Darmstadt Lecture 12 – X-Ray Constraints

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Radiation Radius

The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

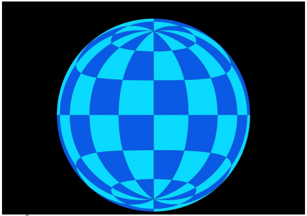
Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition

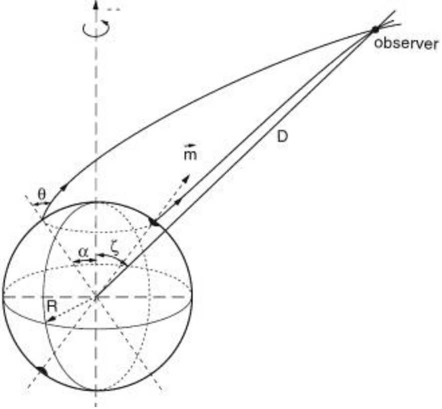
Nearby isolated neutron stars (parallax measurable)

Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmosperes)

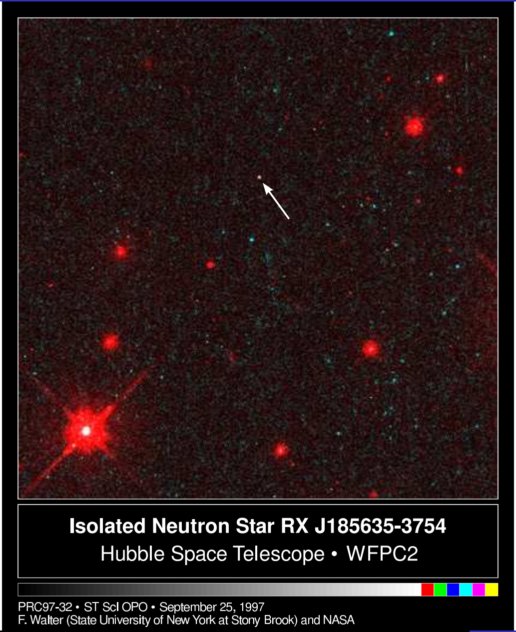
Bursting sources in which Eddington flux is measured

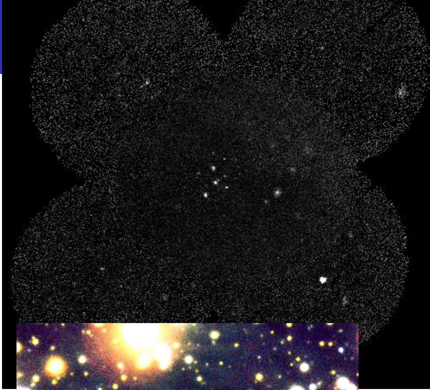
$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1-\frac{2GM}{R_{ph}c^2}}$$

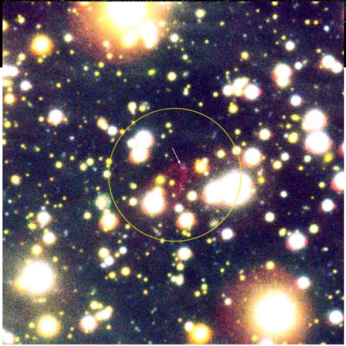




RX J1856-3754







A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

Astrometry of RXJ 1856-3754

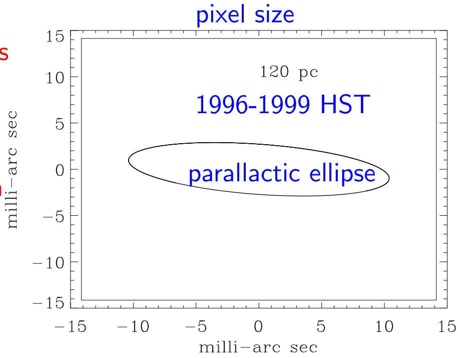
Walter & Lattimer (2002) determined $D = 117 \pm 12$ pc and $v \simeq 190$ km/s from 1996-1999 HST Planetary Camera observations

Star's age is probably 0.5 million years

Walter, Eisenbeiβ, Lattimer, Kim, Hambaryan & Neuhäuser (2010) determined $D \simeq 115 \pm 8$ pc based on 2002-2004 HST Advanced Camera for Surveys observations (double the resolution)

