RUB

Evgeny Epelbaum, RUB

Lecture Week, SFB 1245
TU Darmstadt, September 26-30, 2022

# Chiral effective field theory for nuclear forces

## Summary day 2

- ChPT = EFT for low-energy QCD
- For GB and 1N processes, the amplitude is obtained perturbatively via an expansion in powers of $|\vec{p}| \sim M_\pi \equiv Q$ (thanks to the spontaneously broken chiral symmetry)
- New feature for 2 and more nucleons: low-lying poles in the amplitude $\Rightarrow$ certain non-perturbative resummations are unavoidable…

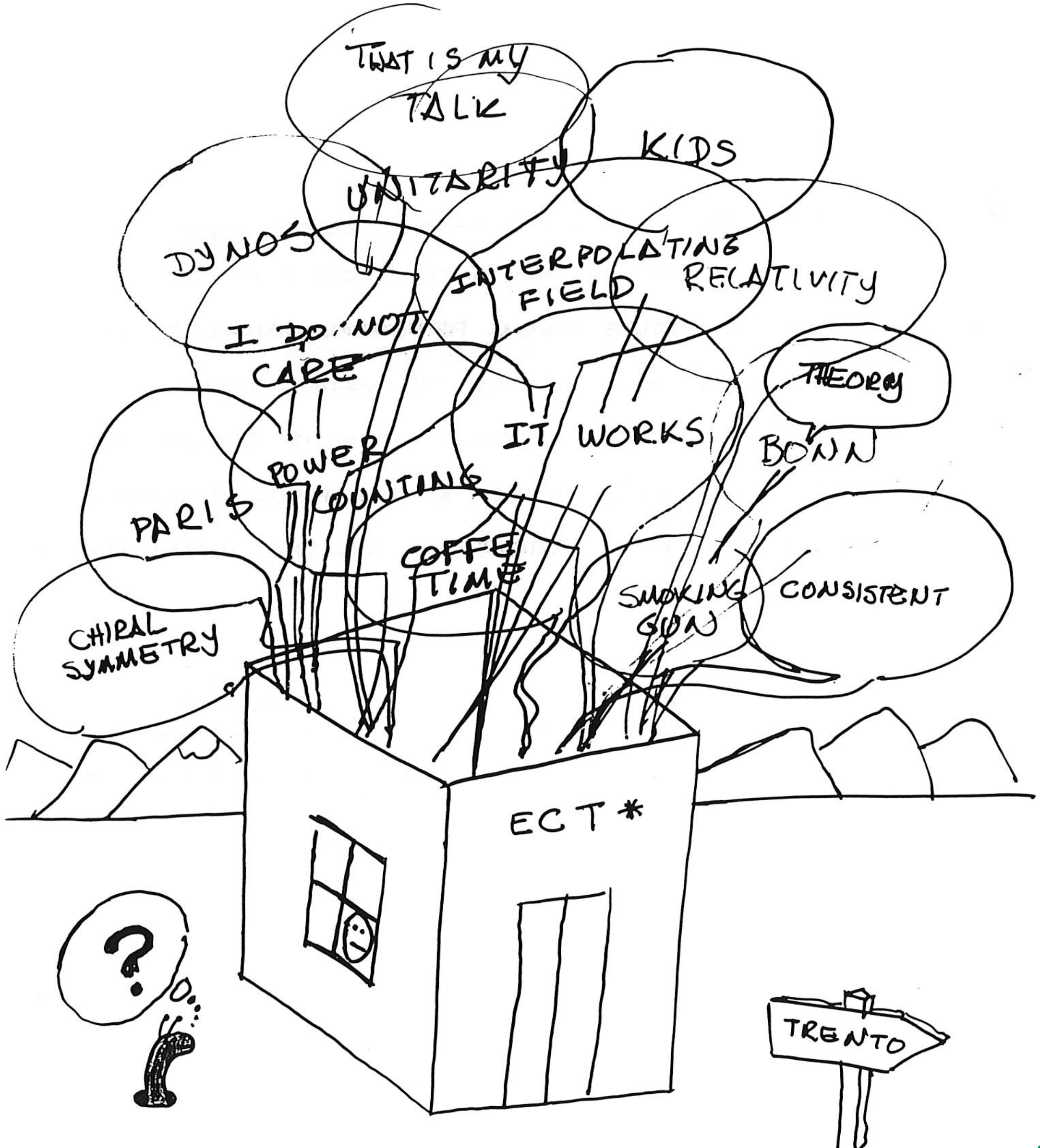
SFB 1245

DFG Deutsche Forschungsgemeinschaft

Bundesministerium für Bildung und Forschung

erc European Research Council

# Renormalization, power counting and all that…




**artwork by Manoel Robilotta at one of the first nuclear-EFT ECT* workshops…**

# Renormalization, power counting and all that…


C. Ordonez and U. van Kolck, Phys. Lett. B **291**, 459 (1992).
N. Kaiser, R. Brockmann and W. Weise, Nucl. Phys. A **625**, 758 (1997), [arXiv:nucl-th/9706045].
N. Kaiser, Phys. Rev. C **61**, 014003 (2000), [arXiv:nucl-th/9910044].
P. F. Bedaque, H. W. Hammer and U. van Kolck, Nucl. Phys. A **676**, 357 (2000), [arXiv:nucl-th/9906032].
P. F. Bedaque, G. Rupak, H. W. Griesshammer and H. W. Hammer, Nucl. Phys. A **714**, 589 (2003), [arXiv:nucl-th/0207034].
H. W. Griesshammer, Nucl. Phys. A **760**, 110 (2005), [arXiv:nucl-th/0502039].
N. Kaiser, Phys. Rev. C **64**, 057001 (2001), [arXiv:nucl-th/0107064].
M. P. Valderrama and E. Ruiz Arriola, Phys. Lett. B **580**, 149 (2004), [arXiv:nucl-th/0306069].
M. P. Valderrama and E. Ruiz Arriola, Phys. Rev. C **72**, 054002 (2005), [arXiv:nucl-th/0504067].
M. P. Valderrama and E. Ruiz Arriola, Phys. Rev. C **74**, 064004 (2006), [arXiv:nucl-th/0507075].
M. C. Birse and J. A. McGovern, Phys. Rev. C **70**, 054002 (2004), [arXiv:nucl-th/0307050].
M. C. Birse, Phys. Rev. C **74**, 014003 (2006), [arXiv:nucl-th/0507077].
J. V. Steele and R. J. Furnstahl, Nucl. Phys. A **645** (1999) 439, [arXiv:nucl-th/9808022].
M. Lutz, Nucl. Phys. A **677**, 241 (2000), [arXiv:nucl-th/9906028].
R. Higa and M. R. Robilotta, Phys. Rev. C **68**, 024004 (2003), [arXiv:nucl-th/0304025].
R. Higa, M. R. Robilotta, C. A. da Rocha, Phys. Rev. C **69**, 034009 (2004), [arXiv:nucl-th/0310011].
M.D. Cozma, O. Scholten, R.G. Timmermans, J.A. Tjon, Phys. Rev. C **75**, 014006 (2007).
L. Girlanda, M. Viviani and W. H. Klink, Phys. Rev. C **76**, 044002 (2007), [nucl-th/0702024 [NUCL-TH]].
E. Epelbaum, U.-G. Meißner, W. Glöckle, Nucl. Phys. **A714**, 535 (2003), nucl-th/0207089.
E. Epelbaum *et al.*, Phys. Rev. Lett. **86**, 4787 (2001), nucl-th/0007057.
E. Epelbaum *et al.*, Phys. Rev. **C66**, 064001 (2002), nucl-th/0208023.
E. Epelbaum, W. Glöckle, U.-G. Meißner, Nucl. Phys. **A637**, 107 (1998), nucl-th/9801064.
E. Epelbaum, W. Glöckle, U.-G. Meißner, Nucl. Phys. A **671**, 295 (2000), [arXiv:nucl-th/9910064].
D. B. Kaplan, M. J. Savage, M. B. Wise, Nucl. Phys. **B478**, 629 (1996), [arXiv:nucl-th/9605002].
D. B. Kaplan, M. J. Savage, M. B. Wise, Phys. Lett. B **424**, 390 (1998), [arXiv:nucl-th/9801034].
D. B. Kaplan, M. J. Savage, M. B. Wise, Nucl. Phys. **B534**, 329 (1998), [arXiv:nucl-th/9802075].
T. D. Cohen and J. M. Hansen, Phys. Rev. C **59**, 3047 (1999), [arXiv:nucl-th/9901065].
S. Fleming, T. Mehen, and I. W. Stewart, Nucl. Phys. **A677**, 313 (2000), [arXiv:nucl-th/9911001].
S. R. Beane, P. F. Bedaque, L. Childress, A. Kryjevski, J. McGuire and U. van Kolck, Phys. Rev. A **64**, 042103 (2001), [quant-ph/0010073].
S. R. Beane, P. F. Bedaque, M. J. Savage, U. van Kolck, Nucl. Phys. **A700**, 377 (2002), [arXiv:nucl-th/0104030].
A. Nogga, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. C **72**, 054006 (2005), [arXiv:nucl-th/0506005].
G. P. Lepage, arXiv:nucl-th/9706029.
T. S. Park, K. Kubodera, D. P. Min, and M. Rho, Nucl. Phys. **A646**, 83 (1999), [arXiv:nucl-th/9807054].
G. P. Lepage, *Conference summary,* Prepared for INT Workshop on Nuclear Physics with EFT, Seattle, Washington, 25-26 Feb 1999.
T. Frederico, V. S. Timoteo and L. Tomio, Nucl. Phys. A **653**, 209 (1999), [nucl-th/9902052].
V. S. Timoteo, T. Frederico, A. Delfino and L. Tomio, Phys. Lett. B **621**, 109 (2005), [nucl-th/0508006].
V. S. Timoteo, T. Frederico, A. Delfino and L. Tomio, Phys. Rev. C **83**, 064005 (2011), [arXiv:1006.1942 [nucl-th]].
C. -J. Yang, C. Elster and D. R. Phillips, Phys. Rev. C **80**, 044002 (2009), [arXiv:0905.4943 [nucl-th]].
M. C. Birse, Phil. Trans. Roy. Soc. Lond. A **369**, 2662 (2011), [arXiv:1012.4914 [nucl-th]].
M. P. Valderrama, Phys. Rev. C **83**, 024003 (2011), [arXiv:0912.0699 [nucl-th]].
M. Pavon Valderrama, Phys. Rev. C **84**, 064002 (2011), [arXiv:1108.0872 [nucl-th]].
B. Long and C. J. Yang, Phys. Rev. C **85**, 034002 (2012), [arXiv:1111.3993 [nucl-th]].
B. Long and U. van Kolck, Annals Phys. **323**, 1304 (2008), [arXiv:0707.4325 [quant-ph]].
E. Epelbaum and J. Gegelia, Eur. Phys. J. A **41**, 341 (2009), [arXiv:0906.3822 [nucl-th]].
D. Djukanovic, J. Gegelia, S. Scherer and M. R. Schindler, Few Body Syst. **41**, 141 (2007), [arXiv:nucl-th/0609055].
J. Gegelia and G. Japaridze, Phys. Lett. B **517**, 476 (2001), [arXiv:nucl-th/0108005].
M. C. Birse, J. A. McGovern and K. G. Richardson, Phys. Lett. B **464**, 169 (1999), [hep-ph/9807302].
J. Gegelia, In *Adelaide 1998, Nonperturbative methods in quantum field theory* 30-35 [nucl-th/9802038].
K. Harada and H. Kubo, Nucl. Phys. B **758**, 304 (2006), [nucl-th/0605004].
M. J. Savage, In *Pasadena 1998, Nuclear physics with effective field theory* 247-267 [nucl-th/9804034].
X. L. Ren, K. W. Li, L. S. Geng, B. W. Long, P. Ring and J. Meng, Chinese Physics C Vol. 42, No. 1 (2018) 014103, [arXiv:1611.08475 [nucl-th]].
M. Pavón Valderrama, M. Sánchez Sánchez, C. J. Yang, B. Long, J. Carbonell and U. van Kolck, Phys. Rev. C **95**, no. 5, 054001 (2017), [arXiv:1611.10175 [nucl-th]].

