Darmstadt Lecture 7 – Proto-Neutron Stars

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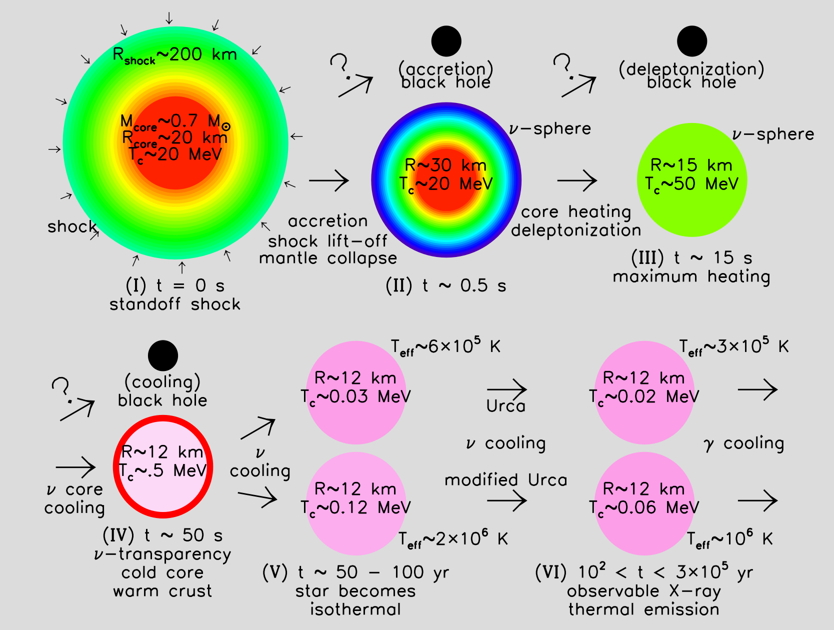
Proto-Neutron Stars



Proto-Neutron Star Evolution

Consider the Newtonian case. The electron neutrino number flux F_ν and ν energy flux L_ν change the lepton number Y_L and entropy s:

$$\begin{aligned} n\frac{dY_L}{dt} &= n\frac{dY_e}{dt}+\frac{d(Y_\nu-Y_{\bar\nu})}{dt}=\frac{1}{r^2}\frac{\partial}{\partial r}r^2F_\nu \\ nT\frac{ds}{dt} &= -\frac{1}{4\pi r^2}\frac{\partial(L_\nu+L_{\bar\nu})}{\partial r}-n\sum_{n,p,e,\nu}\mu_i\frac{dY_i}{dt}. \end{aligned}$$

In the diffusion approximation, fluxes are driven by density gradients:

$$\begin{aligned} F_\nu &= -\int_0^\infty\frac{c}{3}\left(\lambda_N\frac{\partial n_\nu(E)}{\partial r}-\bar\lambda_N\frac{\partial n_{\bar\nu}(E)}{\partial r}\right)dE, \\ L_\nu &= -\int_0^\infty 4\pi r^2\sum_i^6\frac{c\lambda_E^i}{3}\frac{\partial\epsilon_i(E)}{\partial r}dE. \end{aligned}$$

λ_N and λ_E^i's are mean free paths for number and energy transport, respectively. $n_\nu(E)$ and $\epsilon_i(E)$ are the ν_e number and energy densities of species $i=e,\mu,\tau$ and their antiparticles at neutrino energy E.

There are two main sources of opacity:

ν-nucleon absorption, affects only $\nu_e,\bar\nu_e$.
$\lambda_\nu\simeq\bar\lambda_\nu\simeq\lambda_0(E_0/E)^2$; $\lambda_0\approx 5$ cm; $E_0\approx 1$ MeV
$\nu-e$ scattering, affects all flavors. $\lambda_E^i\simeq\lambda_1^i(E_0/E)^2$; $\lambda_1\approx 4$ km

Proto-Neutron Stars – Analytic Analysis

Neutrino fluid

$$n_\nu(E) = \frac{E^2}{2\pi^3(\hbar c)^3} f_\nu(E), \qquad f_\nu(E) = \left[1 + e^{(E-\mu_\nu)/T}\right]^{-1}$$

$$n(Y_\nu - Y_{\bar\nu}) = \frac{\mu_\nu^3 + \pi^2 T^2 \mu}{6\pi^2(\hbar c)^3}$$

$$\epsilon_\nu(E) = E(n_\nu(E) + n_{\bar\nu}(E))$$

Diffusion approximation

$$F_\nu = -\frac{c}{3}\int_0^\infty \left[\lambda_N \frac{\partial n_\nu(E)}{\partial r} - \bar\lambda_N \frac{\partial n_{\bar\nu}(E)}{\partial r}\right] dE$$

$$\simeq -\frac{c\lambda_0 E_0^2 T}{6\pi^2(\hbar c)^3}\frac{\partial T[F_0(\mu_\nu/T) - F_0(-\mu_\nu/T)]}{\partial r} = -\frac{c\lambda_0 E_0^2}{6\pi^2(\hbar c)^3}\frac{\partial \mu_\nu}{\partial r},$$

