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Lecture Week, SFB 1245
TU Darmstadt, September 26-30, 2022

Chiral effective field theory for nuclear forces

Summary day 4

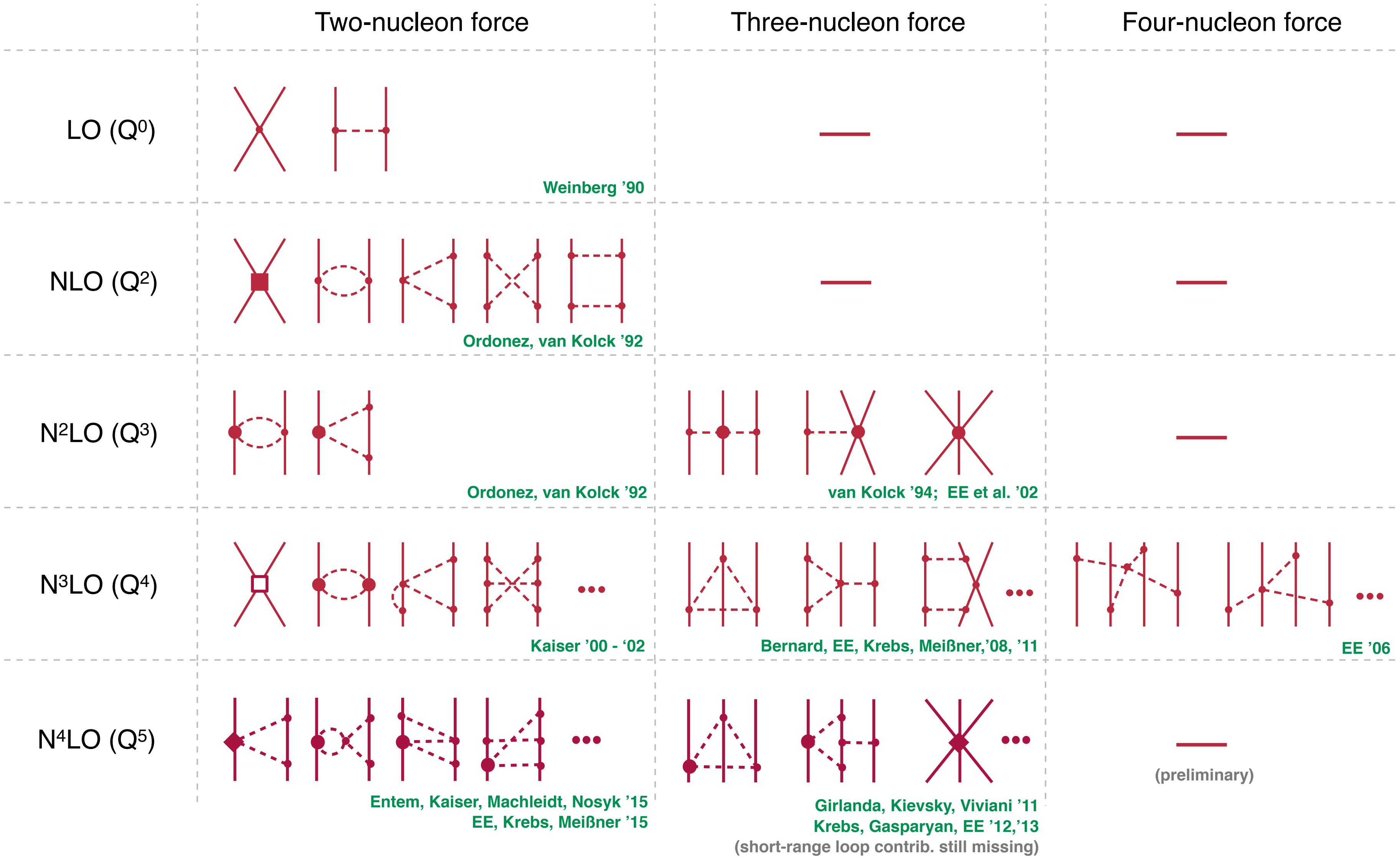
- Pion exchange has to be treated non-perturbatively ⇒ finite-cutoff formulation of chiral EFT
- Nuclear forces and currents = non-iterative contributions to the amplitude. They obey the standard ChPT power counting and can be derived e.g. using the MUT