# Day 3: Pionless EFT for NN scattering

Kaplan, Savage, Wise, van Kolck, Hammer, Barnea, Braaten, …

Motivation: The NN scattering amplitude is calculable analytically (separable interactions) and can be renormalized explicitly in the non-perturbative regime

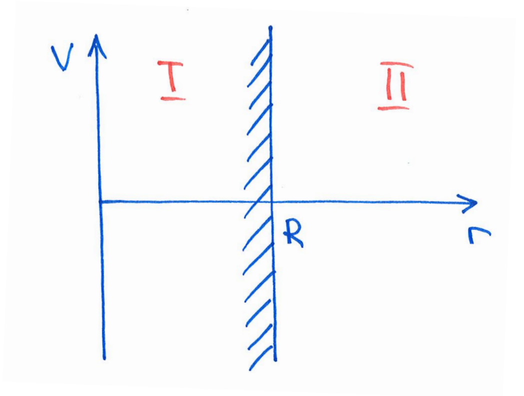
$\Rightarrow$ an ideal playground to test concepts and methods to be used in chiral EFT

1. Preliminaries: Effective Range Expansion
2. Pionless EFT
3. Wilsonian analysis
4. Finite-cutoff EFT for pionless theory
5. Summary of day 3

# Effective range expansion [Landau, Smorodinsky '44, Schwinger '48, Bethe'49]

## 1. Preliminaries: Effective range expansion

Consider hard-sphere scattering $V(r) = \begin{cases} \infty, & r > R \\ 0, & r < R \end{cases}$




Radial wave function:

$$\begin{cases} R^{I}_{l,p}(r) = 0 \\ R^{II}_{l,p}(r) \propto \alpha j_l(pr) + \beta n_l(pr) \end{cases}$$

$$R^{II}_{l,p}(r) \overset{r\to\infty}{\longrightarrow} \propto \frac{\sin(pr - l\pi/2 + \delta_l)}{pr} \longrightarrow R^{II}_{l,p}(r) = A\Big(\cos\delta_l\, j_l(pr) - \sin\delta_l\, n_l(pr)\Big)$$

Matching at the boundary: $R^{II}_{l,p}(R) = 0 \longrightarrow \cot\delta_l(p) = \dfrac{n_l(pR)}{j_l(pR)}$

For s-wave, one then finds: $p\cot\delta_0(p) = \underbrace{-\frac{1}{R}}_{=:\,1/a} + \frac{1}{2}\underbrace{\left(\frac{2R}{3}\right)}_{=:\,r}p^2 + \underbrace{\frac{R^3}{45}}_{=:\,v_2}p^4 + \ldots$

Such an expansion holds for any (regular) short-range potential. The coefficients are specific to the potential. The convergence range is determined by the range of the interaction.

# Effective range expansion [Landau, Smorodinsky '44, Schwinger '48, Bethe'49]

The analytic structure of $T_l(E)$ is well known. In addition to the (right-hand) unitarity cut, one observes left-hand cuts specific to V.

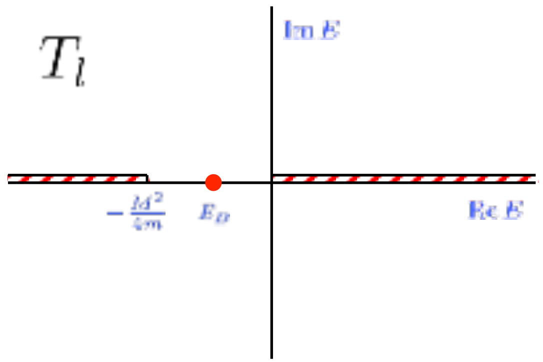
$$V(r) \propto \frac{e^{-Mr}}{r} \quad \Big(\text{or } V(\underbrace{\vec{q}}_{\vec{p}\,'-\vec{p}}) \propto \frac{1}{\vec{q}^{\,2}+M^2}\Big) \longrightarrow \text{left-hand cut starts at } p = iM/2$$

$$\Big(\text{indeed: } T_l(p) \propto \int d(\cos\theta) \frac{1}{2p^2(1-\cos\theta)+M^2} + \ldots \Big)$$

Recall: $S_l(p) = e^{2i\delta_l(p)} = 1 - i\frac{mp}{2\pi}T_l(p)$

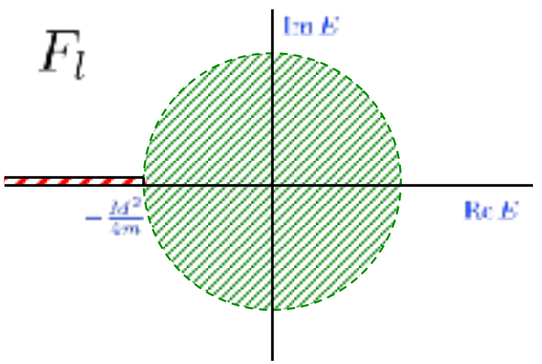
$$\rightarrow \boxed{T_l(p) = -\frac{4\pi}{m}\frac{1}{p\cot\delta_l(p) - ip}} = -\frac{4\pi}{m}\frac{p^{2l}}{\underbrace{F_l(p)}_{p^{2l+1}\cot\delta_l(p)} - ip^{2l+1}}$$

