm{S,V,T,A,P}} \left[\left(\overline{\psi}_{\mathrm{p}}\mathcal{O}_i\psi_{\mathrm{n}}\right)\left(\overline{\psi}_{\mathrm{e}}\mathcal{O}^i(C_i + C_i'\gamma_5)\psi_\nu\right) + \mathrm{h.c.}\right]$$

Goldhaber et al., 1958 measure helicity of neutrino
→ V-A theory, left-handed interaction

Structure of the weak interaction in β decay

β-decay Hamiltonian (Lee & Yang, 1956) :



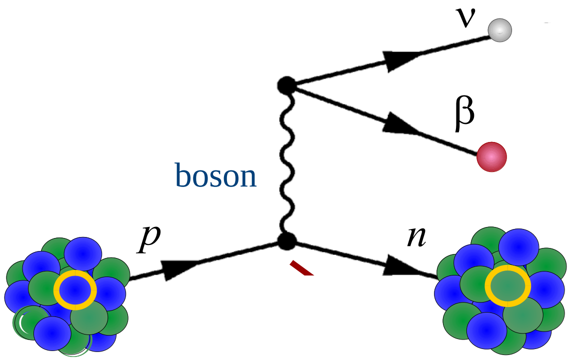
$$\boxed{H_\beta / g \;\propto}\quad (\,\bar p\; 1\; n\,)\;\;[\,\bar e\;\; 1\;\; (\,C_S + C_S^{'}\,\gamma_5)\;\nu\;]$$

$$+\quad (\,\bar p\;\gamma_\mu\; n\,)\;\;[\,\bar e\;\;\gamma_\mu\;\;(\,C_V + C_V^{'}\,\gamma_5)\;\nu\;]$$

$$+\;\frac{1}{2}(\,\bar p\;\sigma_{\mu\nu}\; n\,)\;\;[\,\bar e\;\;\sigma_{\mu\nu}\;(\,C_T + C_T^{'}\,\gamma_5)\;\nu\;]$$

$$-\quad (\,\bar p\;\gamma_\mu\gamma_5\; n\,)\;[\,\bar e\;\;\gamma_\mu\gamma_5(\,C_A + C_A^{'}\,\gamma_5)\;\nu\;]$$

$$+\quad \xcancel{\underset{\approx 0}{\underline{(\,\bar p\;\gamma_5\; n\,)}}\;\;[\,\bar e\;\;\gamma_5\;\;(\,C_P + C_P^{'}\,\gamma_5)\;\nu}\;]$$



with γ_i (i = 1, 2, 3, 4) Dirac matrices $(\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4)$

and $\sigma_{\mu\nu} \;= -\frac{\mathrm{i}}{2}(\gamma_\mu\gamma_\lambda - \gamma_\lambda\gamma_\mu)$

P-violation if $C_i \neq 0$ and $C_i^{'} \neq 0$

T-violation if $\mathrm{Im}(C_i^{(')} / C_j) \neq 0$

the Standard Model:

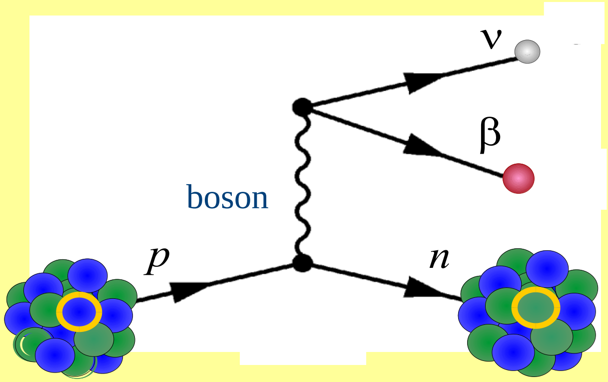
$C_i^{(')}$: coupling constants for the different types of weak interaction

* V-A interaction	$C_V \equiv 1$; $C_A = -1.27$ (C_A/C_V from n-decay)
* maximal P violation	$C_V' = C_V$ & $C_A' = C_A$
* no S, T, or P components	$C_S = C_S' = C_T = C_T' = C_P = C_P' \equiv 0$
* no time reversal violation	all C's are real

(except for the CP-violation included in the CKM matrix)

and Beyond:

experimental upper limits for $\left|C_T^{(')}/C_A\right|$ and $\left|C_S^{(')}/C_V\right|$ at few % level (neutron and nuclear β-decay)



5% level → ~ 350 GeV
per mille level → ~ 2.5 TeV

$$C_i \propto \frac{M_W^2}{M_{new}^2}$$

β decay rate and angular distribution

$$\mathcal{H} = g_F \sum_{i=\mathrm{S,V,T,A,P}} \left[\left(\overline{\psi}_{\mathrm{p}} \mathcal{O}_i \psi_{\mathrm{n}}\right) \left(\overline{\psi}_{\mathrm{e}} \mathcal{O}^i (C_i + C_i' \gamma_5) \psi_\nu\right) + \mathrm{h.c.} \right]$$

Fermi's golden rule:

$$\boxed{\omega = \frac{2\pi}{\hbar} \left|\mathcal{H}_{\mathrm{if}}\right|^2 \rho}$$

Probability of the transition

Density of final states

- In general : decay rate is given by Fermi's Golden rule

$$\boxed{N(E)dE = \frac{2\pi}{\hbar}|H_{fi}|^2 \frac{dN_F}{dE_0} = \frac{2\pi}{\hbar}|H_{fi}|^2 \frac{1}{4\pi^2\hbar^6c^5} p_e E_e (E_0 - E_e)^2 dE_e}$$

$\frac{dN_F}{dE_0}$ density of final states

General Weak interaction Hamiltonian for β-decay

(non relativistic nucleons)

sum over all terms $H_\beta = \sum_i C_i \langle \mathrm{Matrixelement}_i \rangle$

- Scalar

$$C_S(\psi_p^\dagger\psi_n)(\psi_e^\dagger\beta\psi_\nu) + C_S'(\psi_p^\dagger\psi_n)(\psi_e^\dagger\beta\gamma_5\psi_\nu)$$

- Vector

$$C_V(\psi_p^\dagger\psi_n)(\psi_e^\dagger\psi_\nu) + C_V'(\psi_p^\dagger\psi_n)(\psi_e^\dagger\gamma_5\psi_\nu)$$

- Tensor

$$C_T(\psi_p^\dagger\boldsymbol{\sigma}\psi_n)(\psi_e^\dagger\beta\boldsymbol{\sigma}\psi_\nu) + C_T'(\psi_p^\dagger\boldsymbol{\sigma}\psi_n)(\psi_e^\dagger\beta\boldsymbol{\sigma}\gamma_5\psi_\nu)$$

- Axial vector

$$C_A(\psi_p^\dagger\boldsymbol{\sigma}\psi_n)(\psi_e^\dagger\boldsymbol{\sigma}\psi_\nu) + C_A'(\psi_p^\dagger\boldsymbol{\sigma}\psi_n)(\psi_e^\dagger\boldsymbol{\sigma}\gamma_5\psi_\nu)$$

