Darmstadt Lecture 5 – Supernova Types

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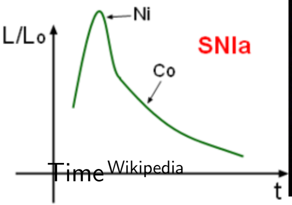
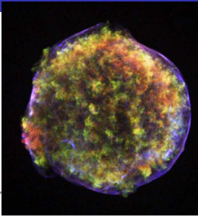
Supernova Types

Thermonuclear

Type Ia — No H, strong Si II in spectra; associated with explosion of C-O degenerate white dwarf near Chandrasekhar limit; accretion in close binary or white dwarf merger.





Cas A

NASA/CXC/Rutgers/J.Warren & J.Hughes

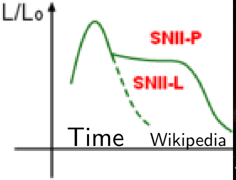
Gravitational Collapse

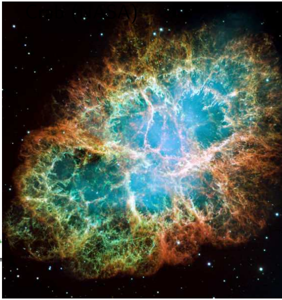
Type Ib — No H, strong He in spectra; loss of outer envelope due to stellar wind or binary companion, collapse of a Wolf-Rayet star; light curve similar to SNIa

Type Ic — No H or He in spectra; loss of outer envelope; may be progenitor of long-hard gamma ray bursts

Type IIP — H in spectra, $9 < M/\mathrm{M}_\odot < 20$, plateau light curve due to high opacity of shock-ionized H

Type IIL — H in spectra, but most of H envelope ejected before explosion, magnitude decreases linearly



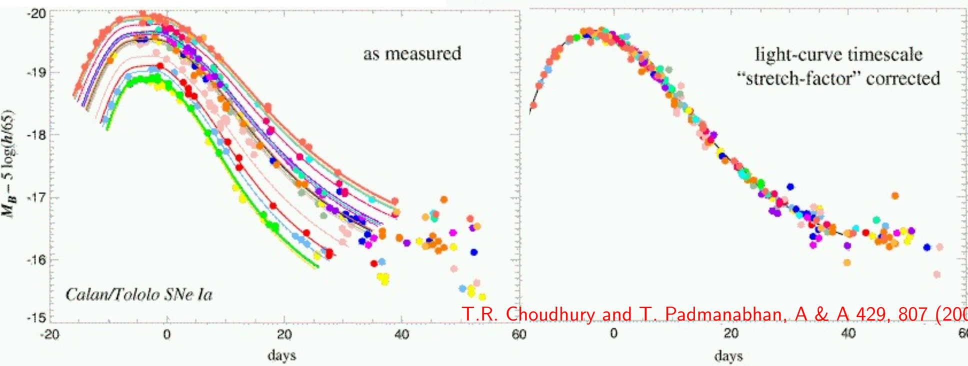


Type Ia

- C + O ⇒ Fe + Ni yields about 1 MeV/baryon, equivalent to 6.1×10^{17} ergs/g or 1.2×10^{51} erg/$M_\odot \equiv 1.2$ bethe/$M_\odot$
- Kinetic energies are consistent with this magnitude of energy release
- Initially rough equipartition of energy between thermal and kinetic, but at late times it is mostly kinetic
- Chandrasekhar-mass is never quite achieved, but increase in density raises temperature of core, producing convection. At some point, a (subsonic) deflagration flame front ensues is, powered by C + C, C + O and O + O.
- C or O are incompletely consumed; burning to Fe-peak incomplete.
- Debatable if the deflagration front transitions into supersonic detonation.
- Even incomplete burning of 1.2 $M_\odot$ of C + O releases 1-2 bethes, more than enough to unbind the star;
- Specific binding energy $GM/R \simeq 2.3 \times 10^{17}$ erg/g; $v/c \simeq \sqrt{2E/M} \simeq 0.03$.
- Nova models are similar, accreting white dwarf in binary, but involve irregular explosions of accreted H material that cannot disrupt the star.
- These are good standard candles since the mass scale is the Chandrasekhar mass and burning is usually more than 50% complete.

