Darmstadt Lecture 4 – Nuclear Experimental Constraints

James Lattimer

Department of Physics & Astronomy
449 ESS Bldg.
Stony Brook University

July 9, 2019

Darmstadt Lecture 4 – Nuclear Experimental Constraints

James.Lattimer@Stonybrook.edu

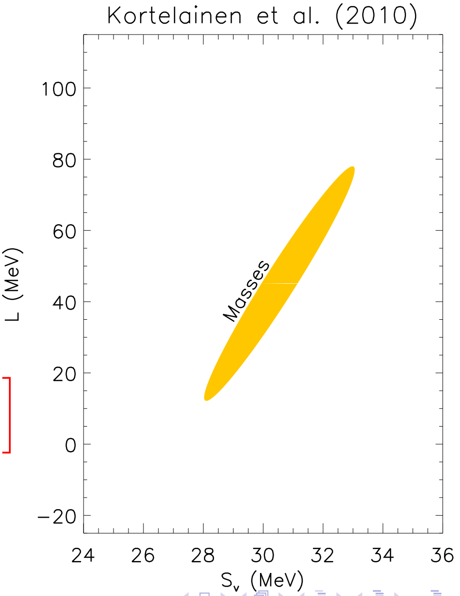
Nuclear Experimental Constraints

Binding Energies

Liquid Droplet Model

$$E_{sym} = AI^2\left[\frac{S_v}{1+S_sA^{-1/3}/S_v} - \frac{Ze^2}{20R}\frac{S_sA^{-1/3}/S_v}{1+S_sA^{-1/3}/S_v} + \cdots\right]$$

$$\frac{S_s}{S_v} \simeq \frac{3a}{2r_o}\left[1 + \frac{L}{3S_v} + \left(\frac{L}{3S_v}\right)^2 \cdots\right]$$



Why Symmetry Parameters are Highly Correlated

Assuming approximate validity of liquid drop model:

$$E_{\rm sym}(N,Z) = (S_v A - S_s A^{2/3}) I^2$$

$$\chi^2 = \frac{1}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}}(E_{ex,i} - E_{sym,i})^2$$

$$\chi_{vv} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}} I_i^4 A_i^2 = 61.6\,\sigma_D^{-2}$$

$$\chi_{vs} = -\frac{2}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}} I_i^4 A_i^{5/3} = -10.7\,\sigma_D^{-2}$$

$$\chi_{ss} = \frac{2}{\mathcal{N}\sigma_D^2}\sum_{i=1}^{\mathcal{N}} I_i^4 A_i^{4/3} = 1.87\,\sigma_D^{-2}$$

$$\sigma_{S_v} = \sqrt{\frac{2\chi_{ss}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 2.3\,\sigma_D$$

$$\sigma_{S_s} = \sqrt{\frac{2\chi_{vv}}{\chi_{vv}\chi_{ss} - \chi_{sv}^2}} \simeq 13.2\,\sigma_D$$

$$\alpha = \frac{1}{2}\tan^{-1}\frac{2\chi_{vs}}{\chi_{vv} - \chi_{ss}} \simeq 9^\circ.8$$

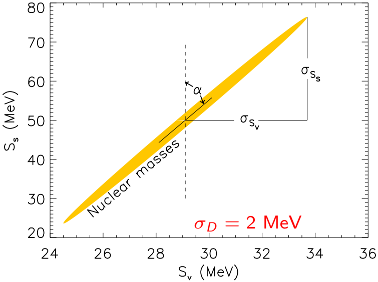
$$r_{vs} = -\frac{\chi_{vs}}{\sqrt{\chi_{vv}\chi_{ss}}} \simeq 0.997$$

Liquid droplet model:

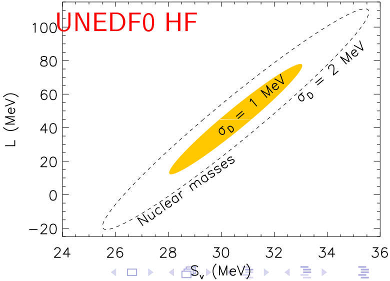
$$E_{\rm sym}(N,Z) = \frac{S_v A I^2}{1 + (S_s/S_v)A^{-1/3}}$$

$$S_s \simeq \frac{3a}{2r_o} S_v\left[1 + (L/3S_v) + (L/3S_v)^2 + \cdots\right]$$