→Calculate correlation by determining $|H_{fi}|^2$

→Make the sum over everything that is not measured

Example: Beta-neutrino correlation in Fermi decays

$\mathbf{p}_e \cdot \mathbf{p}_\nu$

- Choose : z-axis along p_ν
  x-axis : such that the p_e is in the xz-plane ($p_e^y=0$).
- e^- wave functions (par. 2.4.3)

$$\psi_e^{\uparrow} = \frac{1}{\sqrt{1+\eta^2}}\begin{pmatrix} 1 \\ 0 \\ \eta\cos\theta \\ \eta\sin\theta \end{pmatrix} \quad \text{and} \quad \psi_e^{\downarrow} = \frac{1}{\sqrt{1+\eta^2}}\begin{pmatrix} 0 \\ 1 \\ \eta\sin\theta \\ -\eta\cos\theta \end{pmatrix}$$

- θ : angle between p_ν (z-axis) and p_e ; $\eta = \dfrac{c\,p_e}{E_e + m_e c^2}$

- antineutrino ↔ neutrino with $-E$, $-p_\nu$ and spin reversed.
  Wave functions (with $E<0$, $p_z=-p$)

$$\psi_\nu^{\uparrow}\left(\downarrow_{\bar\nu}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \psi_\nu^{\downarrow}\left(\uparrow_{\bar\nu}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

- Hamiltonian for Fermi transitions

$$H_\beta = C_S\left(\psi_p^\dagger\psi_n\right)\left(\psi_e^\dagger\beta\psi_\nu\right) + C_S'\left(\psi_p^\dagger\psi_n\right)\left(\psi_e^\dagger\beta\gamma_5\psi_\nu\right)$$
$$+ C_V\left(\psi_p^\dagger\psi_n\right)\left(\psi_e^\dagger\psi_\nu\right) + C_V'\left(\psi_p^\dagger\psi_n\right)\left(\psi_e^\dagger\gamma_5\psi_\nu\right)$$

$$\left|H_{fi}\right|^2 = \sum_{E_e>0,\sigma_e} \sum_{E_\nu<0,\sigma_\nu} \left| \int_{\text{space}} C_S\left(\psi_f^\dagger\psi_i\right)\left(\psi_e^\dagger\beta\psi_\nu\right) + C_S'\left(\psi_f^\dagger\psi_i\right)\left(\psi_e^\dagger\beta\gamma_5\psi_\nu\right) \right.$$
$$\left. + C_V\left(\psi_f^\dagger\psi_i\right)\left(\psi_e^\dagger\psi_\nu\right) + C_V'\left(\psi_f^\dagger\psi_i\right)\left(\psi_e^\dagger\gamma_5\psi_\nu\right) \right|^2$$

→Sum over all spin states, $E_e>0$ and $E_\nu<0$ (e^- and $\bar{\nu}$)

→Replaced ψ_p and ψ_n with ψ_f and ψ_i

→Allowed decay : lepton wave functions constant over nuclear volume

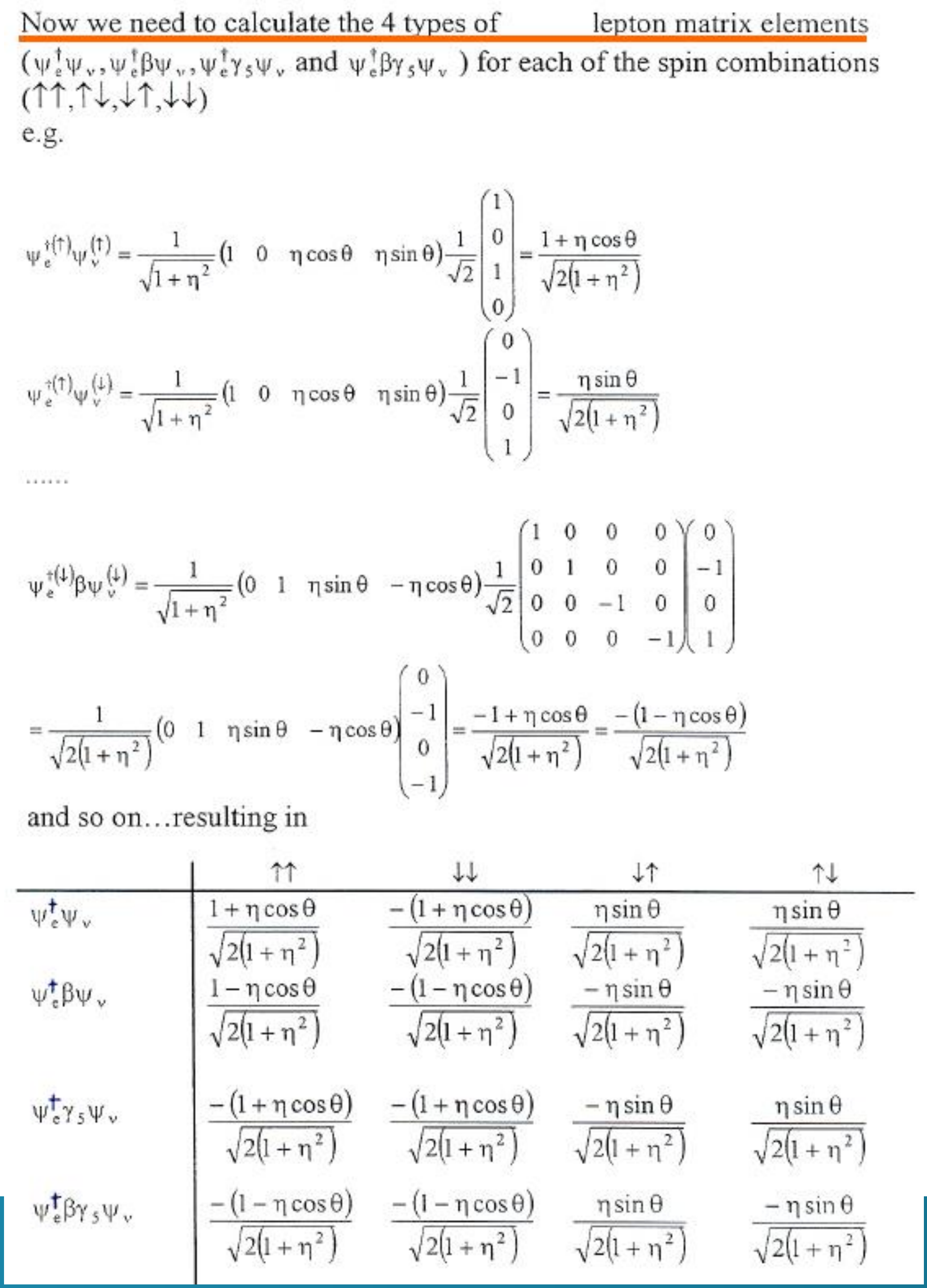
write : $\int_{\text{space}}\left(\psi_f^\dagger\psi_i\right) = M_F$ and note that $\left|H_{fi}\right|^2 = \left\langle\psi_f^\dagger H\psi_i\right\rangle^\dagger \left\langle\psi_f^\dagger H\psi_i\right\rangle$

