

Darmstadt Lecture 8 – Pulsars

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Pulsars: What are they?

- Small source period can only be explained by compact objects – orbital motion, vibrations or spin. The Crab pulsar bursts 30 times a second.
- A binary star with total mass of $M = 1 M_{\odot}$ and a period of $P = 0.033$ s, according to Kepler's Law, would have a separation a equal to

$$a = \left(\frac{M}{M_{\odot}} \right)^{1/3} \left(\frac{P}{1 \text{ year}} \right)^{2/3} = 1.0 \cdot 10^{-6} \text{ AU} = 93 \text{ miles}.$$

This separation is smaller than the radius of any solar mass object except a neutron star or black hole.

- The maximum radius R_{max} an object could have and still spin that fast without shedding mass from the equator is found from

$$\left(\frac{2\pi}{P} \right)^2 R_{max} = \frac{GM}{R_{max}^2} \Rightarrow R_{max} = (GM)^{1/3} \left(\frac{P}{2\pi} \right)^{2/3}.$$

This gives $R_{max} = 15$ km. Again, only a neutron star or black hole works.

- There is no known mechanism for black holes to emit pulses of energy.
- The discovery that the Crab pulsar is slowing down is only consistent with a spinning object and rules out orbiting or vibrating stars.
- Binaries, losing energy, go into tighter orbits with higher frequencies.
- Vibrating objects, losing energy, develop higher oscillation frequencies.

Magnetic Dipole Model for Pulsars

A misaligned magnetic dipole ($\alpha > 0$) emits low-frequency electromagnetic radiation. Larmor formula for electric dipoles (charge q , acceleration \ddot{v}) is

$$P_{rad} = \frac{2q^2 \dot{v}^2}{3c^2} = \frac{2}{3c^3} (q\ddot{r} \sin \alpha)^2 = \frac{2\ddot{p}_\perp^2}{3c^2}$$

where p_\perp is the perpendicular component of the electric dipole moment.

A uniformly magnetized sphere with radius R and surface field B has a magnetic dipole moment $|m| = BR^3$, and if rotating with period $P = 2\pi/\Omega$, has $m = |m|e^{-i\Omega t}$ and $|\ddot{m}| = \Omega^2|m|$. By analogy to an electric dipole,

$$P_{rad} = \frac{2}{3} \frac{\ddot{m}_\perp^2}{c^3} = \frac{2}{3c^2} \left(BR^3 \sin \alpha \right)^2 \left(\frac{2\pi}{P} \right)^4$$

This radiation appears at the low frequency $\nu = P^{-1} < 1$ kHz, too low to propagate through the ionized ISM and be detected.

The total rotational energy and spin-down power, using $I = (2/5)MR^2$, are

$$E_{rot} = \frac{1}{2} I \Omega^2 \simeq 1.6 \cdot 10^{50} \left(\frac{M}{M_\odot} \right) \left(\frac{R}{10 \text{ km}} \right)^2 \left(\frac{10 \text{ ms}}{P} \right)^2 \text{ erg},$$

$$P_{rot} = -I \Omega \dot{\Omega} \simeq 1.6 \cdot 10^{40} \left(\frac{M}{M_\odot} \right) \left(\frac{R}{10 \text{ km}} \right)^2 \left(\frac{10 \text{ ms}}{P} \right)^3 \left(\frac{-\dot{P}}{10^{-12}} \right) \text{ erg s}^{-1}.$$

Magnetic Fields and Ages

Setting $P_{rad} = -P_{rot}$, one finds

$$B = \sqrt{\frac{3c^3 I P \dot{P}}{8\pi^2}} \frac{1}{R^3} \frac{1}{\sin^2 \alpha} \simeq 2.9 \cdot 10^{12} \left(\frac{10 \text{ km}}{R \sin \alpha} \right)^2 \sqrt{\frac{M}{M_\odot} \frac{P}{.01 \text{ s}} \frac{\dot{P}}{10^{-12}}} \text{ G}$$

which is a minimum value since $\sin \alpha < 1$.

The **characteristic age** is estimated by assuming $P \dot{P} \simeq \text{constant}$, or

$$\int_{P_0}^P P dP = P \dot{P} \int_0^\tau dt = P \dot{P} \tau = \frac{P^2 - P_0^2}{2},$$

giving, with $P_0 \gg P$,

$$\tau = \frac{P}{2\dot{P}} \simeq 158 \frac{P}{.01 \text{ s}} \frac{10^{-12}}{\dot{P}} \text{ yr.}$$

A death line exists when the voltage $V \propto B \Omega^2$ near the polar cap drops below that needed to generate $e^+ e^-$ pairs:

$$\Phi = \frac{BR^3 \Omega^2}{2c^2} \simeq 6.6 \cdot 10^{16} \frac{B}{10^{12} \text{ G}} \left(\frac{0.01 \text{ s}}{P} \right)^2 \left(\frac{R}{10 \text{ km}} \right)^3 \text{ V.}$$

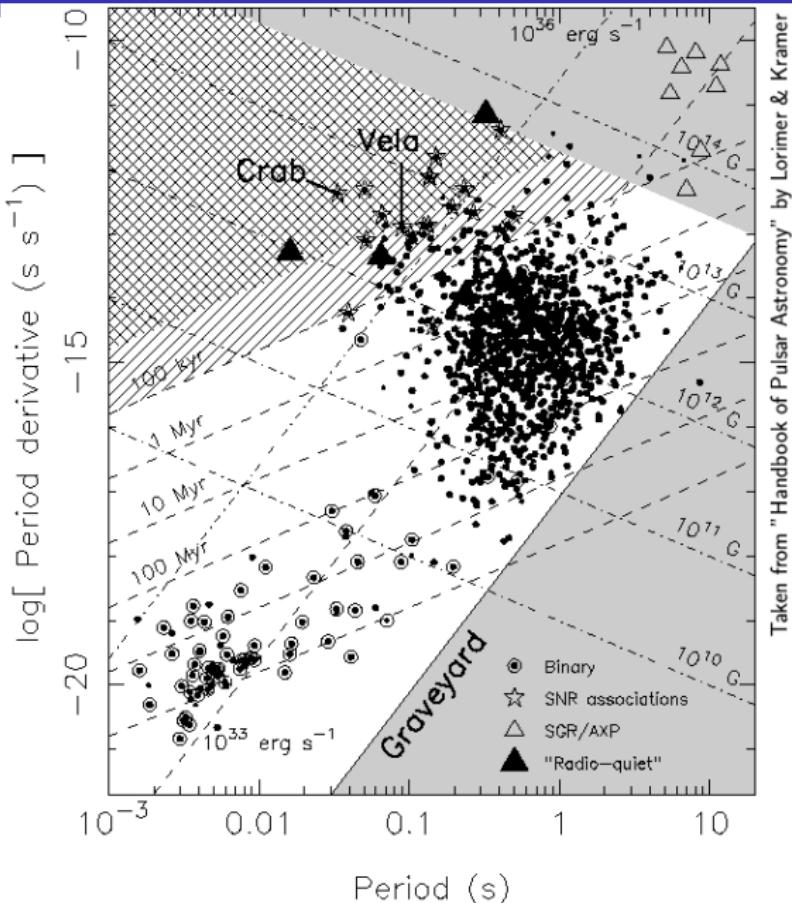
Note $BP^{-2} \propto \sqrt{\dot{P} P^{-3}} \propto P_{rot}$. Death line is where $BP^{-2} \sim 0.2 \cdot 10^{12} \text{ G s}^{-2}$.

A $\log \dot{P} - \log P$ diagram for pulsars is like an H-R diagram for stars.

The $P - \dot{P}$ Diagram

$$B \propto \sqrt{P\dot{P}}$$
$$\tau \propto P/\dot{P}$$
$$P_{\text{rot}} \propto P^{-3}\dot{P}$$

The “death line” is
 $P_{\text{rot}} \sim 10^{30} \text{ erg s}^{-1}$



Distances to Pulsars

Pulsars can probe the ISM, which is a plasma with a refractive index

$$\mu = \sqrt{1 - \left(\frac{\omega_p}{2\pi\nu}\right)^2}, \quad \omega_p = \sqrt{\frac{4\pi e^2 n_e}{m_e}} \simeq 8.97 \sqrt{\frac{n_e}{\text{cm}^{-3}}} \text{ kHz}$$

where ω_p is the plasma frequency.

Typically, $n_e \sim 0.03 \text{ cm}^{-3}$ and $\omega_p/(2\pi) \sim 1.5 \text{ kHz}$.

When $2\pi\nu < \omega_p$, μ is imaginary and photons cannot propagate.

The group velocity $v_g = \mu c < c$.

When $2\pi\omega_p \ll \nu$,

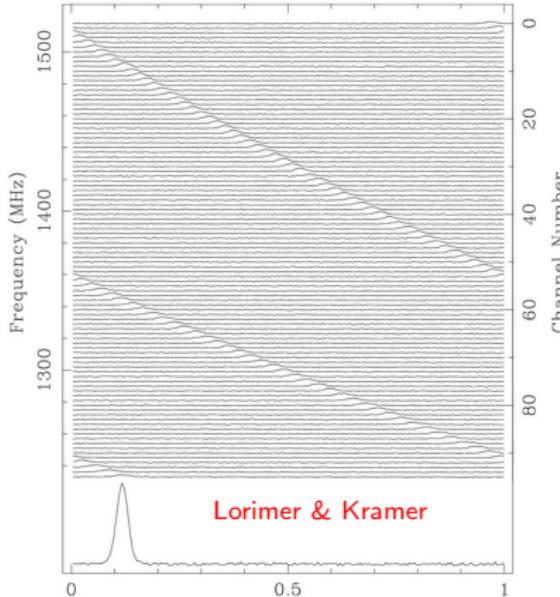
$$v_g \simeq c \left(1 - \frac{2\pi^2 \omega_p^2}{\nu^2}\right),$$

an increasing function of frequency.

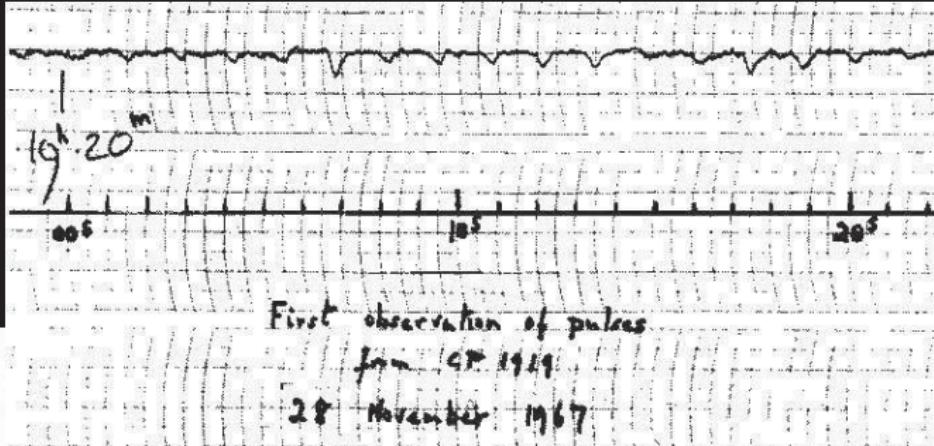
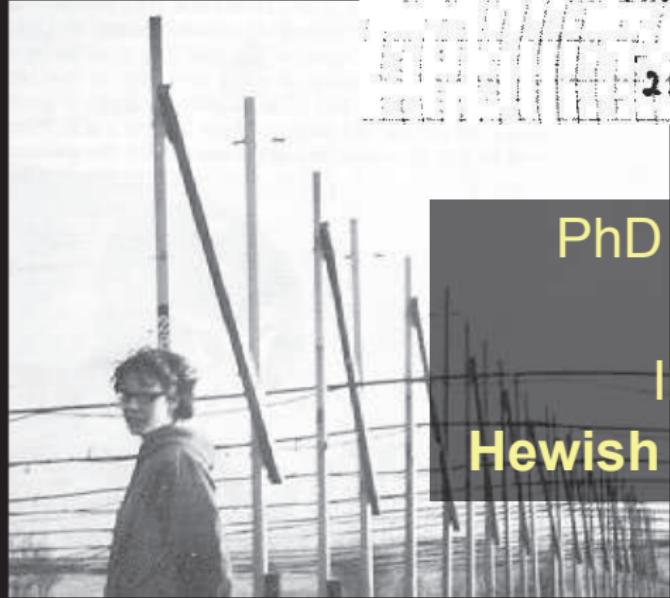
This produces a **dispersion delay**