Liquid drop model



Kortelainen et al. (2010)

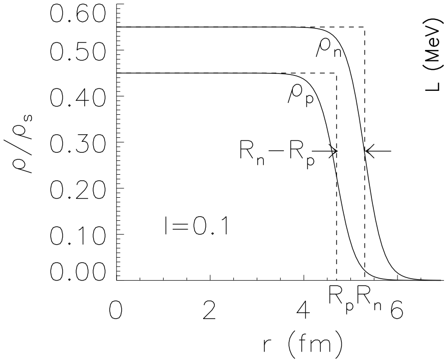


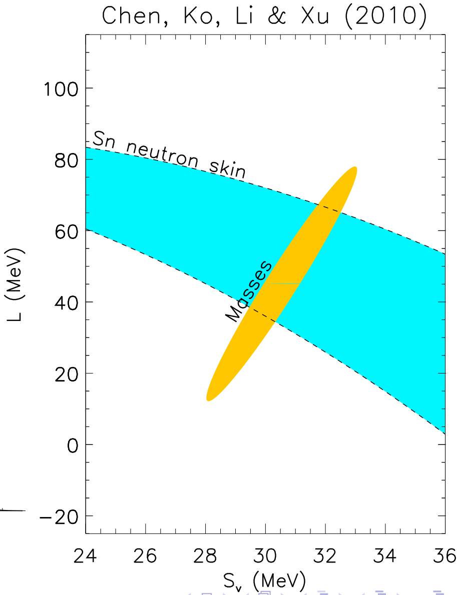
Nuclear Experimental Constraints

Neutron Skin Thicknesses

$$r_{np} = \frac{2r_o}{3S_v}\frac{1}{\sqrt{1-I^2}}\left(1+S_sA^{-1/3}/S_v\right)^{-1} \times\sqrt{\frac{3}{5}}\left[IS_s-\frac{3Ze^2}{140r_o}\left(1+\frac{10}{3}\frac{S_sA^{-1/3}}{S_v}\right)\right]$$

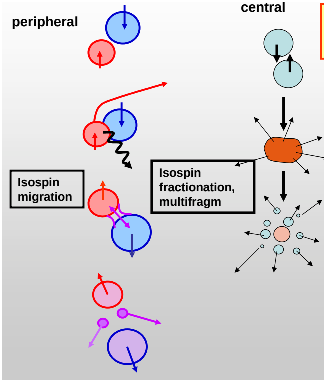
$$r_{np,208} = 0.15 \pm 0.04 \text{ fm}$$



