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TU Darmstadt, September 26-30, 2022

Chiral effective field theory for nuclear forces

Summary day 1

— a key idea of any effective theory: exploit a scale separation to construct a systematically improvable approximation to the underlying theory in a given region of the parameter space (typically, low energies)

— can be applied to both classical and quantum systems

— construction of EFTs: Weinberg's theorem

Day 2: Intro to Chiral Perturbation Theory

Weinberg, Gasser, Leutwyler, Bernard, Kaiser, Meißner, Bijnens, …

Selected review articles

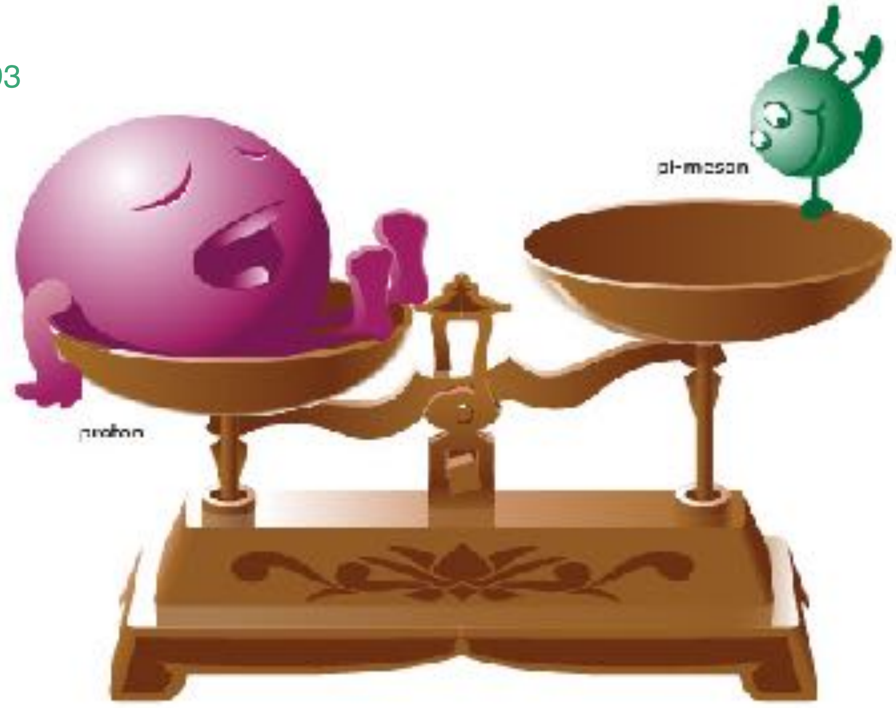
- Bernard, Kaiser, Meißner, Int. J. Mod. Phys. E4 (1995) 193
- Pich, Rep. Prog. Phys. 58 (1995) 563
- Bernard, Prog. Part. Nucl. Phys. 60 (2007) 82
- Scherer, Prog. Part. Nucl. Phys. 64 (2010) 1

Lecture notes

- Scherer, Adv. Nucl. Phys. 27 (2003) 277
- Gasser, Lect. Notes Phys. 629 (2004) 1

Text book

- Scherer, Schindler, A Primer for Chiral Perturbation Theory, Springer, Lecture Notes in Physics, 2012



Day 2: Intro to Chiral Perturbation Theory

Weinberg, Gasser, Leutwyler, Bernard, Kaiser, Meißner, Bijnens, …

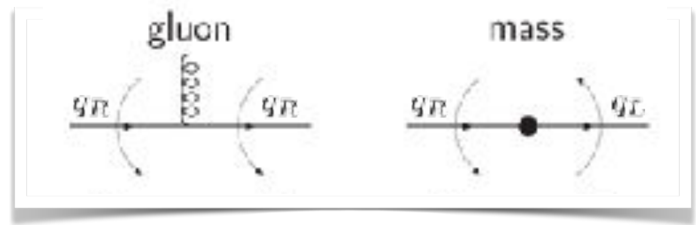
1. Effective Lagrangian for pions
2. From effective Lagrangian to S-matrix
3. Inclusion of nucleons
4. Beyond the heavy-baryon approach
5. Summary of day 2

Effective Lagrangian for pions

1. Effective Lagrangian for pions

- **QCD and chiral symmetry**

$$\mathcal{L}_{\mathrm{QCD}} = \underbrace{\bar{q}(i\not{D} - \mathcal{M})q}_{\underbrace{\bar{q}_L i\not{D} q_L + \bar{q}_R i\not{D} q_R}_{\mathrm{SU(2)_L \times SU(2)_R\ invariant}} - \underbrace{\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M} q_L}_{\text{break chiral symmetry}}} - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$



Light quark masses ($\overline{\mathrm{MS}},\ \mu = 2\ \mathrm{GeV}$):

$$\boxed{\begin{array}{rcl} m_u &=& 1.8 \ldots 2.8\ \mathrm{MeV} \\ m_d &=& 4.3 \ldots 5.2\ \mathrm{MeV} \end{array} \ll \Lambda_{QCD}}$$

⟶ $\mathcal{L}_{\mathrm{QCD}}$ is approx. $SU(2)_L \times SU(2)_R$ invariant

spontaneous breakdown to $SU(2)_V \subset SU(2)_L \times SU(2)_R$ ⟶ Goldston Bosons (pions)