**(effective range function)**



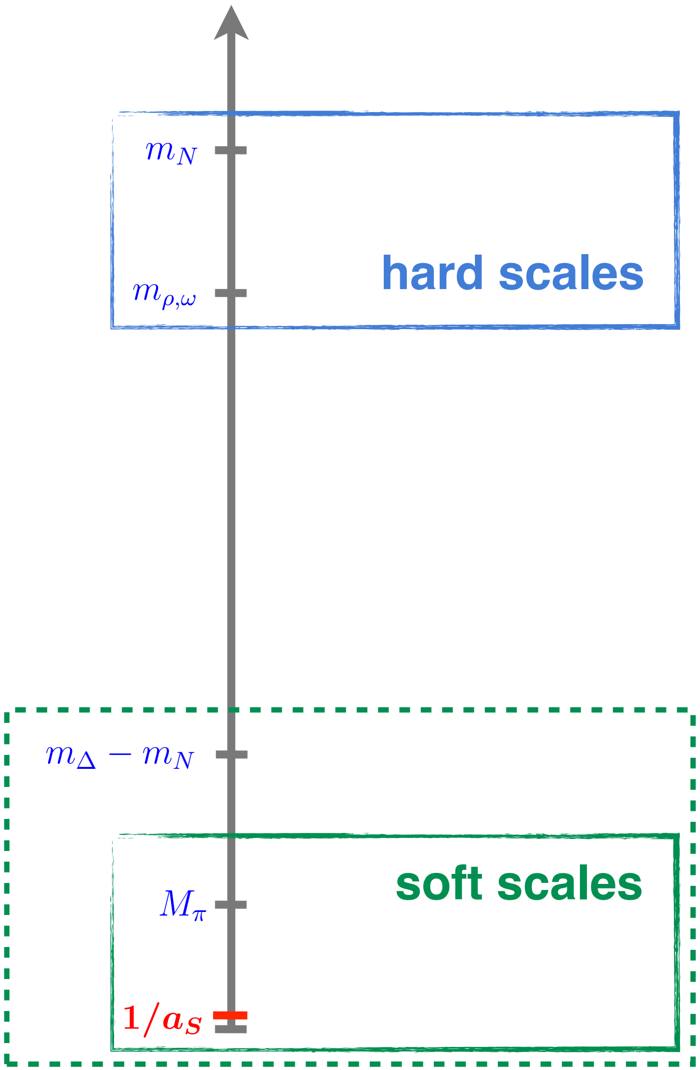

For short-range $V(r)$, $F_l(p)$ is a real meromorphic function of $p^2$ near the origin

ERE: $$\boxed{F_l(p) = -\frac{1}{a} + \frac{1}{2}rp^2 + v_2p^4 + v_3p^6 + \ldots}$$




The (maximal) convergence radius is determined by the range $M^{-1}$ of the interaction.

# Relevant scales



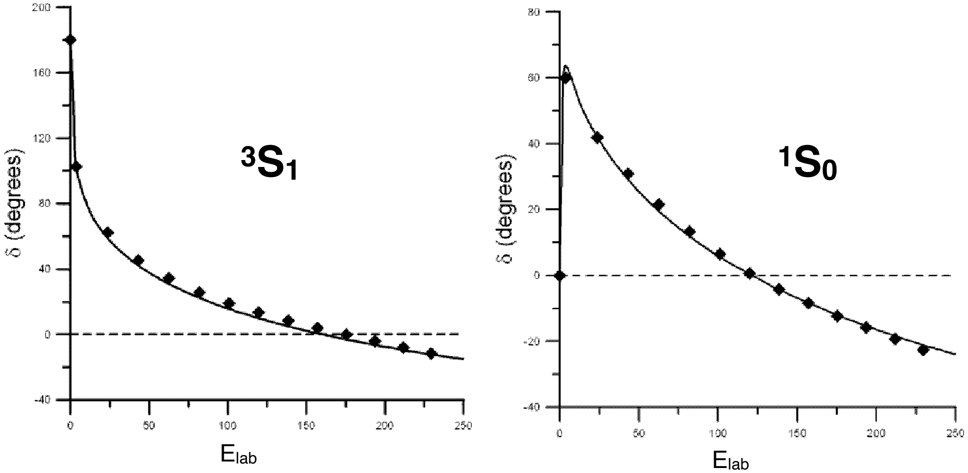

- **Chiral perturbation theory** (0,1 nucleons): perturbative expansion of the amplitude in powers of

$$Q \in \left\{ \frac{M_\pi}{\Lambda}, \frac{|\vec{p}\,|}{\Lambda} \right\}, \quad \Lambda \sim m_\rho \sim 4\pi F_\pi \sim 1 \text{ GeV}$$

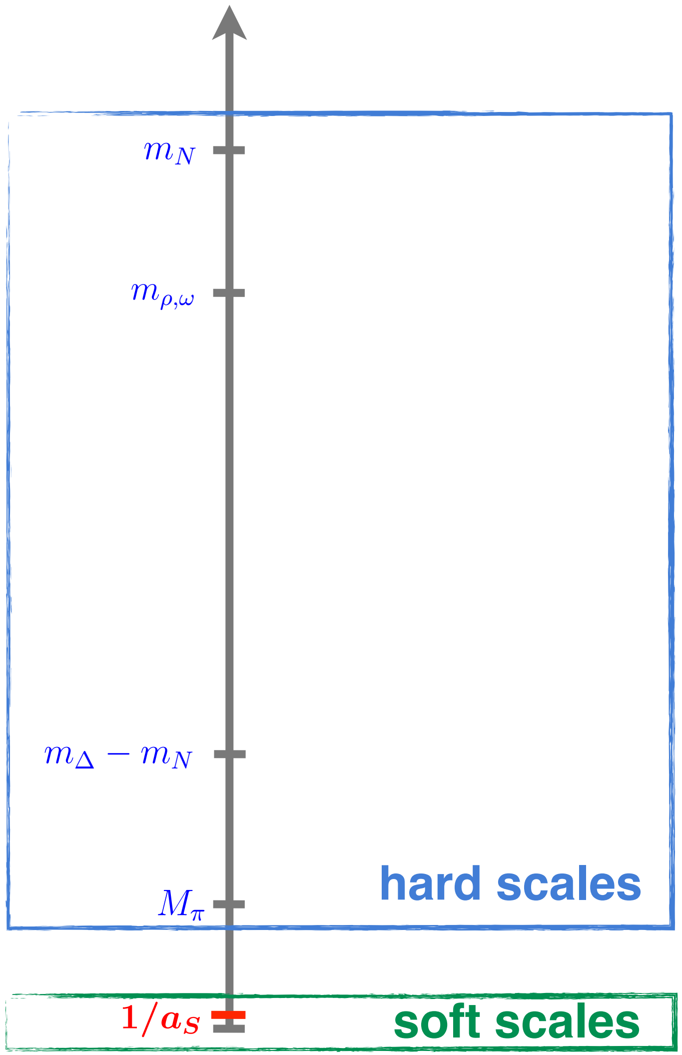
- >1 nucleons: a new very soft scale

$1/a_S \simeq 8.5$ MeV(36 MeV) in $^1S_0$ ($^3S_1$)

has to be generated dynamically ⟶ need nonperturbative resummations: **chiral EFT**



# Relevant scales




- **Chiral perturbation theory** (0,1 nucleons): perturbative expansion of the amplitude in powers of

$$Q \in \left\{ \frac{M_\pi}{\Lambda}, \frac{|\vec{p}\,|}{\Lambda} \right\}, \quad \Lambda \sim m_\rho \sim 4\pi F_\pi \sim 1\ \text{GeV}$$

- >1 nucleons: a new very soft scale

  $1/a_S \simeq 8.5\ \text{MeV} (36\ \text{MeV})$ in $^1S_0$ ($^3S_1$)

  has to be generated dynamically ⟶ need nonperturbative resummations: **chiral EFT**

- **Pionless EFT**:
  expansion of the amplitude in powers of $|\vec{p}\,| \sim 1/a_S$ over $\Lambda \sim M_\pi$

  Both ERE & $\not\pi$-EFT yield an expansion of the amplitude in $|\vec{p}\,|/M_\pi$, have the same validity range and incorporate the same physics ⟶ **ERE ~ $\not\pi$-EFT**