Type Ia

The quality of the standard candle can be improved since it is straightforward to demonstrate that both the peak and the temporal width of the light curve depend on the total explosion energy.

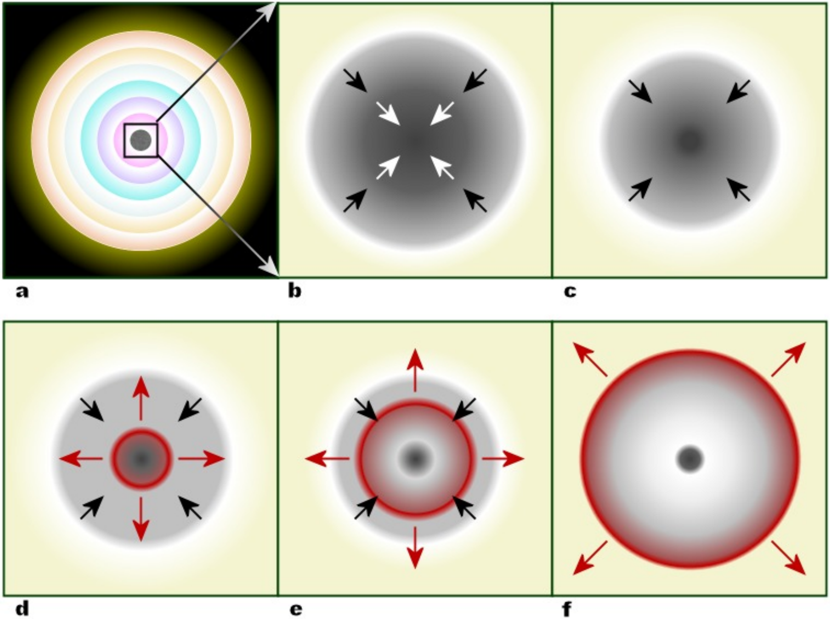


T.R. Choudhury and T. Padmanabhan, A & A 429, 807 (200

Core-Collapse SN (Type II)

- IA massive star has an iron core mass with a maximum size determined by the Chandrasekhar limit, about 1.4 $M_\odot$.
- As the silicon shell surrounding the iron core continues to burn, the iron core mass slowly increases.
- When the core exceeds the Chandrasekhar mass, it has to collapse.
- The collapsing core separates into a sonically cohesive inner core and a supersonic outer core.
- The collapsing core continues to collapse until nuclear saturation density is reached and repulsive forces abruptly halt the collapse.
- The abrupt halt creates a pressure shock wave at the inner core's outer boundary. The shock slows down or reverses the collapse of the overlying infalling matter.
- This shock by itself does not seem capable of exploding the star and ejecting matter into space.
- The inner core plus additional matter falling onto it creates a new neutron star, called a protoneutron star. A protoneutron star differs from a neutron star in having many more protons and electrons as well as being much hotter.
- During collapse, some protons are converted to neutrons. These beta reactions produce neutrinos.

$$p + e^- \rightarrow n + \nu_e$$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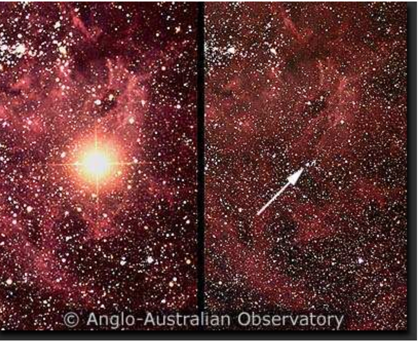
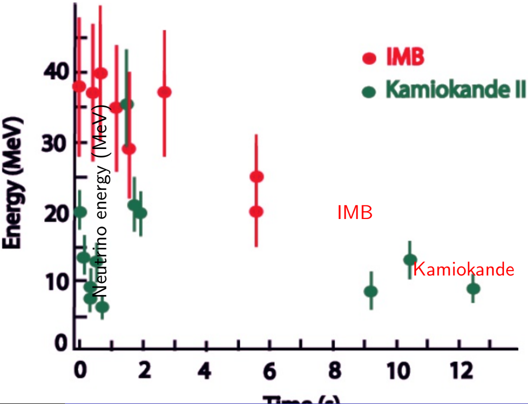
Type II