$$\left|H_{fi}\right|^2 = \sum_{E_e>0,\sigma_e} \sum_{E_\nu<0,\sigma_\nu} \left|M_F\right|^2 \left\{ \left|C_S\right|^2 \left(\psi_e^\dagger\beta\psi_\nu\right)^2 + \left|C_S'\right|^2 \left(\psi_e^\dagger\beta\gamma_5\psi_\nu\right)^2 \right.$$
$$+ \left|C_V\right|^2 \left(\psi_e^\dagger\psi_\nu\right)^2 + \left|C_V'\right|^2 \left(\psi_e^\dagger\gamma_5\psi_\nu\right)^2$$
$$+ \left[\left(C_S C_S'^* + C_S^* C_S'\right)\left(\psi_e^\dagger\beta\psi_\nu\right)\left(\psi_e^\dagger\beta\gamma_5\psi_\nu\right)\right.$$
$$+ \left(C_S C_V^* + C_S^* C_V\right)\left(\psi_e^\dagger\beta\psi_\nu\right)\left(\psi_e^\dagger\psi_\nu\right)$$
$$+ \left(C_S C_V'^* + C_S^* C_V'\right)\left(\psi_e^\dagger\beta\psi_\nu\right)\left(\psi_e^\dagger\gamma_5\psi_\nu\right)$$
$$+ \left(C_S' C_V^* + C_S'^* C_V\right)\left(\psi_e^\dagger\beta\gamma_5\psi_\nu\right)\left(\psi_e^\dagger\psi_\nu\right)$$
$$+ \left(C_S' C_V'^* + C_S'^* C_V'\right)\left(\psi_e^\dagger\beta\gamma_5\psi_\nu\right)\left(\psi_e^\dagger\gamma_5\psi_\nu\right)$$
$$\left.\left. + \left(C_V C_V'^* + C_V^* C_V'\right)\left(\psi_e^\dagger\psi_\nu\right)\left(\psi_e^\dagger\gamma_5\psi_\nu\right)\right] \right\}$$

with $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$, $\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$, $\beta\gamma_5 = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$

The lepton matrix element is a real number so the hermitian conjugate was dropped.

Note that $C_iC_j^* + Ci^*Cj = 2\text{Re}(C_iC_j^*)$



Now we need to calculate the 4 types of lepton matrix elements ($\psi_e^\dagger\psi_\nu, \psi_e^\dagger\beta\psi_\nu, \psi_e^\dagger\gamma_5\psi_\nu$ and $\psi_e^\dagger\beta\gamma_5\psi_\nu$) for each of the spin combinations (↑↑,↑↓,↓↑,↓↓)

e.g.

$$\psi_e^{\dagger(\uparrow)}\psi_\nu^{(\uparrow)} = \frac{1}{\sqrt{1+\eta^2}}\begin{pmatrix}1 & 0 & \eta\cos\theta & \eta\sin\theta\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} = \frac{1+\eta\cos\theta}{\sqrt{2(1+\eta^2)}}$$

$$\psi_e^{\dagger(\uparrow)}\psi_\nu^{(\downarrow)} = \frac{1}{\sqrt{1+\eta^2}}\begin{pmatrix}1 & 0 & \eta\cos\theta & \eta\sin\theta\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix} = \frac{\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$$

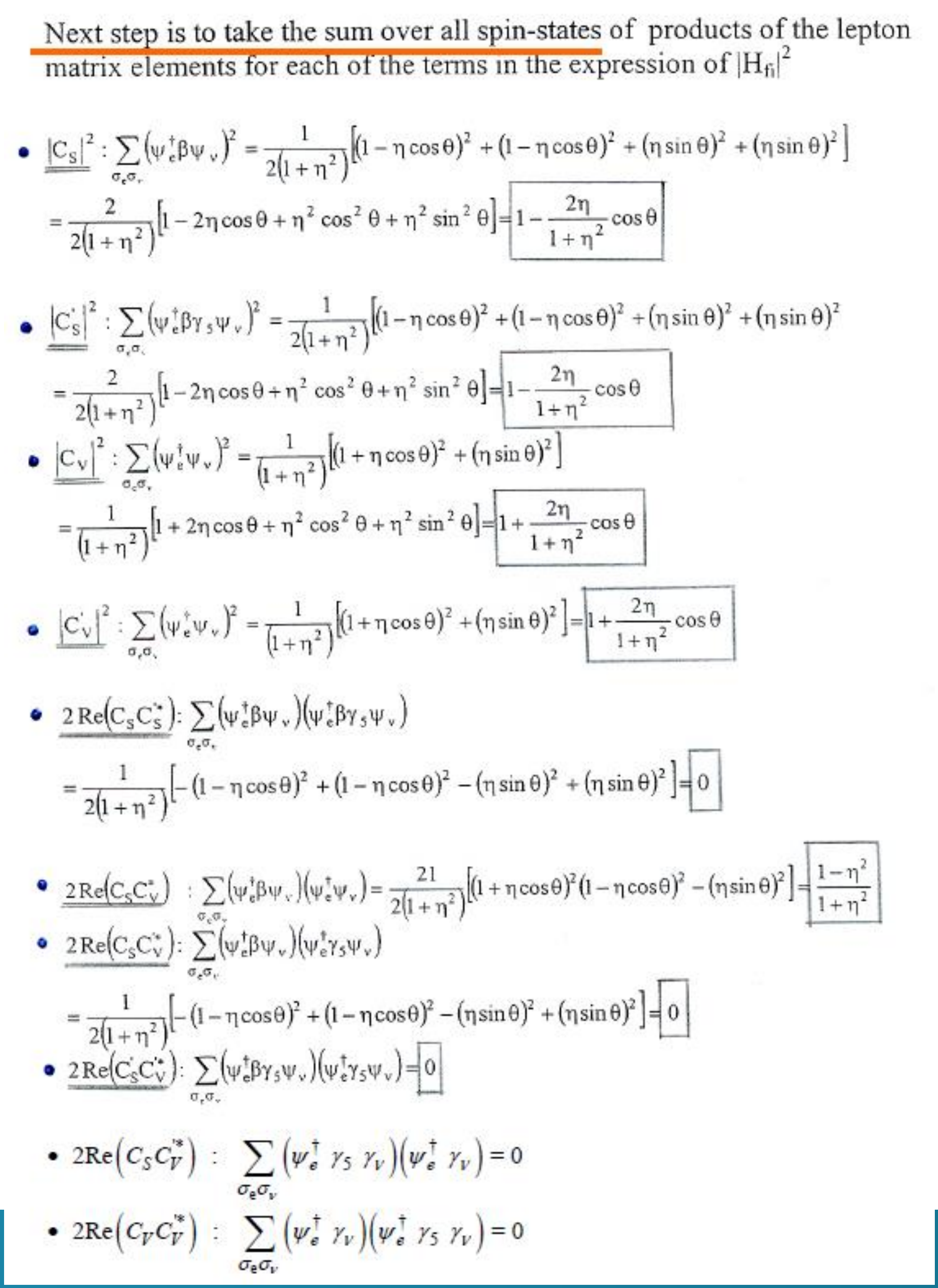
……

$$\psi_e^{\dagger(\downarrow)}\beta\psi_\nu^{(\downarrow)} = \frac{1}{\sqrt{1+\eta^2}}\begin{pmatrix}0 & 1 & \eta\sin\theta & -\eta\cos\theta\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}$$