# Pionless EFT

## 2. Pionless EFT [Kaplan, Savage, Wise, Nucl. Phys. B534 (1998) 617]

**The goal**: design an EFT to match ERE (no predictive power for NN beyond ERE)

**DoF**: nonrelativistic nucleons (use the HB formalism)

**Symmetries**: rotational invariance, isospin symmetry, usual discrete symmetries…

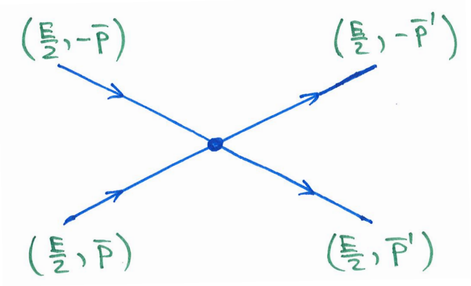
Begin with writing down the most general Lagrangian:

$$\mathcal{L} = N^\dagger \left( i\partial_0 + \frac{\vec{\nabla}^2}{2m_N} \right) N - \frac{1}{2} C_S (N^\dagger N)^2 - \frac{1}{2} C_T (N^\dagger \vec{\sigma} N)^2 + \underbrace{\ldots}_{\text{terms with} \geq 2 \text{ derivatives}}$$

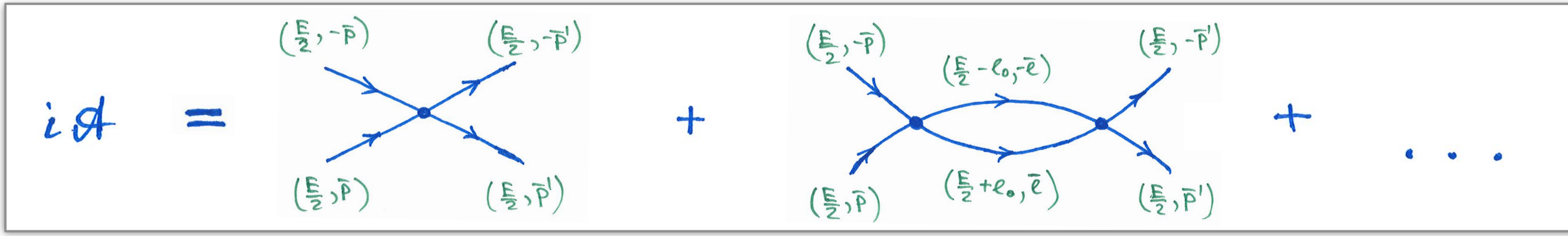
Notice: $(N^\dagger \vec{\tau} N)^2$, $(N^\dagger \vec{\tau}\vec{\sigma} N)^2$ are redundant (Pauli principle). Indeed, in there are only 2 independent s-waves ($^1S_0$ and $^3S_1$) in the isospin limit…

Feynman rule (ignore spin for the moment…):

$$i\mathcal{A}^{\text{tree}} = -i \Big[ \underbrace{C_0}_{\text{linear combination of } C_S,\, C_T} + C_2 (\vec{p}^{\,2} + \vec{p}^{\,\prime\, 2}) + \ldots \Big]$$



# Pionless EFT




$$i\mathcal{A}^{1-\text{loop}} = \int \frac{d^4l}{(2\pi)^4}(-i)\Big[C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \dots\Big] \frac{i}{\frac{E}{2} + l_0 - \frac{\vec{l}^2}{2m_N} + i\epsilon} \frac{i}{\frac{E}{2} - l_0 - \frac{\vec{l}^2}{2m_N} + i\epsilon}$$

$$\times (-i)\Big[C_0 + C_2(\vec{l}^{\,2} + \vec{p}^{\,\prime\,2}) + \dots\Big]$$

$$= (-i)\int \frac{d^3l}{(2\pi)^3}\Big[C_0 + C_2(\vec{p}^{\,2} + \vec{l}^{\,2}) + \dots\Big]\frac{1}{E - \frac{\vec{l}^2}{m_N} + i\epsilon}\Big[C_0 + C_2(\vec{l}^{\,2} + \vec{p}^{\,\prime\,2}) + \dots\Big]$$

Since loop integrals factorize, the results are trivially generalizable to any number of loops. One finds for $E = p^2/m_N$:

$$\mathcal{A}(\vec{p}^{\,\prime}, \vec{p}) = V(\vec{p}^{\,\prime}, \vec{p}) - m_N \int \frac{d^3l}{(2\pi)^3} \frac{V(\vec{p}^{\,\prime}, \vec{l}\,)\,\mathcal{A}(\vec{l}, \vec{p})}{\vec{p}^{\,2} - \vec{l}^{\,2} + i\epsilon}$$

*sign convention for $V$, $\mathcal{A}$*

with the potential $V(\vec{p}^{\,\prime}, \vec{p}) = -(C_0 + C_2(\vec{p}^{\,2} + \vec{p}^{\,\prime 2}) + \dots)$. As expected, the nonrelativistic treatment recovers the quantum mechanical Lippmann-Schwinger equation.

# Pionless EFT

In the following, we focus on s-wave scattering. Utilizing the KSW notation,

$$i\mathcal{A}^{\rm tree} = iV(p', p) = -i(C_0 + \underbrace{C_2(p^2 + p'^2)}_{\substack{\text{terms} \sim pp' \\ \text{contribute to p-waves}}} + \underbrace{\ldots}_{\geq 4 \text{ derivatives}})$$

The LS equation for the half-shell amplitude in the s-waves:

$$\mathcal{A}(p', p) = V(p', p) - m\int \frac{d^3l}{(2\pi)^3} \frac{V(p', l)\, \mathcal{A}(l, p)}{p^2 - l^2 + i\epsilon}, \quad \text{and} \quad S_0 = 1 + i\frac{mp}{2\pi}\mathcal{A}$$

Loop integrals are UV divergent → need to specify renormalization scheme…

- **DR (+PDS):** [KSW '98]

$$J_n(p) = m\int \frac{d^3l}{(2\pi)^3} \frac{l^{2n}}{p^2 - l^2 + i\epsilon} \xrightarrow{\rm DR} m\left(\frac{\mu}{2}\right)^{4-d} \int \frac{d^{d-1}l}{(2\pi)^{d-1}} \frac{l^{2n}}{p^2 - l^2 + i\epsilon}$$

$$= -mp^{2n}(-p^2 - i\epsilon)^{\frac{d-3}{2}}\Gamma\left(\frac{3-d}{2}\right)\frac{(\mu/2)^{4-d}}{(4\pi)^{\frac{d-1}{2}}} \xrightarrow{d=4} -mp^{2n}\frac{ip}{4\pi}$$

The DR result is finite for $d = 4$ (power-like divergences). Instead of using MS, it is advantageous to subtract poles in $d = 3$ (see later). Then: $J_n(p) = -mp^{2n}\frac{\mu + ip}{4\pi}$.

# Pionless EFT

- **Cutoff regularization + subtractions:**

[Gegelia '98]

$$m\int\frac{d^3l}{(2\pi)^3}\frac{l^{2n}}{p^2-l^2+i\epsilon} = \underbrace{-m\int\frac{d^3l}{(2\pi)^3}l^{2n-2}}_{=:\,I_{2n+1}} - \ldots - \underbrace{mp^{2n-2}\int\frac{d^3l}{(2\pi)^3}}_{=:\,p^{2n-2}I_3} + \underbrace{mp^{2n}\int\frac{d^3l}{(2\pi)^3}\frac{1}{p^2-l^2+i\epsilon}}_{=:\,p^{2n}I(p)}$$

Introduce a sharp cutoff to regularize the integrals:

$$I_n \quad\rightarrow\quad I_n^\Lambda = -m\int\frac{d^3l}{(2\pi)^3}\,l^{n-3}\,\theta\,(\Lambda-l) = -\frac{m\,\Lambda^n}{2n\pi^2}$$

$$I(p) \quad\rightarrow\quad I^\Lambda(p) = m\int\frac{d^3l}{(2\pi)^3}\frac{\theta\,(\Lambda-l)}{p^2-l^2+i\eta} = I_1^\Lambda - \frac{i\,m\,p}{4\pi} - \frac{mp}{4\pi^2}\ln\frac{\Lambda-p}{\Lambda+p}$$

Separate the loop integrals into divergent & finite parts and take the limit $\Lambda\rightarrow\infty$ :

$$\lim_{\Lambda\rightarrow\infty} I_n^\Lambda = \lim_{\Lambda\rightarrow\infty}\left(I_n^\Lambda + \frac{m\mu_n^n}{2n\pi^2}\right) - \frac{m\mu_n^n}{2n\pi^2} =:\; \Delta_n(\mu_n) + I_n^R(\mu_n)\,, \qquad n = 3, 5, 7, \ldots$$

$$\lim_{\Lambda\rightarrow\infty} I^\Lambda(p) = \lim_{\Lambda\rightarrow\infty}\left(I_1^\Lambda + \frac{m\mu}{2\pi^2}\right) + \left[-\frac{m\mu}{2\pi^2} - \frac{i\,m\,p}{4\pi}\right] =:\; \underbrace{\Delta(\mu)}_{\text{divergent}} + \underbrace{I^R(\mu,p)}_{\text{finite}}$$

Notice: PDS corresponds to the particular choice $\mu\rightarrow\mu\pi/2,\;\; \mu_i = 0\,.$

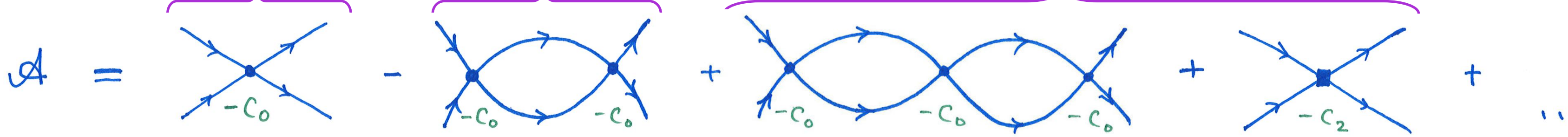
# Pionless EFT

Having defined renormalization scheme, we still need to specify renormalization conditions (i.e. the choice of subtraction scales).