- Observed total photonic energy < 0.1 bethe
- Observed total kinetic energy is about 1 bethe, perhaps less than from a Type Ia supernova
- Observed peak photonic luminosity is about 0.1 that of a Type Ia
- Total available gravitational energy, if a neutron star is formed, is $(3/5)GM_{ns}^2/R_{ns} \simeq 250$ bethe for $M_{ns} = 1.4$ M$_\odot$ and $R_{ns} = 12$ km
- Neutrinos carry off most of this energy
- Peak neutrino luminosity is 10,000 bethes/s; for comparison, optical luminosity of 100 billion galaxies with 100 billion solar-type stars each is 40,000 bethes/s
- Gravitational collapse timescale $\tau_{gc} \sim 1/\sqrt{G\rho} \simeq 0.4$ ms for $\rho = 10^{14}$ g/cm^3
- Average neutrino energy in core: $E_\nu = 3\mu_\nu/4 = 3\hbar c(6\pi^2 n_\nu)^{1/3}/4 \sim 250$ MeV, $n_\nu = \rho Y_\nu N_o \sim 10^{37}$ cm^{-3}
- Neutrino diffusion timescale $\tau_\nu \sim (R_{ns}^2/c\lambda_\nu) \simeq 7.5$ s $\lambda_\nu \sim 1/(\rho N_o \sigma_n) \simeq 10$ cm, $\sigma_n \sim 1.7 \times 10^{-44}(E_\nu/\mathrm{MeV})^2$ cm^2
- Neutrinos become trapped in collapsed remnant
- Average emitted neutrino energy is about 10 MeV, so as neutrinos diffuse out they leave behind degeneracy energy which heats the star.

SN 1987a

- SN 1987a produced average fluence of $F = 5.4 \times 10^{11} (\mathrm{MeV}/E_\nu)$ neutrinos/cm^2
- 5 kton water detector $\Rightarrow N = 3.5 \times 10^{33}$ H atoms
- Predicted number of neutrinos detected is $FN\sigma f/6 \simeq 5f(E_\nu/\mathrm{MeV})$, where f is the detector efficiency and $\sigma = 1.7 \times 10^{-44} (E_\nu/\mathrm{MeV})^2$ cm^2 is the neutrino cross section.





Stellar Explosions

We make four assumptions to produce a tractable analytic model.

- Assume the pressure is dominated by radiation pressure (no neutrinos or e^-e^+ pairs).
- Assume the energy radiated from the surface and direct escape of gamma rays from radioactivity are small compared to the total energy.
- Assume the opacity is primarily due to electron scattering, so $\kappa = \kappa_o$ independent of ρ and T.
- Assume spherical symmetry.

From the first law of thermodynamics $dE + PdV = TdS = dQ$:

$$\dot{E} + P\dot{V} = -\frac{\partial L}{\partial M} + \dot{\epsilon}$$

where $E = aT^4V$ and $P = aT^4/3$ are the radiation energy per gram and pressure. Dots are time derivatives. $V = 1/\rho$ and $\dot{\epsilon}$ is the energy input per gram per second from radioactivity. L is the luminosity and M is the mass interior to r.

Setting up the Problem

Introduce a dimensionless Lagrangian radius containing mass M:
$x = r(M,t)/R(t)$ with R the surface. We have

$$M(x) = 4\pi R(t)^3 \int_0^x \rho(r,t) x'^2 dx'$$

is independent of time, so $\rho(M) \propto R(t)^{-3}$:

$$\rho(r,t) \equiv V^{-1} = \rho_o \eta(x) \left(R_o/R(t)\right)^3$$

with $\rho_o = \rho(0,0)$ and $R_o = R(0)$. Hence

$$\dot{V}/V = 3\dot{R}/R.$$

If radiation and radioactivity losses are small compared to the total energy changes, the system would be adiabatic, giving $\dot{T}/T = -\dot{R}/R$. This suggests

