RUB

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Lecture Week, SFB 1245
TU Darmstadt, September 26-30, 2022

# Chiral effective field theory for nuclear forces

## Concepts

Effective field theory, chiral perturbation theory, renormalization, predictive power, KSW vs Weinberg, power counting…

## Methods

S-matrix matching, method of unitary transformation to derive nuclear forces (and currents), …

## Applications

S-matrix matching, method of unitary transformation to derive nuclear forces (and currents), …

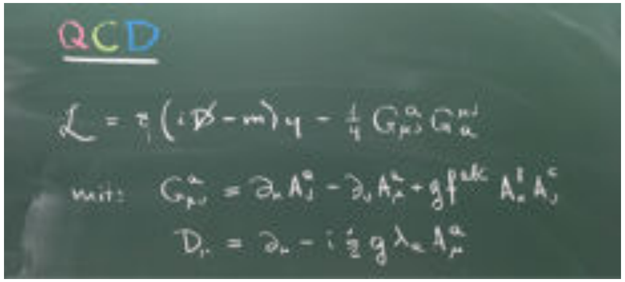
SFB 1245

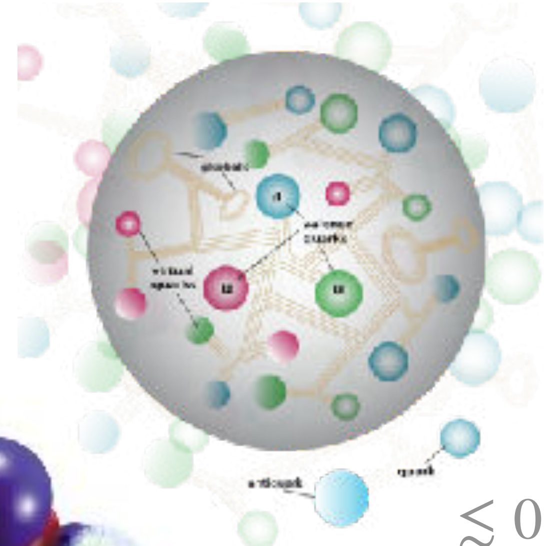
DFG Deutsche Forschungsgemeinschaft

Bundesministerium für Bildung und Forschung

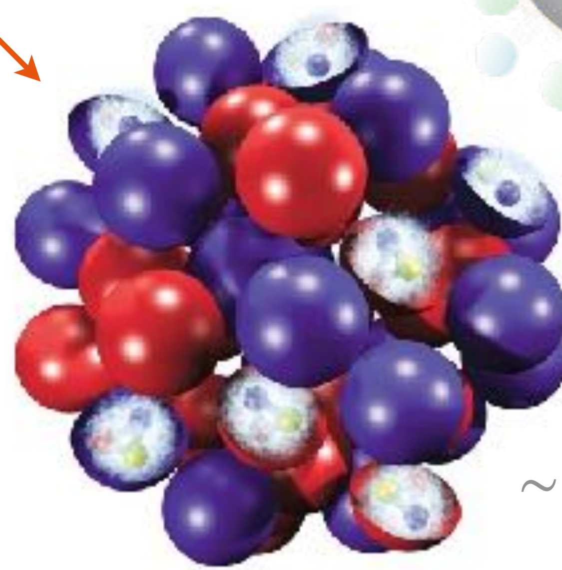
erc European Research Council

# Introductory remarks



$\lesssim 0.1$ fm



$\sim$ few fm

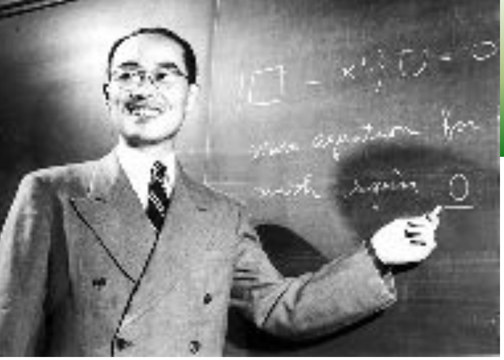
$\sim 10^4$ m

QCD: a beautiful but enormously complex theory…

# QCD methods

- — Perturbation theory in $\alpha_S$ (high-energy processes; factorization)
- — Lattice QCD — truly *ab initio*
- — Effective field theories (effective DoF, symmetries) — systematically improvable
- — Functional methods (Bethe-Salpeter, Schwinger-Dyson, Functional renormalization group)
- — Large-$N_c$ expansion — more of a qualitative nature…
- — Sum rules
- — Models — often useful but model dependent…

…



# Introductory remarks

Yukawa's theory

Proca
Kemmer
Moller
Rosenfeld
Schwinger
Pauli …

discovery of pions

two-pion exchange, meson theory…

discovery of heavy mesons

1930 1940 1950 1960 1970

Okubo, Taketani, Machida, Onuma, Fujita, Miyazawa, …

BE models
inverse scattering
dispersion theory
quark cluster models
phenomenology
…

AV18
CD Bonn
Nijm I,II
Reid93
…

(Chiral) Effective Field Theory
Lattice QCD

1980 1990 2000 2010

# Introductory remarks

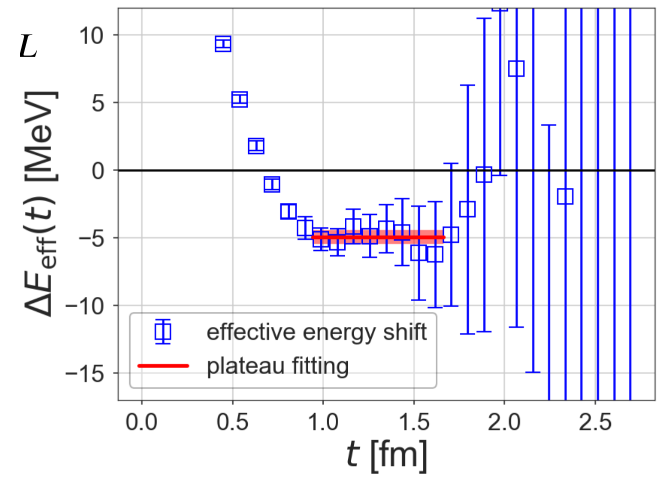
## **Lattice QCD:** brutforce numerical method [HAL QCD, NPLQCD, PACS, CalLat, …]

— truly first-principles approach

— finite-volume energies from 2-point correlators

$$C(t) = \langle 0 | \mathcal{O}(t+t_0)\mathcal{O}^\dagger(t_0) | 0 \rangle = \sum_{n=0}^{\infty} |\langle 0 | \mathcal{O} | n \rangle|^2 \, e^{-E_n t}$$