- **Chiral perturbation theory**

  - Ideal world [$m_u = m_d = 0$], zero-energy limit: non-interacting massless GBs (+ strongly interacting massive hadrons)
  - Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy: weakly interacting light GBs (+ strongly interacting massive hadrons)

⟶ expand about the ideal world (ChPT)

Effective Lagrangian for pions

Pions transform linearly under isospin (isotriplet): $|\pi_1\rangle = \frac{|\pi^+\rangle - |\pi^-\rangle}{\sqrt{2}}, \quad |\pi_2\rangle = \frac{|\pi^+\rangle + |\pi^-\rangle}{\sqrt{2}i}, \quad |\pi^3\rangle = |\pi^0\rangle$

Pions have to transform nonlinearly under chiral rotations
($SU(2)_L$ x $SU(2)_R$ ~ SO(4) $\longrightarrow$ pion fields as coordinates on a 4-dimentional sphere)

Nonlinear field redefinitions of the kind $\vec{\pi} \to \vec{\pi}' = \vec{\pi} F[\vec{\pi}], \; F[0]=1$ do not change physics
$\longrightarrow$ all nonlinear realizations of χ symmetry are equivalent $\longrightarrow$ use most convenient one!
Haag '58; Coleman, Callan, Wess, Zumino '69

Example of an explicit construction:

Infinitesimal SO(4) rotation of the 4-vector $(\pi_1, \pi_2, \pi_3, \sigma)$:
$$\begin{pmatrix} \boldsymbol{\pi} \\ \sigma \end{pmatrix} \xrightarrow{SO(4)} \begin{pmatrix} \boldsymbol{\pi}' \\ \sigma' \end{pmatrix} = \left[\mathbf{1}_{4\times 4} + \sum_{i=1}^{3} \theta_i^V V_i + \sum_{i=1}^{3} \theta_i^A A_i\right] \begin{pmatrix} \boldsymbol{\pi} \\ \sigma \end{pmatrix}$$

where:
$$\sum_{i=1}^{3} \theta_i^V V_i = \begin{pmatrix} 0 & -\theta_3^V & \theta_2^V & 0 \\ \theta_3^V & 0 & -\theta_1^V & 0 \\ -\theta_2^V & \theta_1^V & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \sum_{i=1}^{3} \theta_i^A A_i = \begin{pmatrix} 0 & 0 & 0 & \theta_1^A \\ 0 & 0 & 0 & \theta_2^A \\ 0 & 0 & 0 & \theta_3^A \\ -\theta_1^A & -\theta_2^A & -\theta_3^A & 0 \end{pmatrix}$$

Switch to nonlinear realization: only 3 out of 4 components of the vector $(\boldsymbol{\pi}, \sigma)$ are independent, i.e. $\boldsymbol{\pi}^2 + \sigma^2 = F^2$

$$\boldsymbol{\pi} \xrightarrow{\theta^V} \boldsymbol{\pi}' = \boldsymbol{\pi} + \boldsymbol{\theta}^V \times \boldsymbol{\pi}\,, \qquad \longleftarrow \textit{linear under } \vec{\theta}^V$$
$$\boldsymbol{\pi} \xrightarrow{\theta^A} \boldsymbol{\pi}' = \boldsymbol{\pi} + \boldsymbol{\theta}^A \sqrt{F^2 - \boldsymbol{\pi}^2} \qquad \longleftarrow \textit{nonlinear under } \vec{\theta}^A$$

Effective Lagrangian for pions

Can be rewritten in terms of a 2 x 2 matrix:

$$U = \frac{1}{F}(\sigma\,\mathbf{1}_{2\times 2} + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}) \xrightarrow{\textit{nonlinear realization}} U = \frac{1}{F}\left(\sqrt{F^2-\boldsymbol{\pi}^2}\,\mathbf{1}_{2\times 2} + i\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right)$$

Chiral rotations: $U \longrightarrow U' = LUR^\dagger$ with $L = \exp[-i(\boldsymbol{\theta}^V - \boldsymbol{\theta}^A)\cdot\boldsymbol{\tau}/2]$, $R = \exp[-i(\boldsymbol{\theta}^V + \boldsymbol{\theta}^A)\cdot\boldsymbol{\tau}/2]$

Derivative expansion for the effective Lagrangian $\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$

0 derivatives: $UU^\dagger = U^\dagger U = 1$ - irrelevant $\longleftarrow$ only derivative couplings of GBs

2 derivatives: $\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \overset{g\in G}{\longrightarrow} \mathrm{Tr}(L\partial_\mu U R^\dagger R \partial^\mu U^\dagger L^\dagger) = \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$

$\longrightarrow \mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)$

derivatives act only on the next U

4 derivatives: $[\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger)]^2$, $\mathrm{Tr}(\partial_\mu U \partial_\nu U^\dagger)\mathrm{Tr}(\partial^\mu U \partial^\nu U^\dagger)$, $\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger)$

(terms with $\partial_\mu\partial_\nu U$, $\partial_\mu\partial_\nu\partial_\rho U$, $\partial_\mu\partial_\nu\partial_\rho\partial_\sigma U$ can be eliminated via EOM/partial integration)

…

Chiral symmetry breaking terms

$\delta\mathcal{L}_{\rm QCD} = -\bar{q}_L \mathcal{M} q_R - \bar{q}_R \mathcal{M}^\dagger q_L$ can be made χ-invariant by requiring: $\mathcal{M} \to L\mathcal{M}R^\dagger$