$$T^4(r,t) = \psi(x)\phi(t) T_o^4 R_o^4 / R^4(t)$$

as a solution with $T_o = T(0,0)$. This implies

$$\frac{\dot{T}}{T} = -\frac{\dot{R}}{R} + \frac{\dot{\phi}}{4\phi}.$$

Luminosity and Transport Equations

The stellar structure equation for the luminosity is

$$L(r,t) = -\frac{4\pi r^2}{3}\frac{ac}{\kappa\rho}\frac{\partial T^4}{\partial r} = -\frac{4\pi x^2}{3}\frac{ac}{\kappa\rho_o\eta}\phi(t)T_o^4 R_o\frac{d\psi}{dx},$$

with κ the opacity. The First Law of Thermodynamics becomes

$$-\frac{3\dot{\epsilon}\rho_o^2 R_o^2}{acT_o^4}\left(\frac{\eta}{\phi\psi}\right) + \frac{3R_o^3\rho_o}{c}\frac{\dot{\phi}}{\phi R} = \frac{1}{\psi x^2}\frac{d}{dx}\left(\frac{x^2}{\eta\kappa}\frac{d\psi}{dx}\right).$$

Initially, we ignore the $\dot{\epsilon}$ term. Assuming the opacity is primarily due to electron scattering, $\kappa = \kappa_o \simeq 0.33$ cm^2 g^{-1}, the First Law is separable.

$$\frac{3\rho_o\kappa_o R_o^3}{c}\frac{\dot{\phi}}{\phi R} = -\alpha = \frac{1}{\psi x^2}\frac{d}{dx}\left(\frac{x^2}{\eta}\frac{d\psi}{dx}\right).$$

The time dependence (diffusion timescale τ_o) is:

$$\phi(t) = \exp\left[-\frac{\alpha c}{3\rho_o\kappa_o R_o^3}\int_0^t R(t)dt\right] = \exp\left[-\frac{1}{R_o\tau_o}\int_0^t R(t)dt\right],$$

$$\tau_o = \frac{3R_o^2\rho_o\kappa_o}{\alpha c} \simeq 5.3\cdot 10^6\frac{M}{I_M M_\odot}\frac{10^{14}\ \mathrm{cm}}{R_o}\ \mathrm{s}.$$

The explicit time behavior depends upon $R(t)$.

Spatial Dependence

At origin, $\psi(0) = 1$ and $\psi'(0) = 0$; at surface, Eddington condition is

$$\left(\frac{T}{T_E}\right)^4 = \frac{\psi}{\psi_E} = \frac{3}{4}\left(\tau + \frac{2}{3}\right)$$

where $\tau(r) = -\int_r^\infty \kappa\rho dr$ and $\tau(R_E) = 2/3$. Thus

$$\psi(1) = \psi(\tau = 0) = \psi_E/2, \qquad \psi'(1) = -3\psi_E(\kappa_o\rho R)_{x=1}/4.$$

η can be a decreasing (centrally condensed sphere) or an increasing (shell-like) function. Consider $\eta = e^{Ax}$. If $A = 0$, $\eta = 1$ and the density is uniform, and the spatial solution is a Lane-Emden polytrope of index 1:

$$\psi(x) = (\sqrt{\alpha x})^{-1} \sin\sqrt{\alpha x}.$$

The boundary condition implies

$$\frac{\sin\sqrt{\alpha}}{\sqrt{\alpha}} = -\frac{2}{3}\frac{1}{\kappa_o\rho_o R}\left(\cos\sqrt{\alpha} - \frac{\sin\alpha}{\sqrt{\alpha}}\right)$$

$$\sqrt{\alpha} \simeq \pi\left(1 - \frac{2}{3}\frac{1}{\kappa_o\rho_o R}\right).$$

Surprisingly, non-zero values of A lead to very similar results.

Luminosity

When the shock emerges at the stellar surface $R(0) = R_o$, the energy is equipartitioned between thermal and kinetic energies:

$$E_T(t) = \int_0^R aT(t)^4 4\pi r^2 dr = 4\pi R_o^3 a T_o^4 \frac{R_o}{R(t)} \phi(t) I_T = E_T(0)\phi(t)\frac{R_o}{R(t)},$$

$$E_K(t) = \frac{1}{2}\int_0^R \rho v^2 4\pi r^2 dr = 2\pi\rho_o R_o^3 \dot{R}^2 I_K = E_K(0)\frac{\dot{R}^2}{\dot{R}_0^2},$$

$$I_T = \int_0^1 \psi x^2 dx, \qquad I_K = \int_0^1 \eta x^4 dx, \qquad v = dr/dt = x\dot{R}.$$