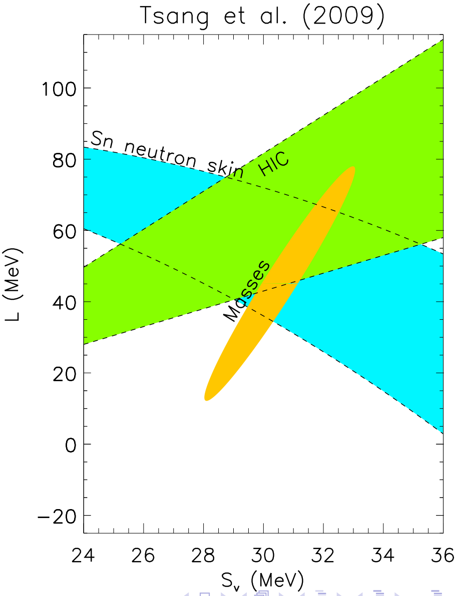


Nuclear Experimental Constraints

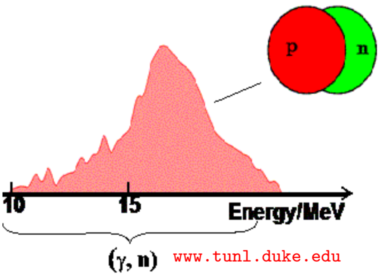
Flows in
Heavy Ion Collisions



Wolter, NuSYM11

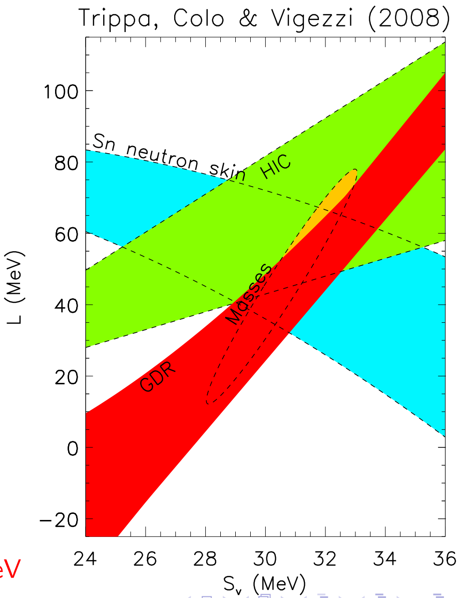


Giant Dipole Resonances



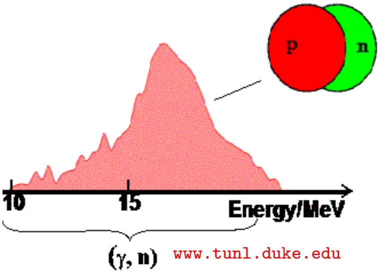
$$E_{-1} \propto \sqrt{\frac{S_v}{1+\frac{5S_s}{3S_v}A^{-1/3}}}$$

23.3 MeV$<S_2(0.1\ \mathrm{fm}^{-3})<$24.9 MeV

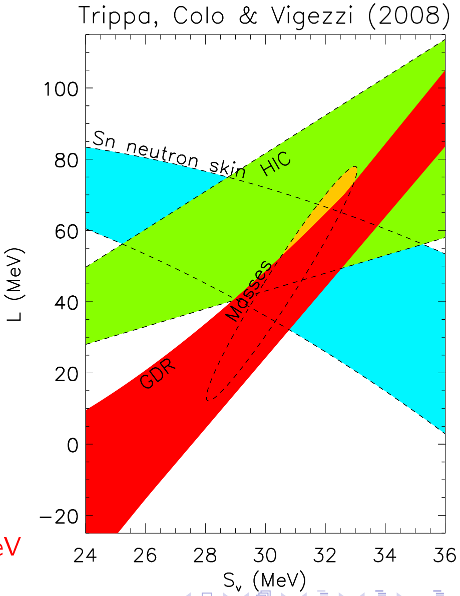


Nuclear Experimental Constraints

Giant Dipole Resonance Centroids



23.3 MeV$< S_2(0.1\ \mathrm{fm}^{-3}) <$24.9 MeV



Nuclear Experimental Constraints

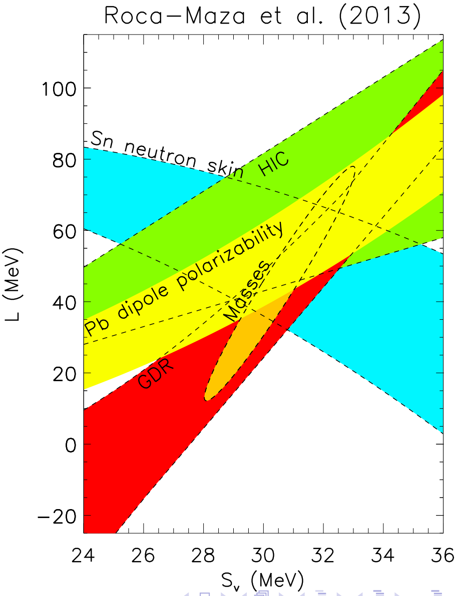
Dipole Polarizabilities

$\alpha_D = 4m_{-1}$

$\simeq \frac{AR^2}{20S_v}\left(1+\frac{5}{3}\frac{S_sA^{-1/3}}{S_v}\right)$

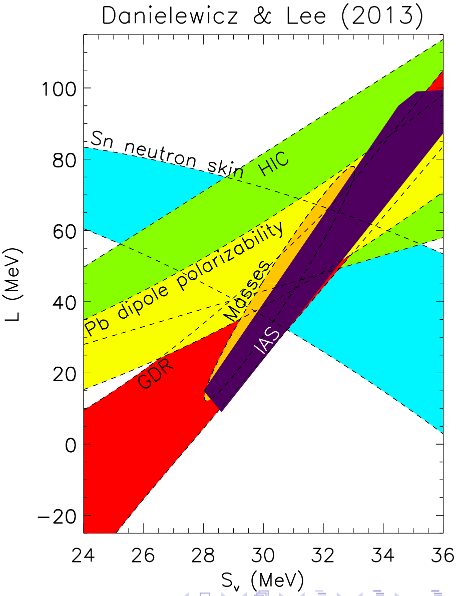
Uses data of Tamii et al. (2011)