$$t_d = \int_0^d \frac{dx}{v_g} - \frac{d}{c} \simeq \frac{e^2 \text{DM}}{2\pi m_e c \nu^2},$$

$$\text{DM} \equiv \int_0^d n_e dx \text{ is the dispersion measure.}$$

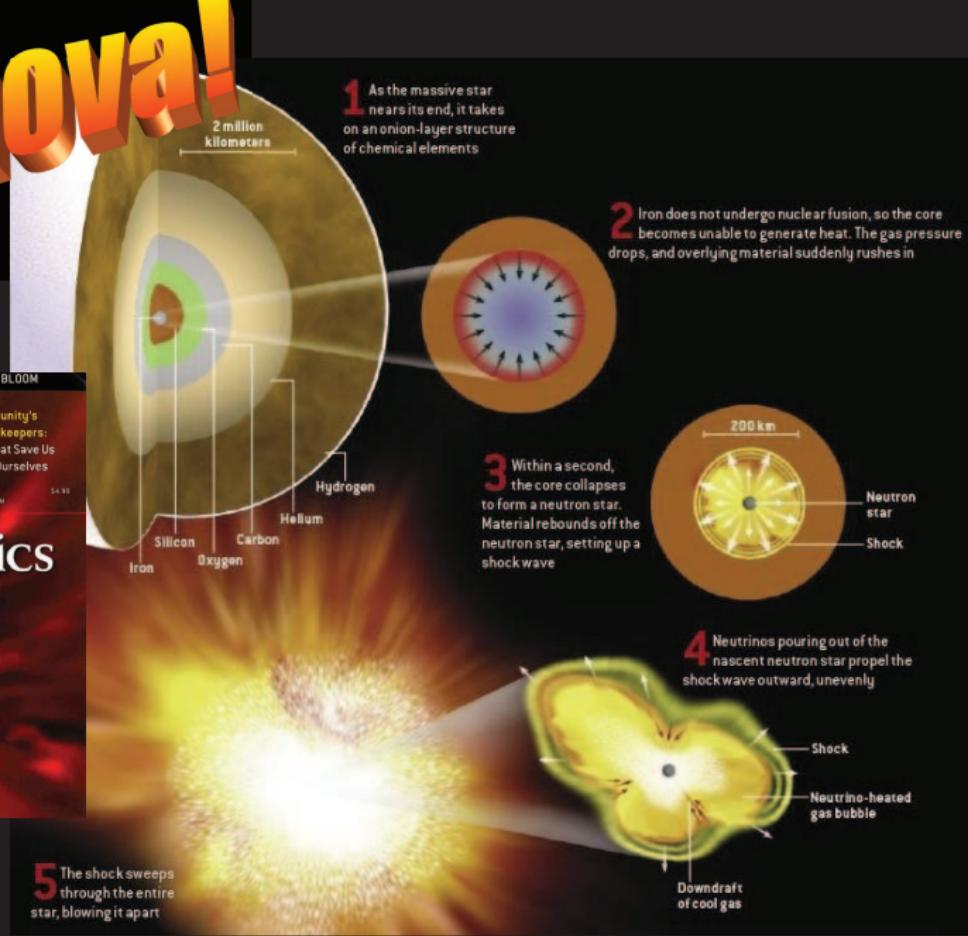
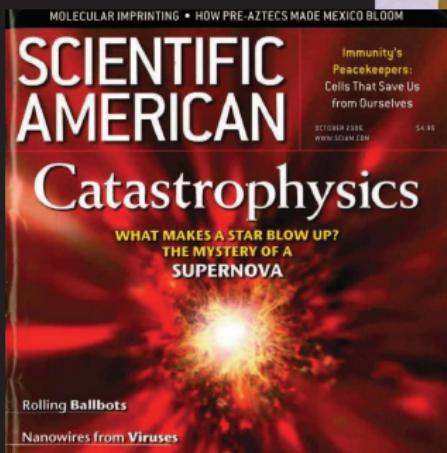


The Discovery of Pulsars



PhD student **Jocelyn Bell** and
Prof. **Antony Hewish**
Initially “Little Green Men”
Hewish won Nobel Prize in 1974

Supernova!



Orion

Betelgeuse



Baade & Zwicky (1934)

*“With all reserve we advance the view
that a super-nova represents the transition
of an ordinary star into a neutron star.
Such a star may possess a very small radius
and an extremely high density...”*

Crab Nebula SN1054AD



Anasazi Indian cave pictogram,
Chaco Canyon, NM

O

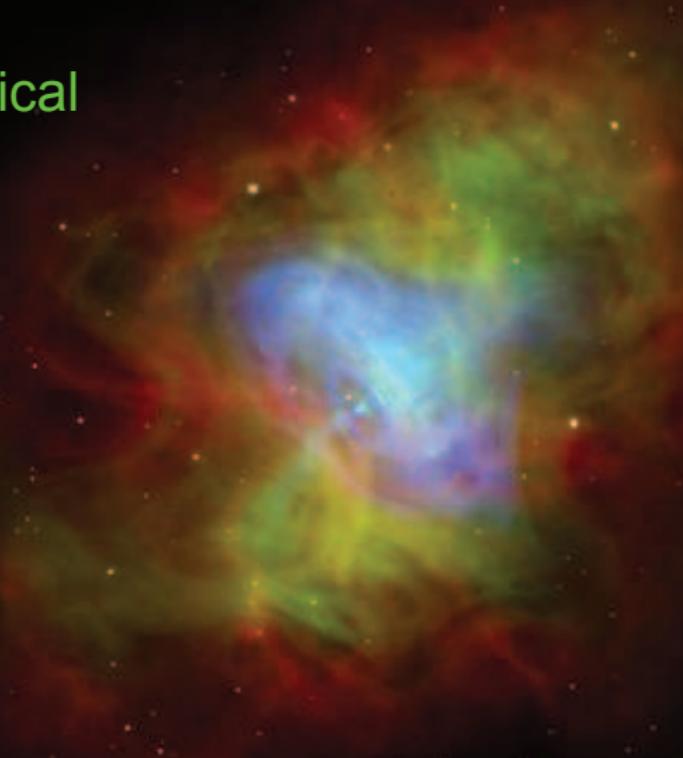
Pulsar rotates
30 times
per second!

The Crab is visible at all energies!

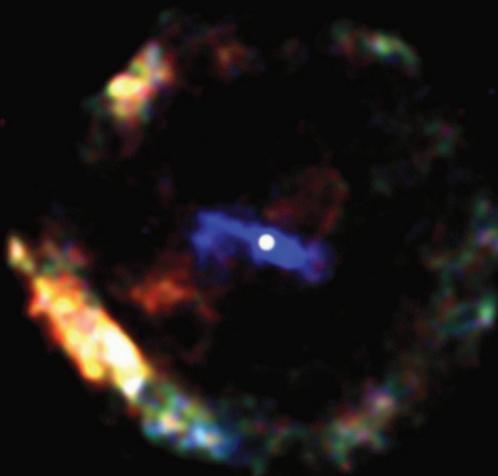
Red = Radio

Green = Optical

Blue = X-ray



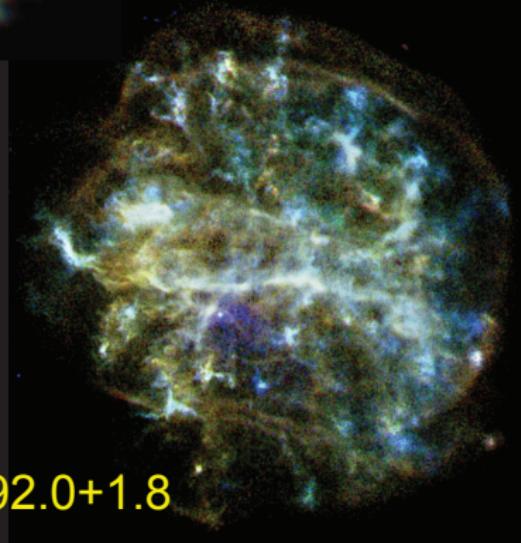
Pulsars!



G11.2-0.3



G21.5-0.9

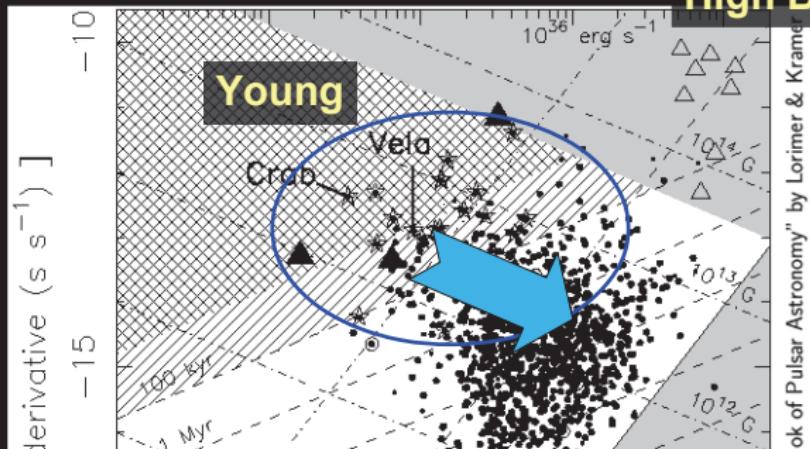


G292.0+1.8

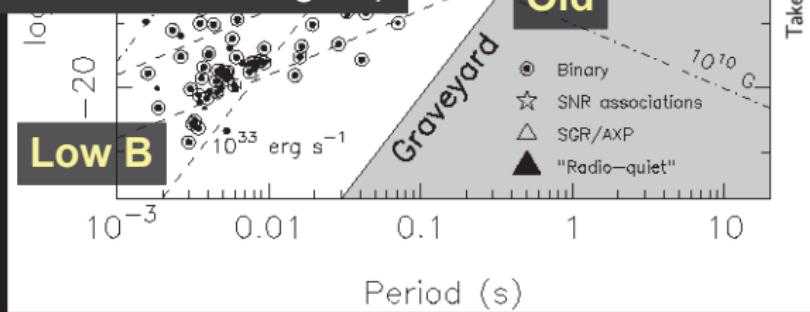
Pulsar Flavors

Young PSRs

(high B, fast spin,
very energetic)



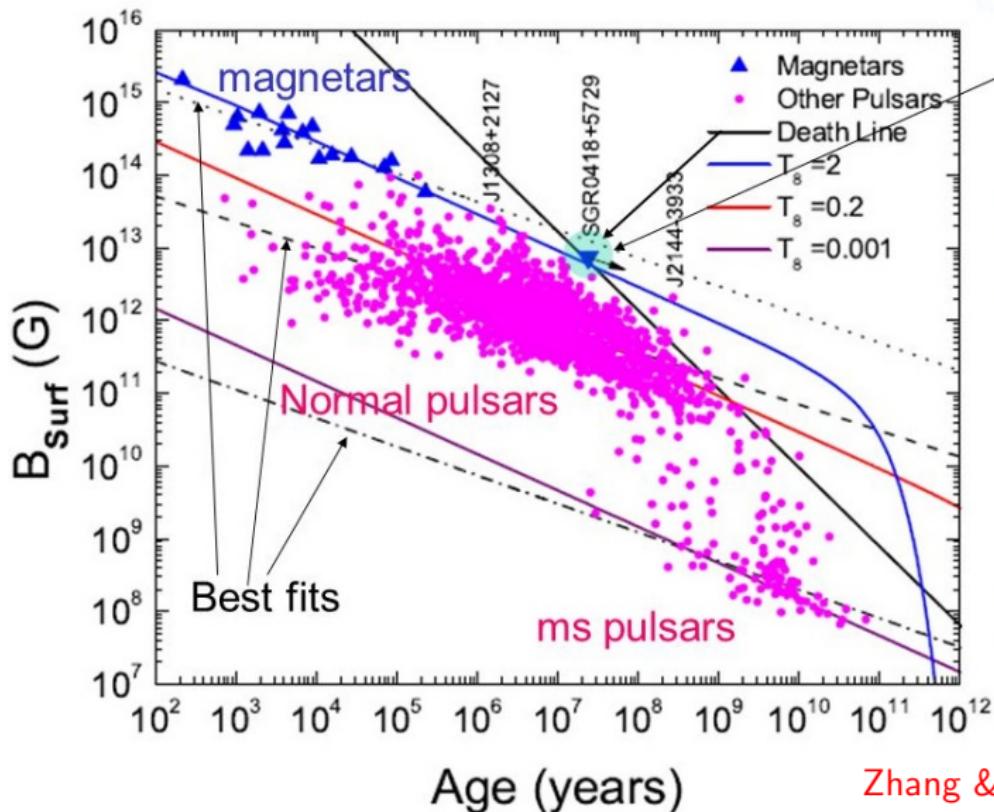
Pulsars move down and right across the diagram as they lose energy (assuming that the magnetic field doesn't change...)



Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Magnetic Field Evolution??

Direct Evidence for B Decay



Pulsar Flavors

Young PSRs

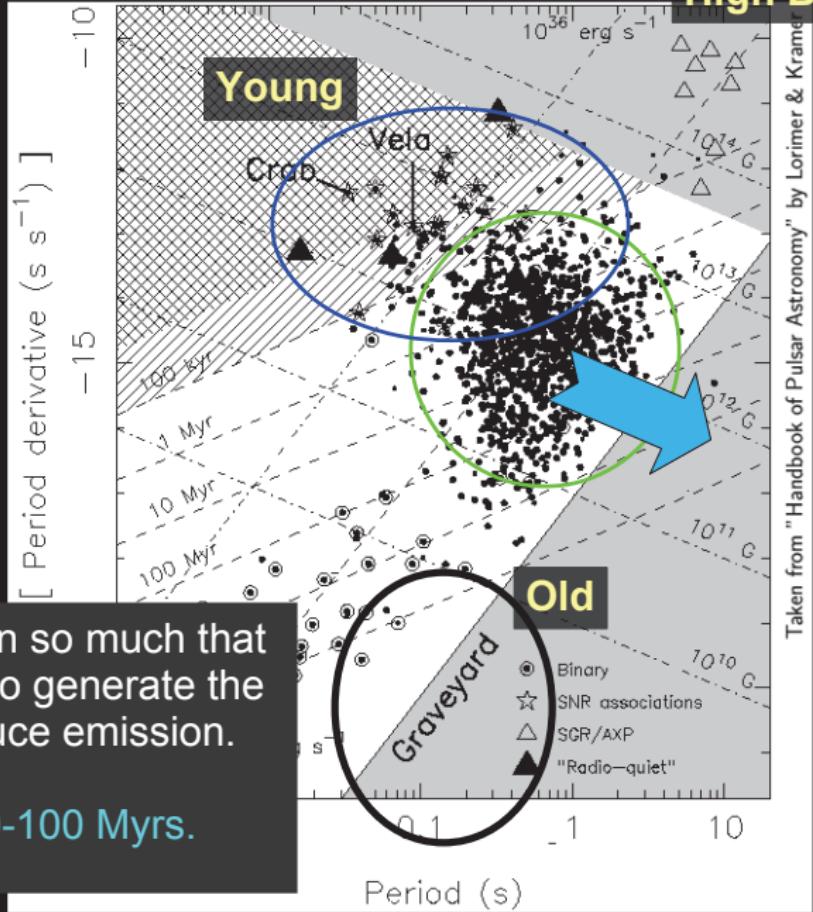
(high B, fast spin,
very energetic)

Normal PSRs

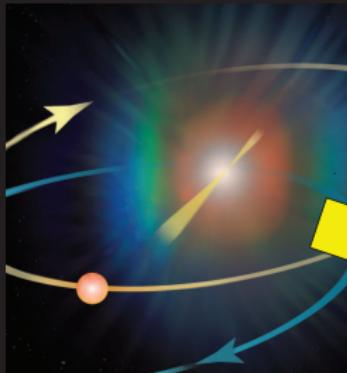
(average B,
slow spin)

Eventually they slow down so much that there is not enough spin to generate the electric fields which produce emission.

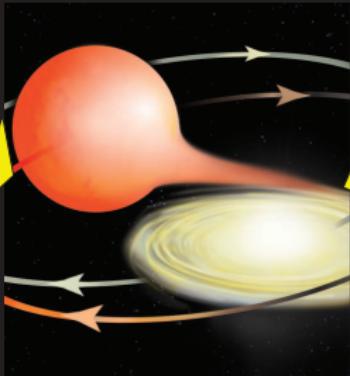
Their lifetimes are 10-100 Myrs.



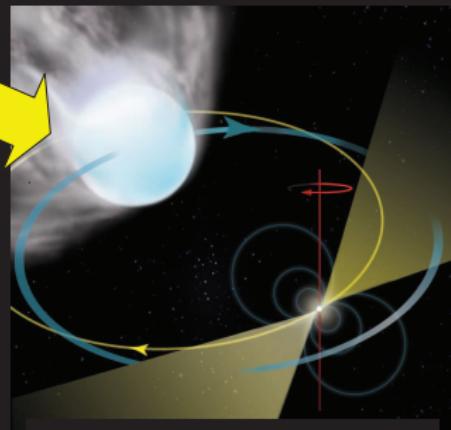
Millisecond Pulsars: via “Recycling”



Supernova produces
a neutron star



Red Giant transfers
matter to neutron star



Millisecond Pulsar
emerges with a white
dwarf companion

Alpar et al 1982
Radhakrishnan & Srinivasan 1984

Picture credits: Bill Saxton, NRAO/AUI/NSF

Pulsar Flavors

Young PSRs

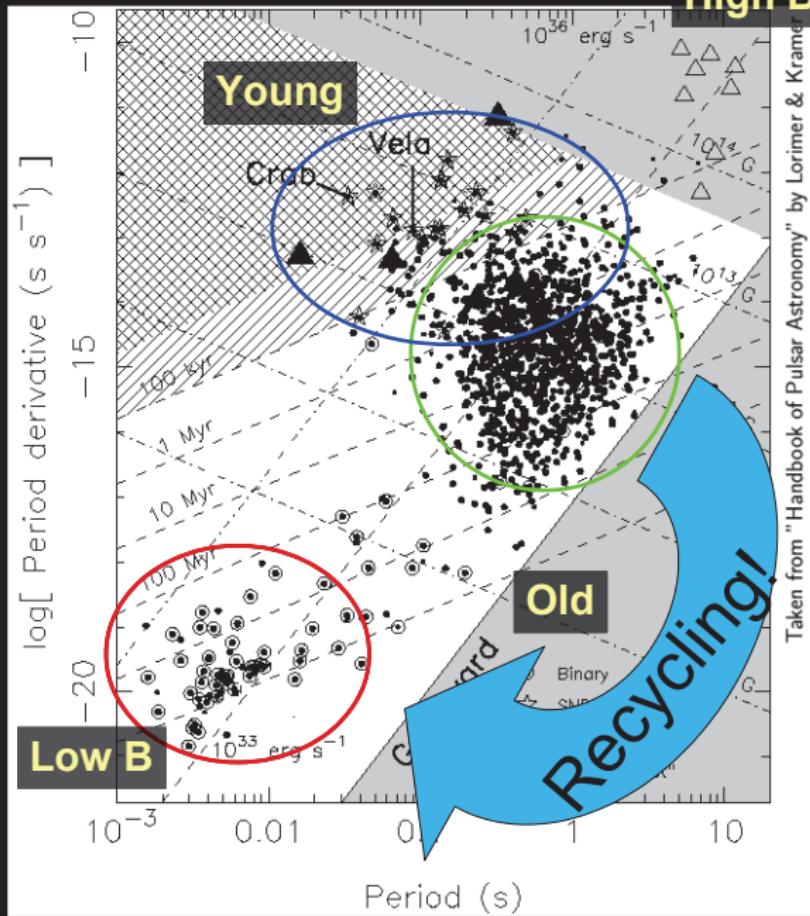
(high B, fast spin,
very energetic)

Normal PSRs

(average B,
slow spin)

Millisecond PSRs

(low B, very fast,
very old, very stable
spin, best for basic
physics tests)



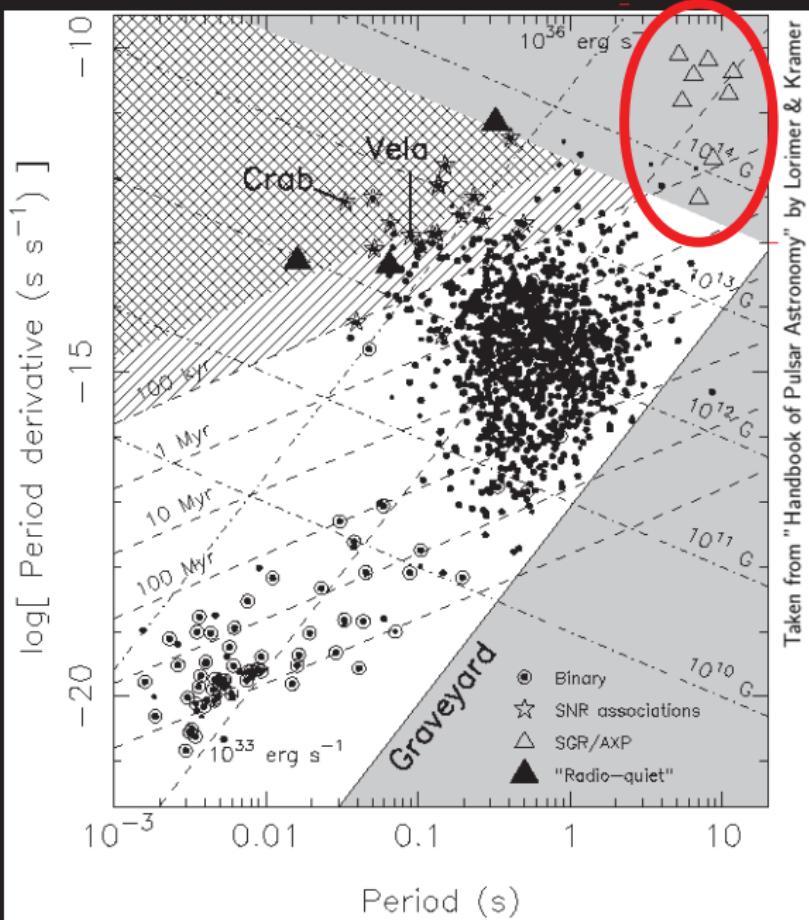
What's a Magnetar?

Neutron stars with **extremely strong** magnetic fields:

$10^{14\text{--}15}$ Gauss

(~ 1000 x stronger than normal PSRs)

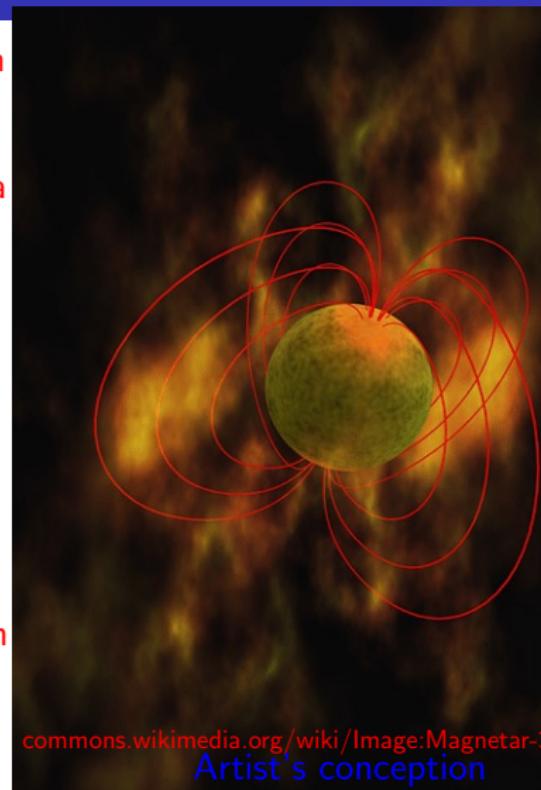
Powered by decay of magnetic field, not rotation!



Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Magnetars – Extreme Neutron Stars

- Anticipated 1992 by Duncan & Thompson
- Characterized by superstrong magnetic fields, $B \sim 10^{15}$ G
- Likely physical explanation for soft gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs)
- Superstrong fields lead to strong braking of the rotation rate, which is observed.
- Estimates are that 1 in 10 supernovae produces a magnetar, not a normal star.
- Strong fields generated by a convection-driven, globally-extending, dynamo in the proto-neutron star if the rotation rate is faster (> 100 Hz) than usual.
- Strong fields decay within 10,000 yrs, then activity (X-ray and γ -ray emission) stop.
- Given the observed number of magnetars, there could be 30 million or more dead magnetars in the Galaxy.
- SGR 1900+14, despite its large distance of 20,000 light years, on the night of August 27, 1998, ionized ionospheric atoms to daytime levels.



commons.wikimedia.org/wiki/Image:Magnetar-3b-45
Artist's conception

Why Highly Magnetized Neutron Stars?

Spin down of star to about 8 s period in the 10,000 yr age of SNR
(magnetic dissipation)

Provide enough energy for flares

Account for short, 0.2 s, duration of the hard spike (timescale of large-scale magnetic field readjustment)

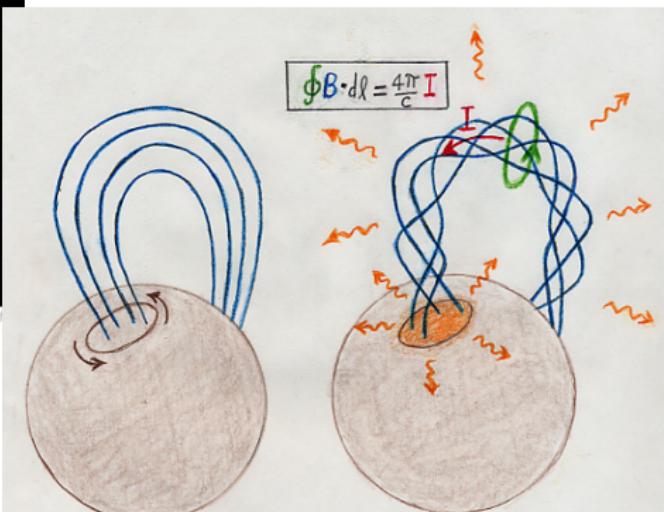
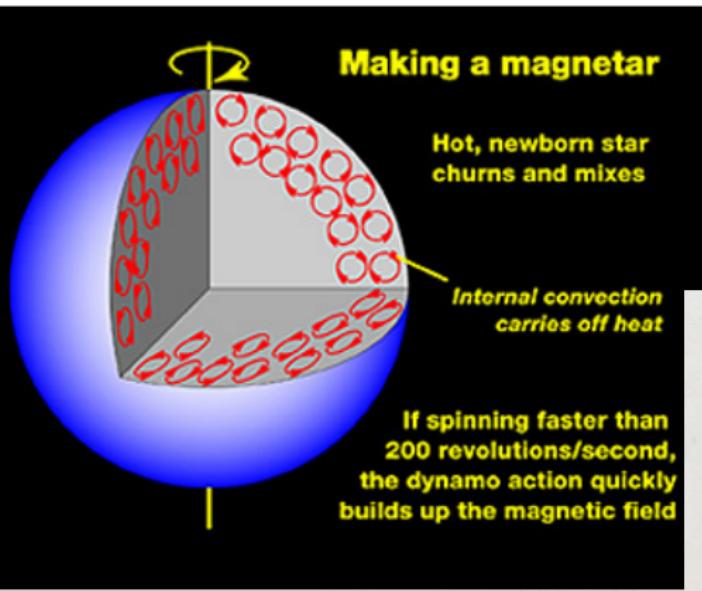
Provide enough energy for steady X-ray glow of SGRs

Make a hot particle gas (fireball) to explain soft tail and intensity of bursts

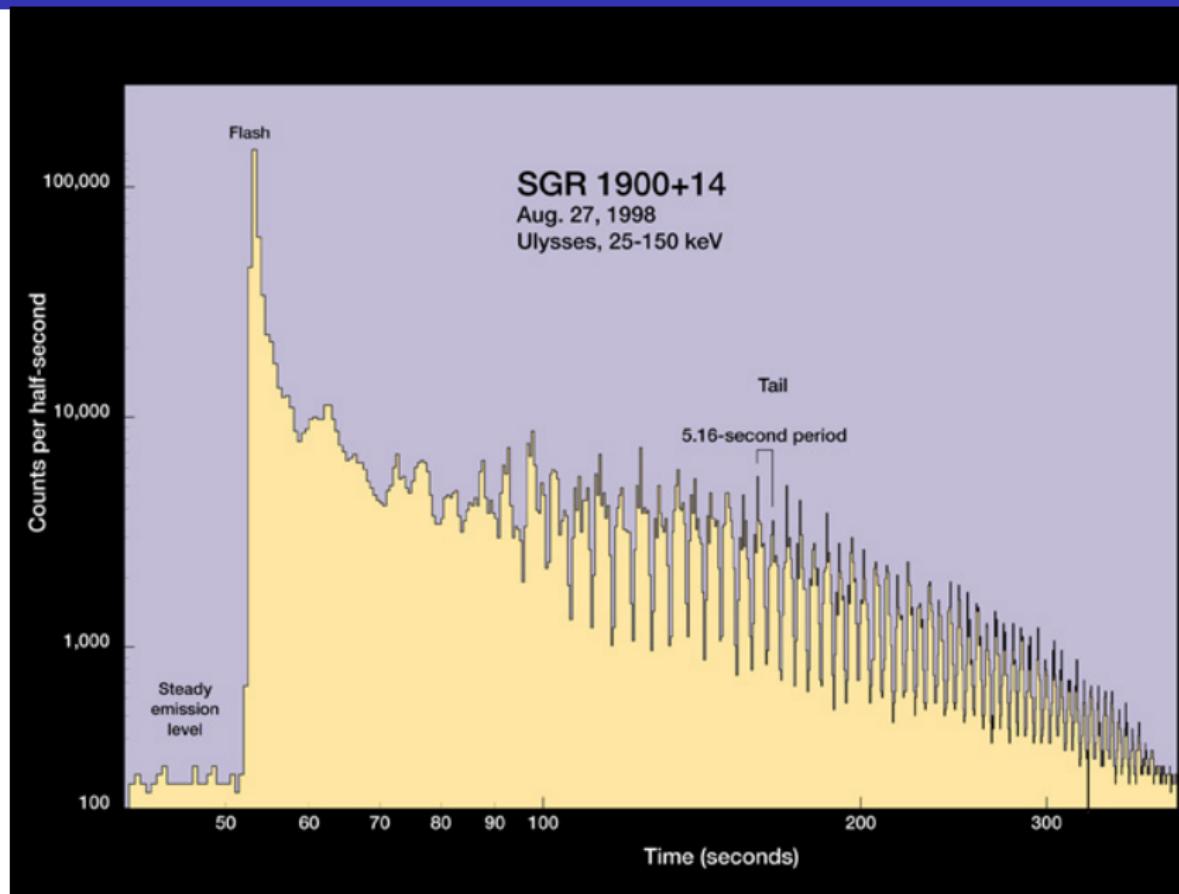
After fireball disperses, makes a residue held down by magnetic forces

Explains periodicity in light curve

Model for SGRs



SGR Light Curve



Some of the 21 Known Magnetars

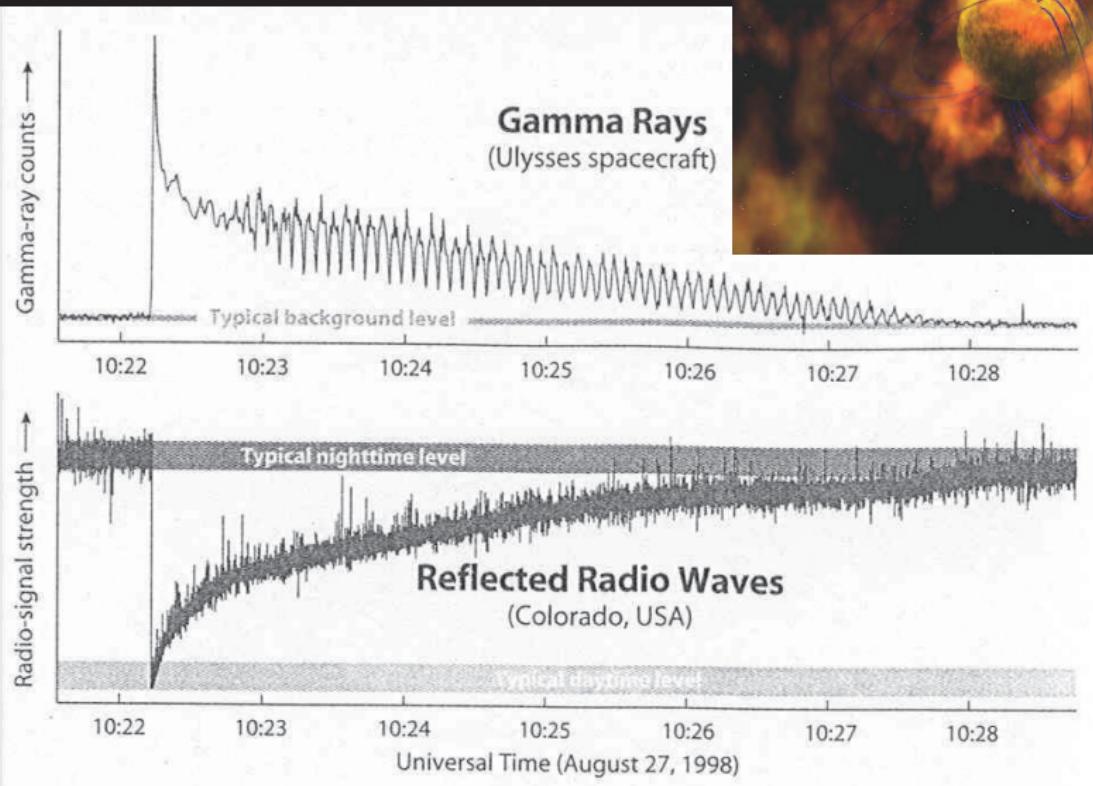
SGR 1806-20, $d = 50,000$ lt-yr
(Sagittarius), $b = 10^{15}$ G. 2004 burst
was brightest event outside solar system
sighted on Earth. Energy released was
 10^{46} erg; from a distance of 10 lt-yr
would have destroyed the ozone layer.
Ionosphere expanded

SGR 1900+14, $d = 20,000$ lt-yr
(Aquila). In 1998, forced NEAR
Shoemaker to shut down and saturated
detectors BeppoSAX, WIND and
RXTE. A ring seen was probably formed
in 1998 burst.

SGR 0501+4516, $d = 15,000$ lt-yr
1E 1048.1-5937, $d = 9000$ lt-yr (Carina) is the nearest known
magnetar (AXP)
SWIFT J195509+261046, formerly GRB 070610
CXO J164710.2-455216 (AXP) in Westerlund 1 (galactic cluster)



Giant X-ray Flares: Magnetar SGR 1900+14



The Primary Pulsar Telescopes

Arecibo



GBT



Parkes



Jodrell Bank



Pulsars are Precise Clocks

PSR J0437-4715

At 00:00 UT Jan 18 2011:

$$P = 5.7574519420243 \text{ ms}$$
$$+/- 0.0000000000001 \text{ ms}$$

The last digit changes by 1 every half hour!