$\longrightarrow$ construct all possible χ-invariant terms involving $\mathcal{M}$ and freeze out $\mathcal{M}$ at the end

LO term: $\delta\mathcal{L}_{\rm SB} = \frac{BF^2}{2}\mathrm{Tr}[\mathcal{M}U^\dagger + U\mathcal{M}^\dagger] = 2BF^2 m_q - Bm_q\vec{\pi}^2 + \ldots \longrightarrow M_\pi^2 = 2m_q B + \mathcal{O}(m_q^2)$

Effective Lagrangian for pions

The leading and subleading effective Lagrangians for pions

$$
\begin{aligned}
\mathcal{L}_\pi^{(2)} &= \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + 2B(\mathcal{M}U + \mathcal{M}U^\dagger)\rangle\,, \\
\mathcal{L}_\pi^{(4)} &= \frac{l_1}{4}\langle \partial_\mu U \partial^\mu U^\dagger\rangle^2 + \frac{l_2}{4}\langle \partial_\mu U \partial_\nu U^\dagger\rangle\langle \partial^\mu U \partial^\nu U^\dagger\rangle + \frac{l_3}{16}\langle 2B\mathcal{M}(U+U^\dagger)\rangle^2 + \ldots \\
&- \frac{l_7}{16}\langle 2B\mathcal{M}(U-U^\dagger)\rangle^2
\end{aligned}
$$

terms involving external fields

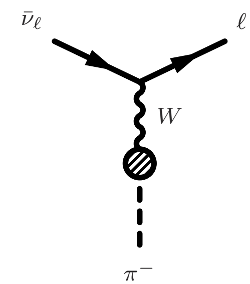
Gasser, Leutwyler ’84

Low-energy constants of $\mathcal{L}_\pi^{(2)}$

- F is related to the pion decay constant F_π: $\langle 0|J^i_{A_\mu}(0)|\pi^j(\vec{p})\rangle = ip_\mu F_\pi \delta^{ij}$

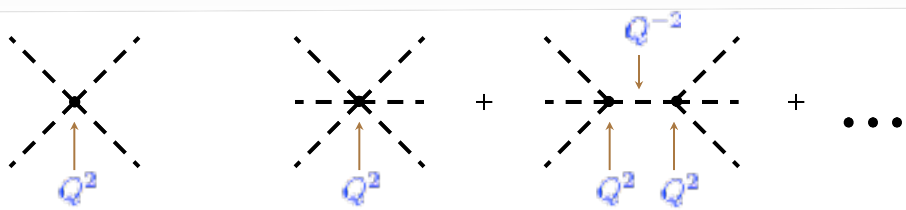
  axial current from $\mathcal{L}_\pi^{(2)}$: $J^i_{A_\mu} = i\mathrm{Tr}[\tau^i(U^\dagger\partial_\mu U - U\partial_\mu U^\dagger)] = -F\partial_\mu\pi^i + \ldots$

  $\longrightarrow$ F is F_π in the chiral limit: $F_\pi = F + \mathcal{O}(m_q) \simeq 92.4\ \mathrm{MeV}$
- B is related to the chiral quark condensate



Tree-level multi-pion connected diagrams from $\mathcal{L}_\pi^{(2)}$

$$
U(\boldsymbol{\pi}) = \mathbf{1}_{2\times 2} + i\frac{\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F} - \frac{\boldsymbol{\pi}^2}{2F^2} - i\alpha\frac{\boldsymbol{\pi}^2\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{F^3} + \mathcal{O}(\boldsymbol{\pi}^4) \longrightarrow \mathcal{L}_\pi^{(2)} = \frac{\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}}{2} - \frac{M^2\boldsymbol{\pi}^2}{2} + \frac{(\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\pi})^2}{2F^2} - \frac{M^2\boldsymbol{\pi}^4}{8F^2} + \ldots
$$



- all diagrams scale as Q^2
- insertions from $\mathcal{L}_\pi^{(4)}$, $\mathcal{L}_\pi^{(6)}$, ... suppressed by powers of Q^2
- remarkable predictive power!

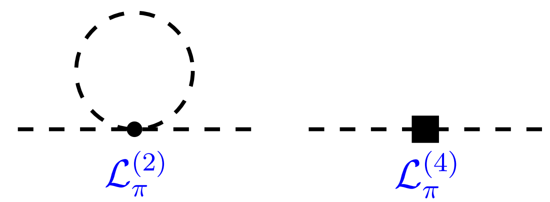
From effective Lagrangian to S-matrix

2. From effective Lagrangian to S-matrix

Tree-level diagrams with higher-order vertices are suppressed at low energy.

The argument can be generalized to quantum corrections (loops) $\longrightarrow$ ChPT Weinberg '79

Typical example of a loop integral:



$$I = \int \frac{d^4 l}{(2\pi)^4} \frac{i}{l^2 - M^2 + i\epsilon} \xrightarrow{\text{DR}} \mu^{4-d} \int \frac{d^d l}{(2\pi)^d} \frac{i}{l^2 - M^2 + i\epsilon}$$

$$= \frac{M^2}{16\pi^2} \ln\left(\frac{M^2}{\mu^2}\right) + 2M^2 L(\mu) + \ldots$$

terms vanishing in d=4

The infinite quantity $L(\mu) = \frac{\mu^{d-4}}{16\pi^2}\left(\frac{1}{d-4} + \text{const}\right)$ can be absorbed into l_i's of $\mathcal{L}_\pi^{(4)}$: $l_i \to l_i^{\text{r}}(\mu)$