Then if radiation and radioactivity are negligible additions/losses, a constant is

$$E_{\rm SN} = E_T(t) + E_K(t) \simeq 2E_T(0) = 2E_K(0).$$

The emergent luminosity and ejected mass are

$$L(t) = -\frac{4\pi}{3}\frac{ac}{\kappa_o\rho_o} R_o T_o^4 \left(\frac{\psi'}{\eta}\right)_{x=1} \phi(t) = \frac{4\pi c I_M R_o E_T(0)}{3\kappa_o M I_T}\left(\frac{\psi'}{\eta}\right)_{x=1} \phi(t).$$

$$M = \int_0^R 4\pi\rho r^2 dr = 4\pi\rho_o R_o^3 I_M, \quad I_M = \int_0^1 \eta x^2 dx, \quad -\alpha I_T = \left(\frac{\psi'}{\eta}\right)_{x=1}.$$

$$L(t) = \frac{2\pi c}{3\kappa_o}\frac{E_{\rm SN}}{M}\alpha I_M R_o \phi(t) = \frac{E_{\rm SN}\phi(t)}{2\tau_o}.$$

Temporal Development

Conservation of energy ($E_{\rm SN}$ constant) gives

$$\dot{R}^2=\dot{R}_o^2\left(2-\phi\frac{R_o}{R}\right),\qquad \dot{R}_o=\sqrt{\frac{E_{\rm SN}I_M}{MI_K}}=10^9\sqrt{\frac{I_M}{I_K}\frac{E_{\rm SN}/M}{10^{18}\ {\rm erg\ g^{-1}}}}\ {\rm cm\ s^{-1}}.$$

At $t=0$, $\phi(t)R_o/R(t)\simeq 1$ so $R(t)$ linearly increases:

$$R(t)\simeq R_o+\dot{R}_o t.$$

The initial expansion timescale is

$$\tau_h=\frac{R_o}{\dot{R}_o}=10^5\frac{R_o}{10^{14}\ {\rm cm}}\sqrt{\frac{I_K}{I_M}\frac{10^{18}\ {\rm erg\ g^{-1}}}{E_{\rm SN}/M}}\ {\rm s}.$$

Note that ϕ changes on the timescale $\tau_o >> \tau_h$. After $t\simeq$ several τ_h (1 week), $\phi(t)R_o << R(t)$, so the expansion, while still linear in time, is $\sqrt{2}$ times faster:

$$R(t)\simeq R_o+\sqrt{2}\dot{R}_o t.$$

Thermal energy has been converted into kinetic energy. We can solve for ϕ:

$$\phi(t)=\exp\left[-\frac{1}{R_o\tau_o}\int_0^t R(t)dt\right]=\exp\left[-\frac{t}{\tau_o}-\frac{t^2}{\sqrt{2}\tau_o\tau_h}\right].$$

The decay is initially exponential but steepens into a Gaussian with decay time

$$\tau_{decay}=\sqrt{\sqrt{2}\tau_o\tau_h}\simeq 1.1\cdot 10^6\sqrt{\frac{M}{M_\odot}}\left(\frac{I_K}{I_M^3}\frac{10^{18}\ {\rm erg\ g^{-1}}}{E_{\rm SN}/M}\right)^{1/4}\ {\rm s}.$$

Further Relations

The maximum observed velocities in the ejecta are

$$v_{max} \simeq \sqrt{2}\dot{R}_o = \sqrt{2\frac{I_M}{I_K}\frac{E_{\rm SN}}{M}} \simeq 1.4\cdot 10^9\sqrt{\frac{I_M}{I_K}\frac{E_{\rm SN}/M}{10^{18}~{\rm erg~g^{-1}}}}~{\rm cm~s^{-1}}.$$

The luminosity becomes

$$L = 1.8\cdot 10^{44}\frac{E_{\rm SN}/M}{10^{18}~{\rm erg~g^{-1}}}\frac{R_o}{10^{14}~{\rm cm}}I_M\phi(t)~{\rm erg~s^{-1}}.$$

The effective temperature $[L/(4\pi\sigma_B R^2)]^{1/4}$ is

$$T_{eff} = 7.2\cdot 10^4\left(\frac{E_{\rm SN}/M}{10^{18}~{\rm erg~g^{-1}}}\frac{10^{14}~{\rm cm}}{R_o}\right)^{1/4}\sqrt{\frac{R_o}{R(t)}}[I_M\phi(t)]^{1/4}~{\rm K}.$$

The largest time dependence in T_{eff} is due to expansion. The distribution of density and opacity in the ejecta only weakly changes observables.