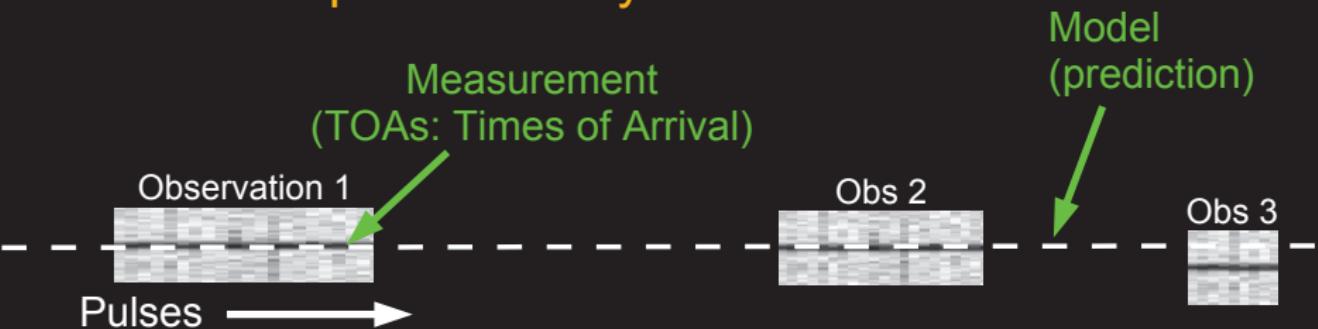
This digit changes by 1 every 500 years!

This extreme precision is what allows us to
use pulsars as tools to do unique physics!

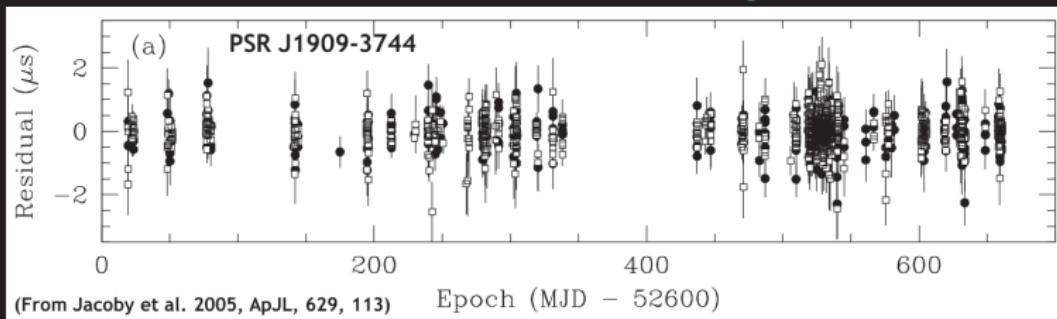
Pulsar Timing:

Pulse Phase Tracking

Unambiguously account for every rotation of a pulsar over years

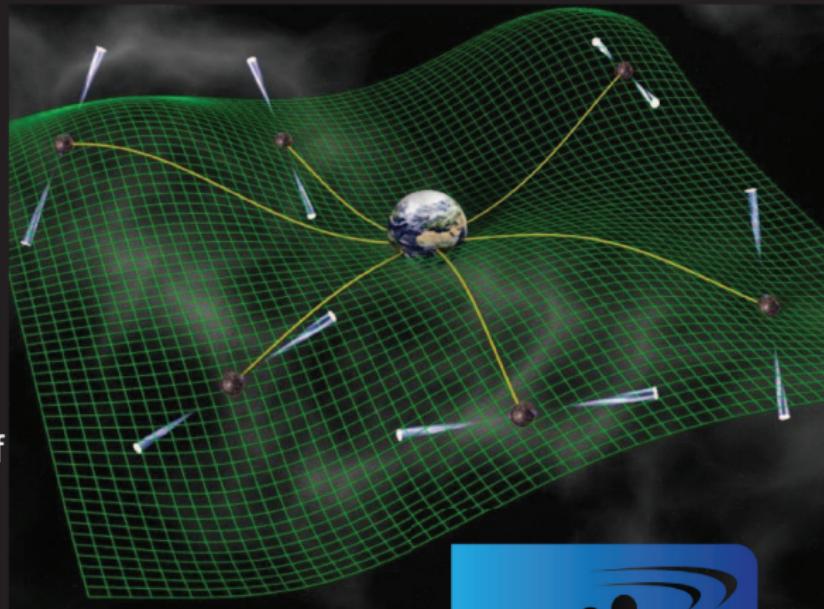


Measurement - Model = Timing Residuals



Gravitational Wave Detection with a Pulsar Timing Array

- Looking for nHz freq gravitational waves from super massive black hole binaries
- Need good MSPs
- Significance scales directly with the number of MSPs being timed.
- Must time the pulsars for 5-10 years at a precision of ~100 nanosec!
- North American (**NANOGrav**), European (EPTA), and Australian (PPTA) efforts



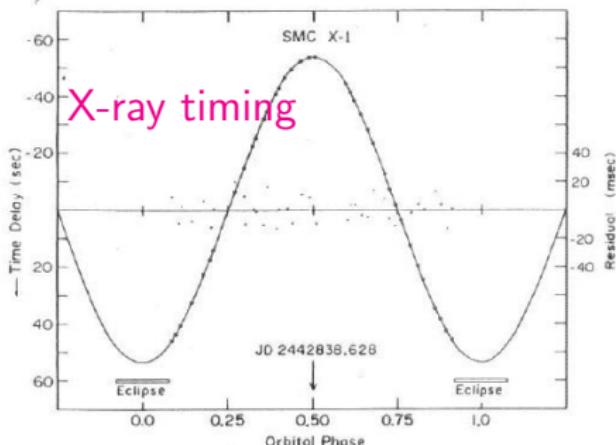
Binary Mass Measurements

Mass function

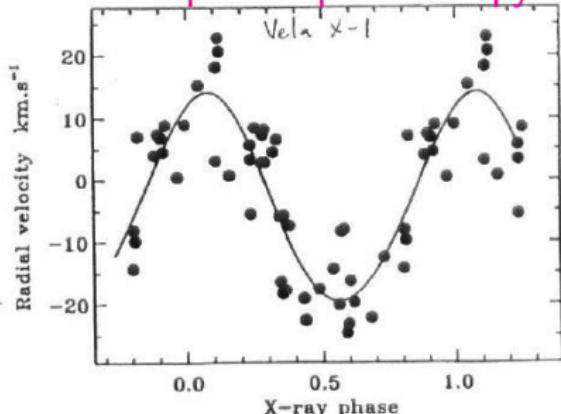
$$f(M_1) = \frac{P(v_2 \sin i)^3}{2\pi G}$$
$$= \frac{(M_1 \sin i)^3}{(M_1 + M_2)^2}$$
$$> M_1$$

$$f(M_2) = \frac{P(v_1 \sin i)^3}{2\pi G}$$
$$= \frac{(M_2 \sin i)^3}{(M_1 + M_2)^2}$$
$$> M_2$$

In an X-ray binary, $v_{optical}$ has the largest uncertainties. In some cases $\sin i \sim 1$ if eclipses are observed. If no eclipses observed, limits to i can be made based on the estimated radius of the optical star.



Optical spectroscopy



Pulsar Mass Measurements

Mass function for pulsar precisely obtained.

It is also possible in some cases to obtain the rate of periastron advance and the combined effects of variations in the transverse Doppler effect and the gravitational redshift around an elliptical orbit:

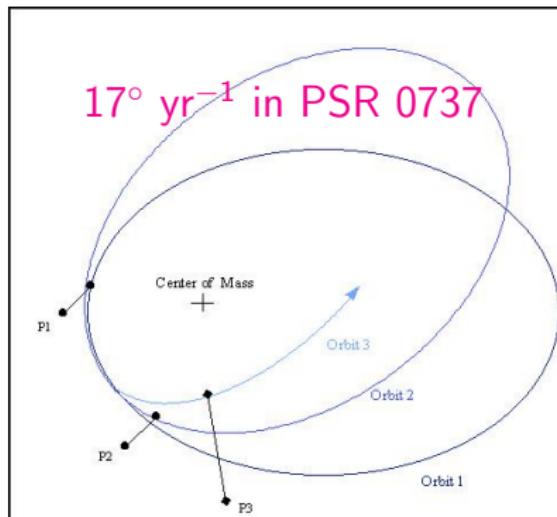
$$\dot{\omega} = \frac{3}{1-e^2} \left(\frac{2\pi}{P} \right)^{5/3} \left(\frac{GM}{c^2} \right)^{2/3}$$

$$\gamma = \left(\frac{P}{2\pi} \right)^{1/3} e M_2 (M + M_2) \left(\frac{G}{M^2 c^2} \right)^{2/3}$$

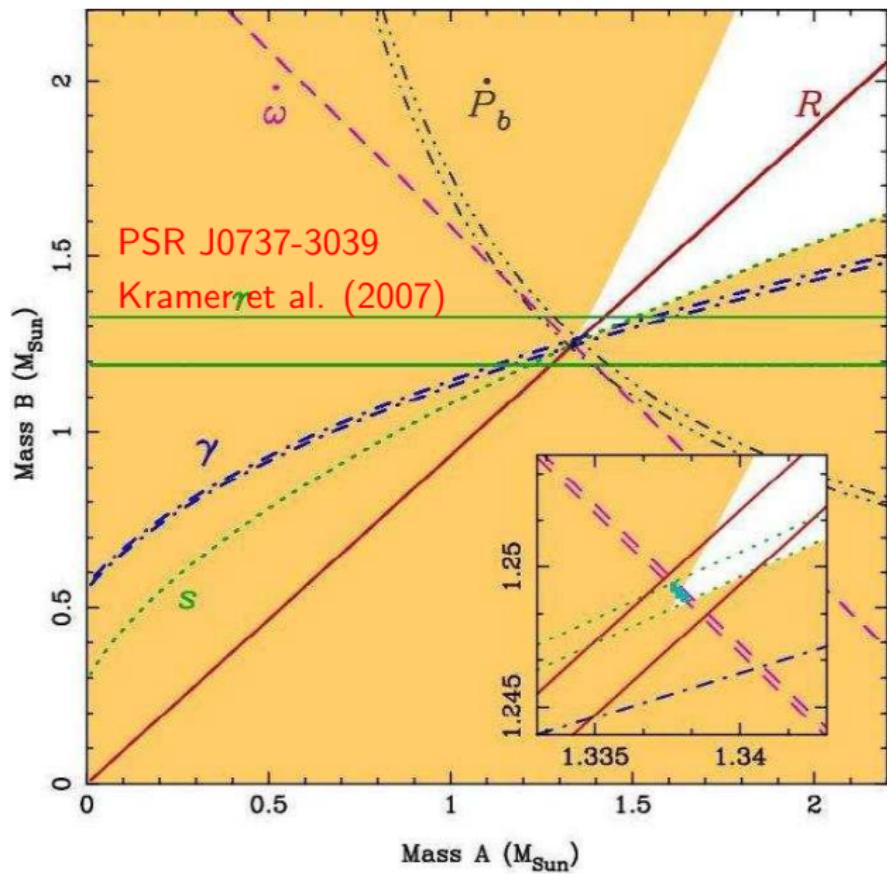
Gravitational radiation leads to orbit decay:

$$\dot{P} = -\frac{192\pi}{5c^5} \left(\frac{2\pi G}{P} \right)^{5/3} (1 - e^2)^{-7/2} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right) \frac{M_1 M_2}{M^{1/2}}$$

In edge-on systems, it's possible to detect Shapiro time delay, with parameters r (imagnitude) and $s = \sin i$ (shape).