The bottom line: after renormalization, all momenta flowing through loop graphs are soft, $\sim Q$

Power counting (Naive Dimensional Analysis)

- pion propagators: $1/(p^2 - M_\pi^2) \sim 1/Q^2$
- momentum integrations: $d^4 l \sim Q^4$
- delta functions: $\delta^4(p - p') \sim 1/Q^4$
- derivatives: $\partial_\mu \sim Q$

# of loops

# of vertices with d derivatives

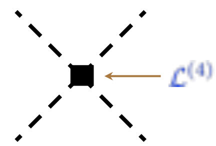
$$D = 2 + 2L + \sum_d N_d (d - 2)$$

power of the soft scale Q for a given diagram

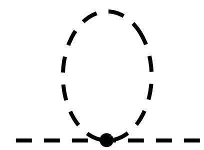
From effective Lagrangian to S-matrix

Examples:

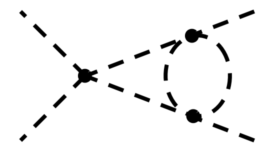
$$\boxed{D = 2 + 2L + \sum_d N_d(d-2)}$$



$D = 2 + 0 + 2 = 4$



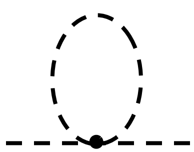
$D = 2 + 2 + 0 = 4$



$D = 2 + 4 + 0 = 6$

Scattering amplitude is obtained via an expansion in Q/Λ_χ. What is the value of Λ_χ?

- Chiral expansion breaks down for $E \sim M_\rho \;\rightarrow\; \Lambda_\chi \sim M_\rho = 770$ MeV
- An upper bound for Λ_χ from pion loops: $\Lambda_\chi \sim 4\pi F_\pi$ Manohar, Georgi '84



$$\frac{M^2}{F^2}\int \frac{d^4 l}{(2\pi)^4}\frac{i}{l^2 - M^2 + i\epsilon} \xrightarrow{\text{DR}} M^2 \frac{M^2}{(4\pi F)^2}\left[\ln\frac{M^2}{\mu^2} + 2\mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

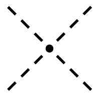
angular integration in 4 dimensions

dimensional arguments

$$\int \frac{d^d l}{(2\pi)^d} = \int \frac{d\Omega_d}{(2\pi)^d}\int l^{d-1} dl = \frac{1}{2^{d-1}\,\pi^{d/2}\,\Gamma(d/2)}\int l^{d-1} dl \xrightarrow{d\to 4} \frac{2}{(4\pi)^2}\int l^3 dl$$

Pion scattering lengths in ChPT

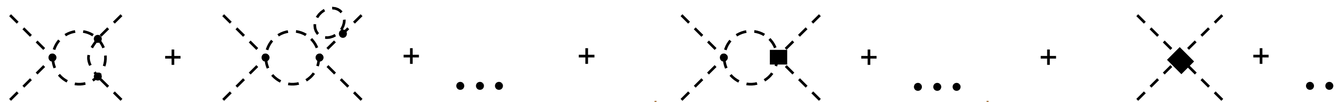
Q^2 :



Q^4 :

$$\underbrace{\qquad\qquad}_{\text{1-loop, all vertices from } \mathcal{L}_\pi^{(2)}} \qquad \underbrace{\qquad\qquad}_{\text{tree, 1 insertion from } \mathcal{L}_\pi^{(4)}}$$

Q^6 :



$$\underbrace{\qquad\qquad}_{\text{2-loops, all vertices from } \mathcal{L}_\pi^{(2)}} \qquad \underbrace{\qquad\qquad}_{\text{1-loop, 1 insertion from } \mathcal{L}_\pi^{(4)}} \qquad \underbrace{\qquad\qquad}_{\substack{\text{tree, 1 insertion from } \mathcal{L}_\pi^{(6)} \\ \text{2 insertions from } \mathcal{L}_\pi^{(4)}}}$$

Predictive power?

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \ldots$$

\# of LECs increasing…

S-wave $\pi\pi$ scattering length

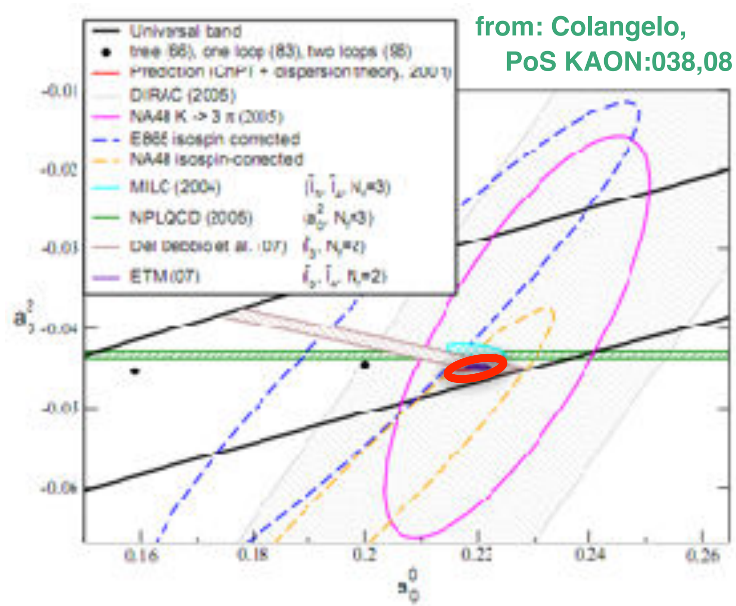
LO: $a_0^0 = 0.16$ [Weinberg '66]