Today: State-of-the-art and applications

Outline Day 5

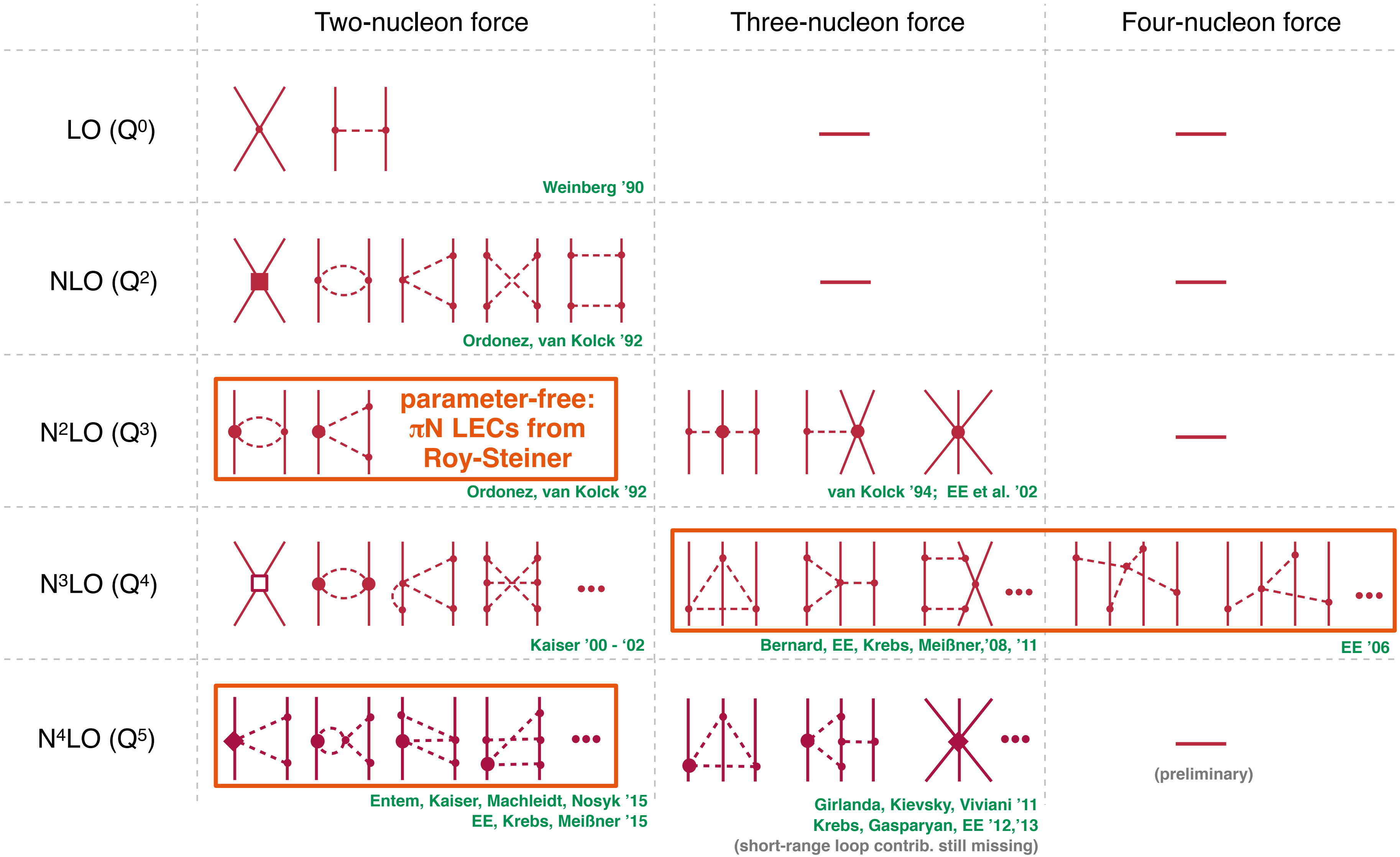
1. The 2N system
   - — regularization, determination of LECs, uncertainty quantification, …
   - — precision determination of the pion-nucleon coupling constants
2. The 3N-force challenge
3. Electroweak currents
4. Chiral EFT and lattice QCD
5. Nuclear lattice simulations
6. Summary Day 5