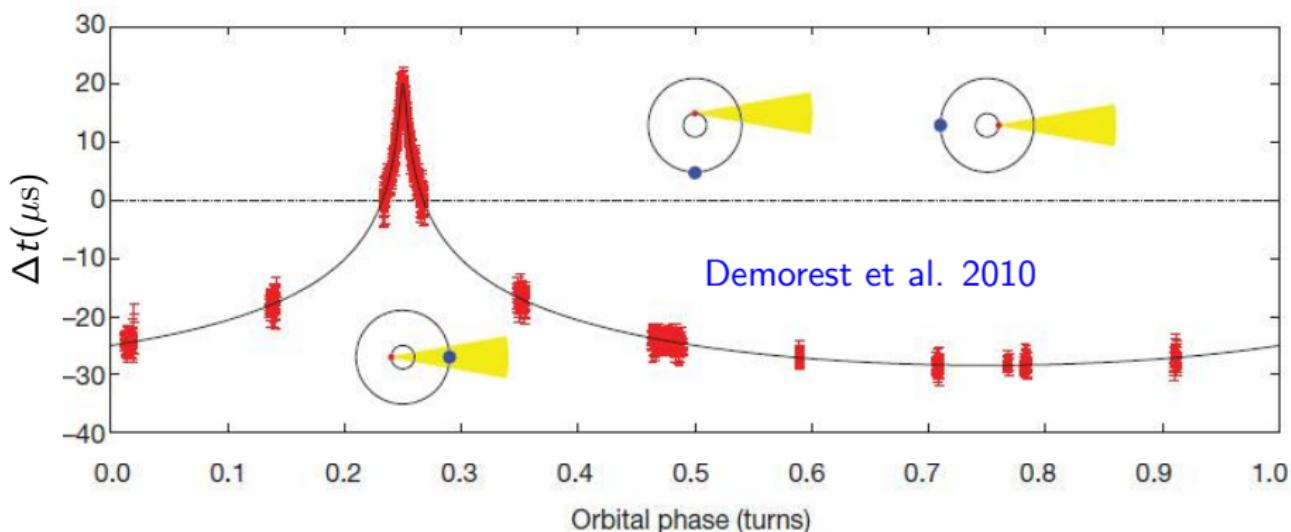


PSR J0737-3039



PSR J1614-2230

3.15 ms pulsar in 8.69d orbit with $0.5 M_{\odot}$ white dwarf companion.
Shapiro delay tightly confines the edge-on inclination: $\sin i = 0.99984$
Pulsar mass is $1.928 \pm 0.017 M_{\odot}$
Distance > 1 kpc, $B \simeq 1.8 \times 10^8$ G



Shapiro Time Delay

Shapiro delay produces a delay in pulse arrival times

$$\frac{\delta s(\phi)}{2M_2 T_\odot} = \ln \left[\frac{1 + e \cos \phi}{1 - \sin(\omega + \phi) \sin i} \right]$$

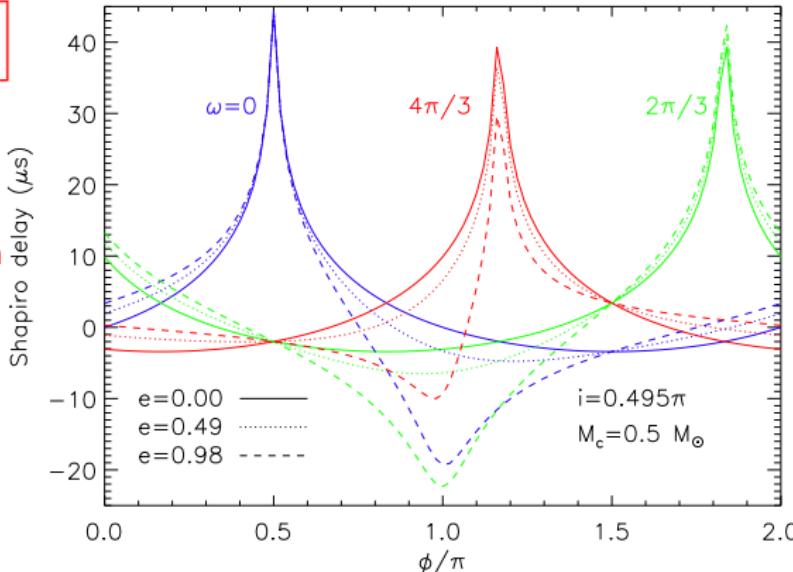
with ϕ the **true anomaly**, the orbital angular parameter defining the position of the pulsar relative to the periastron position ω .

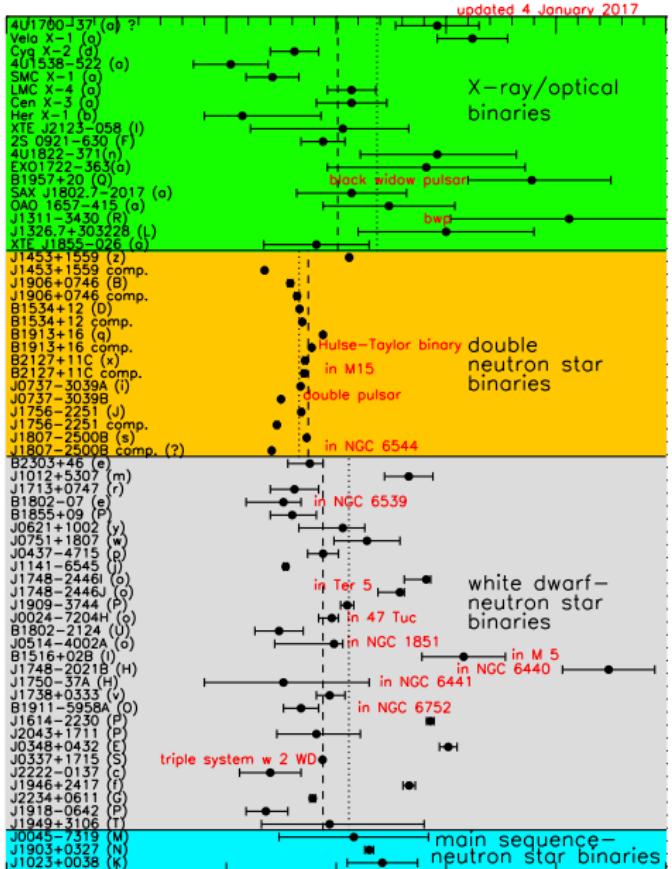
$$T_\odot = \frac{GM_\odot}{c^3} = 4.9255 \text{ } \mu\text{s}$$

δs is a periodic function with approximate amplitude

$$\Delta s \simeq 2M_2 T_\odot \left| \ln \left[\left(\frac{1 + e \sin \omega}{1 - e \sin \omega} \right) \left(\frac{1 + \sin i}{1 - \sin i} \right) \right] \right|.$$

This is large only if $\sin i \sim 1$ or if both e and $\sin \omega$ are nearly unity.





vanKerkwijk 2010

Romani et al. 2012

Although simple average mass of w.d. companions is $0.23 M_{\odot}$ larger, weighted average is $0.04 M_{\odot}$ smaller

Demorest et al. 2010

Fonseca et al. 2016

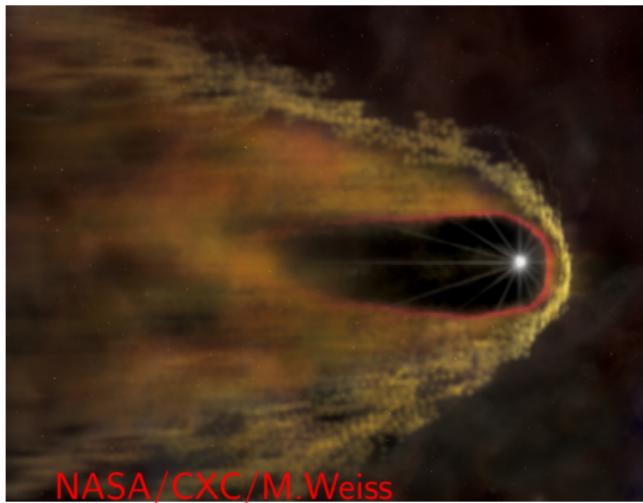
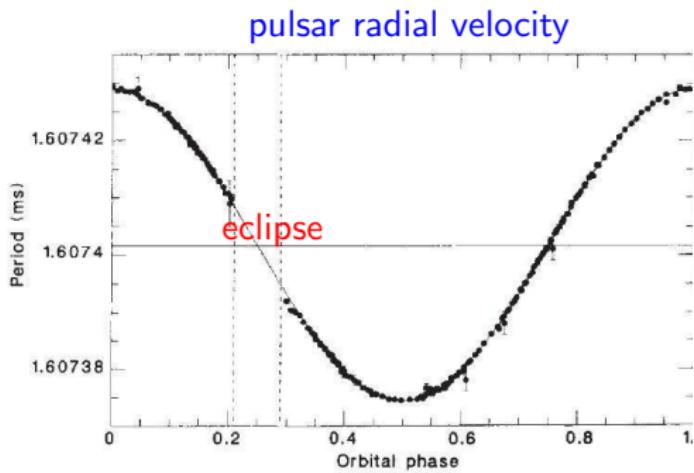
Antoniadis et al. 2013

Barr et al. 2016

Champion et al. 2008

Black Widow Pulsar PSR B1957+20

A 1.6ms pulsar in circular 9.17h orbit with $\sim 0.03 M_{\odot}$ companion. The pulsar is eclipsed for 50-60 minutes each orbit; the eclipsing object has a volume much larger than the secondary or its Roche lobe. The pulsar is ablating the companion leading to mass loss and the eclipsing plasma cloud. The secondary may nearly fill its Roche lobe. Ablation by the pulsar leads to secondary's eventual disappearance. The optical light curve tracks the motion of the secondary's irradiated hot spot rather than its center of mass motion.



Black Widow Pulsar PSR B1957+20

The peak radial velocity of the center of mass of component i is:

$$K_i = 2\pi \frac{a_i \sin i}{P}$$

$$q = \frac{M_P}{M_*} = \frac{a_*}{a_P} = \frac{K_*}{K_P}$$

$$K_* = K_{\text{obs}} \left(1 + \frac{R_*}{a_*} \right)$$

Companion's hot spot has a smaller orbit than its center of mass.

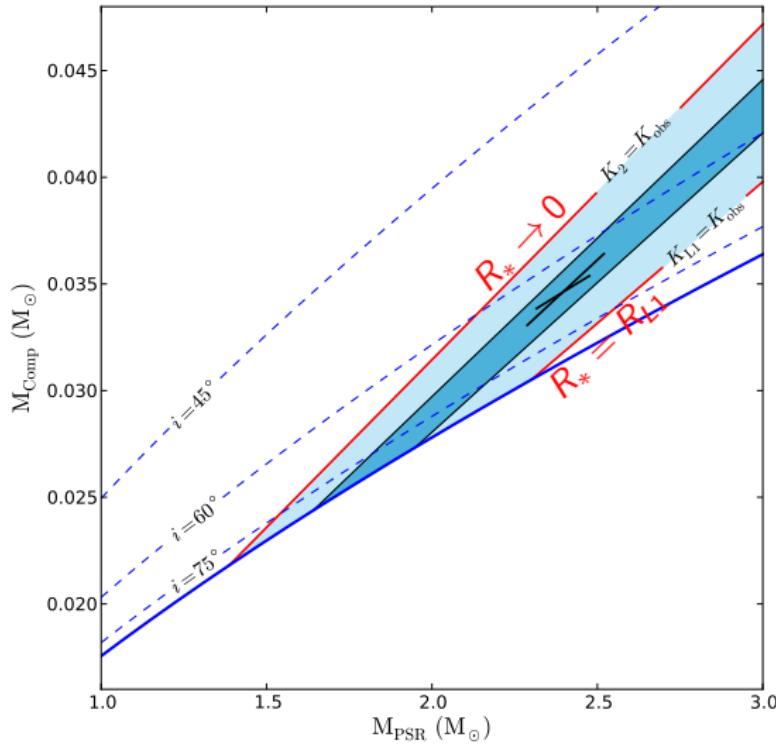
$$M_P = q(1+q)^2 \frac{P}{2\pi G} \left(\frac{K_P}{\sin i} \right)^3$$