NLO: $a_0^0 = 0.20$ [Gasser, Leutwyler '83]

NNLO: $a_0^0 = 0.217$ [Bijnens et al. '95]

NNLO + disp. relations: [Colangelo et al.]

$$a_0^0 = 0.217 \pm 0.008\,(\mathrm{exp}) \pm 0.006\,(\mathrm{th})$$



from: Colangelo, PoS KAON:038,08

Summary of Mesonic ChPT

- QCD features approximate SU(2)$_L$ x SU(2)$_R$ symmetry spontaneously broken down to SU(2)$_V$. Pions are Goldstone Bosons of the broken axial generators.

- ChPT = EFT to describe QCD at low energy using GBs & matter fields as DOF. It provides perturbative (GBs!) expansion of the amplitude in powers of p ~ M_π over Λ_χ ~ M_ρ ~ 1 GeV.

- $\mathcal{L}_{\rm eff}$ for GBs: Expansion in ∂_μ and M_π, $M_\pi^2 = 2Bm_q + \mathcal{O}(m_q^2)$; pions transform nonlinearly under SU(2)$_L$ x SU(2)$_R$ [CCWZ]: $U \to LUR^\dagger$.

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_\pi^{(2)} = \underbrace{\frac{F^2}{4}}_{\text{LECs}} \mathrm{Tr}(\partial_\mu U \partial^\mu U) + \frac{F^2}{2} \underbrace{B}_{\text{LECs}} \mathrm{Tr}(\underbrace{\mathcal{M}}_{= \mathrm{diag}(m_q)} U + \mathcal{M} U^\dagger)$$

- Use DR to calculate Feynman diagrams. Connected diagrams scale as q^ν,

$$\boxed{\nu = 2 + 2\underbrace{L}_{\text{\# of loops}} + \sum_i \underbrace{V_i}_{\text{\# of vertices of type } i} \Delta_i\,, \qquad \Delta_i = \underbrace{d_i}_{\text{\# of derivatives \& } M_\pi} - 2}$$

- At each order q^ν, a finite number of LECs contribute (exp., lattice, models, …)

Inclusion of nucleons

3. Inclusion of the nucleons

In the baryon sector, it is more convenient to work with u defined via $U =: u^2$. Then:

$$u \longrightarrow u' = \sqrt{RUL^\dagger} =: RuK^{-1} \quad \Rightarrow \quad K = (\sqrt{RUL^\dagger})^{-1} R\sqrt{U}$$

Notice: the transformation property of u can also be written as $u \longrightarrow u' = KuL^\dagger$.

The so-called compensator field K is a complicated SU(2)-valued function of θ^L, θ^R, U (and thus of space-time), $K = K(L,R,U)$, except for isospin (i.e. vector) rotations with $\theta^L = \theta^R = \theta^V$:

$$K(V,V,U) = V$$

Then, one **defines** the transformation properties of the nucleon fields via: $N \longrightarrow N' = KN$

The Coleman-Callan-Wess-Zumino (CCWZ) nonlinear realization of the chiral group:

$$\boxed{\begin{pmatrix} U \\ N \end{pmatrix} \xrightarrow{g} \begin{pmatrix} U' \\ N' \end{pmatrix} = \begin{pmatrix} RUL^\dagger \\ K(L,R,U)N \end{pmatrix}}$$

[see e.g. the book by Scherer and Schindler]

To construct the effective Lagrangian, one uses building blocks which transform covariantly with respect to $SU(2)_R \times SU(2)_L$

$$N \longrightarrow N' = KN, \quad O_i \longrightarrow O_i' = KO_iK^{-1} = KO_iK^\dagger$$

to write terms like: $\bar{N}O_1 \ldots O_n N \;\; \mathrm{Tr}\,(O_{n+1} \ldots O_m) \;\; \ldots \;\; \mathrm{Tr}\,(O_{m+1} \ldots O_k)$

Inclusion of nucleons

Covariant derivatives of the nucleon and pion fields:

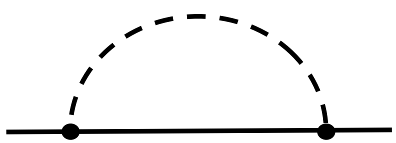
$$D_\mu N := (\partial_\mu + \Gamma_\mu)\, N, \qquad \Gamma_\mu := \frac{1}{2}\left(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right)$$

$$u_\mu := i u^\dagger \left(\partial_\mu U\right) u^\dagger$$

$$\rightarrow \quad \mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \ldots\,, \qquad \mathcal{L}_{\pi N}^{(1)} = \bar{N}\left(i\gamma^\mu D_\mu - m + \frac{\mathring{g}_A}{2}\gamma^\mu \gamma_5 u_\mu\right) N$$

$$g_A = G_A(0): \quad \langle N(p')|A_i^\mu(0)|N(p)\rangle =: \bar{u}(p')\left[\gamma^\mu G_A(t) + \frac{(p'-p)^\mu}{2m} G_p(t)\right]\gamma_5 \frac{\tau_i}{2} u(p)$$

Problem (?): new hard mass scale m $\rightarrow$ power counting ??