$\Rightarrow$ phase shifts using the Lüscher method



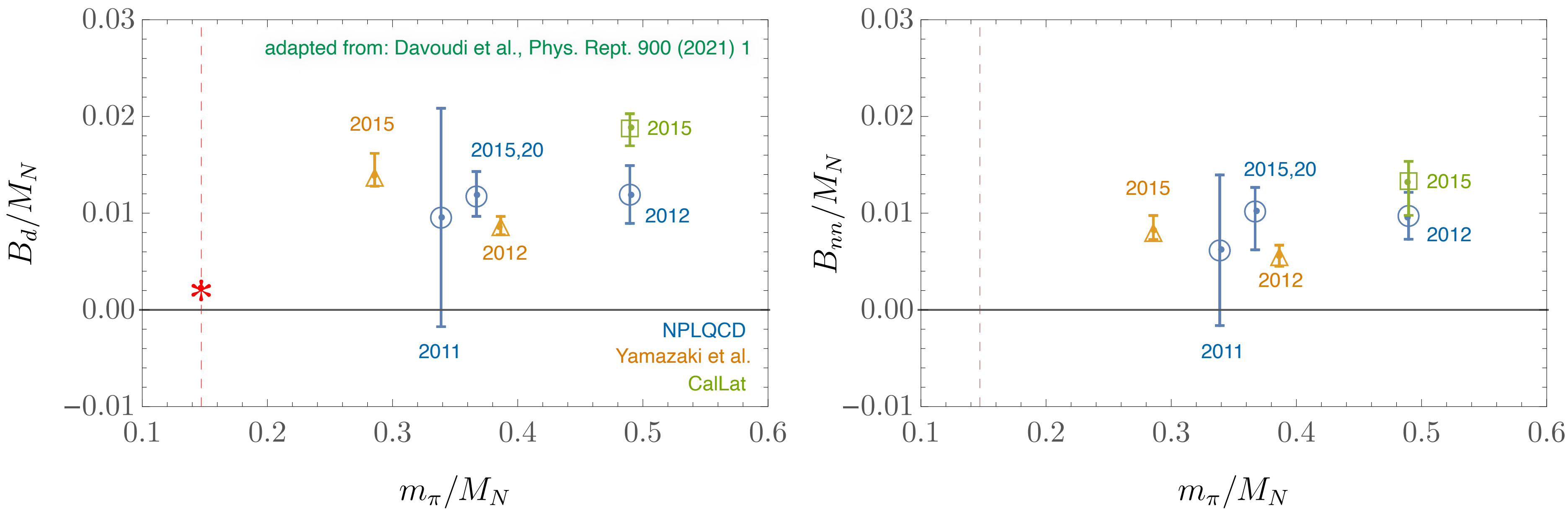

from: Iritani, LATTICE2018

— Signal-to-Noise ratio: $\sim \exp\left[-A\left(m_B - \frac{3}{2}m_M\right)t\right]$ Parisi ’84

— An alternative method [HAL QCD]: potentials from Nambu-Bethe-Salpeter wave functions (derivative expansion under control? see e.g. 1808.06299)

# Introductory remarks

**Lattice QCD:** brutforce numerical method [HAL QCD, NPLQCD, PACS, CalLat, …]




— HAL QCD sees NO bound states for all $M_\pi$—values

— also CalLat [2009.11825] strongly disfavors bound states at $M_\pi \sim 714$ MeV

— Both methods have issues/systematics; situation in NN is highly controversial…

— In any case, highly inefficient degrees of freedom to describe nuclei; unlikely to provide precise results for heavier nuclei/nuclear interactions in the near/midterm future…

# Introductory remarks

Weinberg's 3rd law of progress in theoretical physics:

*you may use any **degrees of freedom** you like to describe a physical system, but if you use the wrong ones, you will be sorry…*

in Asymptotic Realms of Physics, MIT Press, Cambridge, 1983

## Typical scales in nuclear physics:

— binding momentum of the deuteron: $p \sim \sqrt{m_N B_d} \sim 45$ MeV

— binding momentum of heavier nuclei: $p \sim \sqrt{2 m_N B_d / A}$. E.g., for $^4$He, $p \sim 115$ MeV

— Fermi-momentum at the saturation density: $p_F = (3/2\pi^2 \rho)^{1/3} \sim 270$ MeV