$\alpha_{D,208} = 20.1 \pm 0.6 \text{ fm}^2$



Nuclear Experimental Constraints

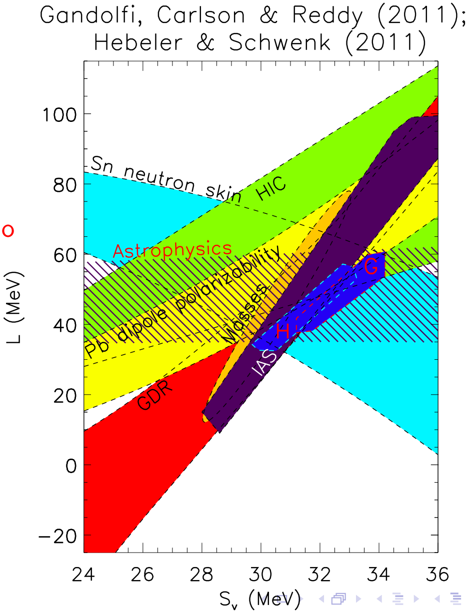
Isobaric Analog States



Theoretical Neutron Matter Calculations

H&S: Chiral Lagrangian

GC&R: Quantum Monte Carlo



The Nuclear Incompressibility Parameter

Chief experimental constraint is the giant isoscalar monopole resonance, or breathing mode. In the *scaling model* the finite-nucleus incompressibility is related to the breathing mode energy E_{br} by $K_A = MR^2(E_{br}/\hbar)^2$.

$$\frac{E_A}{A} = \frac{H_B(n,0)}{n} + \frac{4\pi R^2}{A}\left(\sigma + \sigma_I I^2\right) + \frac{3}{5}\frac{Z^2e^2}{AR} + S_v h(n) I^2$$

$$K_A = \frac{R^2}{A}\left(\frac{\partial^2 E_A}{\partial R^2}\right)_A = 9\frac{n^2}{A}\left(\frac{\partial^2 E_A}{\partial n^2}\right)_A$$

$$\frac{1}{A}\left(\frac{\partial E_A}{\partial R}\right)_A = -\frac{3n}{R}\left[(H_B(n,0)/n)' + \frac{4\pi R^2}{A}\left(\sigma' + \sigma_I' I^2\right) + S_v h' I^2\right] + \frac{8\pi R}{A}\left(\sigma + \sigma_I I^2\right) - \frac{3}{5}\frac{Z^2e^2}{AR^2}$$

Setting this to zero fixes the density and yields

$$p(n,0) = \frac{n}{3}\left[\frac{8\pi R^2}{A}\left(\sigma + \sigma_I I^2 - 3n\left[\sigma' + \sigma_I' I^2\right]\right) - \frac{3}{5}\frac{Z^2e^2}{AR} - 3S_v n h' I^2\right] = n^2(H_B(n,0)/n)' \simeq \frac{n-n_s}{n_s}\frac{K_s}{9}$$

Incompressibility and Skewness

$$\frac{dp(n,0)}{dn} = 2n(H_B(n,0)/n)' + n^2(H_B(n,0)/n)''$$
$$\simeq K_s + \frac{n-n_s}{n_s}\left(4K_s + 9n_s^3(H_B(n,0)/n)_s'''\right)$$
$$\simeq K_s + p(n,0)\left(36 - 3n_s^3 K_s'/K_s\right)$$

Skewness $K_s' = -27n_s^3(H_B(n,0)/n)_s'''$

$$K_A = \frac{R^2}{A}\left(\frac{\partial^2 E_A}{\partial R^2}\right)_A = \frac{dp(n,0)}{dn} + \frac{12}{5}\frac{Z^2e^2}{AR}$$
$$+ \frac{8\pi R^2}{A}\left[-\sigma - \sigma_I I^2 + \frac{9}{2}n_s^2\left(\sigma'' + \sigma_I'' I^2\right)\right] + 9S_v I^2\left(n_s^2 h_s'' + 2n_s h_s'\right)$$
$$= K_s + K_{surf}A^{-1/3} + K_C Z^2 A^{-4/3} + K_I I^2$$