$$L_\nu = -4\pi r^2 \int_0^\infty \sum_i^6 \frac{c\lambda_E^i}{3}\frac{\partial \epsilon_i(E)}{\partial r} dE$$

$$\simeq -\frac{r^2 c}{3\pi(\hbar c)^3}\sum_i^3 \lambda_1^i E_0^{i2} \frac{\partial T^2[F_1(\mu_\nu/T) + F_1(-\mu_\nu/T)]}{\partial r}$$

$$= -\frac{r^2 c}{3\pi(\hbar c)^3}\sum_i^3 \lambda_1^i E_0^{i2} \frac{\partial}{\partial r}\left(\mu_\nu^2 + \frac{\pi^2 T^2}{3}\right).$$

The Deleptonization of a Proto-Neutron Star

Energy transport dominated by degenerate electron neutrinos propagating through degenerate matter.

Number transport equation

$$\frac{\partial n Y_L}{\partial t} = \frac{\partial Y_L}{\partial Y_\nu}\frac{1}{6\pi^2(\hbar c)^3}\frac{\partial \mu_\nu^3}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 F_\nu$$
$$= -\frac{c\lambda_0 E_0^2}{6\pi^2(\hbar c)^3}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mu_\nu}{\partial r}\right)$$

$$\frac{\partial Y_L}{\partial Y_\nu} \simeq 5, \quad \mu_\nu = \mu_{\nu,0}\psi(x)\phi(t), \qquad x = x_1 r/R$$

$$\tau_D \frac{d\phi^2}{dt} = -\frac{1}{x^2\psi^3}\frac{\partial}{\partial x}\left[x^2\frac{\partial \psi}{\partial x}\right] = -1$$

Spatial solution is Lane-Emden function of index 3, ψ_3.

$$\phi = \sqrt{1-\frac{t}{\tau_D}}, \quad \psi_3(x_1) = 0 \implies x_1 = 6.897$$
$$\tau_D = \frac{6}{c\lambda_0}\frac{\partial Y_L}{\partial Y_\nu}\left(\frac{\mu_{\nu,0}}{E_0}\right)^2\left(\frac{R}{x_1}\right)^2 \approx 5 - 10 \text{ s}$$

Heating During Deleptonization

Beta equilibrium:

$$\sum_j \mu_j dY_j = (-\mu_n + \mu_p + \mu_e - \mu_\nu) dY_e + \mu_\nu dY_L = \mu_\nu dY_L$$

Energy transport equation:

$$nT\frac{\partial s}{\partial t} = -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r} - n\sum_j \mu_j \frac{\partial Y_j}{\partial t} \simeq -n\mu_\nu \frac{\partial Y_L}{\partial t}$$

$s \simeq aT, \qquad a \simeq 0.06\ \mathrm{MeV}^{-1}$

$$\frac{s}{a}\frac{ds}{dt} \simeq -\mu_\nu \frac{\partial Y_L}{\partial t} \simeq -3\frac{\partial Y_L}{\partial Y_\nu} Y_{\nu,0}\mu_{\nu,0}\left(\frac{\mu_\nu}{\mu_{\nu,0}}\right)^3 \frac{\partial \mu_\nu/\mu_{\nu,0}}{\partial t}$$

$$s_f^2 - s_i^2 \simeq \frac{3a}{2}\left(\frac{\partial Y_L}{\partial Y_\nu}\right)\mu_{\nu,0} Y_{\nu,0}$$

$$s_i \sim 1, \qquad s_f \sim 2.5, \qquad T_f \simeq 50\ \mathrm{MeV}$$

Core Cooling

Following deleptonization, $\mu_\nu << T$

$$L_\nu = -4\pi r^2 \frac{c}{6\pi^2(\hbar c)^3}\frac{\partial}{\partial r}\int_0^\infty \sum_i^6 \lambda_E^i E_0^{i2} E f_\nu dE$$
$$= -4\pi r^2 \frac{c\lambda_1 E_0^2}{\pi^2(\hbar c)^3}\frac{\partial}{\partial r}\frac{\pi^2 T^2}{3}.$$

Energy transport equation:

$$nT\frac{\partial s}{\partial t} = -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r} - n\sum_j \mu_j \frac{dY_j}{dt} \simeq -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r}$$
$$naT\frac{\partial T}{\partial t} = \frac{c\lambda_1 E_0^2}{3(\hbar c)^3}\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial T^2}{\partial r}.$$

Assume $T = T_0\phi(t)\psi(x), T_0 \sim 50$ MeV.