Thus, measurements of largest ejecta velocities yields $E_{\rm SN}/M$. The initial effective temperature yields the initial radius R_o. The peak luminosity can then give the distance to the supernova. Measurement of the decay timescale provides an estimate of the ejected mass.

Energetics Check

We assumed that radiation losses were small compared to the total energy,

$$\int_0^\infty L(t)dt = \frac{E_{\rm SN}}{2\tau_o}\int_0^\infty \phi(t)dt << E_{\rm SN}.$$

This integral involves

$$I_t = \int_0^\infty \phi(t)dt = \frac{\sqrt{\pi}}{2}\tau_{decay}\exp\left(\frac{\tau_{decay}^2}{4\tau_o^2}\right)\left[1-{\rm erf}\left(\frac{\tau_{decay}}{2\tau_o}\right)\right].$$

We have

$$\frac{\tau_{decay}}{2\tau_o} \simeq 0.103\sqrt{\frac{M_\odot}{M}}\frac{R_o}{10^{14}~{\rm cm}}\left(I_M I_K\frac{10^{18}~{\rm erg~g}^{-1}}{E_{\rm SN}/M}\right)^{1/4} << 1,$$

for both Type Ia and II supernovae. When $z << 1$,

$${\rm erf}(z) \simeq 2\pi^{-1/2} z e^{-z^2} << 1.$$

Therefore

$$\frac{\int_0^\infty L(t)dt}{E_{\rm SN}} = \frac{I_t}{2\tau_o} \simeq \frac{\sqrt{\pi}}{2}\frac{\tau_{decay}}{2\tau_o} << 1$$

which validates the assumption that radiation losses are small.

Radioactive Heating

Supernova radioactivity is dominated by the decays ^{56}Ni$\rightarrow$^{56}Co$\rightarrow$^{56}Fe which have energy release and decay times

$$\dot{\epsilon}_{Ni} = 4.78 \cdot 10^{10} \text{ erg g}^{-1} \text{ s}^{-1}, \qquad \tau_{Ni} = 7.6 \cdot 10^{5} \text{ s},$$

$$\dot{\epsilon}_{Co} = 7.97 \cdot 10^{9} \text{ erg g}^{-1} \text{ s}^{-1}, \qquad \tau_{Co} = 9.82 \cdot 10^{6} \text{ s}.$$

A simple generic parametrization is

$$\dot{\epsilon} = \xi(x) \left[\dot{\epsilon}_{Ni} e^{-t/\tau_{Ni}} + \dot{\epsilon}_{Co} \left(1 - e^{-t/\tau_{Ni}}\right) e^{-t/\tau_{Co}} \right]$$

where $\xi(x)$ is the spatial distribution of the radioactivities. In SN Ia, the expanding envelope is partially transparent to the Co decay γs and e^+s, which led to an early mistaken identification with the decay time of Cf.

The energy release from Co decay exceeds that from Ni decay when

$$\frac{t}{\tau_{Ni}} \simeq \left[\ln\left(\frac{\dot{\epsilon}_{Ni}}{\dot{\epsilon}_{Co}}\right) - \ln\left(1 - e^{-t/\tau_{Ni}}\right) \right] \left(1 - \frac{\tau_{Ni}}{\tau_{Co}}\right)^{-1} \simeq 2.08,$$

about 18 days. After $4\tau_{Ni}$, Ni decay provides only 1/8 of the total energy release, and one can use

$$\dot{\epsilon} = \xi(x) \dot{\epsilon}_{Co} e^{-t/\tau_{Co}} \equiv \dot{\epsilon}_o \xi(x) f(t).$$

The net energy release is about 0.12 MeV/b or $2 \cdot 10^{50}$ erg $M_\odot^{-1} << E_{\rm SN}/M$ and does not therefore alter the envelope's homologous expansion.