Modeling of light curve shape suggests that

$$M_P > 1.8M_\odot, \quad \sin i < 66^\circ$$

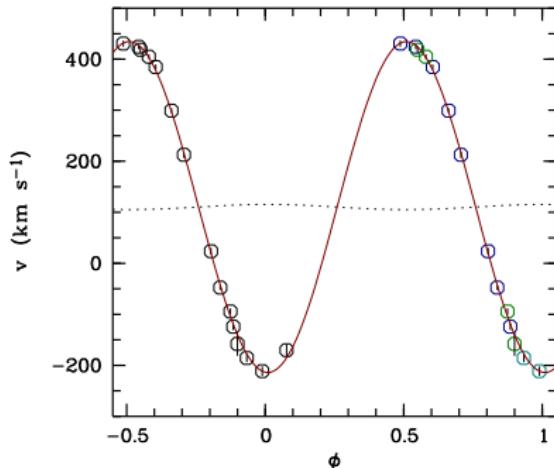
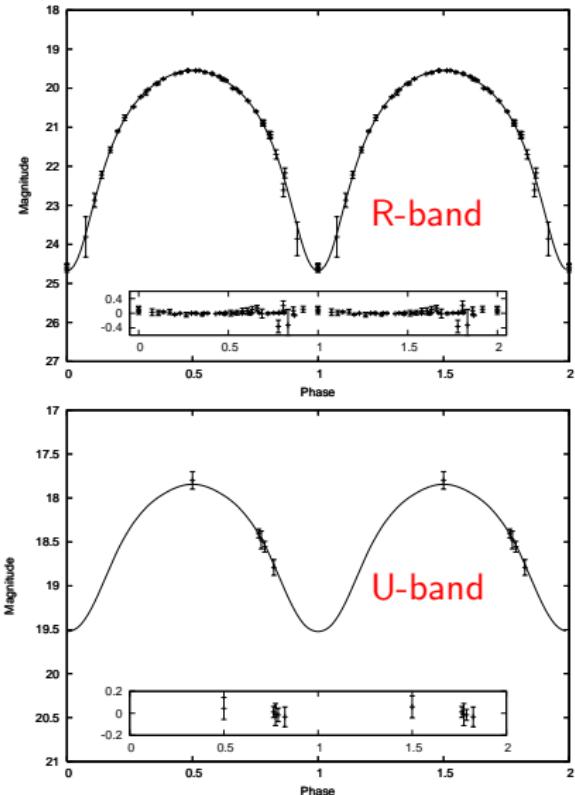
Most probable values:

$$2.20M_\odot < M_P < 2.55M_\odot$$



Companion Light Curves and Radial Velocity

Reynolds et al. 2007



System Masses

Radial velocity amplitudes:

$$K_i = 2\pi a_i \sin i / P$$

Light curve shape $\Rightarrow i \simeq 65 \pm 2^\circ$

$$K_1 = 5.093 \text{ km/s}, K_{obs} = 324 \pm 3 \text{ km/s}$$

$$q = M_p/M_* = K_*/K_p = a_*/a_p$$

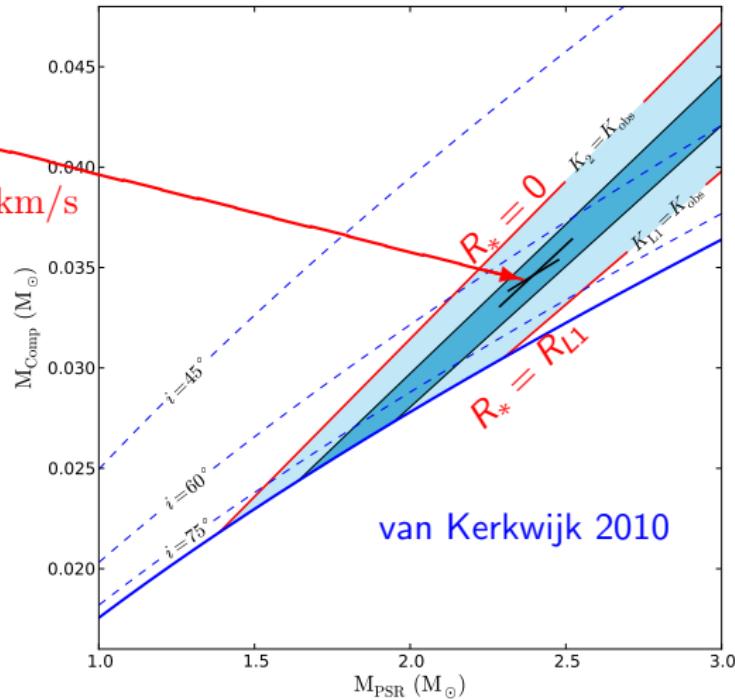
$$K_* \simeq K_{obs}(1 + R_*/a_*)$$

$$G(M_p + M_*)P^2 = (4\pi)^2(a_p + a_*)^3$$

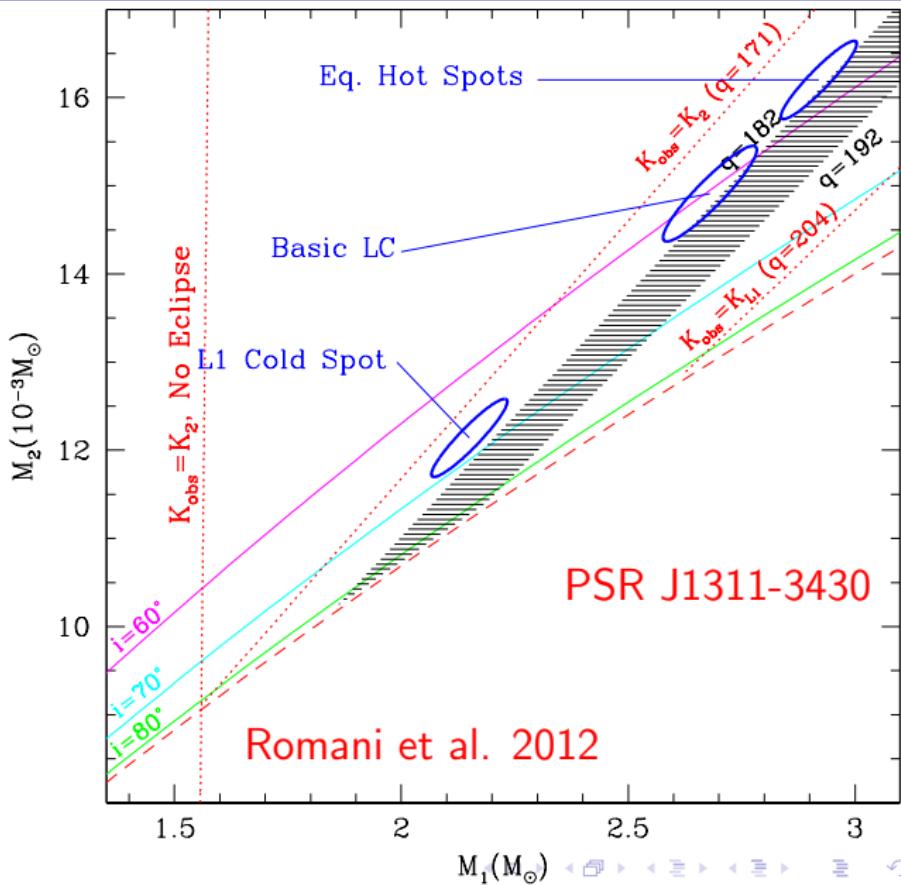
$$R_{L1} = 0.46a_p(1 + q)^{2/3}$$

$$M_p = q(1 + q)^2 \frac{P}{2\pi G} \left(\frac{K_p}{\sin i} \right)^3$$

$$\left(\frac{2\pi}{P} \right)^2 \frac{(a_p \sin i)^3}{G} = \frac{(M_* \sin i)^3}{(M_p + M_*)^2} = 5.2 \times 10^{-6} M_\odot$$