Conventional wisdom suggests: $\mu,\ \mu_i \sim$ soft scale $\sim p \ll M_\pi$ . [i.e. loop momenta of the order of the soft scale after renormalization…]

**Let's calculate the amplitude to two loops assuming NDA scaling of LECs:**

*order $p^0$* — *order $p^1$* — *order $p^2$*




$$-C_0 \qquad \hbar\,(-C_0)^2\, I(p) \qquad \hbar^2\,(-C_0)^3\,\big(I(p)\big)^2 \qquad (-C_2)2p^2$$

Recall: $$I(p) = \underbrace{\lim_{\Lambda\to\infty}\left(\frac{m}{2\pi^2}(\mu-\Lambda)\right)}_{=\,\Delta(\mu)} \underbrace{-\,\frac{m}{4\pi}\left(ip+\frac{2}{\pi}\mu\right)}_{=\,I^{\rm R}(\mu,p)}$$

Renormalization: $$C_0 = C_0^{\rm R}(\mu) + \hbar\,\underbrace{\delta C_{0,1}}_{-(C_0^{\rm R})^2\,\Delta} + \hbar^2\,\underbrace{\delta C_{0,2}}_{(C_0^{\rm R})^3\,\Delta^2} + \mathcal{O}(\hbar^3)$$

Thus, finally: $$\boxed{\mathcal{A} = -C_0^{\rm R}(\mu) - \big(C_0^{\rm R}(\mu)\big)^2 I^R(\mu,p) - \big(C_0^{\rm R}(\mu)\big)^3\big(I^R(\mu,p)\big)^2 - 2C_2p^2 + \ldots}$$

[If all counter terms are included, renormalization amounts to just replacing $C_i \to C_i^{\rm R}(\mu)\,,\ I_{[n]} \to I_{[n]}^{\rm R}(\mu)$.]

# Pionless EFT

To determine LECs, we have to match the amplitude to the ERE:

$$\mathcal{A} = \frac{4\pi}{m}\frac{1}{p\cot\delta - ip} = \frac{4\pi}{m}\frac{1}{\left[-\frac{1}{a} + \frac{1}{2}rp^2 + v_2 p^4 + \ldots\right] - ip}$$

Such matching is, however, only possible for $a, r, v_i \sim \mathcal{O}(1)$. One then has:

$$\mathcal{A} = \frac{4\pi}{m}\left(-a + ia^2 p + a^3 p^2 - \frac{a^2 r}{2}p^2 + \ldots\right) \stackrel{!}{=} -C_0^{\rm R} - C_0^{\rm R\,2}\underbrace{I^R(\mu,p)}_{-m/(4\pi)(2\mu/\pi + ip)} - C_0^{\rm R\,3}\left(I^R(\mu,p)\right)^2 - 2C_2 p^2 + \ldots$$

$$\rightarrow \begin{cases} C_0^{\rm R} = \dfrac{4\pi a}{m}\left[1 + \mathcal{O}(a\mu)\right] \\ C_2 = \dfrac{\pi a^2}{m} r \end{cases}$$

[Choosing $\mu = 0$, one reproduces exactly the first four terms in the expansion of $\mathcal{A}$ …]

However, in reality, the scattering lengths are large:

$$a_{^1S_0} = -23.714 \text{ fm} \sim -16.6\, M_\pi^{-1}, \qquad a_{^3S_1} = 5.42 \text{ fm} \sim 3.8\, M_\pi^{-1}$$

Thus, it seems more appropriate to count $a \sim p^{-1}$. This leads to the expansion:

$$\mathcal{A} = -\frac{4\pi}{m}\Bigg[\underbrace{\frac{1}{a^{-1} + ip}}_{\text{order } p^{-1}} + \underbrace{\frac{rp^2}{2(a^{-1} + ip)^2}}_{\text{order } p^0} + \underbrace{\frac{r^2p^4}{4(a^{-1} + ip)^3}}_{\text{order } p} + \ldots\Bigg]$$

# Pionless EFT

The large scattering length signals non-perturbative physics. In order to accommodate for it, some fine tuning has to be built in the EFT.

The resulting power counting depends on the choice of renormalization conditions!
**[for more details see: EE, Gegelia, Meißner, Nucl. Phys. B925 (2017) 161]**

Consider a general expansion for the potential: $V = V^{\rm LO} + V^{\rm NLO} + V^{\rm N^2LO} + \ldots$

Want to assign powers of $p$ to match: $\mathcal{A} = \mathcal{A}^{(-1)} + \mathcal{A}^{(0)} + \mathcal{A}^{(1)} + \ldots$

LS equation: $\hat{\mathcal{A}}^{(-1)} = \hat{V}^{\rm LO} - \hat{V}^{\rm LO}\hat{G}_0\hat{\mathcal{A}}^{(-1)} \quad \rightarrow \quad \hat{\mathcal{A}}^{(-1)} = (1+\hat{V}^{\rm LO}\hat{G}_0)^{-1}\,\hat{V}^{\rm LO}$

Let: $\hat{V}^{\rm LO} \sim \mathcal{O}(p^x) \quad \rightarrow \quad 1+\hat{V}^{\rm LO}\hat{G}_0 \sim \mathcal{O}(p^{1+x}) \quad \rightarrow \quad \begin{cases} \hat{G}_0 \sim \mathcal{O}(p), & x \le -1 \\ \hat{G}_0 \sim \mathcal{O}(p^{-x}), & x > -1 \end{cases}$

A desired scaling of $\hat{G}_0$ can be realized by choosing the renormalization conditions:

- **Weinberg:** $\mu \sim \mathcal{O}(1), \quad \mu_i \sim \mathcal{O}(p) \quad \rightarrow \quad x = 0 \quad \rightarrow \quad V^{\rm LO}_{\rm Weinberg} \sim \mathcal{O}(1)$
- **KSW:** $\mu, \mu_i \sim \mathcal{O}(p) \quad \rightarrow \quad x = -1 \quad \rightarrow \quad V^{\rm LO}_{\rm KSW} \sim \mathcal{O}(p^{-1})$

Scaling of $V^{\rm NLO}$ can be read off from:

$$\hat{\mathcal{A}}^{(0)} = \hat{V}^{\rm NLO} - \hat{V}^{\rm NLO}\hat{G}_0\hat{\mathcal{A}}^{(-1)} - \hat{\mathcal{A}}^{(-1)}\hat{G}_0\hat{V}^{\rm NLO} + \hat{\mathcal{A}}^{(-1)}\hat{G}_0\hat{V}^{\rm NLO}\hat{G}_0\hat{\mathcal{A}}^{(-1)} \rightarrow \begin{cases} V^{\rm NLO}_{\rm Weinberg} \sim \mathcal{O}(p^2) \\ V^{\rm NLO}_{\rm KSW} \sim \mathcal{O}(1) \end{cases}$$

# Pionless EFT

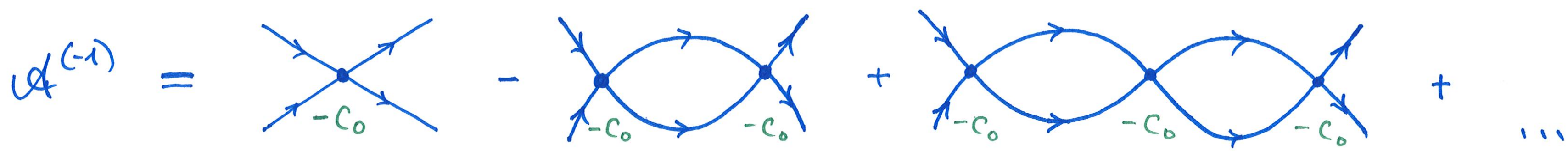
Both choices of the renormalization conditions

— lead to self-consistent approaches,

— are equivalent for pionless EFT (but yield different results in chiral EFT…),

— involve some fine tuning beyond NDA [see: EE, Gegelia, Meißner, NPB 925 (2017) 161]