Incorporating Radioactivity

Note that the combination $\xi(x)\eta(x)/\psi(x) = b$ is insensitive to x if radioactivity is centrally concentrated, a good assumption.

The initial mass of radioactive nuclei is

$$M_r = 4\pi R_o^3 \rho_o \int_0^1 \xi(x)\eta(x)x^2 dx = b\frac{I_T}{I_M}M.$$

The temporal development equation becomes

$$\dot{\phi} + \phi\frac{R(t)}{\tau_o R_o} = f(t)\frac{R(t)}{R_o}\frac{2\dot{\epsilon}_o M_r}{E_{\rm SN}}.$$

Let

$$\dot{u} = \frac{R(t)}{R_o\tau_o}, \qquad u = \frac{t}{\tau_o} + \frac{t^2}{\tau_{decay}^2}.$$

Then

$$\phi(u) = e^{-u} + \bar{\epsilon}_r e^{-u}\int_0^u e^u f(u) du$$

where, for Ni (Co) decay,

$$\bar{\epsilon}_{Ni(Co)} = \frac{2\dot{\epsilon}_{Ni(Co)}M_r\tau_o}{E_{\rm SN}} = \frac{M_r\dot{\epsilon}_{Ni(Co)}}{L(0)} \simeq 3.1(0.52)\frac{M_r}{M_\odot}\frac{10^{14}~{\rm cm}}{R_o}\frac{10^{18}~{\rm erg~g}^{-1}}{E_{\rm SN}/M}.$$

Time Development

The quantity $\bar{\epsilon}$ measures the importance of radioactivity to the light curve. Consider early times $u << 1$. Since $\tau_{Ni} << \tau_o$ and $\tau_{decay} << \tau_o$ (especially for SN Ia), we have

$$\phi \simeq 1 + (\bar{\epsilon}_r - 1)u \qquad\qquad u << 1.$$

In the case of SN Ia, $R_o \sim 10^8$ cm. Therefore $\bar{\epsilon} >> 1$ so we expect the bolometric luminosity to initially increase with time. In the case of core-collapse SN with red giant progenitors and small M_r, $\bar{\epsilon}_r < 1$ and the luminosity falls from the start.

Over long times, we have

$$\phi \simeq e^{-u} + \bar{\epsilon}_r(f - e^{-u}), \qquad\qquad u >> 1.$$

Even if $\bar{\epsilon}_r << 1$ the radioactivity term becomes important, as f is an exponential decay while e^{-u} is a Gaussian decay.

The function ϕ has the property that it is a maximum when

$$\dot{\phi}_{peak} = 0, \qquad \phi_{peak} = f_{peak}\bar{\epsilon}_r,$$

The light curve peaks when it has the same value as the radioactivity term. When $\bar{\epsilon}_r < 1$, the peak is at $t = 0$.

Light Curve Behavior

Because $\tau_o >> \tau_{decay}$, one can safely approximate $u \to (t/\tau_{decay})^2$ when $t >> \tau_{decay}^2/\tau_o = \sqrt{2}\tau_h << \tau_{Ni}$. On shorter timescales, transient effects from shock breakout make simple light-curve modelling impossible.

When $\bar{\epsilon}_r >> 1$ (Type Ia SN and blue supergiant Type II SN) the light curve has a peak. The width of the peak is given by $\Delta_{peak} = \sqrt{\phi_{peak}/\ddot{\phi}_{peak}}$. Taking the derivative of $\dot{\phi} + \phi\dot{u} = f\bar{\epsilon}_r\dot{u}$, one finds

$$\ddot{\phi}_{peak} = \dot{f}_{peak}\bar{\epsilon}_r\dot{u}_{peak}, \qquad \Delta_{peak} = \sqrt{\frac{\phi_{peak}}{\ddot{\phi}_{peak}}} = \sqrt{\frac{-f_{peak}}{\dot{f}_{peak}\dot{u}_{peak}}}$$