$h_s = 1$, $n_s h_s' = L/(3S_v)$, $n_s^2 h_s'' = K_{sym}/(9S_v)$, $\sigma' = 0$ (due to optimization)

$$K_C = -\frac{3e^2}{5r_o}\left[8 - \frac{K_s'}{K_s}\right], \qquad K_I = K_{sym} - 6L + L\frac{K_s'}{K_s}$$
$$K_{surf} = 4\pi r_o^2\sigma\left[9\frac{n^2}{\sigma}\sigma'' + 22 - \frac{2K_s'}{K_s}\right]$$

An Analytic Model

A useful nuclear force model that allows for finite skewness ($d \neq 1$) is

$$H_B + Bn_s u = Bn_s u(1 - u^{1/d})^2, \qquad K_s = \frac{18B}{d^2}, \qquad K_s' = -\frac{162B}{d^3}(d-1)$$

$$u' = -\sqrt{\frac{B}{Q}}u\left(1 - u^{1/d}\right), \; u = \left(1 + e^{z-y}\right)^{-d}, \; z = \sqrt{\frac{B}{Q}}\frac{r}{d}, \; \sigma = \frac{2\sqrt{QB}n_s}{1+d}$$

$$t_{90-10} = 2.3 \text{ fm} = -\int_{0.1}^{0.9}\frac{du}{u'} = \sqrt{\frac{Q}{B}}\left[2\ln 3 + d\ln\left(\frac{1-0.1^{1/d}}{1-0.9^{1/d}}\right)\right]$$

Blaizot (1976) has shown

$$n^2\frac{\partial^2\sigma}{\partial n^2} = \int_{-\infty}^{\infty}\left[\frac{\partial^2 H_B(n,0)}{\partial n^2}n^2 - n_s n\left(\frac{\partial^2 H_B(n,0)}{\partial n^2}\right)_s\right]dx$$

$$= -\int_0^1 \frac{1}{u'}\left[\frac{\partial^2 H_B(u,0)}{\partial u^2}u^2 - u\left(\frac{\partial^2 H_B(u,0)}{\partial u^2}\right)_1\right]du$$

$$\frac{n^2}{\sigma}\frac{\partial^2\sigma}{\partial n^2} = -\frac{1+3d+d^2}{d^2}, \qquad K_{surf} = -4\pi r_o^2\sigma\frac{9+45d-31d^2}{d^2}$$

$$K_C = -\frac{3e^2}{5r_o}\frac{17d-9}{d}, \qquad K_I = K_{sym} - 6L - \frac{9L}{d}(d-1)$$

Comparison With Experiment

The model prediction

$$K_A = K_s + K_{surf} A^{-1/3} + K_C Z^2 A^{-4/3} + K_I I^2$$

is now a function of d. We can compare with measurements of K_A.

Monopole resonance data from J.M. Pearson 1991, Phys. Lett. B271, 12.

Nucleus	E_{br} (MeV)	R (fm)	K_A (MeV)	d	K_s (MeV)
^{112}Sn	15.88 ± 0.14	4.56	126.4 ± 2.2	1.171	213.9
^{114}Sn	15.80 ± 0.14	4.59	126.8 ± 2.2	1.169	215.8
^{116}Sn	15.69 ± 0.16	4.62	126.7 ± 2.6	1.168	217.2
^{120}Sn	15.52 ± 0.15	4.67	126.7 ± 2.4	1.165	221.1
^{124}Sn	15.35 ± 0.16	4.72	126.6 ± 2.6	1.161	225.7
^{144}Sm	15.13 ± 0.14	4.93	134.2 ± 2.5	1.134	229.9
^{208}Pb	13.90 ± 0.30	5.54	143.0 ± 6.2	1.095	254.4

The predicted values are $K_s \simeq 225^{+30}_{-10}$ MeV and $K_s' = -235^{+100}_{-30}$ MeV. They are relatively insensitive to assumed values for L and K_{sym}.