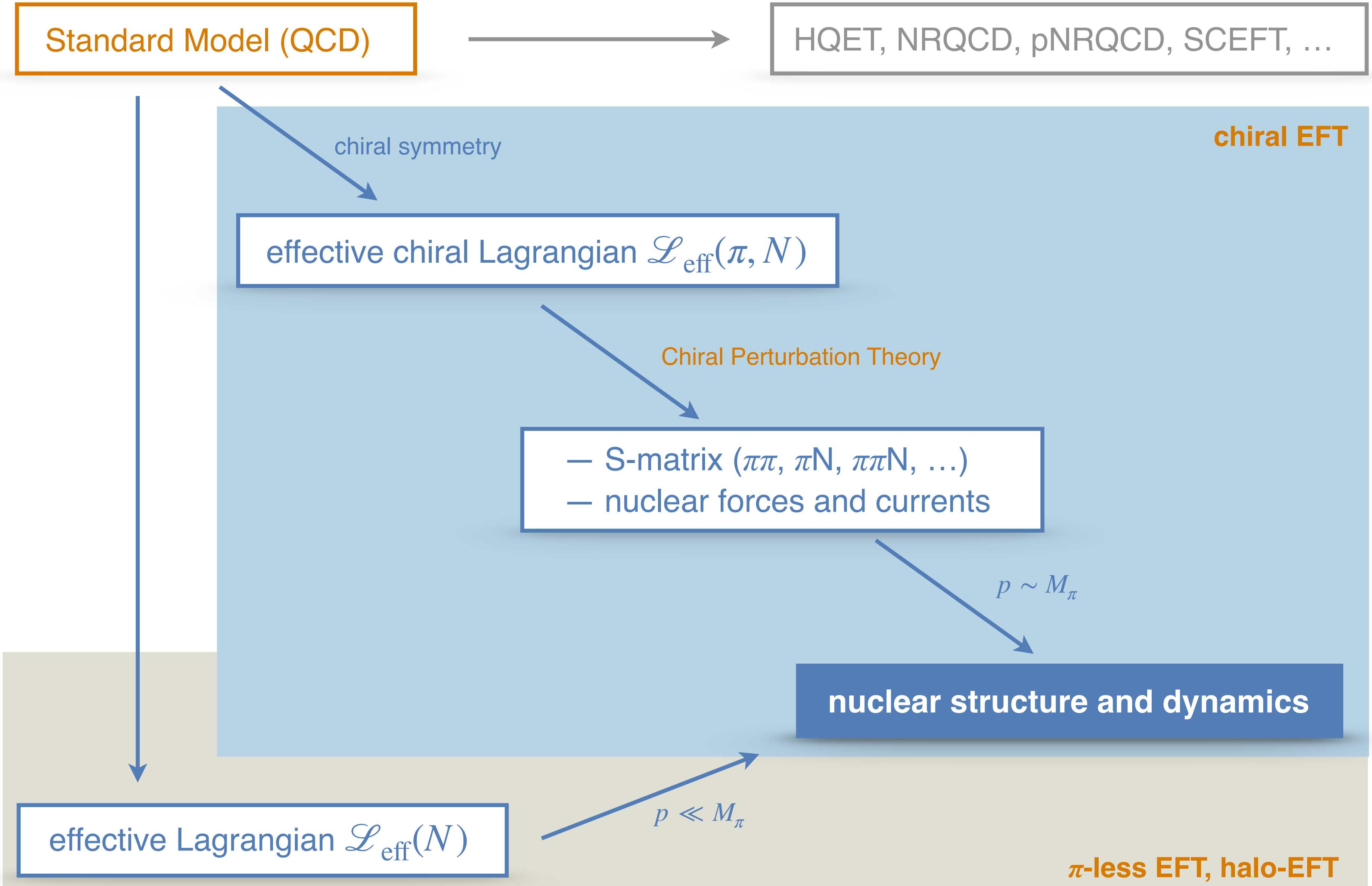
$\Rightarrow$

1. Non-relativistic description in the framework of the Schrödinger theory

$$\left[ \left( \sum_i \frac{-\vec{\nabla}^2}{2m_N} + \mathcal{O}(m_N^{-3}) \right) + V_{2N} + V_{3N} + V_{4N} + \ldots \right] |\Psi\rangle = E\,|\Psi\rangle$$

2. EFT for nuclear interactions should involve π, N (and possibly Δ) DoF

# Introductory remarks



# Lecture Syllabus

## Day 1: General introduction to EFT

EFT philosophy, renormalization, power counting, construction principles…

## Day 2: QCD and ChPT

Chiral symmetry, effective Lagrangian, chiral expansion, loops, inclusion of nucleons, …

## Day 3: Pionless EFT

Resummation of the amplitude, fine tuning, renormalization conditions, RG analysis,…

## Day 4: Chiral EFT for nuclear forces and currents

Inclusion of pions, derivation of the nuclear potentials from the effective chiral Lagrangian…

## Day 5: Applications

Determination of LECs, truncation uncertainty, state of the art, challenges and perspectives…

# What to expect

At the end of the week you should know:

## Why

— using EFTs in general

— using chiral EFT for low-energy nuclear systems

## How

— to derive nuclear forces and currents

— to renormalize the Schrödinger equation (and how not)

## What

— is the role of chiral symmetry

— can and cannot be predicted in $\chi$EFT

— does the $\chi$EFT expansion of nuclear forces correspond to

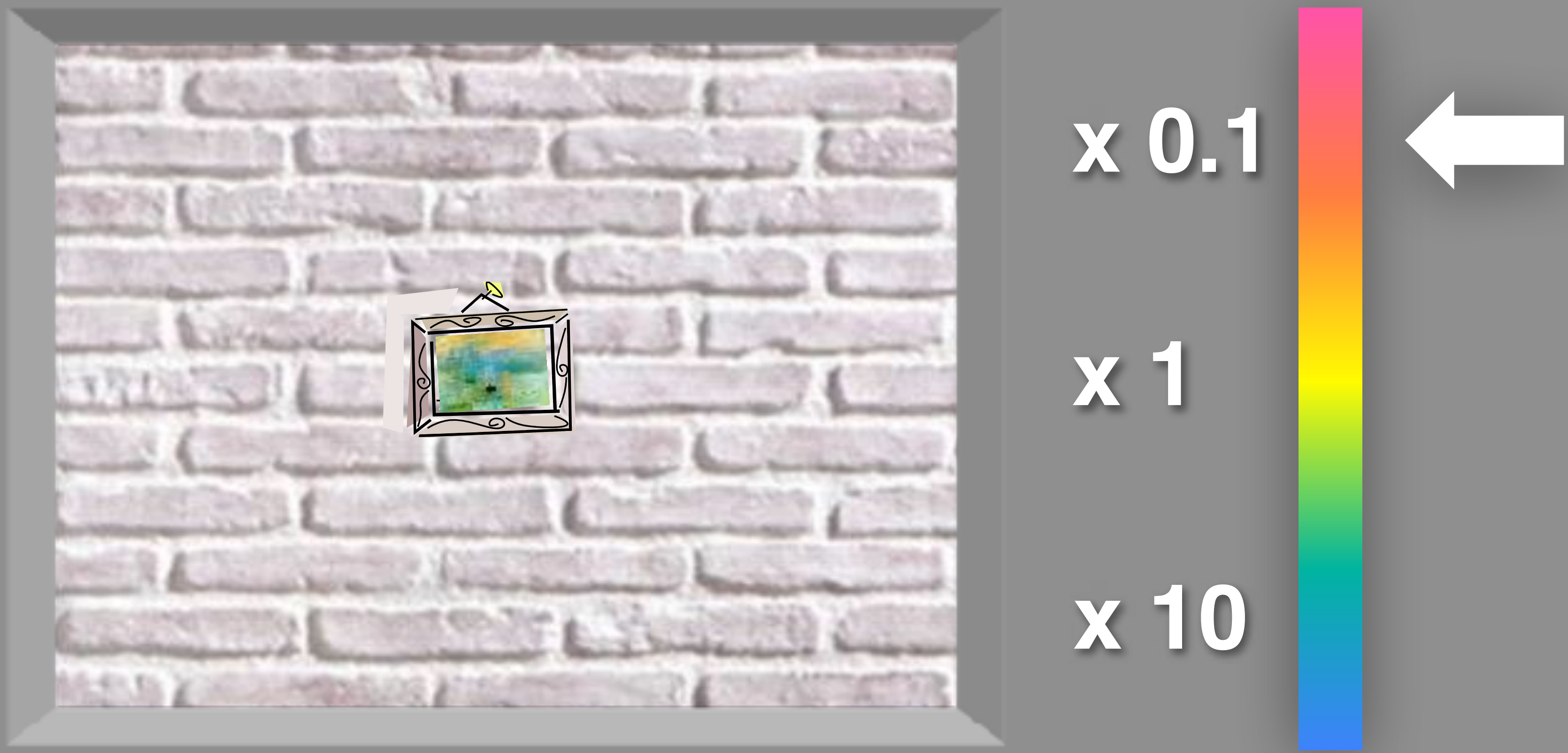
— is the current status and future perspectives of $\chi$EFT for nuclear systems

# General introduction to EFT

Some lecture notes (free access):

- — Antonio Pich, Effective Field Theory, hep-ph/9806303
- — Ira Rotstein, TASI lectures on effective field theories, hep-ph/0308266
- — David Kaplan, Five lectures on effective field theory, nucl-th/0510023
- — Aneesh Manohar, Introduction to Effective Field Theories, arXiv:1804.05863 [hep-ph]
- — Matthias Neubert, Renormalization Theory and EFTs, arXiv:1901.06573 [hep-ph]