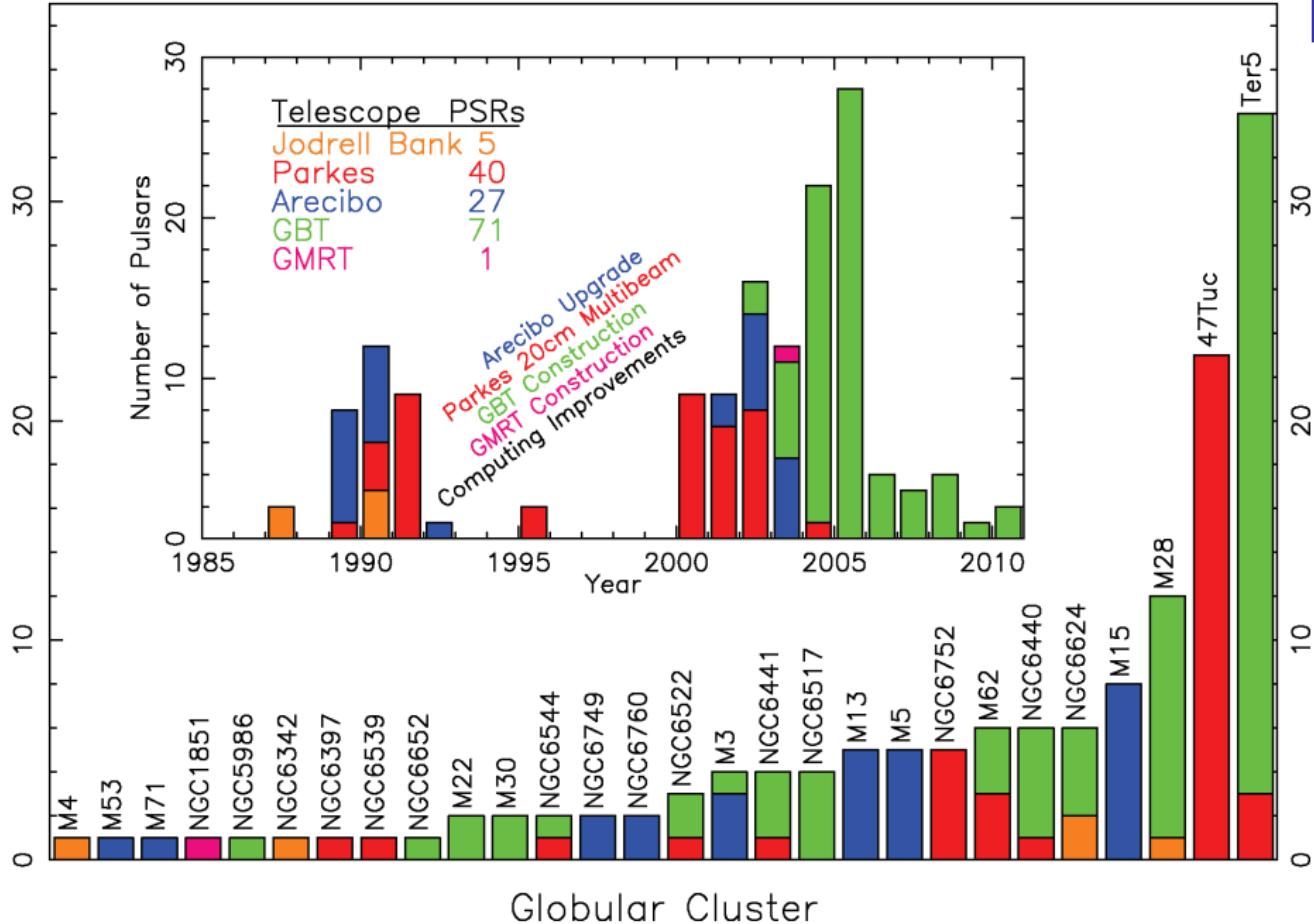


Another Black Widow Pulsar



144 pulsars in 28 clusters

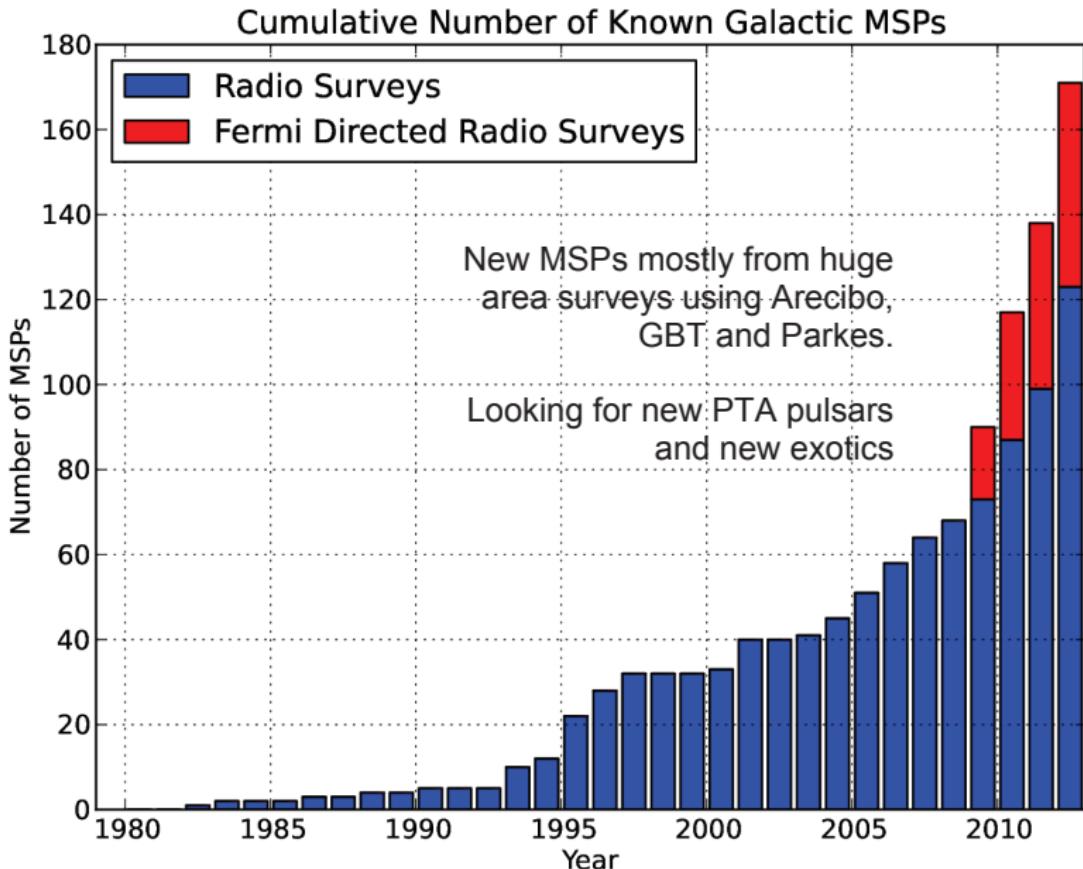
Number of Pulsars



Precession in 15+ PSRs in Clusters

Name	P(ms)	Pb(d)	E	Mcmin	Mtot	Mpmmed
Ter5ai	21.228	0.85	0.440	0.49	1.887(1)	1.32
Ter5J	80.338	1.10	0.350	0.34	2.205(3)	1.74
Ter5I	9.570	1.33	0.428	0.21	2.1660(5)	1.87
Ter5Z	2.463	3.49	0.761	0.22	1.743(3)	1.48
Ter5U	3.289	3.57	0.605	0.39	2.246(2)	1.73
Ter5W	4.205	4.88	0.016	0.25	2.09(7)	1.69
Ter5X	2.999	5.00	0.302	0.25	1.92(1)	1.60
M5B	7.947	6.85	0.138	0.13	2.3(1)	2.12
M28C	4.158	8.08	0.847	0.26	1.631(1)	1.33
NGC6544B	4.186	9.96	0.747	1.22	2.567(2)	1.17
NGC6441A	111.601	17.33	0.712	0.59	2.0(2)	1.35
NGC1851A	4.991	18.79	0.888	0.92	2.44(5)	1.34
NGC6440B	16.760	20.55	0.570	0.08	2.8(3)	2.68
Ter5Q	2.812	30.30	0.722	0.46	2.422(9)	1.79
M28D	79.835	30.41	0.776	0.38	1.2(7)	

SMR, PCCF, Freire et al 2007, 2008a+b, Lynch et al 2011



Black Widows and Redbacks in Galactic Field

Old
BWs

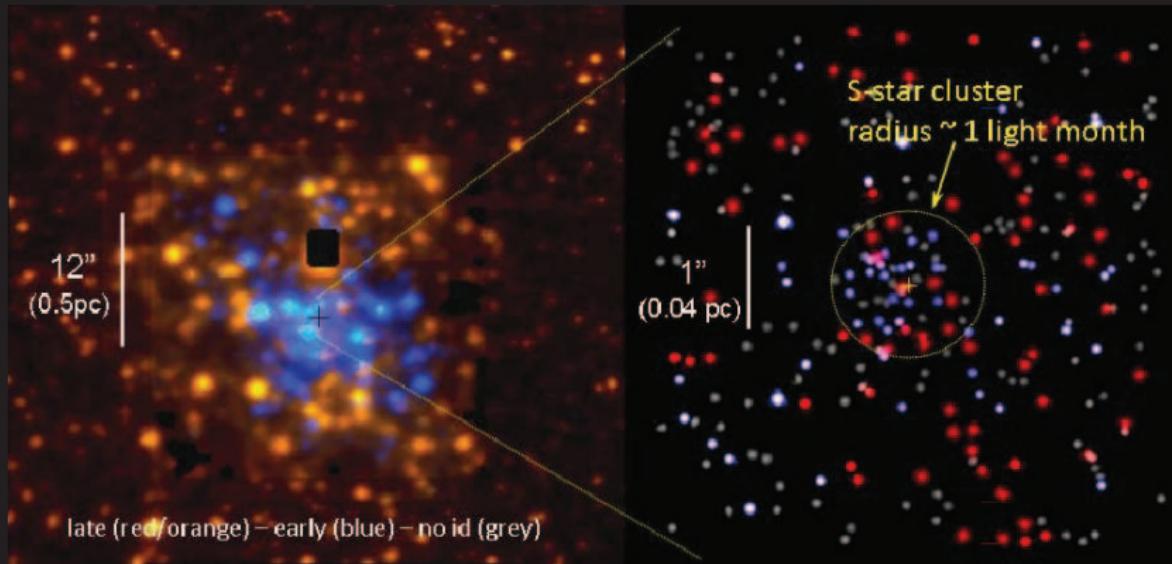
Pulsar	P_s (ms)	$E/10^{34}$ (erg/s)	d_{NE2001} (kpc)	P_B (hrs)	M_C (min. solar)
B1957+20 F	1.61	5.5	5.5	9.2	0.021
J0610-2100 F	3.86	0.23	3.5	6.9	0.025
J2051-0827	4.51	0.33	1.0	2.4	0.027
J2241-5236 ^P F	2.19	2.5	0.5	3.4	0.012
J2214+3000 ^{G8} F	3.12	1.9	3.6	10.0	0.014
J1745+1017 ^N F	2.65	1.3	1.3	17.5	0.014
J2234+09 ^P F	3.63	9/11 new Black	0	10	0.015
J0023+09 ^{G3} F	3.05	Widows in past 3 yrs	0.7	3.3	0.016
J1301+08 ^{G8} F	1.84	from Fermi!	0.7	6.5	0.024
J1124-36 ^{G3} F	2.41	0.7	1.7	5.4	0.027
J2256-1024 ³ F	2.29	5.2	0.6	5.1	0.034
J2047+10 ^{G8} F	4.29	??	2.0	3.0	0.035
J1731-1847 ^I	2.3	7.6	2.5	7.5	0.04
J1810+17 ^{G3} F	1.66	4.0	2.0	3.6	0.044
J1628-32 ^{G8} F	3.21	??	1.2	5.0	0.16
J1816+45 ⁴ F	3.19	??	2.1	8.7	0.16
J1023+0038 ³ F	1.69	6	6	4.8	0.2
J2215+51 ^{G3} F	2.61	0.3	3	4.2	0.22
J1723-28 ²	1.86	??	0.75	14.8	0.24
J2129-04 ^{G3} F	7.61	??	0.9	15.2	0.37

F=Fermi detected; I. HTRUPS Keith et al. 2010 2. PMB pulsar, Crawford et al. 2010 3. GBT Drift Scan 4. GBNCC
 Fermi targeted discoveries: G8=GBT 820 MHz, G3=GBT 350 MHz, N=Nancay, P=Parkes

Table from Mallory Roberts

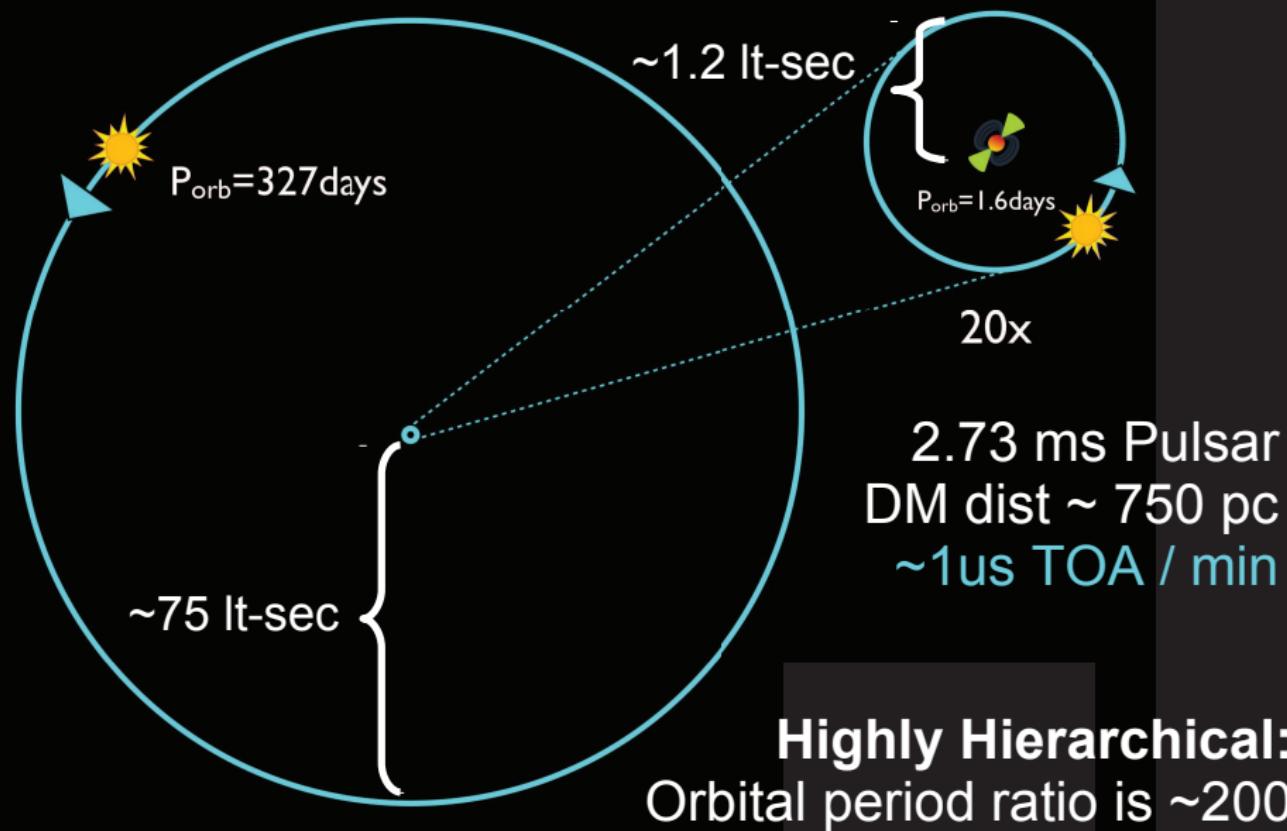
Pulsars around SgrA*?

- 100s of massive young stars within ~ 0.1 pc
- 10s-100s of PSRs with orbits < 100 yrs? (e.g. Pfahl & Loeb 2004)
- PSR timing much more precise than IR imaging and astrometry



Genzel, Eisenhauer, Gillesen 2010

J0337+1715: Stellar Triple System



A fully solved system:

(thanks to Anne Archibald)

- Full three-body, high-precision model
- All masses and inclinations fully determined to high precision (10^{-4} for masses):
 - $M_{\text{psr}} = 1.442 \text{ Msun}$
 - $M_{\text{c_inner}} = 0.198 \text{ Msun}$
 - $M_{\text{c_outer}} = 0.411 \text{ Msun}$ (another WD!)
- Orbit inclinations are co-planar at $39.18(4)$ deg
- Inner mass ratio perfectly matches optical value
- Apsides are aligned (despite inner orbits $e \sim 7 \times 10^{-4}$!)
- Osculating orbital elements are obvious