A magnetic hydrogen atmosphere model (Ho et al. 2007) suggests $M \simeq 1.29$ $M_\odot$ and $R \simeq 11.6$ km

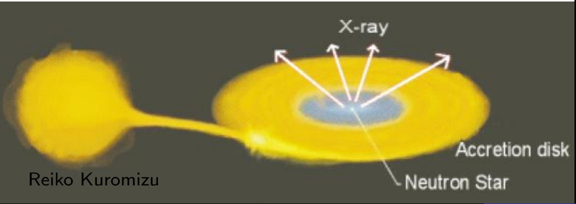
Redshift or gravity measurements, which would allow simultaneous M and R determinations, are not yet possible.



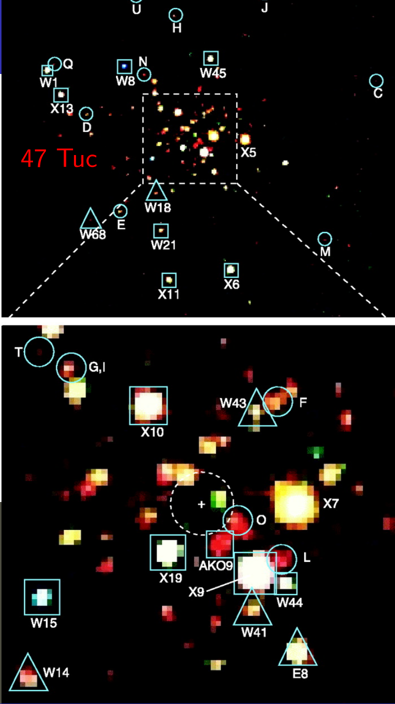
Quiescent Sources in Globulars

Hot neutron stars in globular clusters

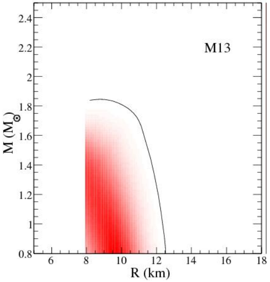
- Globular clusters evolve: more massive stars, including binaries, sink to center via long-range stellar encounters.
- Close binaries formed in encounters.
- Episodes of accretion in close binaries heats neutron stars: they are reborn.
- Following accretion, they become quiescent, low-mass X-ray sources.
- Accretion suppresses surface B fields.
- Atmospheric composition is H.