# Effective Theories



# Effective Theories

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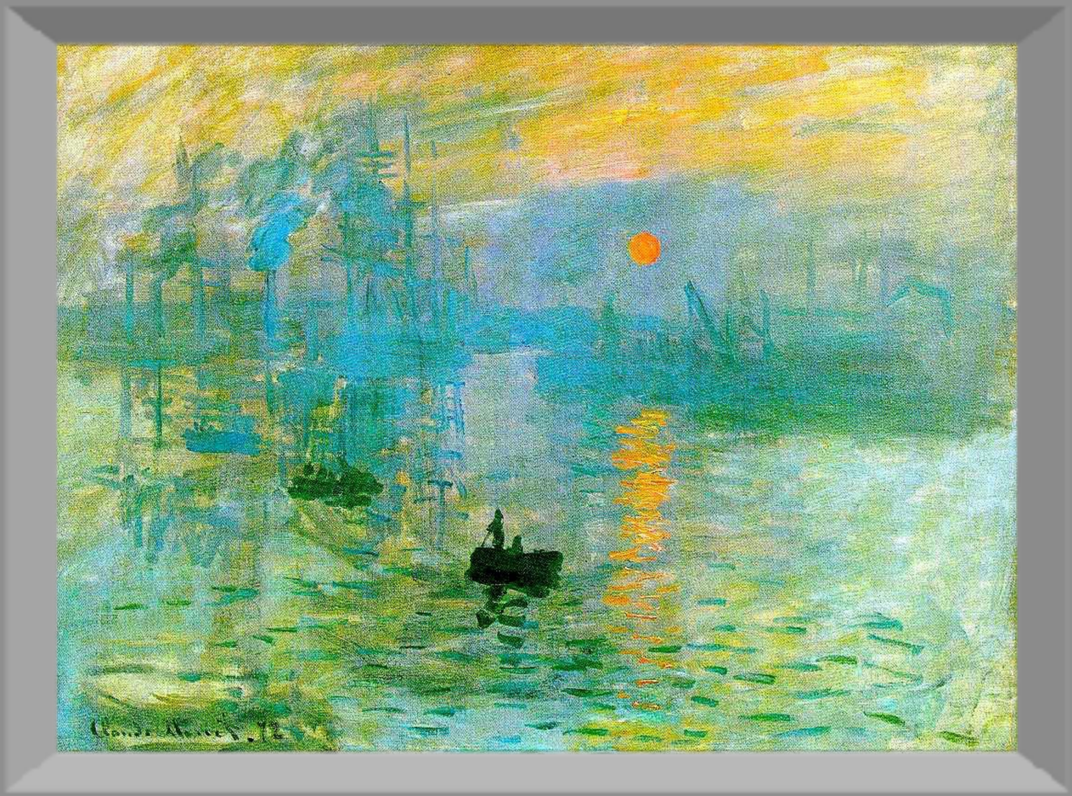


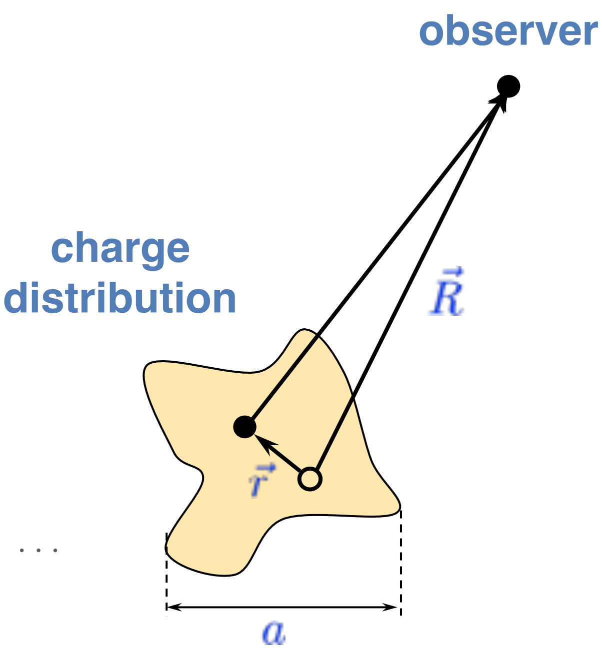

→ **it is crucial to choose a proper resolution !**

# What is an effective theory?

## 1. Main idea using a classical example

The goal: compute electric potential generated by a localized charge distribution $\rho(\vec{r})$

The answer is $V(\vec{R}) \propto \int d^3r \, \frac{\rho(\vec{r})}{|\vec{R}-\vec{r}|}$




An effective theory for $R \gg a$: The Top-Down approach

$$\frac{1}{|\vec{R}-\vec{r}|} = \frac{1}{R} + r_i \left[\frac{\partial}{\partial r_i}\frac{1}{|\vec{R}-\vec{r}|}\right]_{\vec{r}=0} + \frac{1}{2!} r_i r_j \left[\frac{\partial^2}{\partial r_i \partial r_j}\frac{1}{|\vec{R}-\vec{r}|}\right]_{\vec{r}=0} + \ldots$$

$$= \frac{1}{R} + \frac{R_i}{R^3} r_i + \frac{1}{2!}\frac{R_i R_j}{R^5}(3 r_i r_j - r^2 \delta_{ij}) + \ldots$$

$$\Rightarrow \boxed{V(\vec{R}) = \frac{q}{R} + \frac{R_i}{R^3} P_i + \frac{1}{2}\frac{R_i R_j}{R^5} Q_{ij} + \ldots}$$

with $q = \int d^3r \, \rho(\vec{r}), \qquad P_i = \int d^3r \, \rho(\vec{r})\, r_i, \qquad Q_{ij} = \int d^3r \, \rho(\vec{r})\,(3 r_i r_j - r^2 \delta_{ij})$

We have just „integrated out“ short-distance physics. For $R \gg a$, the only information needed about $\rho(\vec{r}\,)$ is hidden in the moments $q$, $P_i$, $Q_{ij}$, …

# What is an effective theory?

An effective theory for $R \gg a$: The Bottom-Up approach

What if we cannot „integrate out“ short-distance physics or don't even now $\rho(\vec{r})$, apart from the fact that it is localized in the volume $\sim a^3$?

Solution: write down the **most general expression for** $V$ using the **long-distance DoF** (i.e., $\vec{R}$) compatible with the **symmetry principles** (rotational invariance)

$$V(\vec{R}) = \sum \left[ \begin{array}{l} \textit{rotational tensors} \\ \textit{constructed from } \vec{R} \end{array} \right] \cdot \left[ \begin{array}{l} \textit{rotational tensors characterizing} \\ \textit{the system, independent of } \vec{R} \end{array} \right]$$

$$= \underbrace{\frac{1}{R}}_{[V] = \text{length}^{-1}} \text{const} + \frac{1}{R^3} R_i \underbrace{X_i}_{\sim a \text{ (NDA)}} + \frac{1}{R^5} R_i R_j \underbrace{X_{ij}}_{\sim a^2 \text{ (NDA)}} + \ldots$$

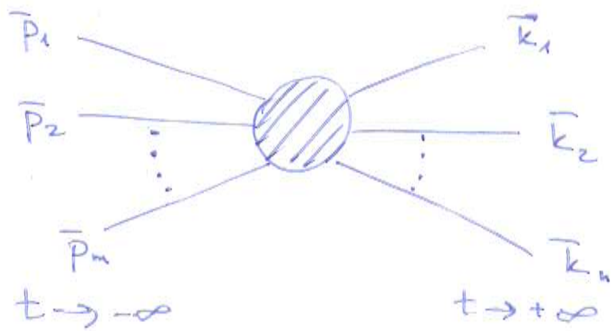
symmetric and traceless (otherwise redundant structures)

The $(2n+1)$ components of $X_{i_1 \ldots i_n}$ are called in the EFT language LECs and can be determined from experimental data.