## Leading order ($p^{-1}$):




$$\mathcal{A}^{(-1)} = -C_0 - C_0^2 I(p) - C_0^3 (I(p))^2 + \ldots = -\frac{1}{C_0^{-1} - I(p)} = -\frac{1}{(C_0^{\rm R}(\mu))^{-1} - I^{\rm R}(\mu, p)}$$

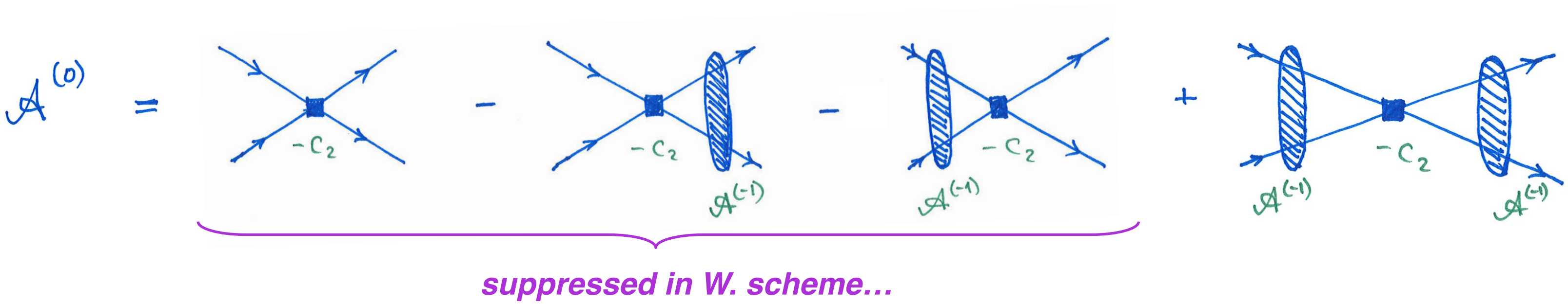
$$= -\frac{4\pi}{m} \frac{1}{\frac{4\pi}{m}(C_0^{\rm R}(\mu))^{-1} + \frac{2}{\pi}\mu + ip} \overset{!}{=} -\frac{4\pi}{m}\frac{1}{a^{-1} + ip} \quad \longrightarrow \quad C_0^{\rm R} = \frac{4\pi}{m}\frac{1}{a^{-1} - \frac{2}{\pi}\mu}$$

One recovers the Weinberg/KSW scaling of $C_0^{\rm R}$ depending on the choice of $\mu$ :

$C_0^{\rm R} \sim \mathcal{O}(1)$ for $\mu \sim \mathcal{O}(1)$; $\quad C_0^{\rm R} \sim \mathcal{O}(p^{-1})$ for $\mu \sim \mathcal{O}(p)$ .

# Pionless EFT

## Subleading order (p⁰):




The subleading amplitude (including terms suppressed in W. scheme) reads:

$$\mathcal{A}^{(0)} = -2C_2p^2 - 2\Big(-C_2(p^2I(p) + \underbrace{J_1(p)}_{=-I_3+p^2I(p)})\Big)\mathcal{A}^{(-1)} - 2C_2J_1(p)I(p)\Big(\mathcal{A}^{(-1)}\Big)^2$$

For the sake of simplicity, choose $\mu_3 = 0$ (so that $I_3^{\rm R} = 0$ ).

After renormalization ($I(p) \to I^{\rm R}(\mu, p)$, $C_2 \to C_2^{\rm R}(\mu)$), one finds:

$$\mathcal{A}^{(0)} = -2\,C_2^{\rm R}\,p^2\,\frac{\left(a^{-1} - \frac{2}{\pi}\mu\right)^2}{(a^{-1}+ip)^2} \overset{!}{=} -\frac{4\pi}{m}\,r\,p^2\,\frac{1}{2(a^{-1}+ip)^2} \longrightarrow C_2^{\rm R} = \frac{\pi}{m}\,\frac{r}{\left(a^{-1} - \frac{2}{\pi}\mu\right)^2}$$

Again, we recover: $C_2^{\rm R} \sim \mathcal{O}(1)$ for $\mu \sim \mathcal{O}(1)$; $C_2^{\rm R} \sim \mathcal{O}(p^{-2})$ for $\mu \sim \mathcal{O}(p)$.

# Pionless EFT

## 3. Wilsonian RG analysis Birse et al.; Harada et al.

Lippmann-Schwinger equation for the off-shell K-matrix

$$K(k',k,p) = V(k',k,p,\Lambda) + 2M\,\mathcal{P}\int \frac{d^3l\,\theta(\Lambda-l)}{(2\pi)^3}\,\frac{V(k',l,p,\Lambda)\,K(l,k,p)}{p^2-l^2}$$

$$\frac{\partial V}{\partial \Lambda} = \frac{M}{\pi^2}\,V(k',\Lambda,p,\Lambda)\,\frac{\Lambda^2}{\Lambda^2-p^2}\,V(\Lambda,k,p,\Lambda)$$

Express all low-energy scales in units of Λ, $\hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda) := \frac{M\,\Lambda}{\pi^2}\,V(\Lambda\hat{k}',\Lambda\hat{k},\Lambda\hat{p},\Lambda)$,

to obtain the RG equation for the rescaled potential:

$$\Lambda\frac{\partial \hat{V}}{\partial \Lambda} = \hat{k}'\frac{\partial \hat{V}}{\partial \hat{k}'} + \hat{k}\frac{\partial \hat{V}}{\partial \hat{k}} + \hat{p}\frac{\partial \hat{V}}{\partial \hat{p}} + \hat{V}(\hat{k}',\hat{k},\hat{p},\Lambda) + \hat{V}(\hat{k}',1,\hat{p},\Lambda)\,\frac{1}{1-\hat{p}^2}\,\hat{V}(1,\hat{k},\hat{p},\Lambda)$$

For Λ → 0, the system approaches a fixed point (scale-invariant solutions):

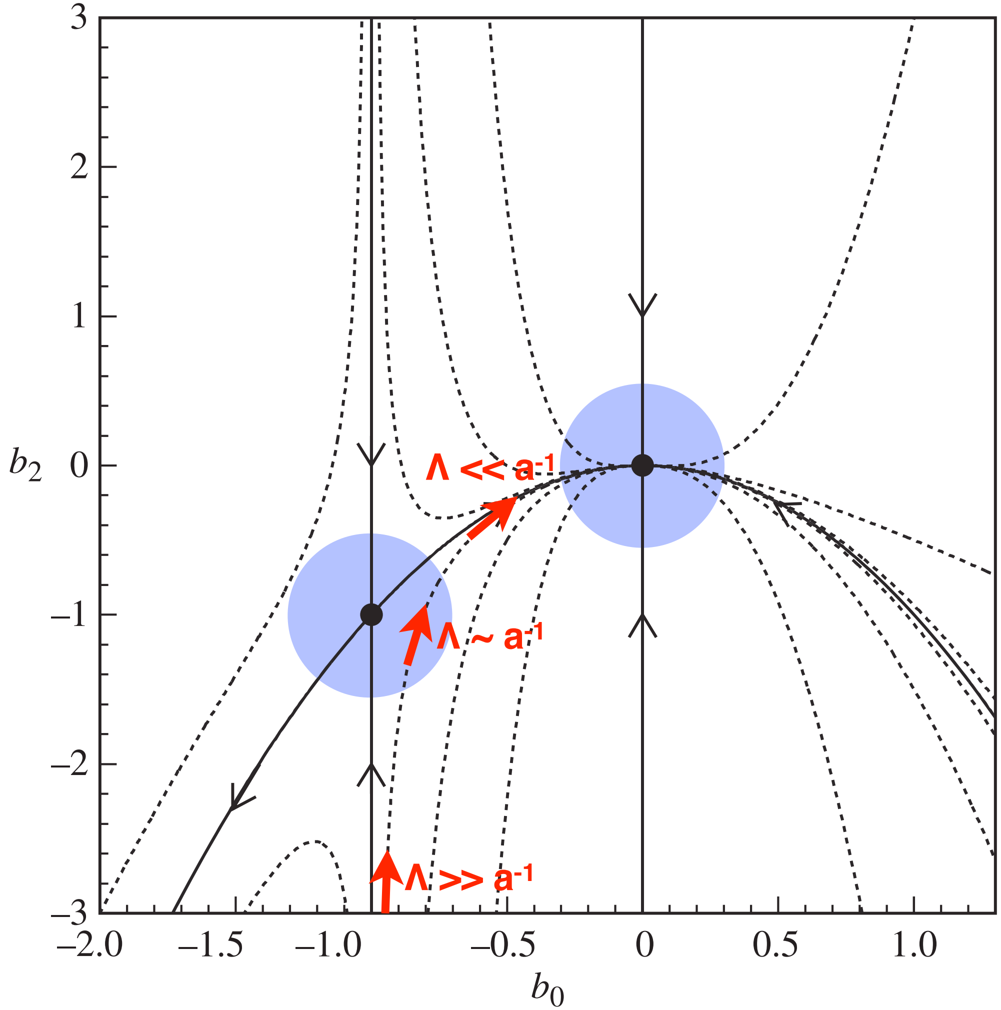
— trivial: $\hat{V} = 0$ (stable, no scattering)

— non-trivial, p-dependent: $\frac{1}{\hat{V}} = -1 + \frac{\hat{p}}{2}\ln\frac{1+\hat{p}}{1-\hat{p}}$ (unstable, unitary limit)

Notice: there exist infinitely many other fixed points [Birse, EE, Gegelia '16]

# Pionless EFT

Power counting can be identified by considering perturbations around the fixed points, e.g. of the form ~ $p^{2n}$.