Core Cooling

$$\frac{\tau_c}{\phi}\frac{d\phi}{dt} = -1 = \frac{1}{\psi^2 x^2}\frac{\partial}{\partial x}x^2\frac{\partial \psi^2}{\partial x}.$$

ψ^2 is Lane-Emden function of index 1: $\psi^2 = \psi_1 = \sin x/x$

$$\psi_1(x_1) = 0 \implies x_1 = \pi, \qquad \left(x\frac{\partial\psi_1}{\partial x}\right)_{x_1} = -1.$$

$$\phi = \exp\left(\frac{t_0 - t}{\tau_c}\right), \quad \tau_c = \frac{3(\hbar c)^3 an}{c\lambda_1 E_0^2}\left(\frac{R}{x_1}\right)^2 \approx 10 - 30 \mathrm{~s}$$

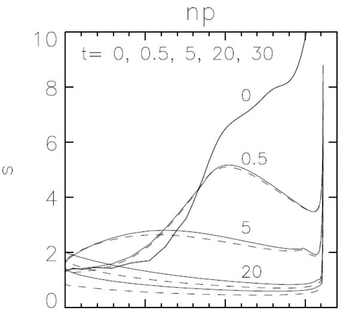
Emergent Luminosity:

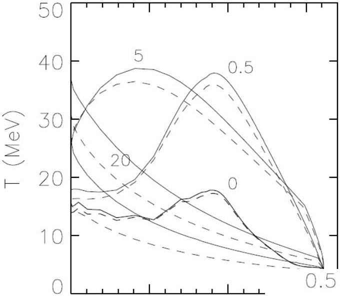
$$L_\nu(R,t) = -4\pi R\frac{c\lambda_1 E_0^2 T_0^2}{3(\hbar c)^3}\left(x\frac{\partial\psi^2}{\partial x}\right)_{x_1}\phi^2(t) = \frac{cF_3(0)}{2(\hbar c)^3}R^2 T_{\nu,eff}(t)^4$$

$$= L_0\phi(t)^2 \approx 1.2\cdot 10^{51}\phi(t)^2 \mathrm{~erg~s}^{-1}$$

$$T_{\nu,eff}(t) = \left(\frac{2L_0(\hbar c)^3}{cF_3(0)R^2}\right)^{1/4}\sqrt{\phi(t)} \simeq 4\sqrt{\phi(t)} \mathrm{~MeV}, \quad < E_\nu > \sim 3T_\nu.$$

Model Simulations





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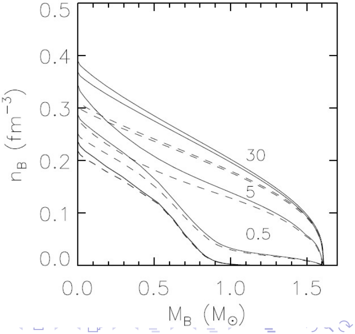
Pons et al.



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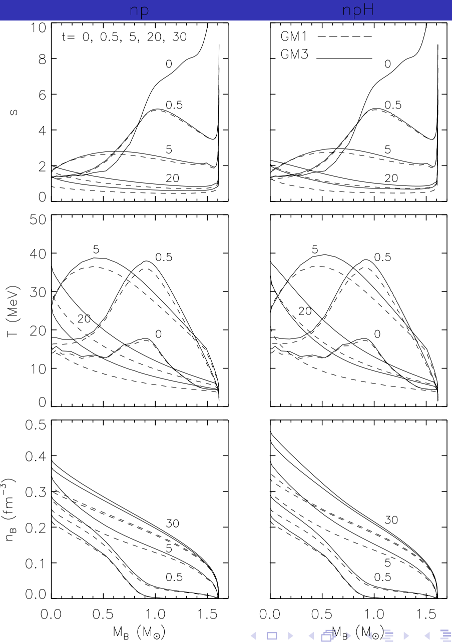

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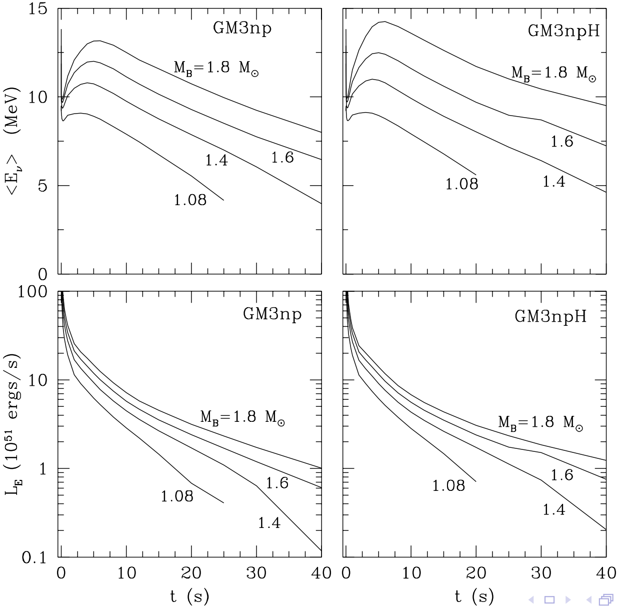

Model Signal