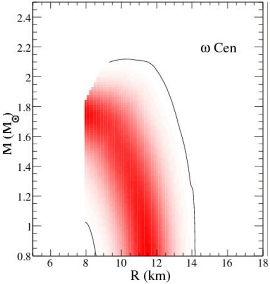


Reiko Kuromizu

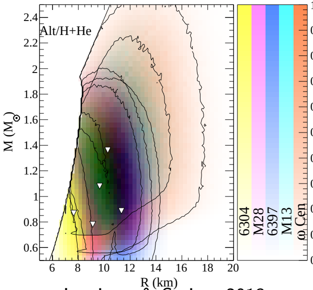


Inferred M-R Probability Distributions

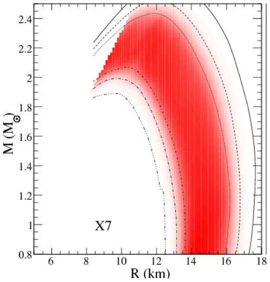


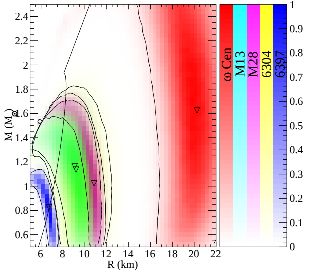


Steiner, Lattimer & Brown 2010



Lattimer & Steiner 2013





Guillot et al. 2013

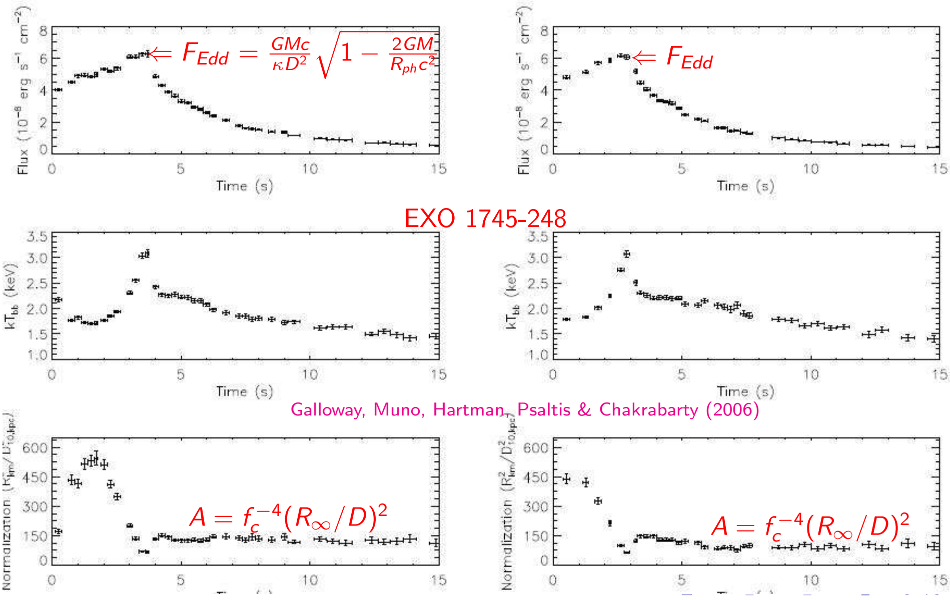
QLMXB Analysis (Guillot et al. 2013)

Source	D (kpc)	$N_{H,21}$	$N_{H,21}^{\dagger}$	M ($M_\odot$)	R (km)
M28	5.5 ± 0.3	2.52	1.89	$1.25^{+0.54}_{-0.63}$	$10.5^{+2.0}_{-2.9}$
NGC 6397	2.02 ± 0.18	0.96	1.4	$0.84^{+0.30}_{-0.28}$	$6.6^{+1.2}_{-1.1}$
M13	6.5 ± 0.6	0.08	0.145	$1.27^{+0.71}_{-0.63}$	$10.2^{+3.7}_{-2.8}$
ω Cen	4.8 ± 0.3	1.82	1.04	$1.78^{+1.03}_{-1.07}$	$23.6^{+7.6}_{-7.1}$
NGC 6304	6.22 ± 0.6	3.46	2.66	$1.16^{+0.96}_{-0.56}$	$9.6^{+4.9}_{-3.4}$
optimized with fixed R				0.86 – 2.42	$9.1^{+1.3}_{-1.5}$
optimized assuming crust EOS + causality‡				0.78 – 1.19	10.4 ± 0.9
optimized assuming $N_{H,21}^{\dagger}$ + crust EOS + causality + H/He‡				0.96 – 1.39	11.8 ± 0.9

† Dickey & Lockman (1990)

‡Lattimer & Steiner (2013)

Photospheric Radius Expansion X-Ray Bursts



Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} = \frac{GMc}{\kappa D^2}\sqrt{1-2\beta}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm^2 g^{-1}}$$

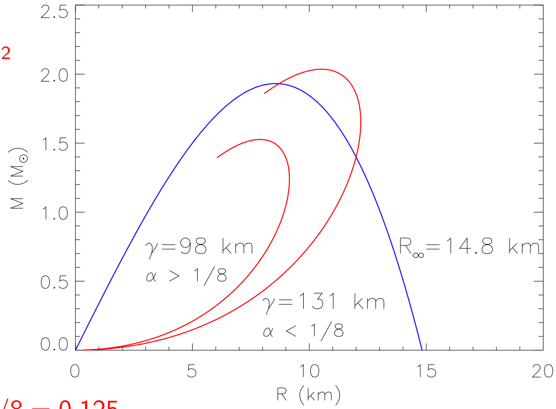
$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta(1-2\beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)^{3/2}}$$

$$\beta = \frac{1}{4} \pm \frac{1}{4}\sqrt{1-8\alpha}$$

$$R_\infty = \alpha\gamma$$



Real solutions require $\alpha < 1/8 = 0.125$

Systematics with $R_{ph} >> R$

$$F_{Edd} = \frac{GMc}{\kappa D^2}\sqrt{1 - 2\frac{GM}{R_{ph}c^2}} \simeq \frac{GMc}{\kappa D^2}$$

$$\kappa \simeq 0.2(1+X)\ \mathrm{cm}^2\mathrm{g}^{-1}$$

$$A = \frac{F_\infty}{\sigma T_\infty^4} = f_c^{-4}\left(\frac{R_\infty}{D}\right)^2$$