$$= \frac{1}{\sqrt{2(1+\eta^2)}}\begin{pmatrix}0 & 1 & \eta\sin\theta & -\eta\cos\theta\end{pmatrix}\begin{pmatrix}0\\-1\\0\\-1\end{pmatrix} = \frac{-1+\eta\cos\theta}{\sqrt{2(1+\eta^2)}} = \frac{-(1-\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$$

and so on…resulting in

	↑↑	↓↓	↓↑	↑↓
$\psi_e^\dagger\psi_\nu$	$\frac{1+\eta\cos\theta}{\sqrt{2(1+\eta^2)}}$	$\frac{-(1+\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$	$\frac{\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$	$\frac{\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$
$\psi_e^\dagger\beta\psi_\nu$	$\frac{1-\eta\cos\theta}{\sqrt{2(1+\eta^2)}}$	$\frac{-(1-\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$	$\frac{-\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$	$\frac{-\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$
$\psi_e^\dagger\gamma_5\psi_\nu$	$\frac{-(1+\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$	$\frac{-(1+\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$	$\frac{-\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$	$\frac{\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$
$\psi_e^\dagger\beta\gamma_5\psi_\nu$	$\frac{-(1-\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$	$\frac{-(1-\eta\cos\theta)}{\sqrt{2(1+\eta^2)}}$	$\frac{\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$	$\frac{-\eta\sin\theta}{\sqrt{2(1+\eta^2)}}$



Next step is to take the sum over all spin-states of products of the lepton matrix elements for each of the terms in the expression of $|H_{fi}|^2$

- $|C_S|^2 : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \beta \psi_\nu\right)^2 = \frac{1}{2(1+\eta^2)}\left[(1-\eta\cos\theta)^2 + (1-\eta\cos\theta)^2 + (\eta\sin\theta)^2 + (\eta\sin\theta)^2\right]$

  $= \frac{2}{2(1+\eta^2)}\left[1 - 2\eta\cos\theta + \eta^2\cos^2\theta + \eta^2\sin^2\theta\right] = \boxed{1 - \frac{2\eta}{1+\eta^2}\cos\theta}$

- $|C_S'|^2 : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \beta\gamma_5 \psi_\nu\right)^2 = \frac{1}{2(1+\eta^2)}\left[(1-\eta\cos\theta)^2 + (1-\eta\cos\theta)^2 + (\eta\sin\theta)^2 + (\eta\sin\theta)^2\right.$

  $= \frac{2}{2(1+\eta^2)}\left[1 - 2\eta\cos\theta + \eta^2\cos^2\theta + \eta^2\sin^2\theta\right] = \boxed{1 - \frac{2\eta}{1+\eta^2}\cos\theta}$

- $|C_V|^2 : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \psi_\nu\right)^2 = \frac{1}{(1+\eta^2)}\left[(1+\eta\cos\theta)^2 + (\eta\sin\theta)^2\right]$

  $= \frac{1}{(1+\eta^2)}\left[1 + 2\eta\cos\theta + \eta^2\cos^2\theta + \eta^2\sin^2\theta\right] = \boxed{1 + \frac{2\eta}{1+\eta^2}\cos\theta}$

- $|C_V'|^2 : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \psi_\nu\right)^2 = \frac{1}{(1+\eta^2)}\left[(1+\eta\cos\theta)^2 + (\eta\sin\theta)^2\right] = \boxed{1 + \frac{2\eta}{1+\eta^2}\cos\theta}$

- $2\,\mathrm{Re}(C_S C_S'^*) : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \beta \psi_\nu\right)\left(\psi_e^\dagger \beta\gamma_5 \psi_\nu\right)$

  $= \frac{1}{2(1+\eta^2)}\left[-(1-\eta\cos\theta)^2 + (1-\eta\cos\theta)^2 - (\eta\sin\theta)^2 + (\eta\sin\theta)^2\right] = \boxed{0}$

- $2\,\mathrm{Re}(C_S C_V^*) : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \beta \psi_\nu\right)\left(\psi_e^\dagger \psi_\nu\right) = \frac{21}{2(1+\eta^2)}\left[(1+\eta\cos\theta)^2(1-\eta\cos\theta)^2 - (\eta\sin\theta)^2\right] = \boxed{\frac{1-\eta^2}{1+\eta^2}}$
- $2\,\mathrm{Re}(C_S C_V'^*) : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \beta \psi_\nu\right)\left(\psi_e^\dagger \gamma_5 \psi_\nu\right)$

  $= \frac{1}{2(1+\eta^2)}\left[-(1-\eta\cos\theta)^2 + (1-\eta\cos\theta)^2 - (\eta\sin\theta)^2 + (\eta\sin\theta)^2\right] = \boxed{0}$
- $2\,\mathrm{Re}(C_S' C_V'^*) : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger \beta\gamma_5 \psi_\nu\right)\left(\psi_e^\dagger \gamma_5 \psi_\nu\right) = \boxed{0}$

- $2\mathrm{Re}\left(C_S C_V'^*\right) : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger\ \gamma_5\ \gamma_V\right)\left(\psi_e^\dagger\ \gamma_V\right) = 0$

- $2\mathrm{Re}\left(C_V C_V'^*\right) : \sum_{\sigma_e \sigma_\nu} \left(\psi_e^\dagger\ \gamma_V\right)\left(\psi_e^\dagger\ \gamma_5\ \gamma_V\right) = 0$

now $\eta = \frac{c\,p_e}{E_e + m_e c^2}$ and we know that $E^2 = p^2c^2 + m^2c^4$

so

$$\frac{2\eta}{1+\eta^2} = \frac{2\left(\frac{c\,p_e}{E_e + m_e c^2}\right)}{1+\left(\frac{c\,p_e}{E_e + m_e c^2}\right)^2} = \frac{2\left(E_e + m_e c^2\right)\left(c\,p_e\right)}{\left(E_e + m_e c^2\right)^2 + \left(c\,p_e\right)^2} = + \frac{2\left(E_e + m_e c^2\right)\left(c\,p_e\right)}{E_e^2 + 2E_e m_e c^2 + E_e^2} = \frac{c\,p_e}{E_e}$$

and

$$\frac{1-\eta^2}{1+\eta^2} = \frac{1-\left(\frac{c\,p_e}{E_e + m_e c^2}\right)^2}{1+\left(\frac{c\,p_e}{E_e + m_e c^2}\right)^2} = \frac{\left(E_e + m_e c^2\right)^2 - \left(c\,p_e\right)^2}{\left(E_e + m_e c^2\right)^2 + \left(c\,p_e\right)^2} = \frac{2E_e m_e c^2 + 2m_e^2 c^4}{E_e^2 + 2E_e m_e c^2 + E_e^2} = \frac{m_e c^2}{E_e}$$