$\Rightarrow$ systematically improvable approximation for $V(\vec{R})$ at $R \gg a$ without knowing $\rho(\vec{r})$!

## 2. Basic QFT terminology

The LSZ formula (only connected contributions)




$$
{}_{\text{out}}\langle \vec{k}_1 \ldots \vec{k}_n | \vec{p}_1 \ldots \vec{p}_m \rangle_{\text{in}} \; = \; \langle \vec{k}_1 \ldots \vec{k}_n | S | \vec{p}_1 \ldots \vec{p}_m \rangle
$$

$$
= \prod_{i,j} \frac{p_i^2 - m_{\text{ph}}^2}{\underbrace{i\sqrt{Z}}_{\langle p | \hat{\phi}(0) | 0 \rangle}} \frac{k_j^2 - m_{\text{ph}}^2}{i\sqrt{Z}} \underbrace{G_{n+m}(-k_1, \ldots, -k_n, \, p_1 \ldots, p_m)}_{\text{momentum-space Green's function}}
$$

(Here and in what follows, use natural units with $\hbar = c = 1$ unless stated otherwise.)

The Green's functions are defined as:

$$
G_l(x_1, \ldots, x_l) \; \equiv \; \int \frac{d^4 q_1}{(2\pi)^4} \cdots \frac{d^4 q_l}{(2\pi)^4} \, e^{i \sum_l q_l \cdot x_l} \, G_l(q_1, \ldots, q_l) \; = \; \langle 0 | T \{ \underbrace{\hat{\phi}(x_1) \ldots \hat{\phi}(x_l)}_{\text{Heisenberg-picture operators}} \} | 0 \rangle
$$

Switching to the interaction picture for field operators, one derives the master formula:

$$
G_l(x_1, \ldots, x_l) \; = \; \frac{\langle \Omega | T \left\{ \hat{\phi}_I(x_1) \ldots \hat{\phi}_I(x_l) e^{i \int d^4x \hat{\mathcal{H}}^I_{\text{int}}} \right\} | \Omega \rangle}{\langle \Omega | T e^{i \int d^4x \hat{\mathcal{H}}^I_{\text{int}}} | \Omega \rangle}
$$

⟵ *basis for perturbation theory in powers of $\mathscr{H}^I_{\text{int}}$; can be cast into a set of rules (Feynman diagrams)*

For those not familiar with QFT see: K. Kumeric, *Feynman Diagrams for Beginners*, arXiv:1602.04182 [physics.ed-ph]

# Quantum Field Theory

Alternatively, Green's functions can be calculated using the path integral approach (no operators!)

$$G_l(x_1, \dots, x_l) \;=\; \frac{\int \mathcal{D}\phi \; \phi(x_1)\dots\phi(x_l)\; e^{iS[\phi]}}{\int \mathcal{D}\phi \; e^{iS[\phi]}}$$

It is convenient to introduce external sources $J(x)$ (auxiliary quantities) and define the generating functional via:

$$Z[J] \;=\; \int \mathcal{D}\phi \;\; e^{iS[\phi]\,+\,i\int d^4x J(x)\phi(x)}$$

so that: $G_l(x_1, \dots, x_l) \;=\; \frac{1}{Z[0]}(-i)^l \frac{\delta^n Z[J]}{\delta J(x_1)\dots\delta J(x_l)}\bigg|_{J=0}$

Perturbative expansion for the generating functional:

$$Z[J] \;=\; e^{i\int d^4x \mathcal{L}_{\rm int}\left(\frac{\delta}{i\delta J(x)}\right)} \underbrace{e^{-\frac{1}{2}\int d^4x d^4y J(x) G_F(x-y) J(y)} \; Z_0[0]}_{Z_0[J]}$$

To refresh the basic concepts, see e.g. (free access):

— canonical quantization: chapters 4, 5 of M. Dasgupta, An *Introduction to Quantum Field Theory*

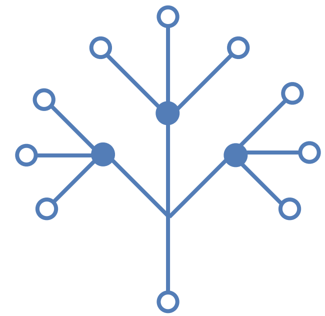
— path integral quantization: chapters 3, 4 of J. Cardy, *Introduction to Quantum Field Theory*

# Trees and loops

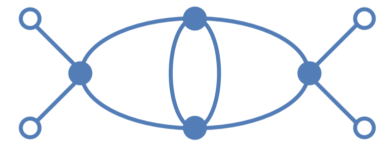
Two types of diagrams. E.g. in the $\phi^4$- theory:

— tree-level diagrams for Green's functions emerge when (pertirbatively) solving the EOM in *classical field theory*

— loop diagrams represent *quantum corrections*; loop expansion = expansion in $\hbar$



tree-level diagram



loop diagram

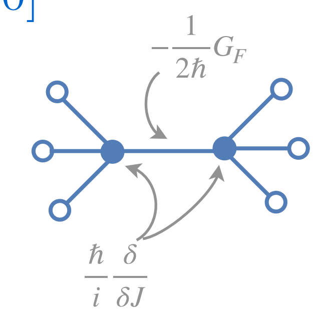
Indeed, retaining the powers of $\hbar$ in the expression for the generating functional one has:

$$Z[J] \;=\; e^{\frac{i}{\hbar}\mathcal{S}_{\rm int}\left(\frac{\hbar}{i}\frac{\delta}{\delta J(x)}\right)}\; e^{-\frac{1}{2\hbar}\int d^4x d^4y J(x) G_F(x-y) J(y)}\; Z_0[0]$$

— each vertex brings $\hbar^{-1}$

— each internal line brings $\hbar$ (external lines bring no powers of $\hbar$)

$\Rightarrow$ each diagram scales as $\hbar^{I-V}$ (with $I$ & $V$ = # of int. lines & vertices)

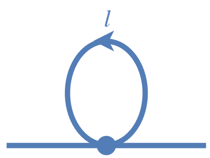



Connected diagrams: $L = I - \underbrace{(V-1)}$ $\Rightarrow$ a contribution to $Z[J]$ scales as $\hbar\, Z[J] \sim \hbar^L$