The Unitary Gas

The **unitary gas** is an idealized system consisting of fermions interacting via a pairwise zero-range s-wave interaction with an infinite scattering length:

As long as the scattering length $a >> k_F^{-1}$ (interparticle spacing), and the range of the interaction $R << k_F^{-1}$, the properties of the gas are universal in the sense they don't depend on the details of the interaction.

The sole remaining length scale is $k_F = (3\pi^2 n)^{1/3}$, so the unitary gas energy is a constant times the Fermi energy $\hbar^2 k_F^2/(2m)$:

$$E_{\rm UG} = \xi_0 \frac{3\hbar^2 k_F^2}{10m}.$$

$\xi_0 \simeq 0.37$ is known as the **Bertsch** parameter, measured in cold-atom experiments.

The Unitary Gas as Analogue of the Neutron Gas

A pure neutron matter (PNM) gas differs from the unitary gas:

- $|a| \simeq 18.5$ fm; $|ak_F|^{-1} \simeq 0.03$ for $n = n_s$.
- $R \simeq 2.7$ fm; $Rk_F \approx 4.5$ for $n = n_s$.
- Repulsive 3-body interactions are additionally necessary for neutron matter to fit the energies of light nuclei.
- Neutron matter has potentially atractive p-wave and higher-order interactions.

The first three imply $E_{\rm PNM} > E_{\rm UG}$:

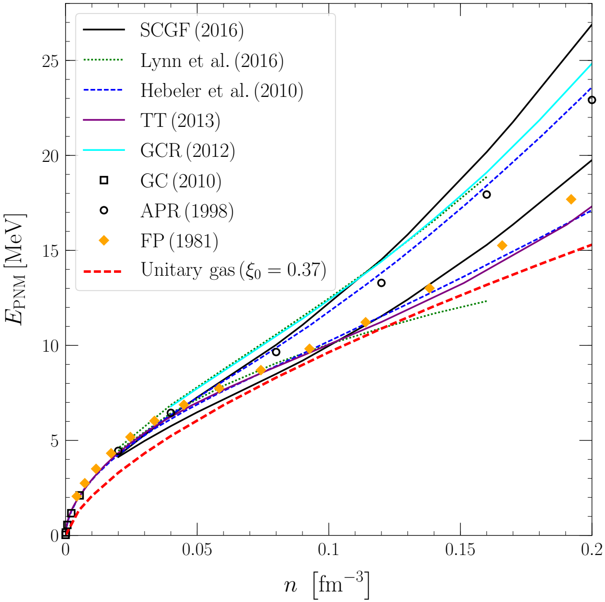
$$\xi \simeq \xi_0 + 0.6|ak_F|^{-1} + \dots \qquad |ak_F|^{-1} << 1$$

$$\xi \simeq \xi_0 + 0.12Rk_F + \dots \qquad Rk_F << 1$$

A reasonable conjecture would appear to be ($u = n/n_s$)

$$E_{\rm PNM}(u) = E(u, Y_p = 0) \geq E_{\rm UG,0}u^{2/3} \simeq 12.6u^{2/3}\ {\rm MeV}$$

Comparison to Neutron Matter Calculations



Consequences for the Nuclear Symmetry Energy

$$S(u) = E_{\rm PNM} - E(u, Y_p = 1/2).$$

A good approximation for the Y_p-dependence of E is

$$S(u) \simeq \frac{1}{8}\frac{\partial^2 E(u, Y_p)}{\partial Y_p^2}.$$

Near n_s,

$$S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_{sym}}{18}(u-1)^2 + \cdots$$