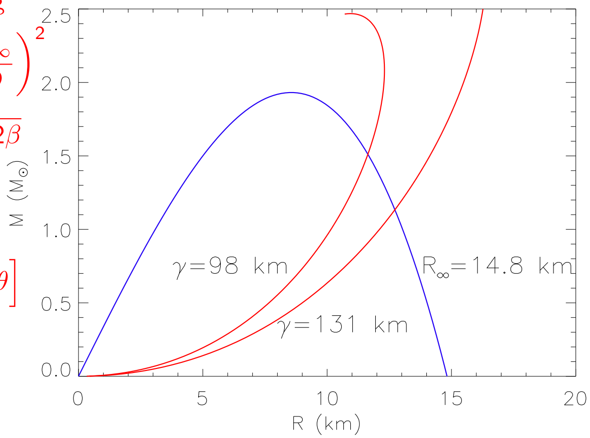
$$\alpha = \frac{F_{Edd}}{\sqrt{A}}\frac{\kappa D}{c^3 f_c^2} = \beta\sqrt{1-2\beta}$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd}\kappa} = \frac{R}{\beta(1-2\beta)}$$

$$\beta = \frac{1}{6}\left[1 + \sqrt{3}\sin\theta - \cos\theta\right]$$

$$\theta = \frac{1}{3}\cos^{-1}(1 - 54\alpha^2)$$

$$R_\infty = \alpha\gamma$$



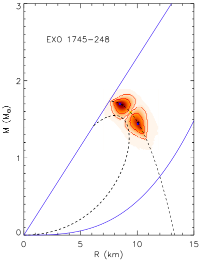
Real solution with $\beta < 1/3$ require $\alpha < 1/\sqrt{27} \simeq 0.192$

PRE Burst $M - R$ Predictions

$R_{ph} = R$

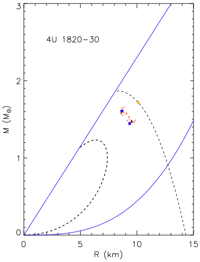
EXO 1745-248

$\alpha = 0.14 \pm 0.02$



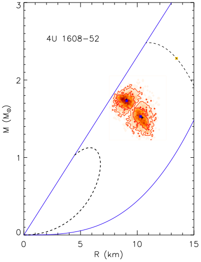
4U 1820-30

$\alpha = 0.18 \pm 0.02$



4U 1608-52

$\alpha = 0.26 \pm 0.11$

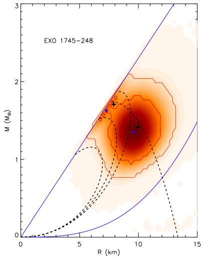


Özel, Baym & Güver 2010

$R_{ph} > R$

EXO 1745-248

$\alpha = 0.14 \pm 0.02$



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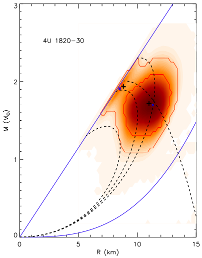