$$\rightarrow \quad \delta m \xrightarrow{M \to 0} -m\frac{3g_A^2 m^2}{(4\pi F)^2}\left[\log\frac{m}{\mu} + \mu^{d-4}\left(\frac{1}{d-4} + \text{const}\right)\right]$$

$$\frac{m_N}{4\pi F_\pi} \sim 1$$

divergence has to be absorbed by m from the LO Lagrangian…

Inclusion of nucleons

Making power counting manifest: The Heavy-Baryon approach

Jenkins & Manohar '91; Bernard, Keiser, Meißner '92; Mannel, Roberts, Ryzak '92

Write the nucleon momentum as $p^\mu = m v^\mu + l^\mu$ with $v^2 = 1$ and $l_\mu \ll m$

Split the nucleon fields into $N_v = e^{imv\cdot x} P_v^+ N, \;\; h_v = e^{imv\cdot x} P_v^- N$ with $P_v^\pm := \dfrac{1 \pm \not{v}}{2}$

For the free Dirac Lagrangian, one then obtains:

$$\bar N (i\not{\partial} - m) N = \ldots = \bar N_v i v\cdot\partial N_v - \bar h_v (i v\cdot\partial + 2m) h_v + \underbrace{\bar N_v i\partial_\perp h_v + \bar h_v i \partial_\perp N_v}_{A_\perp := A - (v\cdot A) v}$$

$\longrightarrow$ the small component h_v behaves as a heavy field and can be integrated out:

$$\boxed{\mathcal{L}_{\pi N}^{(1)} = \bar N_v \left(i v\cdot D + g_A S\cdot u\right) N_v \; + \; \mathcal{O}(m^{-1})}$$ where $S_\mu \equiv i \gamma_5 \sigma_{\mu\nu} v^\nu$

The HB propagator of the nucleon $S(p) = \dfrac{i}{v\cdot p + i\epsilon}$ has an obvious interpretation:

$$S(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p_0 + i\epsilon} e^{-ip\cdot(x-y)} = \theta(x_0 - y_0)\delta^3(\vec x - \vec y)$$

Inclusion of nucleons

In the HB formulation, the nucleon mass does not appear in the nucleon propagator and contribute only through 1/m corrections to vertices $\Rightarrow$ power counting is manifest!

It is easy to derive the power counting formula for the scattering amplitude $T \sim Q^D$:

number of internal pion & nucleon lines

$$D \;=\; \sum_i V_i^\pi d_i \;+\; \sum_i V_i^N d_i \;-\; 2I_\pi \;-\; I_N \;+\; 4L$$

number of loops

number of vertices from $\mathscr{L}_\pi^{(d_i)}$

number of vertices from $\mathscr{L}_{\pi N}^{(d_i)}$

To eliminate I_π, use $L = I_\pi + I_N - \left[\sum_i (V_i^\pi + V_i^N) - 1\right]$. For connected processes with a single nucleon, one can eliminate I_N using $I_N = \sum_i V_i^N - 1$. This then leads to:

$$D \;=\; 1 \;+\; 2L \;+\; \sum_i V_i^\pi (d_i - 2) \;+\; \sum_i V_i^N \underbrace{(d_i - 1)}_{(d_i + n_i/2 - 2),\ \text{where } n_i = \#\text{ of N-fields}}$$

$$\Rightarrow \quad \boxed{D \;=\; 1 \;+\; 2L \;+\; \sum_i V_i \Delta_i, \qquad \Delta_i = -2 + \frac{n_i}{2} + d_i}$$

Chiral expansion of m_N in the HB approach

Dressed HB propagator: $\dfrac{i}{v\cdot k - \Sigma(k) + i\epsilon}$ $\longleftarrow$ recall the definition of the HB momentum: $p = mv + k$
$\Rightarrow \quad k = (m_N - m)v$

Up to the order Q³, the physical mass ($p =: m_N v$) is given by $m_N = m + \Sigma(0)$.

Chiral expansion of the nucleon self energy: $\Sigma(k) =$  $+$  $+ \ldots$

$$\underbrace{-4c_1M^2 - \vec{k}^{\,2}/(2m)}_{Q^2} \qquad \underbrace{\Sigma_{\rm loop}(k) =: -3g_A^2/(4F^2)\, I(k)}_{Q^3} \qquad \underbrace{}_{Q^4}$$

$$I(0) = -i\int\frac{d^4l}{(2\pi)^4}\frac{\vec{l}\cdot\vec{l}}{(l_0+i\epsilon)(l^2-M^2+i\epsilon)} \xrightarrow{\text{DR}} \frac{M^3}{8\pi} \quad\Rightarrow\quad m_N = \underbrace{m - 4c_1M^2}_{\text{not fixed by } \chi\text{-symmetry}} \underbrace{-\frac{3g_A^2}{32\pi F^2}M^3}_{\text{ChPT prediction}} + \ldots$$

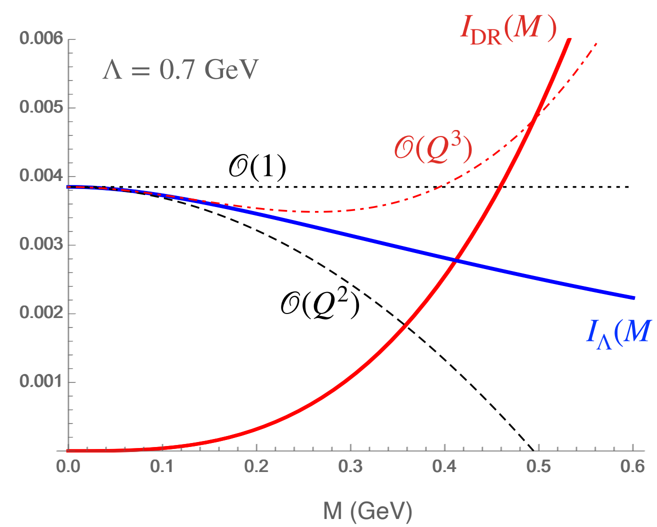
How about cutoff regularization?