$$E(u, Y_p = 1/2) \simeq -B + \frac{K_s}{18}(u-1)^2 + \cdots$$

In this case, the unitary gas conjecture is

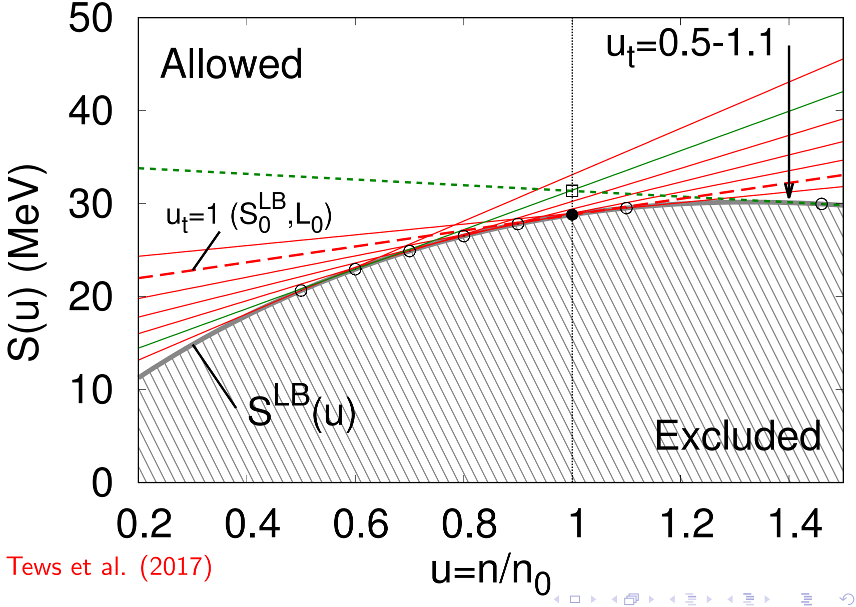
$$S(u) > E_{\rm UG,0}u^{2/3} - \left[-B + \frac{K_s}{18}(u-1)^2 + \cdots\right] \equiv S^{\rm LB}(u)$$

Thus, the symmetry energy parameters S_0 and L must satisfy

$$S(u=1) = S_0 \geq S_0^{\rm LB} = E_{\rm UG,0} + B \simeq 28.5 \text{ MeV}$$

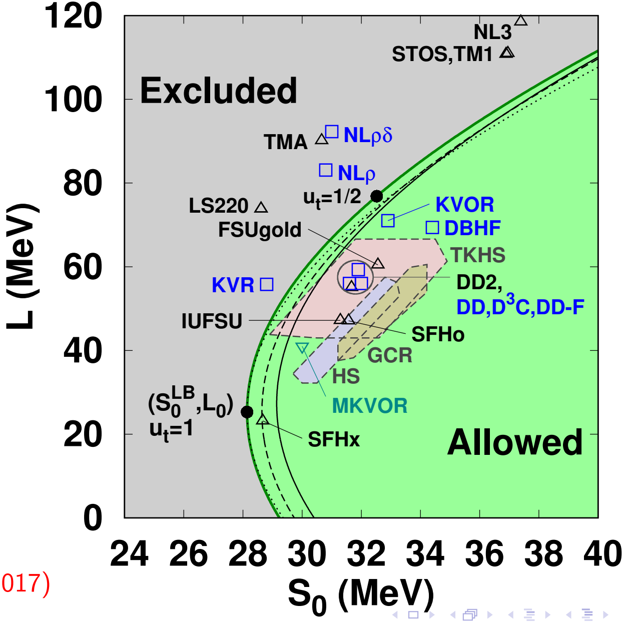
$$L(u=1) = L_0 = 3\left(udS/du\right)_{u=1} = 2E_{\rm UG,0} \simeq 25.2\text{MeV}$$

$S_0 > S_0^{\rm LB}: \ \left(S = S^{\rm LB}\right)_{u_t}, \left(dS/du = dS^{\rm LB}/du\right)_{u_t}$



Tews et al. (2017)