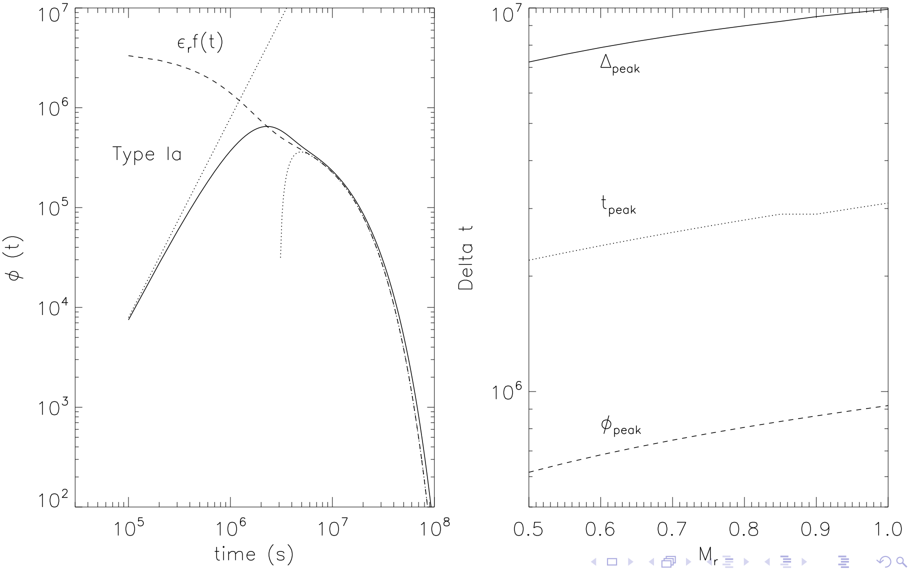
$$\frac{-f_{peak}}{\dot{f}_{peak}} \simeq \sqrt{\tau_{Co}\tau_{Ni}}, \qquad \dot{u}_{peak} = \frac{2t_{peak}}{\tau_{decay}^2}, \qquad \Delta_{peak} = \frac{\tau_{decay}}{\sqrt{2t_{peak}}}(\tau_{Ni}\tau_{Co})^{1/4}.$$

When $\bar{\epsilon}_r >> 1$, the peak is reached when $u_{peak} \sim 1$ and $t_{peak} \sim \tau_{decay}$.

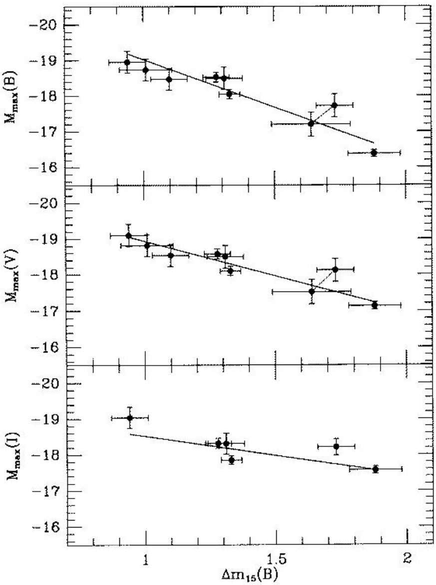
Thus $\Delta_{peak} \propto \sqrt{\tau_{decay}} \propto E_{SN}^{1/4}$, assuming $E_{SN}/M \sim$ constant.

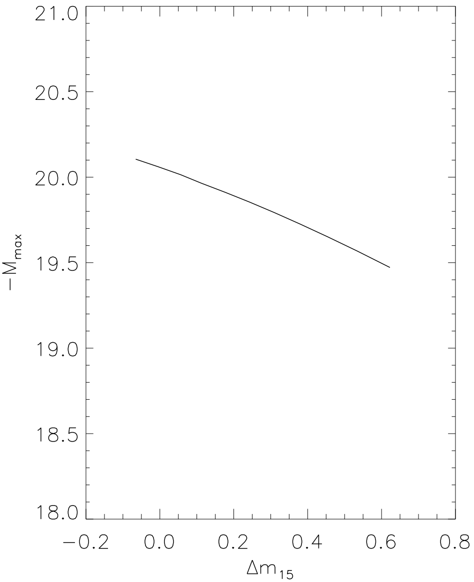
This is, essentially, the Philips relation made famous as the key step in the calibration of Type Ia SN for cosmological purposes.

Computed Correlations for Type Ia SN



The Philips Relation





Typical Examples