RG flow of the potential

$$\hat{V}(\hat{p}, \Lambda) = b_0(\Lambda) + b_2(\Lambda)\hat{p}^2 + \dots$$

**(from: Birse, Phil. Trans. R. Soc. A (2011) 369, 2662-2678)**

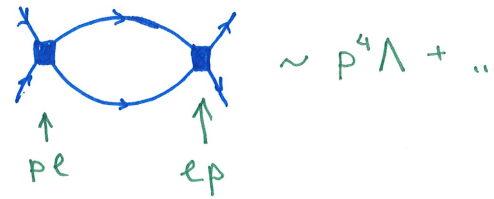
The power counting near the nontrivial fixed point is found to agree with the KSW one (same choice of renorm. conditions…)

# Pionless EFT

## 4. Finite-cutoff EFT for pionless theory

Suppose, one needs to include the range term nonperturbatively (e.g. one wants describe p-wave resonance).

Problem: need infinite number of counter terms…



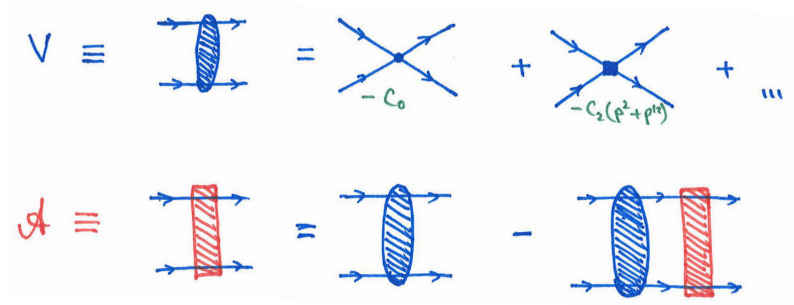

Fortunately, that's not the end of EFT!

„The theory is fully specified by the values of the bare constants once a suitable regularization procedure is chosen. In principle, the renormalization program is straightforward: one calculates quantities of physical interest in terms of the bare parameters at given, large value of (ultraviolet cutoff) $\Lambda$. Once a sufficient number of physical quantities have been determined as functions of the bare parameters one inverts the result and expresses the bare parameters in terms of physical quantities, always working at some given, large value of $\Lambda$. Finally, one uses these expressions to eliminate the bare parameters in all other quantities of physical interest."

Gasser, Leutwyler, Phys. Rep. **87** (1982) 77

# Pionless EFT

Define the contact potential; introduce a UV cutoff $\Lambda \sim M_\pi$; solve the LS equation; tune **bare** LECs $C_0(\Lambda)$, $C_2(\Lambda)$ to a, r.




For a sharp cutoff, one finds at NLO:

$$mC_0 = \frac{6\pi^2\left(\beta - 6\sqrt{3}\sqrt{\alpha(\pi - 2a\Lambda)^2}\right)}{5\alpha\Lambda}\,, \qquad mC_2 = \frac{6\pi^2\left(\sqrt{3}\sqrt{\alpha(\pi - 2a\Lambda)^2} - \alpha\right)}{\alpha\Lambda^3}$$

where I have introduced:

$$\alpha \equiv 16a^2\Lambda^2 - \pi a\Lambda\left(a\Lambda^2 r + 12\right) + 3\pi^2\,, \qquad \beta \equiv 64a^2\Lambda^2 - \pi a\Lambda\left(3a\Lambda^2 r + 62\right) + 18\pi^2$$

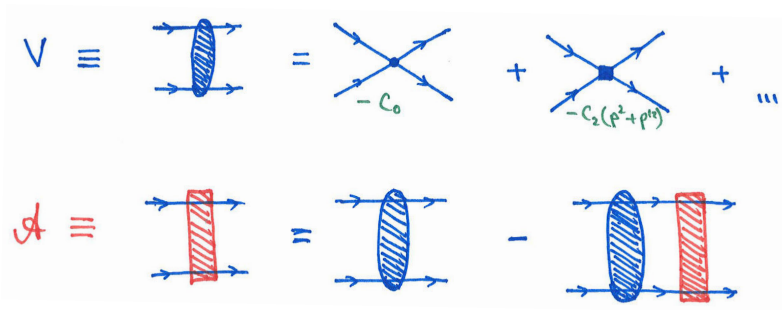
Implicitly renormalized expression for the inverse amplitude:

$$\frac{4\pi}{m}\frac{1}{\mathcal{A}(p)} = \left[-\frac{1}{a} + \frac{1}{2}rp^2 + \frac{\pi\left(8 - 3a\Lambda^2 r(\pi\Lambda r - 8)\right) - 64a\Lambda}{12\pi\Lambda^3(\pi - 2a\Lambda)}p^4 + \mathcal{O}(p^6)\right] - ip$$

⟶ well-defined & correct (up to higher-order terms) result for $\Lambda \sim r^{-1} \sim M_\pi$;
things may (and generally will!) go wrong for $\Lambda \gg r^{-1}$
(complex $C_i$, Wigner bound, peratization…)

# Pionless EFT

Define the contact potential; introduce a UV cutoff $\Lambda \sim M_\pi$; solve the LS equation; tune **bare** LECs $C_0(\Lambda)$, $C_2(\Lambda)$ to a, r.




For a sharp cutoff, one finds at NLO:

$$mC_0 = \frac{6\pi^2\left(\beta - 6\sqrt{3}\sqrt{\alpha(\pi - 2a\Lambda)^2}\right)}{5\alpha\Lambda}\,, \qquad mC_2 = \frac{6\pi^2\left(\sqrt{3}\sqrt{\alpha(\pi - 2a\Lambda)^2} - \alpha\right)}{\alpha\Lambda^3}$$

where I have introduced:

$$\alpha \equiv 16a^2\Lambda^2 - \pi a\Lambda\left(a\Lambda^2 r + 12\right) + 3\pi^2\,, \qquad \beta \equiv 64a^2\Lambda^2 - \pi a\Lambda\left(3a\Lambda^2 r + 62\right) + 18\pi^2$$

**Notice:**

- Contrary to the previous cases, not all c.t. needed to remove UV divergences are included ⟶ it is not legitimate to take the limit $\Lambda \to \infty$.
- Higher-order terms are indeed small provided Λ ~ hard scale (NDA…).
- Implicit renormalization (i.e. no explicit splitting of $C_i$ into $C_i^{\rm R}(\mu)$ and $\Delta(\mu)$).
- The bare LECs $C_i(\Lambda)$ must be re-fitted at every order.

# Pionless EFT

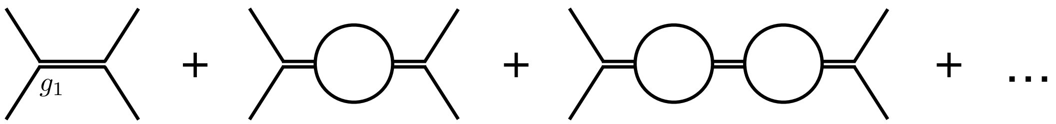
## 4. Application to fine-tuned P-wave systems

EE et al., Few-Body Syst. 62 (2021) 51

Scattering amplitude: $T(k) \propto \dfrac{k^2}{(-1/a + rk^2/2 + v_2 k^4 + \ldots) - ik^3}$

Single fine tuning: $1/a \sim M_{\rm lo}^2 M_{\rm hi}\,, \qquad r \sim M_{\rm hi}\,, \qquad v_n \sim M_{\rm hi}^{3-2n}$

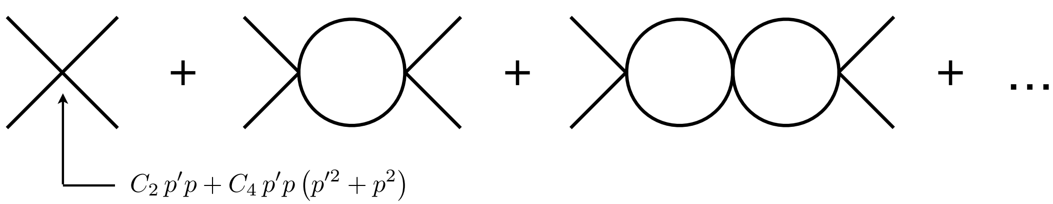
Double fine tuning: $1/a \sim M_{\rm lo}^3\,, \qquad r \sim M_{\rm lo}\,, \qquad v_n \sim M_{\rm hi}^{3-2n}$




EFT with an auxiliary dimer field

Bedaque, Hammer, van Kolck '03

How to set up an EFT without the dimer field?