Donoghue, Holstein '98

$$I_\Lambda(0) = -i\int\frac{d^4l}{(2\pi)^4}\frac{\vec{l}\cdot\vec{l}\; e^{-\vec{l}^{\,2}/\Lambda^2}}{(l_0+i\epsilon)(l^2-M^2+i\epsilon)}$$

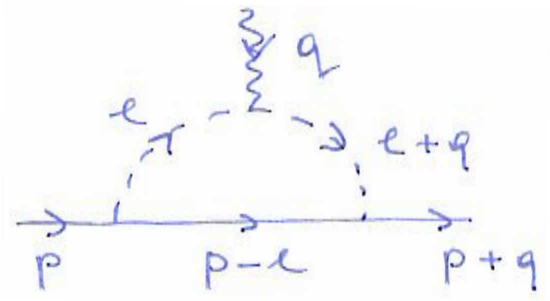
$$= \frac{1}{16\pi^{3/2}}\left[\Lambda^3 - 2\Lambda M^2 + 2\sqrt{\pi}M^3\, e^{\frac{M^2}{\Lambda^2}}\,\text{erfc}\left(\frac{M}{\Lambda}\right)\right]$$

$$\xrightarrow{\Lambda\to\infty} \underbrace{\alpha\Lambda^3 + \beta\Lambda M^2}_{\substack{\text{can be absorbed in} \\ m(\Lambda),\ c_1(\Lambda)}} + \frac{M^3}{8\pi} + \mathcal{O}(\Lambda^{-1})$$



Limitations of the HB approach

4. Beyond the HB approach

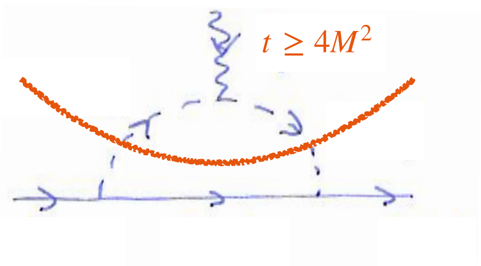


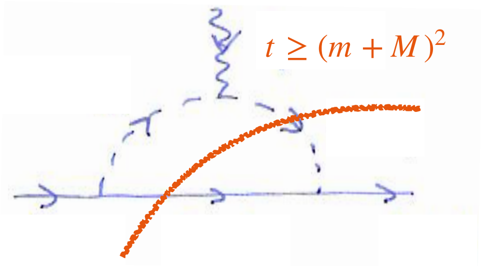
Consider the scalar form factor of the nucleon. Using relativistic nucleon propagators (i.e., no 1/m-expansion) and ignoring the vertex structure, the amplitude is:

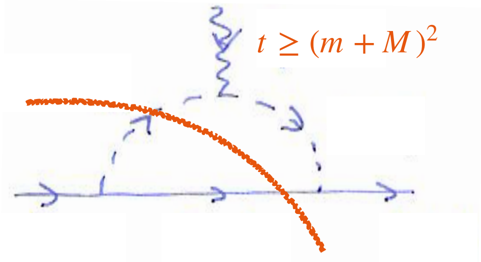
$$J = i \int \frac{d^4 l}{(2\pi)^4} \frac{1}{[l^2 - M^2 + i\epsilon][(l+q)^2 - M^2 + i\epsilon][(p-l)^2 - m^2 + i\epsilon]}$$

For on-shell nucleons ($p^2 = (p+q)^2 = m^2$), we have $J = J(t)$ with $t = q^2$.

The amplitude develops an imaginary part when two intermediate particles can become on-shell:







Instead of calculating $J(t)$ (tedious…), one can compute $\mathrm{Im}[J(t)]$ using Cutkosky cutting rules (i.e., replace the cut propagators by the δ-functions enforcing the on-shell relation, see, e.g., Schröder & Peskin)

Limitations of the HB approach

For $t \geq 4M^2$ (1st graph): $$\mathrm{Im}[J(t)] = \frac{1}{8\pi}\frac{1}{\sqrt{t(4m^2-t)}}\arctan\left[\frac{\sqrt{(t-4M^2)(4m^2-t)}}{t-2M^2}\right]$$

We now perform the $1/m$-expansion of this result for $t = \mathcal{O}(M^2) \equiv \mathcal{O}(Q^2)$:

$$\frac{\sqrt{(t-4M^2)(4m^2-t)}}{t-2M^2} \sim \mathcal{O}\left(\frac{m}{Q}\right) \gg 1 \quad \Rightarrow \quad \arctan(x) \xrightarrow{x\gg 1} \underbrace{\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} + \ldots}_{\text{converges for } x > 1}$$

This expansion (+ pre-factor) is the HB expansion: $$\mathrm{Im}[J(t)] \overset{\mathrm{HB}}{=} \frac{1}{8\pi}\frac{1}{2m\sqrt{t}}\frac{\pi}{2} + \mathcal{O}(m^0)$$