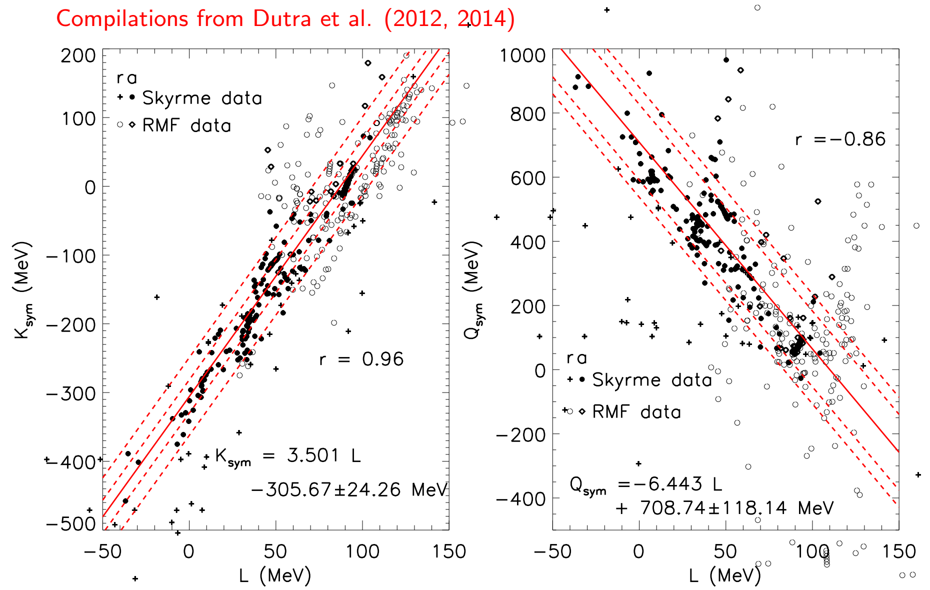
Symmetry Parameter Exclusions



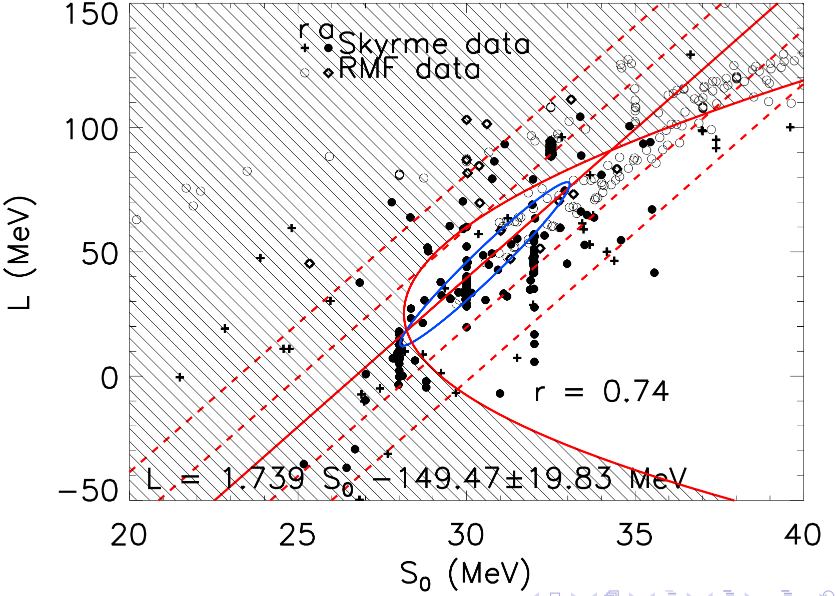
Tews et al. (2017)

Symmetry Parameter Correlations

Compilations from Dutra et al. (2012, 2014)



More Realistic Exclusion Region



Analytic Approximation for the Boundary

$$S(u_t) = S^{\rm LB}(u_t), \qquad \left(\frac{dS}{du}\right)_{u_t} = \left(\frac{dS^{\rm LB}}{du}\right)_{u_t}$$

gives

$$S_0 + \frac{L}{3}(u_t - 1) + \frac{K_{sym}}{18}(u_t - 1)^2 = E_{\rm UG,0}\,u_t^{2/3} + B - \frac{K_s}{18}(u_t - 1)^2$$

$$L + \frac{K_{sym}}{3}(u_t - 1) = 2E_{\rm UG,0}\,u_t^{-1/3} - \frac{K_s}{3}(u_t - 1)$$

Assume $K_n = 3L$ (i.e., $K_{\rm sym} \approx 3L - K_s$). Then

$$S_0 = \frac{E_{\rm UG,0}}{3u_t^{4/3}}(1 + 2u_t^2) - E_0, \qquad L = \frac{2E_{\rm UG,0}}{u_t^{4/3}}$$

or after eliminating u_t,

$$S_0 = \frac{L}{6}\left[1 + 2\left(\frac{2E_{\rm UG,0}}{L}\right)^{3/2}\right] - E_0$$

Experimental Constraints

Isovector Skins and
Isobaric Analog States
from Danielewicz et al. (2017)

Other experimental constraints from Lattimer & Lim (2013)

Unitary gas constraints from
Tews et al. (2017)

Experimental and neutron matter constraints are compatible with unitary gas bounds.