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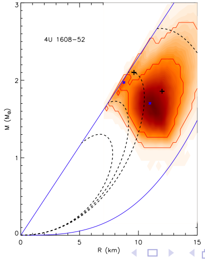
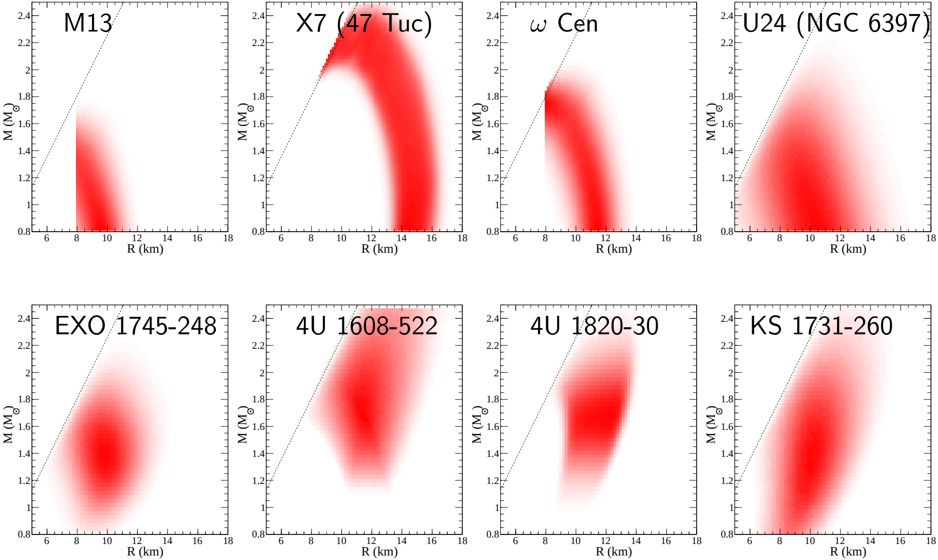


Steiner, Lattimer & Brown 2010

$M - R$ Probability Estimates



Model-Independent Radius Limits?

We've already observed that GR + causality + largest measured mass lead to an absolute lower limit to the neutron star radius:
$M_{obs} > 2M_\odot \implies R_{1.4,\mathrm{min}} = 8.15$ km.

This is an extreme limit, because the EOS almost certainly approaches causality gradually at high densities.

An upper limit to the radius of typical stars is apparently set by symmetry properties of nuclear matter: $L < 80$ MeV $\implies R_{1.4} < 13.5$ km.

We can establish more realistic theoretical limits by using the 'piecewise polytrope' method.

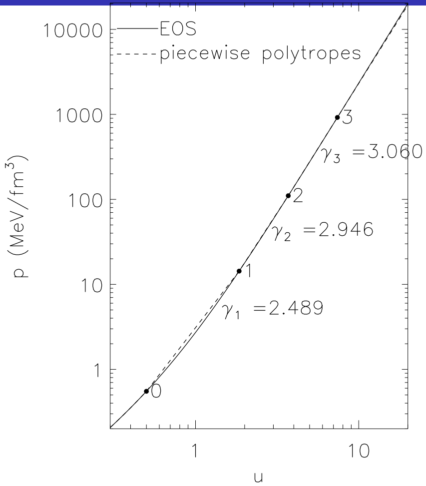
Read, Lackey, Owen & Friedman (2009) demonstrated that this can approximate the pressure of any published EOS to within a few percent.

Piecewise Polytropes, modified

The EOS below $n_0 \simeq 0.4 n_s$ is well-established, being that of the neutron star crust.

The EOS between n_0 and $n_1 \simeq 1.85 n_s$ is estimated from neutron matter theory, with predicted nuclear matter symmetry properties consistent with nuclear experiment.

Divide higher density matter with boundaries n_2 and n_3.



$$p(n) = p_{i-1}\left(\frac{n}{n_{i-1}}\right)^{\gamma_i} \qquad n_{i-1} \le n \le n_i; \quad 1 \le i \le 2$$

$$p(n) = p_2\left(\frac{n}{n_2}\right)^{\gamma_3} \qquad n \ge n_2$$

Piecewise Polytrope EOS

$$\gamma_i = \frac{\ln(p_i/p_{i-1})}{\ln(n_i/n_{i-1})}, \qquad \varepsilon_{i-1} = n_{i-1}\left(m_b c^2 + E_{i-1}\right)$$

$$\varepsilon = \frac{p}{\gamma_{i-1}} + \left(\varepsilon_{i-1} - \frac{p_{i-1}}{\gamma_i - 1}\right)\left(\frac{p}{p_{i-1}}\right)^{1/\gamma_i}, \qquad p_{i-1} \le p \le p_i, \qquad 1 \le i \le 2$$

$$\varepsilon = \frac{p}{p_2} + \left(\varepsilon_2 - \frac{p_2}{\gamma_3 - 1}\right)\left(\frac{p}{p_2}\right)^{1/\gamma_3}, \qquad p \ge p_2$$