*number of $\delta$-functions for momentum conservation apart from the overall one*

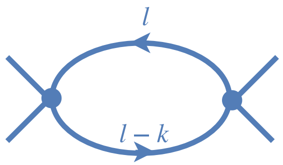
# UV divergences

Loop diagrams are typically UV divergent. E.g., for $\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$




$$\frac{1}{2}(-i\lambda)\int\frac{d^4l}{(2\pi)^4}\,\frac{i}{l^2-m^2+i\epsilon}$$

⟵ quadratically divergent



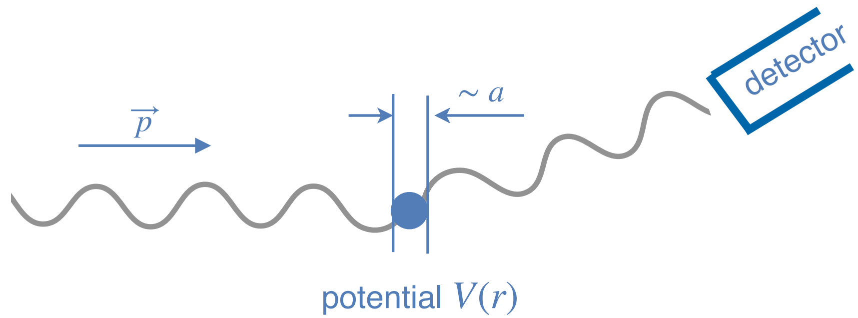

$$\frac{1}{2}(-i\lambda)^2\int\frac{d^4l}{(2\pi)^4}\,\frac{i}{l^2-m^2+i\epsilon}\,\frac{i}{(l-k)^2-m^2+i\epsilon}$$

⟵ logarithmically divergent

## What is the origin of UV divergences?

Consider quantum mechanical scattering

off some potential $V(r)$, e.g. $V(r) = e^{-r^2/(2a^2)}$




At $p \ll 1/a$ can approximate: $V(r) \propto \delta^3(r)$

$\Rightarrow V(q) = \text{const} \equiv C$

$\Rightarrow$ the Lippmann-Schwinger eq. becomes divergent: $T = V + \underbrace{VG_0V}_{\int\frac{d^3l}{(2\pi)^3}C\frac{m}{\vec{p}^{\,2}-\vec{l}^{\,2}+i\epsilon}C} + VG_0VG_0V + \ldots$

The basic principles of a QFT (causality, unitarity, relativity & cluster separability) require local Lagrangian densities…

# Regularization, renormalization and all that…

**How to deal with UV divergences in QFT?**

1. Regularize (DimReg, Pauli-Villars, cutoff, lattice, …)

2. Renormalize: express the (generally infinite) bare parameters in $\mathscr{L}$ (masses, fields, coupling constants) in terms of finite, physical quantities. Notice: this is ambiguous $\Rightarrow$ dependence on renormalization conditions/subtraction scales. An inappropriate choice may spoil convergence of the loop expansion…

3. Remove the regulator to restore the original theory (this is optional for EFTs)

**Example: the $\phi^4$-theory** $\mathcal{L} = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$

— rewrite $\mathscr{L}$ using renormalized quantities $\phi_0 =: \sqrt{Z}\phi,\ Zm_0^2 =: Z_m m^2$ and $Z^2\lambda_0 =: Z_\lambda \lambda$:

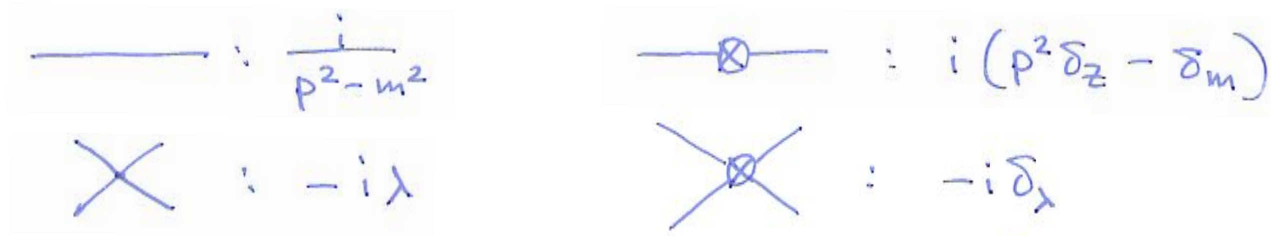
$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4}_{\text{renormalized Lagrangian}} + \underbrace{\frac{1}{2}\delta_Z\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\delta_m\phi^2 - \frac{\delta_\lambda}{4!}\phi^4}_{\text{counter terms } (\Delta\mathscr{L})}$$

with $\delta_Z = Z - 1$, $\delta_m = m^2(Z_m - 1)$, $\delta_\lambda = \lambda(Z_\lambda - 1)$

Notice: counter terms are ***not*** free parameters (and not observable) and determined from the requirement to cancel the UV divergences: $\delta_i = \hbar\delta_i^{(1)} + \hbar^2\delta_i^{(2)} + \ldots$

# Regularization, renormalization and all that…

Feynman rules:

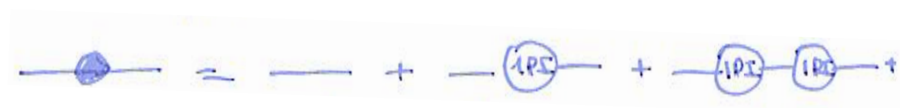



*renormalized Lagrangian* *counter terms ($\Delta\mathscr{L}$)*

2-point function to 1 loop: $-i\Sigma(p^2) =$


Using e.g. cutoff regularization one finds: $\Sigma_{\text{loop}}(p^2) = \alpha\Lambda^2 + \beta m^2 \ln\frac{\Lambda}{m} + \gamma m^2$

Dressed propagator:




$$= \frac{i}{p^2 - m^2 - \Sigma(p^2)} \overset{!}{=} \frac{i}{p^2 - m^2} + \text{non pole terms}$$

$$\Rightarrow \quad \Sigma(p^2)\Big|_{p^2=m^2} = 0, \quad \frac{d}{dp^2}\Sigma(p^2)\Big|_{p^2=m^2} = 0$$

← *on-shell renormalization conditions (renorm. m = physical mass)*

$\Rightarrow$ read off: $\delta_Z^{(1)} = 0, \quad \delta_m^{(1)} = -(\alpha\Lambda^2 + \beta m^2 \ln\frac{\Lambda}{m} + \gamma m^2)$

$\delta_X^{(n)}$ depend on both the regulator and renorm. cond., while renormalized result is unambiguous…

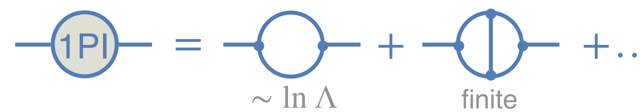
# Regularization, renormalization and all that…

For $\phi^4$-theory in 4 dimensions, $\forall$ divergences in n-point functions are cancelled by $\delta_Z$, $\delta_m$ and $\delta_\lambda$ at any loop order, so that the theory is renormalizable. (Perturbative) renormalizability is generally determined by the mass dimension $[\lambda]$, $(\lambda \sim \text{mass}^{[\lambda]})$ of the coupling.