Putting all terms together we find :

$$\left|H_{fi}\right|^2 = \left|M_F\right|^2\left[\left(\left|C_S\right|^2 + \left|C_S'\right|^2\right)\left(1 - \frac{cp_e}{E_e}\cos\theta\right) + \left(\left|C_V\right|^2 + \left|C_V'\right|^2\right)\left(1 + \frac{cp_e}{E_e}\cos\theta\right)\right.$$

$$\left. + 2\,\mathrm{Re}\left(C_S C_V^* + C_S' C_V'^*\right)\left(\frac{m_e c^2}{E_e}\right)\right]$$

$$= \left|M_F\right|^2\left[\left(\left|C_S\right|^2 + \left|C_S'\right|^2 + \left|C_V\right|^2 + \left|C_V'\right|^2\right) + \left(-\left|C_S\right|^2 - \left|C_S'\right|^2 + \left|C_V\right|^2 + \left|C_V'\right|^2\right)\frac{cp_e}{E_e}\cos\theta\right.$$

$$\left. + 2\,\mathrm{Re}\left(C_S C_V^* + C_S' C_V'^*\right)\left(\frac{m_e c^2}{E_e}\right)\right]$$

So the decay rate is

$$\boxed{N(E_e,\Omega_e,\Omega_\nu)dE_e\, d\Omega_e\, d\Omega_\nu = \boxed{|H_{fi}|^2} \frac{1}{(2\pi)^5 \hbar^7 c^5} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e\, d\Omega_\nu}$$

$$= \frac{1}{(2\pi)^5 \hbar^7 c^5} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e\, d\Omega_\nu$$

$$|M_F|^2 \left[\left(|C_S|^2 + |C_S^{'}|^2 + |C_V|^2 + |C_V^{'}|^2 \right) + \left(-|C_S|^2 - |C_S^{'}|^2 + |C_V|^2 + |C_V^{'}|^2 \right) \frac{cp_e}{E_e} \cos\theta \right.$$

$$\left. + 2\,\mathrm{Re}\left(C_S C_V^{*} + C_S^{'} C_V^{'*} \right) \left(\frac{m_e c^2}{E_e} \right) \right]$$

$$\boxed{\begin{aligned} &= \frac{1}{(2\pi)^5 \hbar^7 c^5} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e\, d\Omega_\nu \\ &\xi \left[1 + a \frac{c^2 \mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} \cos\theta + 2\,\mathrm{Re}\, b \left(\frac{m_e c^2}{E_e} \right) \right] \end{aligned}}$$

with

$$\xi = |M_F|^2 \left(|C_S|^2 + |C_S^{'}|^2 + |C_V|^2 + |C_V^{'}|^2 \right)$$

$$a\xi = |M_F|^2 \left(-|C_S|^2 - |C_S^{'}|^2 + |C_V|^2 + |C_V^{'}|^2 \right)$$

$$b\xi = 2\,\mathrm{Re}\left[|M_F|^2 \left(C_S C_V^{*} + C_S^{'} C_V^{'*} \right) \right]$$

Beta decay transition probability :

distribution in energy, emission angle and polarization of β-particles for allowed β-decay of polarized nuclei

$$dW = \boxed{dW_0}\,\boxed{\xi}\left\{ 1 + \frac{\bar{p}\cdot\bar{q}}{E_e E_\nu}\,a + \frac{\Gamma m}{E_e}\,b\right.$$

βν-correlation

Fierz interference term

$$+ \bar{J}\cdot\left[\frac{\bar{p}}{E_e}A + \frac{\bar{q}}{E_\nu}B + \frac{\bar{p}\,\mathrm{x}\,\bar{q}}{E_e E_\nu}D\right]$$

β-asymmetry ν-asymmetry

$$\left. + \bar{\sigma}\cdot\left[\frac{\bar{p}}{E_e}G + \hat{p}\,(\bar{J}\cdot\hat{p})\,Q' + \bar{J}\,\mathrm{x}\,\frac{\bar{p}}{E_e}R\right] \ldots\right\}$$

longitudinal polarization transversal polarization

$\bar{p}$: momentum of beta particle

$\bar{q}$: momentum of neutrino

E: energy

m: electron rest mass

$\bar{J}$: spin of nucleus

$\Gamma = \sqrt{(1-(\alpha Z)^2}$

(Z of daughter nucleus)

with

$$\boxed{dW_0} \propto G_F^2\ \underbrace{F(\pm Z, E_e)}_{\text{Fermi function}}\ \underbrace{(E_e - E_0)^2\ p\,E_e\,dE\,d\Omega_e\,d\Omega_\nu}_{\text{phase space}}$$

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$$\boxed{\xi} = M_F^2\left[|C_V|^2 + |C'_V|^2 + |C_S|^2 + |C'_S|^2\right] + M_{GT}^2\left[|C_A|^2 + |C'_A|^2 + |C_T|^2 + |C'_T|^2\right]$$

$$\boxed{a}\,\xi = M_F^2\left[|C_V|^2 + |C'_V|^2 - |C_S|^2 - |C'_S|^2\right] - \frac{M_{GT}^2}{3}\left[|C_A|^2 + |C'_A|^2 - |C_T|^2 - |C'_T|^2\right]$$

$$\boxed{b}\,\xi = \pm 2\,\mathrm{Re}\left[M_F^2\left(C_S C_V^* + C'_S C'^*_V\right) + M_{GT}^2\left(C_T C_A^* + C'_T C'^*_A\right)\right]$$

$$\boxed{A}\,\xi = 2\,\mathrm{Re}\left[\mp\lambda_{J'J}\; M_{GT}^2\left(C_A C'^*_A - C_T C'^*_T\right)\right.$$
$$\left. - \delta_{J'J}\sqrt{\frac{J}{J+1}}\; M_F M_{GT}\left(C_V C'^*_A + C'_V C_A^* - C_S C'^*_T - C'_S C_T^*\right)\right]$$