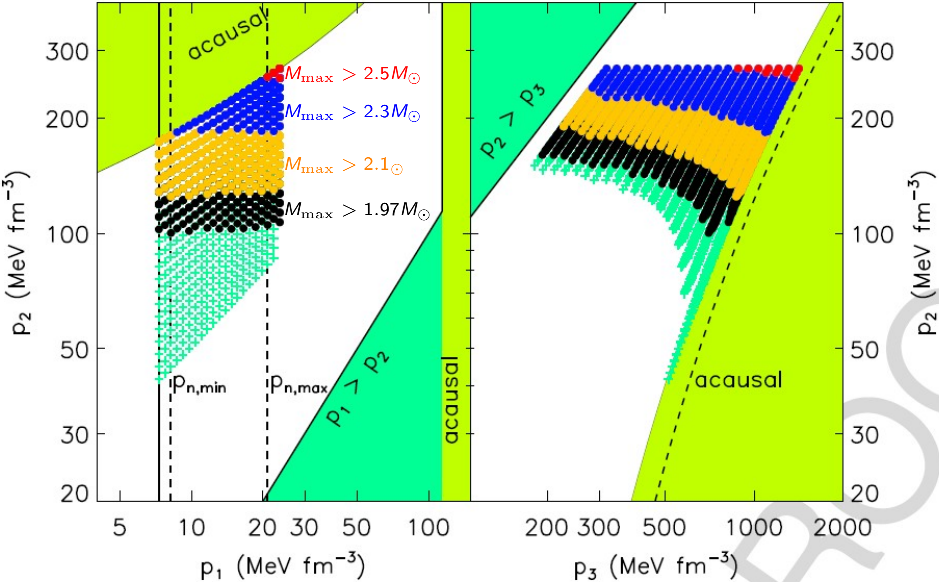
Read et al. (2009) found that a large number of EOSs are collectively best approximated with $u_1 = 1.85$, $u_2 = 3.7$, $u_3 = 7.4$.

p_0 fixed by the core-crust boundary.

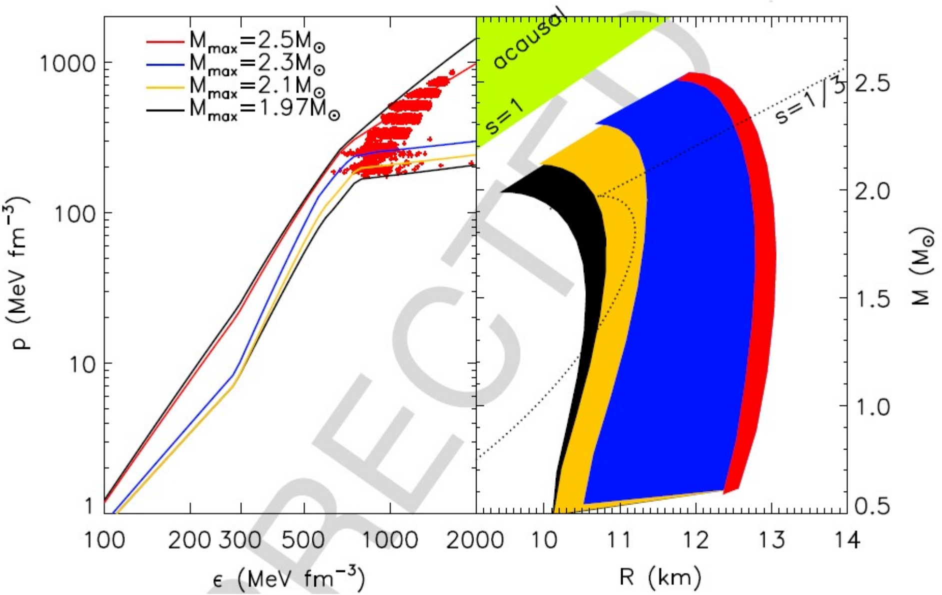
The remaining parameters $[\gamma_1, \gamma_2, \gamma_3]$ or $[p_1, p_2, p_3]$ are limited by