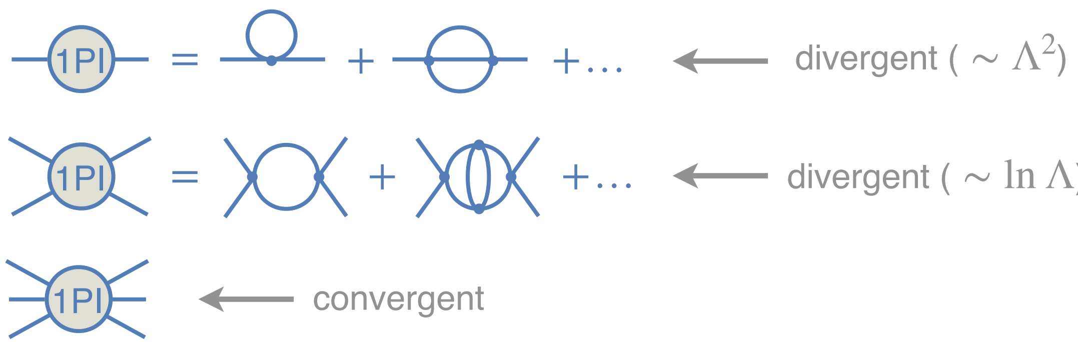
Consider e.g. $\mathscr{L}_{\text{int}} = \dfrac{\lambda}{3!}\phi^3$ in 4 dimensions: $[S] = 0 \;\Rightarrow\; [\mathscr{L}] = 4 \;\Rightarrow [\phi] = 1 \;\Rightarrow\; [\lambda] = 1$

Using NDA, one can show:

$[\lambda] > 0$: super-renormalizable (only few divergent diagrams)




$[\lambda] = 0$: renormalizable (QED, QCD)




$[\lambda] < 0$: non-renormalizable (starting from some loop order, $G_n$ become divergent for all $n$)

Notice: obviously, only a very limited number of possible interactions in 4 dimensions are renormalizable!

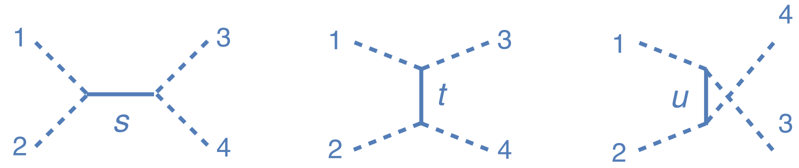
# Example of an EFT

## 3. First example of an EFT

Consider a QFT for two scalar fields ($M \gg m$) interacting with a Yukawa-like coupling:

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2}_{\textit{light}} + \underbrace{\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}M^2\Phi^2}_{\textit{heavy}} - \frac{\lambda}{2}\phi^2\Phi$$

$\phi\phi \to \phi\phi$ scattering at LO (i.e., $\mathcal{O}(\lambda^2)$):




$$i\mathcal{A} = -i\lambda^2\left(\frac{1}{s-M^2} + \frac{1}{t-M^2} + \frac{1}{u-M^2}\right)$$ where $s = (p_1+p_2)^2, \quad t = (p_1-p_3)^2, \quad u = (p_1-p_4)^2$

At low energies ($s \sim t \sim u \sim m^2 \sim E^2 \ll M^2$): $i\mathcal{A} \approx -i\lambda^2\left(-\frac{1}{M^2}\right)\left(3 + \frac{4m^2}{M^2} + \frac{s^2+t^2+u^2}{M^4} + \ldots\right)$

But this looks like the tree-level amplitude obtained from the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{\lambda^2}{8M^2}\phi^4 + \underbrace{\frac{\lambda^2}{8M^4}\phi^2\Box\phi^2 + \ldots}_{\text{an infinite tower of non-renormalizable interactions suppressed by powers of } M}$$

*an infinite tower of non-renormalizable interactions suppressed by powers of $M$*

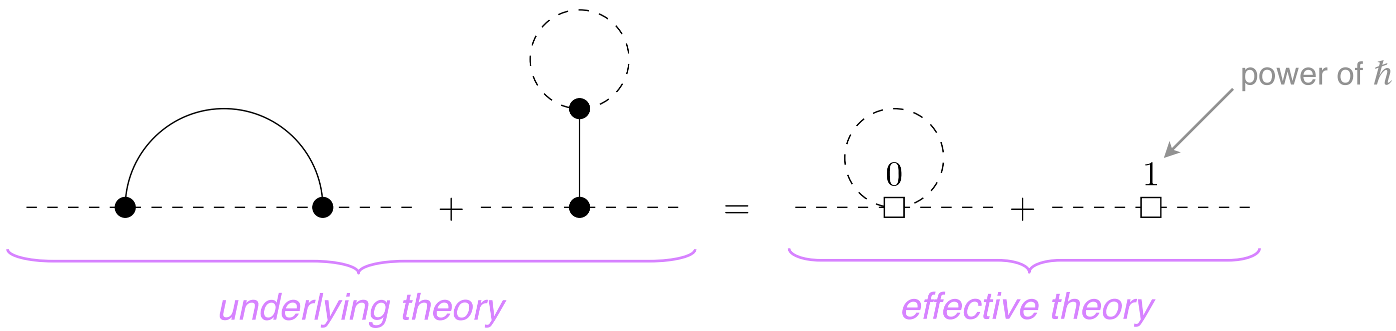
# Example of an EFT

So far, the LECs in $\mathscr{L}_{\text{eff}}$ were determined by matching at tree level. This can be extended to higher orders in $\hbar$ in a systematic way!

$$\mathcal{L}_{\text{eff}} = -\frac{1}{2} l_{\text{kin}} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} l_m^2 \phi^2 - \frac{l_1}{4!} \phi^4 - \frac{l_2}{4} (\partial_\mu \phi)(\partial^\mu \phi) \phi^2 + \ldots$$

Tree level matching: $l_{\text{kin}}^{(0)} = -1, \quad l_m^{(0)} = m^2, \quad l_1^{(0)} = -\frac{3\lambda^2}{M^2}, \quad l_2^{(0)} = -\frac{2\lambda^2}{M^4}$

Matching the 2-point function at the 1-loop order:




Using MS with $\mu = M$, one finds: $l_{\text{kin}}^{(1)} = -\frac{\lambda^2}{32\pi^2 M^2}\left(1 + 5\frac{m^2}{M^2}\right), \quad l_m^{(1)} = -\frac{\lambda^2}{16\pi^2}\left(1 + \frac{m^2}{M^2}\right)$

Notice: all non-analytic terms (like e.g. $\ln m^2/M^2$) are exactly the same on both sides!