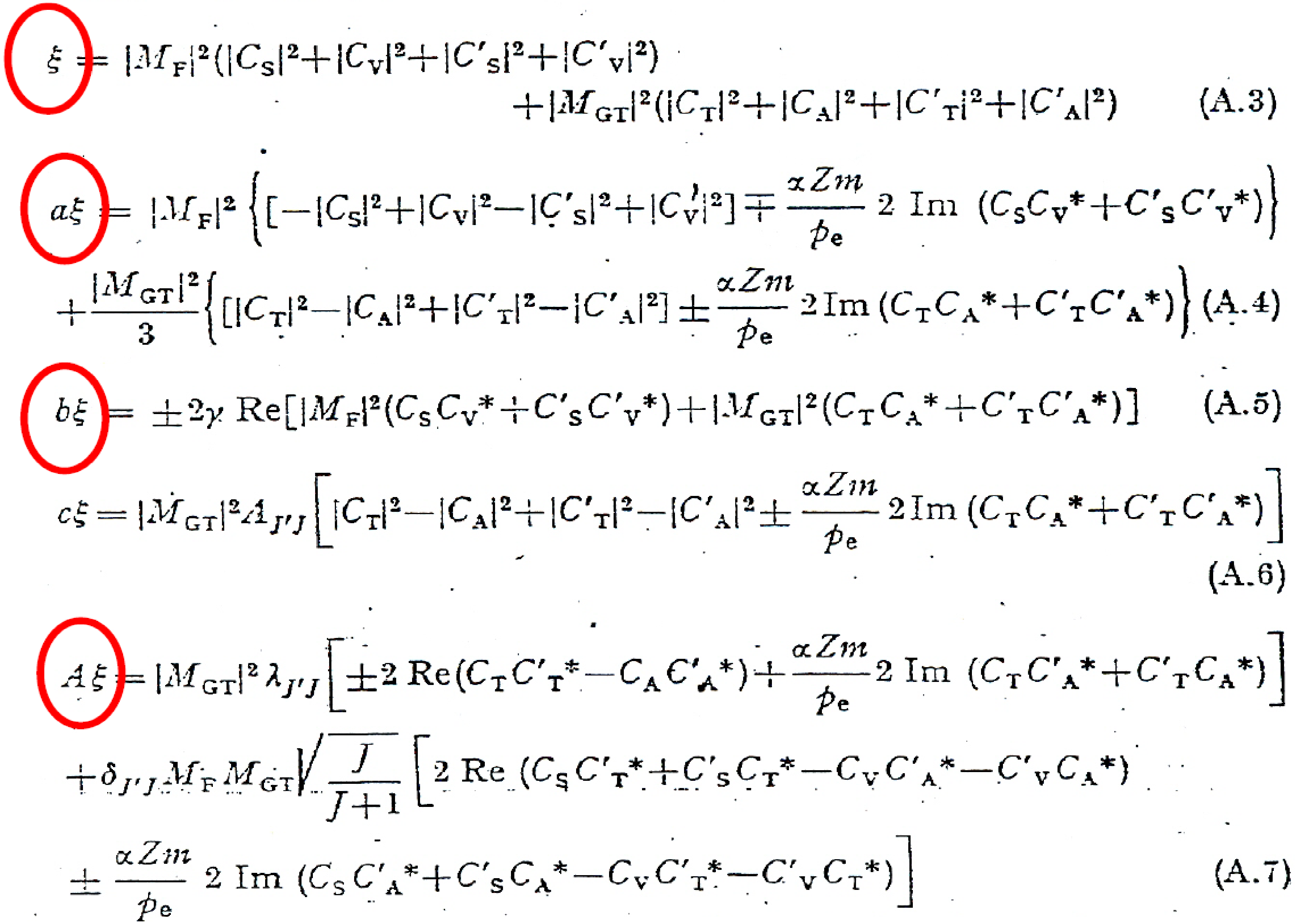
$$\lambda_{J'J} = \begin{cases} 1 & J \rightarrow J' = J-1 \\ \dfrac{1}{J+1} & J \rightarrow J' = J \\ -\dfrac{1}{J+1} & J \rightarrow J' = J+1 \end{cases}$$

with: $M_{F(GT)}$ **= Fermi(Gamow-Teller) nuclear matrix element,**

C_i **= coupling constants of the S, V, A, T weak interactions,**

and assuming time-reversal invariance (i.e. all C_i real).

Full equations for the beta decay correlation coefficients



$$\xi = |M_F|^2(|C_S|^2+|C_V|^2+|C'_S|^2+|C'_V|^2) + |M_{GT}|^2(|C_T|^2+|C_A|^2+|C'_T|^2+|C'_A|^2) \qquad \text{(A.3)}$$

$$a\xi = |M_F|^2\left\{[-|C_S|^2+|C_V|^2-|C'_S|^2+|C'_V|^2] \mp \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_S C_V^* + C'_S C'^*_V)\right\} + \frac{|M_{GT}|^2}{3}\left\{[|C_T|^2-|C_A|^2+|C'_T|^2-|C'_A|^2] \pm \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_T C_A^* + C'_T C'^*_A)\right\} \qquad \text{(A.4)}$$

$$b\xi = \pm 2\gamma\,\mathrm{Re}[|M_F|^2(C_S C_V^* + C'_S C'^*_V) + |M_{GT}|^2(C_T C_A^* + C'_T C'^*_A)] \qquad \text{(A.5)}$$

$$c\xi = |M_{GT}|^2 \Lambda_{J'J}\left[|C_T|^2-|C_A|^2+|C'_T|^2-|C'_A|^2 \pm \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_T C_A^* + C'_T C'^*_A)\right] \qquad \text{(A.6)}$$

$$A\xi = |M_{GT}|^2 \lambda_{J'J}\left[\pm 2\,\mathrm{Re}(C_T C'^*_T - C_A C'^*_A) + \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_T C'^*_A + C'_T C_A^*)\right] + \delta_{J'J} M_F M_{GT}\sqrt{\frac{J}{J+1}}\left[2\,\mathrm{Re}\,(C_S C'^*_T + C'_S C_T^* - C_V C'^*_A - C'_V C_A^*) \pm \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_S C'^*_A + C'_S C_A^* - C_V C'^*_T - C'_V C_T^*)\right] \qquad \text{(A.7)}$$

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$$\boxed{B\xi} = 2\,\mathrm{Re}\left\{ |M_{\mathrm{GT}}|^2 \lambda_{J'J} \left[\frac{\gamma m}{E_e}(C_T C'^*_A + C'_T C_A^*) \pm (C_T C'^*_T + C_A C'^*_A)\right]\right.$$
$$- \delta_{J'J} M_F M_{\mathrm{GT}} \sqrt{\frac{J}{J+1}} \left[(C_S C'^*_T + C'_S C_T^* + C_V C'^*_A + C'_V C_A^*)\right. \qquad \text{(A.8)}$$
$$\left.\left.\pm \frac{\gamma m}{E_e}(C_S C'^*_A + C'_S C_A^* + C_V C'^*_T + C'_V C_T^*)\right]\right\}$$

$$\boxed{D\xi} = \delta_{J'J} M_F M_{\mathrm{GT}} \sqrt{\frac{J}{J+1}} \left[2\,\mathrm{Im}\,(C_S C_T^* - C_V C_A^* + C'_S C'^*_T - C'_V C'^*_A)\right.$$
$$\left.\mp \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Re}\,(C_S C_A^* - C_V C_T^* + C'_S C'^*_A - C'_V C'^*_T)\right] \qquad \text{(A.9)}$$

$$\boxed{G\xi} = |M_F|^2 \left[\pm 2\,\mathrm{Re}\,(C_S C'^*_S - C_V C'^*_V) + \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_S C'^*_V + C'_S C_V^*)\right]$$
$$+ |M_{\mathrm{GT}}|^2 \left[\pm 2\,\mathrm{Re}\,(C_T C'^*_T - C_A C'^*_A) + \frac{\alpha Z m}{p_e}\, 2\,\mathrm{Im}\,(C_T C'^*_A + C'_T C_A^*)\right] \text{(A.10)}$$