- Hydrodynamic stability: $p_i \ge p_{i-1}$ or $\gamma_i \ge 0$
- Causality: $$\frac{c_{s,i}^2}{c^2} = \left(\frac{\partial p}{\partial \varepsilon}\right)_i = \frac{\gamma_{i,max} p_{i,max}}{\varepsilon_{i,max} + p_{i,max}} \le 1$$
- Neutron matter: 7.56 MeV fm$^{-3} < p_1 < 21$ MeV fm^{-3}
- Minimum value of the neutron star maximum mass

Limits to p_1, p_2 and p_3



Limits to the EOS and $M - R$



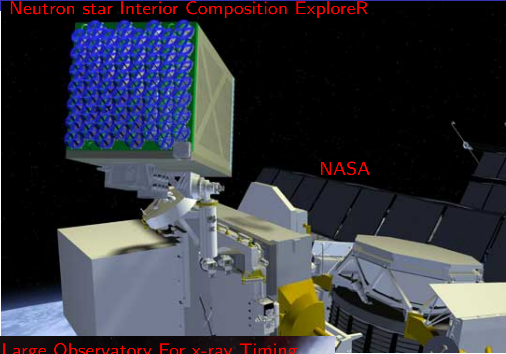
Additional Proposed Radius and Mass Constraints

Pulse profiles
Hot or cold regions on rotating neutron stars alter pulse shapes: NICER and LOFT will enable timing and spectroscopy of thermal and non-thermal emissions. Light curve modeling $\rightarrow M/R$; phase-resolved spectroscopy $\rightarrow R$.

Supernova neutrinos
Millions of neutrinos detected from a Galactic supernova will measure BE$= m_B N - M$, $< E_\nu >$, τ_ν.

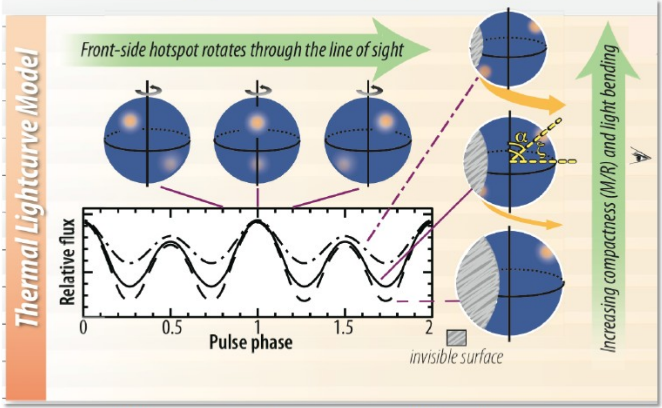
QPOs from accreting sources
ISCO and crustal oscillations

Gravitational radiation
Mergers of neutron stars with neutron stars or black holes
'Mountains' on spinning stars
R-mode instabilities



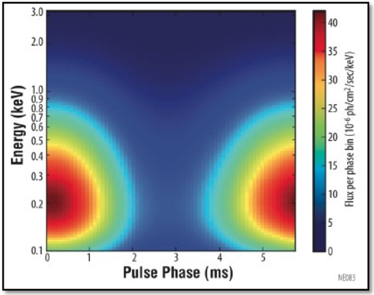
Science Measurements

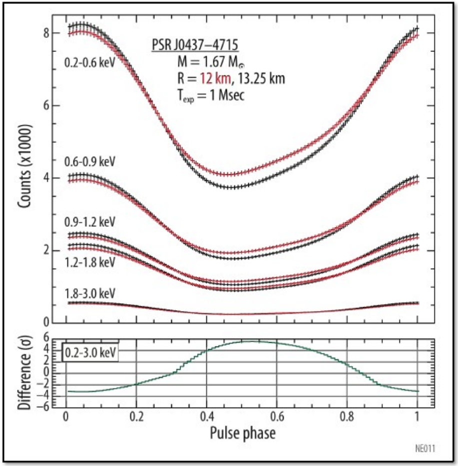
Reveal stellar structure through lightcurve modeling, long-term timing, and pulsation searches



Lightcurve modeling constrains the compactness (M/R) and viewing geometry of a non-accreting millisecond pulsar through the depth of modulation and harmonic content of emission from rotating hot-spots, thanks to gravitational light-bending…

Science Measurements (cont.) NICER





… while phase-resolved spectroscopy promises a direct constraint of radius *R*.