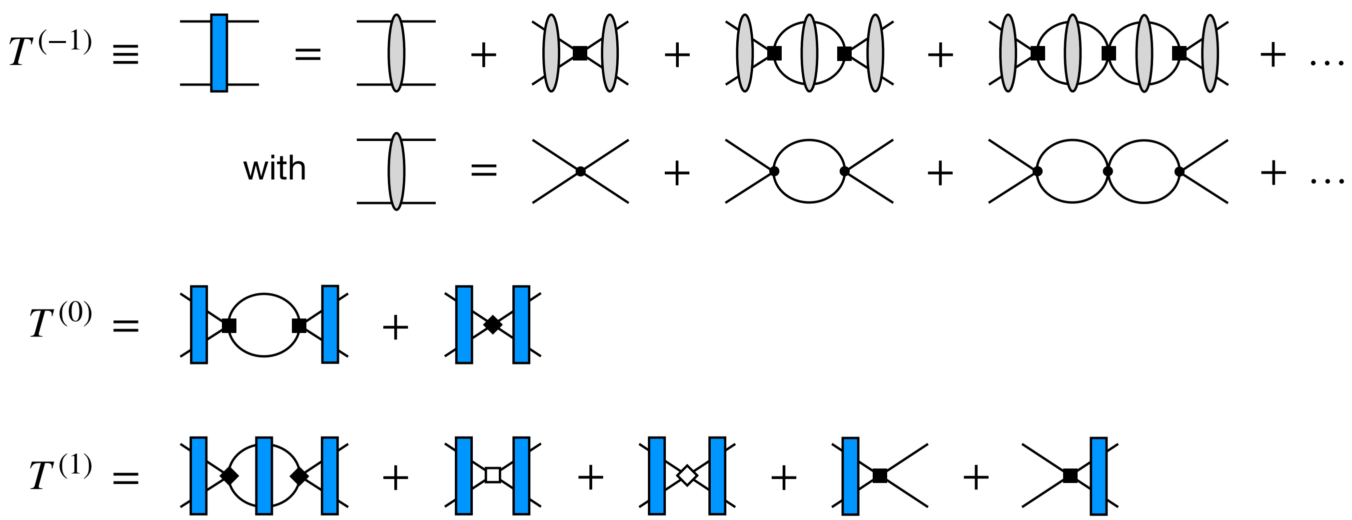
# Pionless EFT

The choice of renormalization conditions leading to a *consistent* EFT (i.e., the size of all contributions beyond the fine-tuned one is determined *a priori* by the power counting)

$$\mu_1 \sim \mu_3 \sim M_{\rm hi}, \qquad \mu_5 \sim \mu_7 \sim \mu_9 \sim \ldots \sim M_{\rm lo}$$

← Weinberg-like setup with all renormalized LECs scaling according to NDA

EFT expansion for the amplitude for the double-fine-tuned scenario:




← fine-tuning for $C_4$: $T^{(0)} \to T^{(-1)}$

← fine-tuning for $C_2$: $T^{(2)} \to T^{(0)}$

For explicit results up to order $\mathcal{O}(Q^6)$, also for the less fine-tuned scenario, see the paper.

# Pionless EFT

**Further observations:**

— the obtained results are equivalent to the formulation with the dimer field

— EFT works for both energy dependent and energy-independent interactions (as it should)

— contrary to the case of S-waves, the KSW-like formulation ($\mu_i \sim M_{\rm lo}$, $\forall i$) is not possible for the considered scenario

— The RG analysis reproduces the scaling of the amplitude (double fine tuned case = expansion about the unitary fixed point)

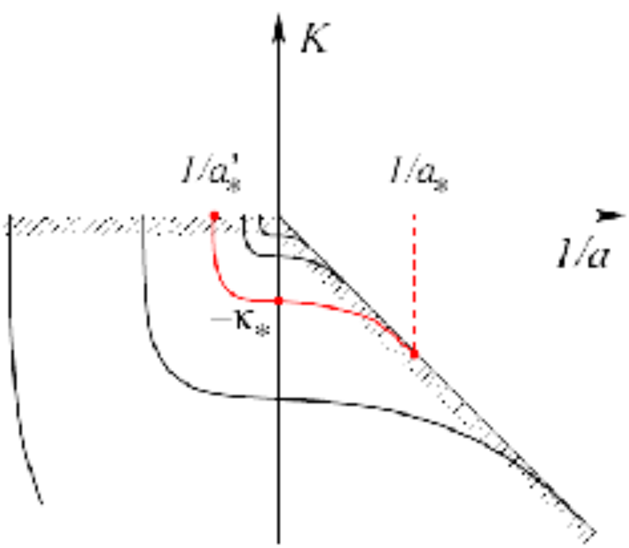
$$\hat V(\hat p', \hat p, \hat k, \Lambda) = \hat p' \hat p\, \hat\omega(\hat k, \Lambda) \qquad \hat\omega_{\rm U}(\hat k) = \frac{-6}{2 + 6\hat k^2 - 3\hat k^3 \ln\frac{1+\hat k}{1-\hat k}}$$

— The finite-cutoff EFT formulation can reproduce the less fine tuned scenario if $\Lambda \sim M_{\rm hi}$. For $\Lambda \gg M_{\rm hi}$, this is not possible…

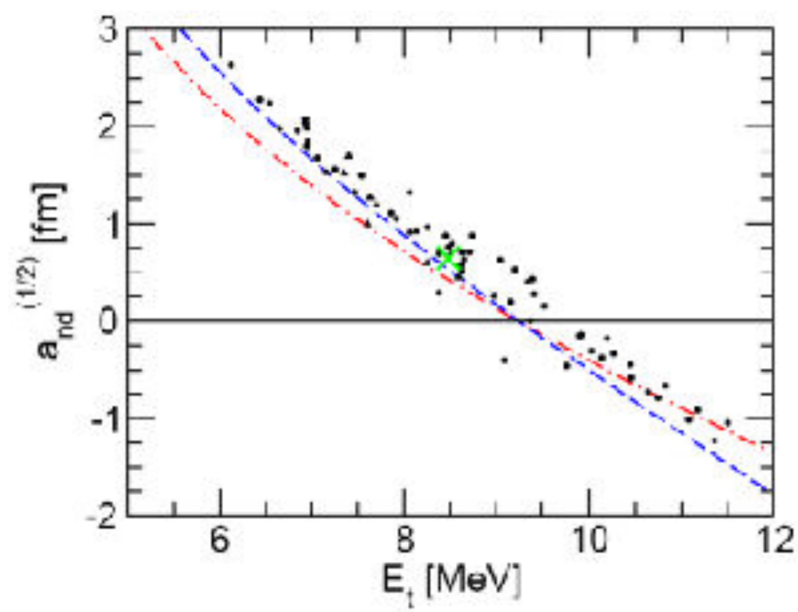
# Pionless EFT: (some) applications

- Astrophysical reactions **Butler, Chen, Kong, Ravndal, Rupak, Savage, …**
- Efimov physics and universality in few-body systems with large 2-body scatt. length (e.g. Phillips/Tjon „lines“) **Braaten, Hammer, Meißner, Platter, von Stecher, Schmidt, Moroz, …**
- Halo-nuclei **Bedaque, Bertulani, Hammer, Higa, van Kolck, Phillips, …**
- Parity violation **Schindler, Springer, Vanasse, …** any many other topics…

## Efimov effect (3-body spectrum)




## Phillips line




**Braaten, Hammer, Phys. Rept 428 (06) 259**

# Summary Day 3

In the NN sector, pionless EFT = effective range expansion

Power counting applies to renormalized diagrams and depends on the choice of renormalization conditions!

Nucleon-nucleon scattering is fine-tuned (blame QCD…)

EFT can be set up to discussed fine-tuned NN S-wave scattering. We discussed the KSW-choice (fine tuning realized in LECs enhanced beyond NDA) and Weinberg-like choice (all LECs are natural). Both choices are equivalent in that case but suggest different extensions to pionful EFT.

RG analysis gives one the scaling of the amplitude (already known in the case of pionless EFT).

Also discussed finite-cutoff EFT a la Lepage and clarified the meaning of (implicit) renormalization.