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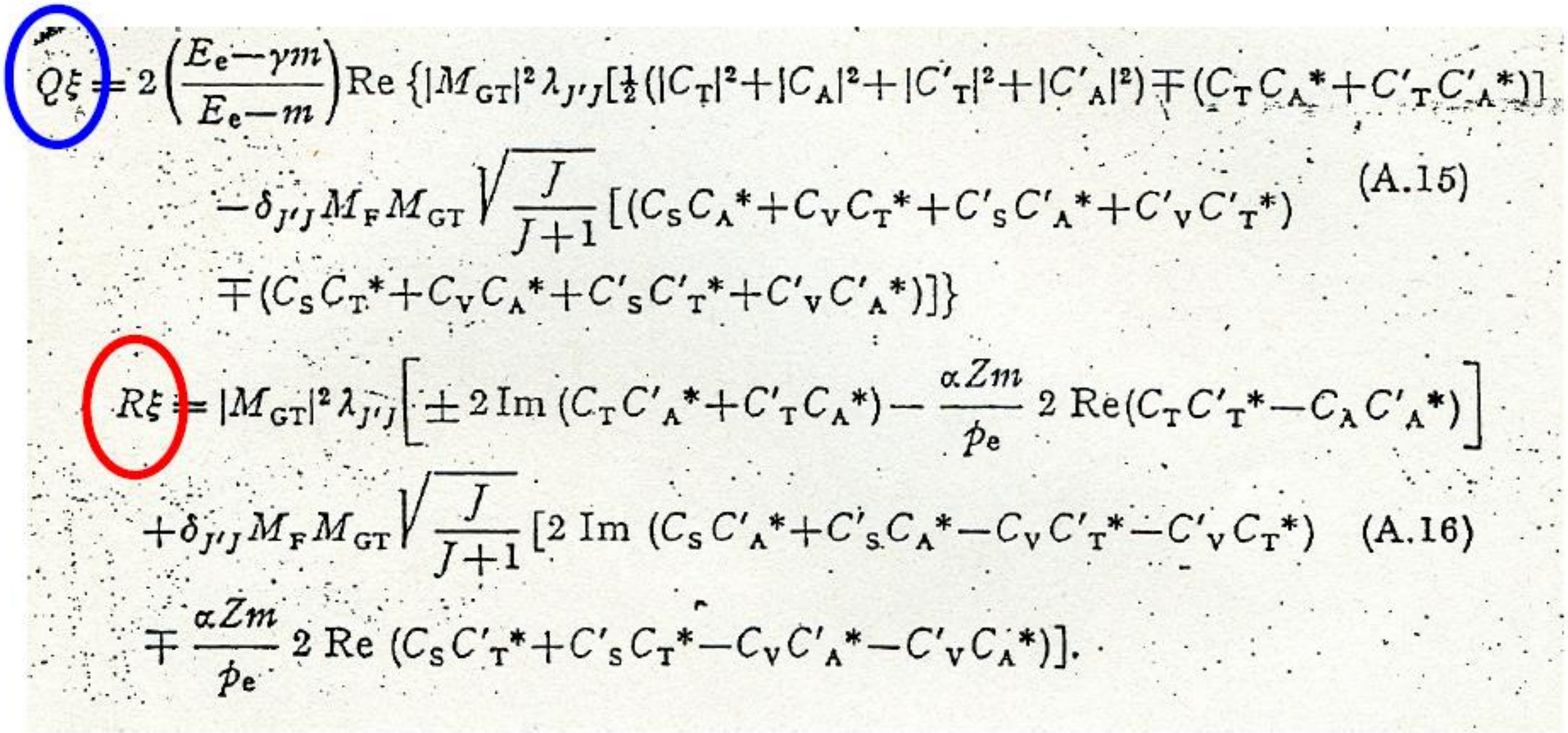

$$
\begin{aligned}
Q\xi = 2\left(\frac{E_e-\gamma m}{E_e-m}\right)\mathrm{Re}\,\{&|M_{\mathrm{GT}}|^2\lambda_{J'J}[\tfrac{1}{2}(|C_{\mathrm{T}}|^2+|C_{\mathrm{A}}|^2+|C'_{\mathrm{T}}|^2+|C'_{\mathrm{A}}|^2)\mp(C_{\mathrm{T}}C_{\mathrm{A}}{}^*+C'_{\mathrm{T}}C'_{\mathrm{A}}{}^*)] \\
&-\delta_{J'J}M_{\mathrm{F}}M_{\mathrm{GT}}\sqrt{\frac{J}{J+1}}[(C_{\mathrm{S}}C_{\mathrm{A}}{}^*+C_{\mathrm{V}}C_{\mathrm{T}}{}^*+C'_{\mathrm{S}}C'_{\mathrm{A}}{}^*+C'_{\mathrm{V}}C'_{\mathrm{T}}{}^*) \\
&\mp(C_{\mathrm{S}}C_{\mathrm{T}}{}^*+C_{\mathrm{V}}C_{\mathrm{A}}{}^*+C'_{\mathrm{S}}C'_{\mathrm{T}}{}^*+C'_{\mathrm{V}}C'_{\mathrm{A}}{}^*)]\}
\end{aligned}
\tag{A.15}
$$

$$
\begin{aligned}
R\xi = {}&|M_{\mathrm{GT}}|^2\lambda_{J'J}\left[\pm 2\,\mathrm{Im}\,(C_{\mathrm{T}}C'_{\mathrm{A}}{}^*+C'_{\mathrm{T}}C_{\mathrm{A}}{}^*)-\frac{\alpha Zm}{p_e}2\,\mathrm{Re}(C_{\mathrm{T}}C'_{\mathrm{T}}{}^*-C_{\mathrm{A}}C'_{\mathrm{A}}{}^*)\right] \\
&+\delta_{J'J}M_{\mathrm{F}}M_{\mathrm{GT}}\sqrt{\frac{J}{J+1}}[2\,\mathrm{Im}\,(C_{\mathrm{S}}C'_{\mathrm{A}}{}^*+C'_{\mathrm{S}}C_{\mathrm{A}}{}^*-C_{\mathrm{V}}C'_{\mathrm{T}}{}^*-C'_{\mathrm{V}}C_{\mathrm{T}}{}^*) \\
&\mp\frac{\alpha Zm}{p_e}2\,\mathrm{Re}\,(C_{\mathrm{S}}C'_{\mathrm{T}}{}^*+C'_{\mathrm{S}}C_{\mathrm{T}}{}^*-C_{\mathrm{V}}C'_{\mathrm{A}}{}^*-C'_{\mathrm{V}}C_{\mathrm{A}}{}^*)].
\end{aligned}
\tag{A.16}
$$

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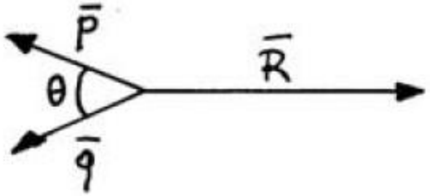
Spin and momentum vectors indicate geometry required for maximal sensitivity:

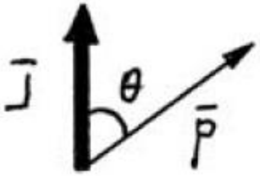
β-ν correlation

$$\frac{\bar{p}\cdot\bar{q}}{E_\beta\, E_\nu}\quad a$$



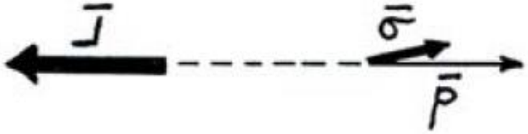
β-asymmetry

$$\bar{J}\cdot\frac{\bar{p}}{E_\beta}\quad A$$



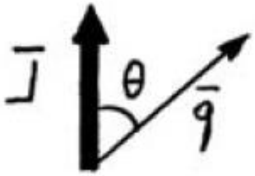
longitudinal β-polarisation

$$\bar{\sigma}\cdot\left[\frac{\bar{p}}{E_\beta}\,G + \hat{p}\,(\bar{J}\cdot\hat{p})\,Q'\right]$$