Problem: it breaks down in the vicinity of $t \approx 4M^2$: $$x < 1 \;\Rightarrow\; 4M^2 \leq t \lesssim 4M^2\left(1 + \frac{M^2}{m^2}\right)$$

Solution 1: Infrared Regularization Becher, Leutwyler '99

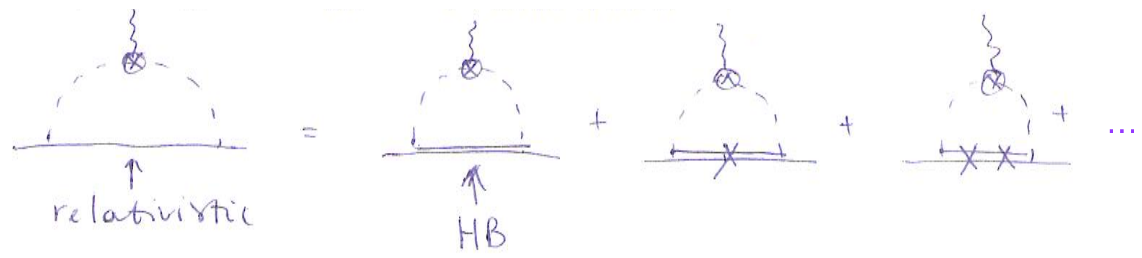
$$i\frac{\not{p}+m}{p^2-m^2+i\epsilon} = i\frac{\not{p}+m}{2mv\cdot l + l^2 + i\epsilon} = i\frac{\not{p}+m}{2mv\cdot l + i\epsilon}\frac{1}{1+\frac{l^2}{2mv\cdot l + i\epsilon}}$$

$$= \frac{\not{p}+m}{2m}\frac{i}{v\cdot l + i\epsilon}\left[1 + i\underbrace{\frac{l^2}{2m}}_{\text{vertex from } \mathscr{L}_{\pi N}^{(2)}}\frac{i}{v\cdot l + i\epsilon} + \left(i\frac{l^2}{2m}\frac{i}{v\cdot l + i\epsilon}\right)^2 + \ldots\right]$$

use: $p = mv + l$

Limitations of the HB approach

The IR approach: expand the integrand in 1/m, compute the integrals using DR (power counting manifest) and resum all contributions… Schindler et al.'03



(At the 1-loop level can also be realized by selecting out the infrared singular parts of the integrals Becher, Leutwyler '99.)

Disadvantage: violates the analytic structure of the amplitude (for hard momenta).

Solution 1: The Extended On-Mass-Shell (EOMS) Renormalization scheme

Gegelia, Japaridze '99; Fuchs et al. '03

Main idea: work in the manifestly covariant approach and restore chiral power counting by using appropriate renormalization conditions (i.e., perform additional finite subtractions of PC violating terms involving positive powers of the nucleon mass)

This is nowadays the standard approach for baryon ChPT…

(Some) topics in and beyond ChPT

- **Resummation of leading Log's**

  Weinberg, Bijnens, Colangelo, Bissiger, Fuhrer, Kivel, Polyakov, Vladimirov, …

  Leading logs can be computed for higher loops, all orders possible in certain cases

- **Combining ChPT and dispersion theory**

  Colangelo, Gasser, Leutwyler, Bernard, Meißner, Descotes Genon, Knecht, Pelaez, Hoferichter, Kubis, Ruiz de Elvira, …

- **Covariant baryon ChPT**

  Becher, Leutwyler, Bernard, Meißner, Kubis, Gegelia, Scherer, Camalich, Geng, Ren, …

- **ChPT with explicit spin-3/2 degrees of freedom**

  Hemmert, Bernard, Fettes, Meißner, Pascalutsa, Vanderhaeghen, Kaiser, Gegelia, EE, Gasparyan, Krebs, Siemens, …

  $\Delta(1232)$ has low excitation energy ~ 300 MeV $\rightarrow$ better to include as an explicit DOF…

- **ChPT and/for lattice QCD**

  Colangelo, Beane, Savage, Jiang, Tiburzi, Procura, Weise, Walker Loud, Bernard, Meißner, Rusetsky, Hemmert, …

  Chiral extrapolations, finite volume corrections, quenched ChPT, …

- **Unitarized ChPT and resonance physics**

  Oeller, Meißner, Dobado, Pelaez, Oset, Hanhart, Llanes-Estrada, Kaiser, Weise, ,…

Summary of part I

- QCD is approximately $SU(2)_L \times SU(2)_R$ invariant. The chiral symmetry is spontaneously broken down to $SU(2)_V$ (isospin group). Pions are Goldstone Bosons of the broken axial generators. They would be massless in the chiral limit.

- It is easy to write down the most general chiral invariant effective Lagrangian for pions. The choice of a particular realization of the chiral group is irrelevant (provided proper imbedding of the isospin group). Thanks to the Goldstone Bosons nature, only derivative interactions are allowed → suppression at low energy! Can incorporate explicit breaking due to the quark masses.

- Feynman calculus using DR: every loop is suppressed by Q^2 (power counting) → quantum corrections can be calculated in perturbation theory.

- It is straightforward to extend the effective Lagrangian to nucleons. Because of the nucleon mass, loops calculated with just DR are not suppressed. Either use additional finite subtractions (EOMS) or perform nonrelativistic expansion of the Lagrangian (the HB approach).