Similar matching can be performed for $\geq 2$ point functions…

# Example of an EFT

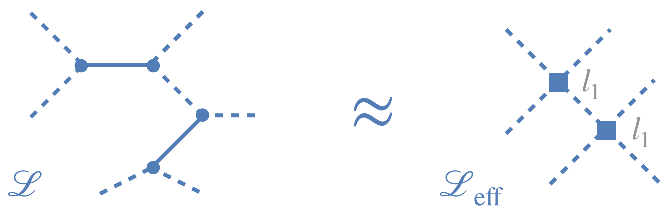
What if we were not able to determine $\mathscr{L}_{\text{eff}}$ by matching (e.g., the underlying theory not known or non-perturbative)?

$\Rightarrow$ write down **all possible terms** in $\mathscr{L}_{\text{eff}}(\phi)$ compatible with the **symmetries** (what not a $\phi^3$-interaction?) and **fix LECs from experimental data**

What about predictive power?

— at tree level, $\mathscr{A}(s,t)$ is determined by a single LEC from $l_1/(4!)\,\phi^4$ (up to corrections $\sim E^2/M^2$)

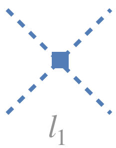
— this interaction also determines the LO contribution to processes with more $\phi$'s, e.g.:



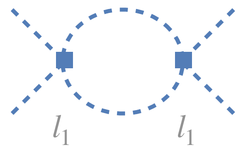

— obviously, contributions of terms with derivatives (e.g., $l_2\phi^2\,\Box\,\phi^2$) are suppressed, at tree level, by powers of $M$ („irrelevant“ interactions). But inside loop diagrams, we integrate over arbitrarily high momenta! Can one expect irrelevant operators be suppressed beyond tree level?

# Example of an EFT

Let's do power counting (NDA):



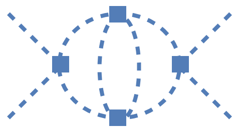

$\sim$ const



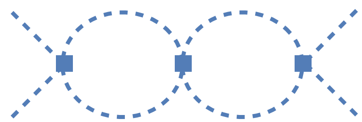

$$\sim \int \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - m^2)((l-p)^2 - m^2)} \sim \mathcal{O}(1)$$

(we count powers of **soft scales** $Q$ like $p \sim m \sim \mu_i$)

Similarly:

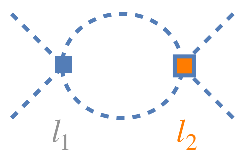


$\sim$



$\sim \mathcal{O}(1)$

On the other hand:




$$\sim \int \frac{d^4 l}{(2\pi)^4} \frac{l^2}{(l^2 - m^2)((l-p)^2 - m^2)} \sim \mathcal{O}(Q^2)$$

(after renormalization!)

The suppression appears automatically using DR, but it also holds in general (e.g., using $\Lambda$) for proper renormalization conditions (all subtraction scales $\mu_i \sim Q$).

$\Rightarrow$ power counting:

LO ( $\sim Q^0$): $\forall$ diagrams made out of $l_1$-vertices

NLO ( $\sim Q^2$): $\forall$ diagrams made out of $l_1$-vertices and 1 insertion of dim-6 vertex ($l_2$)

…

The birth of ChPT (and an EFT in general): Steven Weinberg, *Phenomenological Lagrangians*, Physica A96 (79) 327 (about 3700 citations…)

# EFT vs Multipole Expansion

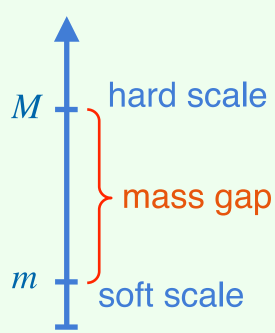
## Effective Field Theory

- Most general effective Lagrangian for light DoF compatible with the symmetries of the underlying theory

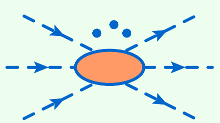
$$\mathcal{L}_{\rm eff} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{l_1}{4!}\phi^4 - \frac{l_2}{4}(\partial_\mu\phi)(\partial^\mu\phi)\phi^2 + \ldots$$

- The size of (renormalized) LECs governed by the hard scale M. LECs carry information about short-range dynamics. They can be calculated from matching or determined from experiment

- Separation of scales: [soft] $Q \sim m \ll M$ [hard]




- Energy expansion of the amplitude (Feynman graphs, power counting, renormalization)



## Electric potential

Most general expression for the electric potential (rotational invariance)

LECs (multipoles) governed by the size $a$ of $\rho(\vec{r})$, they can be calculated or determined from exp.

[soft] $1/R \ll 1/a$ [hard]

Multipole expansion for $V(\vec{R})$ in powers of $a/R$

# The principles of an EFT

## Construction of QFTs (~1930 … 1980)

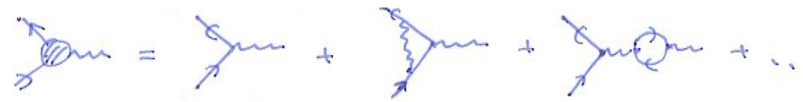
1) Construct the action respecting some symmetries. E.g., gauge invariance of QED:

$$\Psi \to \Psi' = e^{-i\alpha(x)}\Psi, \quad A_\mu \to A'_\mu = A_\mu - \partial_\mu \alpha(x)$$

2) Retain only renormalizable interactions ($D \leq 4$), e..g. in QED:

$$\underbrace{\bar{\Psi}\gamma_\mu(\partial^\mu + ieA^\mu)\Psi,}_{D\,=\,4} \qquad \underbrace{\cancel{\bar{\Psi}S_{\mu\nu}\Psi F^{\mu\nu},}}_{D\,=\,5} \qquad \underbrace{\cancel{(F_{\mu\nu}F^{\mu\nu})^2,}}_{D\,=\,8} \qquad \ldots$$

3) Quantize, compute the amplitude



4) Fix parameters from data (in QED, only $e$ and fermion masses) and make predictions…

## Modern view is based on Weinberg's Theorem:

„if one writes down the most general possible Lagrangian, including all terms consistent with the assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition, and the assumed symmetry properties"

S. Weinberg, Physica 96A (1979) 327; see also H. Leutwyler, Annals Phys. (1994) 165

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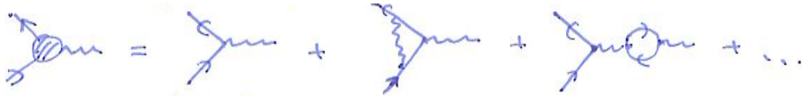

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# Summary

**This lecture:**

— general introduction, literature, syllabus, …

— the idea of an effective theory using classical multipole expansion as an example

— a quick reminder on QFT (LSZ formula, Feynman diagrams, trees and loops, UV divergences and their origin, regularization, renormalization, counter terms, … )

— first example of an EFT (effective Lagrangian, LECs, power counting, …)

**Coming next:**

— Chiral Perturbation Theory as an EFT of QCD