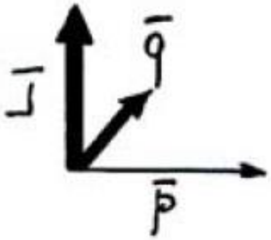
ν- asymmetry

$$\bar{J}\cdot\frac{\bar{q}}{E_\nu}\,B$$



D- triple correlation

$$\bar{J}\cdot\frac{\bar{p}\times\bar{q}}{E_\beta E_\nu}\,D$$



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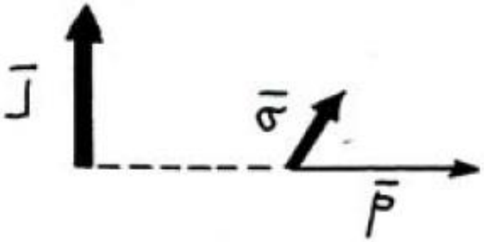
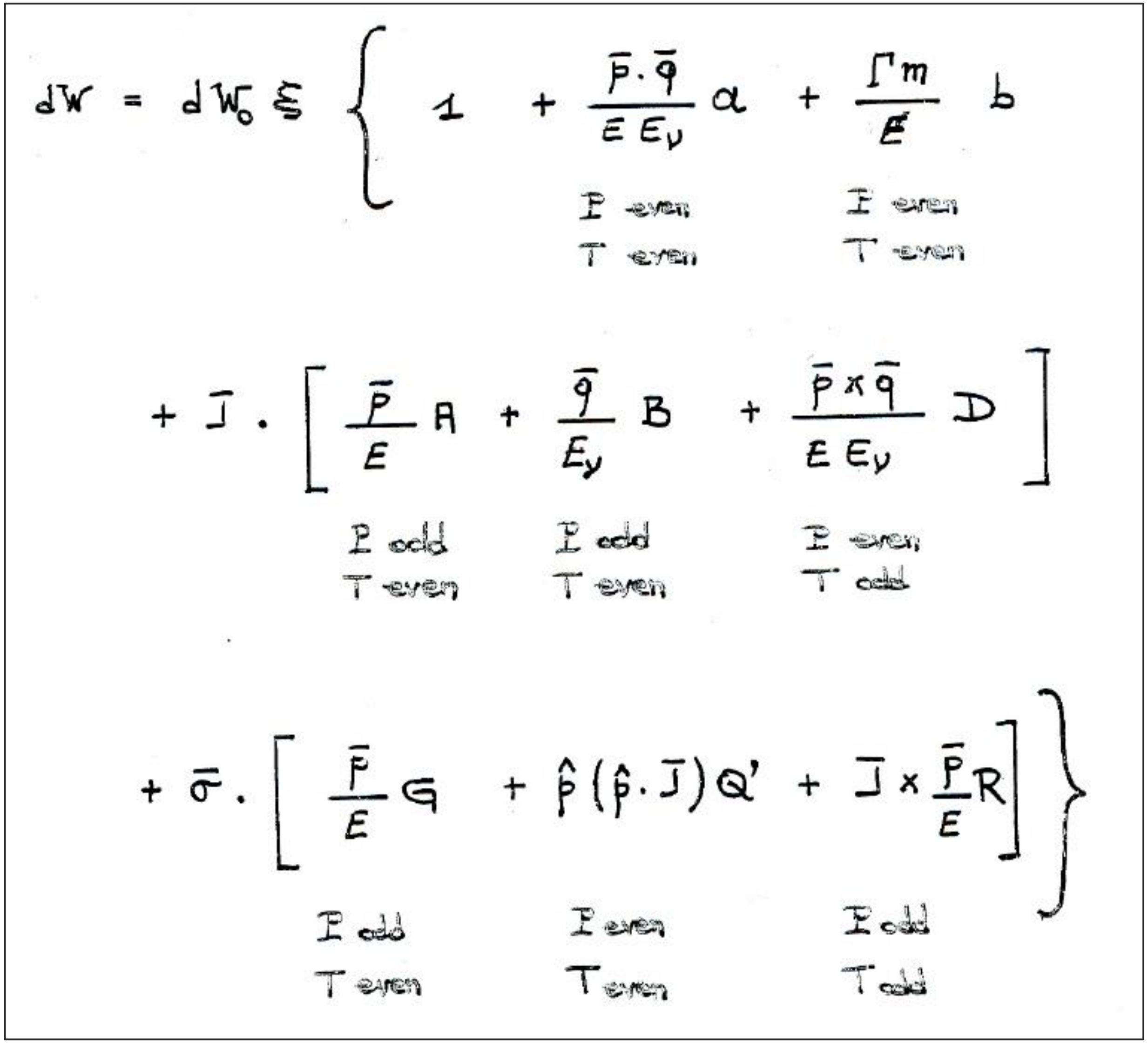
transversal β-polarisation

$$\bar{J}\cdot\frac{\bar{p}}{E_\beta}\times\bar{\sigma}\,R$$



Behavior of correlation coefficients under P(parity) and T(time-reversal) operations



$$dW = dW_0\, \xi \left\{ 1 + \frac{\bar{p}\cdot\bar{q}}{E\, E_\nu}\, a + \frac{\Gamma m}{E}\, b \right.$$

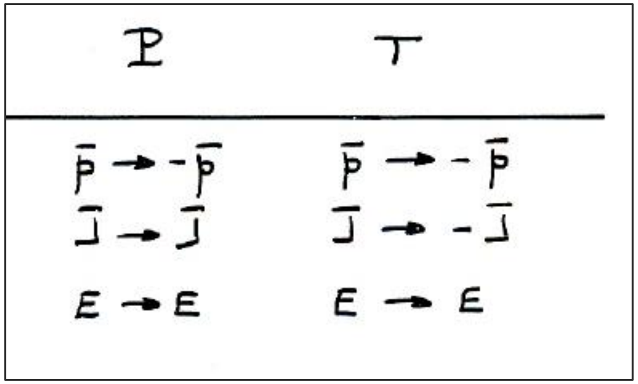
P even, T even (term a); P even, T even (term b)

$$+ \bar{J} \cdot \left[\frac{\bar{p}}{E} A + \frac{\bar{q}}{E_\nu} B + \frac{\bar{p}\times\bar{q}}{E\, E_\nu} D \right]$$

P odd, T even (term A); P odd, T even (term B); P even, T odd (term D)

$$\left. + \bar{\sigma} \cdot \left[\frac{\bar{p}}{E} G + \hat{p}\,(\hat{p}\cdot\bar{J})\, Q' + \bar{J}\times\frac{\bar{p}}{E} R \right] \right\}$$

P odd, T even (term G); P even, T even (term Q'); P odd, T odd (term R)



P	T
$\bar{p} \rightarrow -\bar{p}$	$\bar{p} \rightarrow -\bar{p}$
$\bar{J} \rightarrow \bar{J}$	$\bar{J} \rightarrow -\bar{J}$
$E \rightarrow E$	$E \rightarrow E$