Nuclear Hamiltonian: State-of-the-art [W-counting]

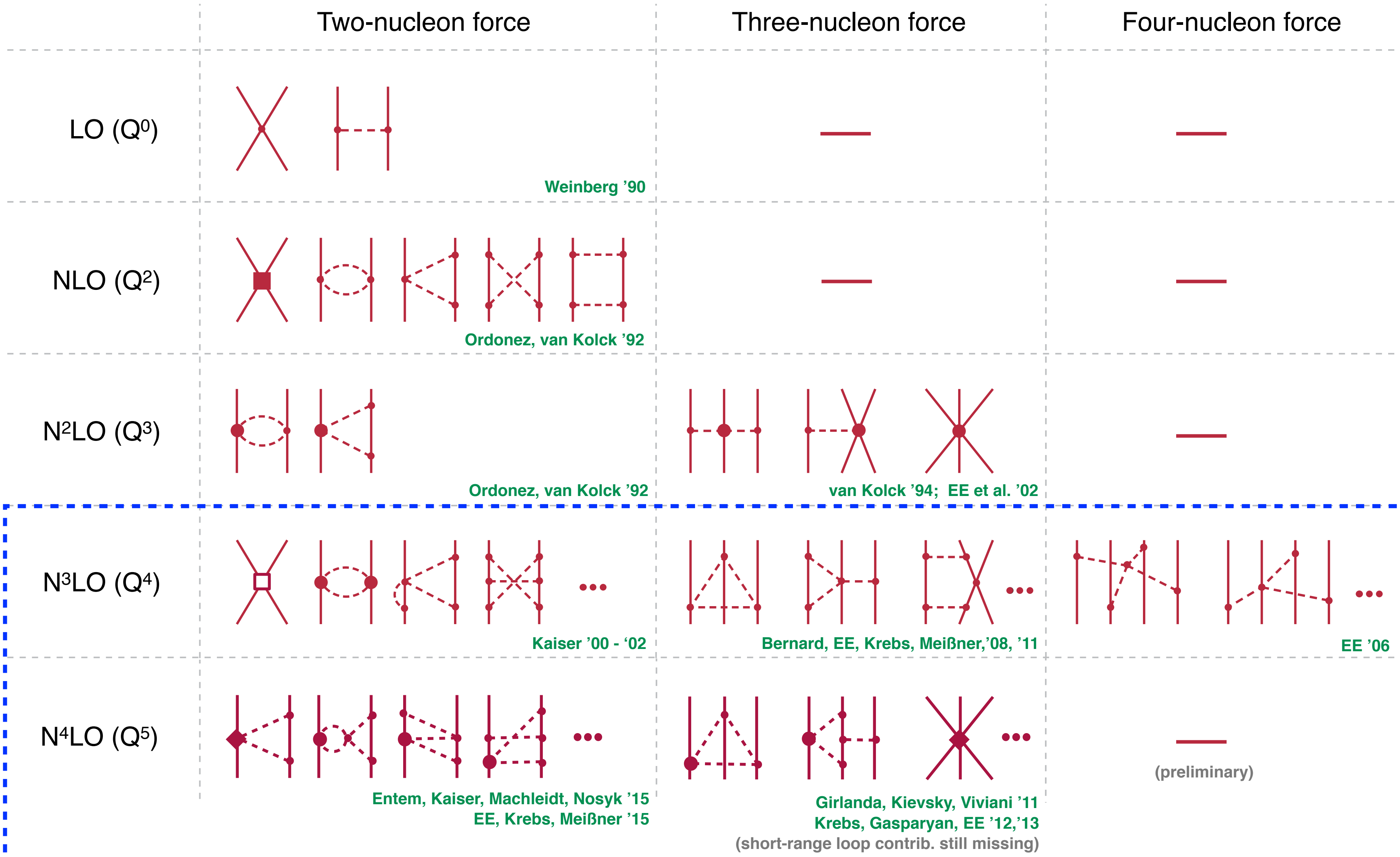


Chiral power counting provides a natural explanation for the observed hierarchy of nuclear forces!
Similar program is being carried out in chiral EFT with Δ(1232) **[Ordonez et al., Kaiser et al., Krebs, EE, Meißner].**

Nuclear Hamiltonian: State-of-the-art [W-counting]

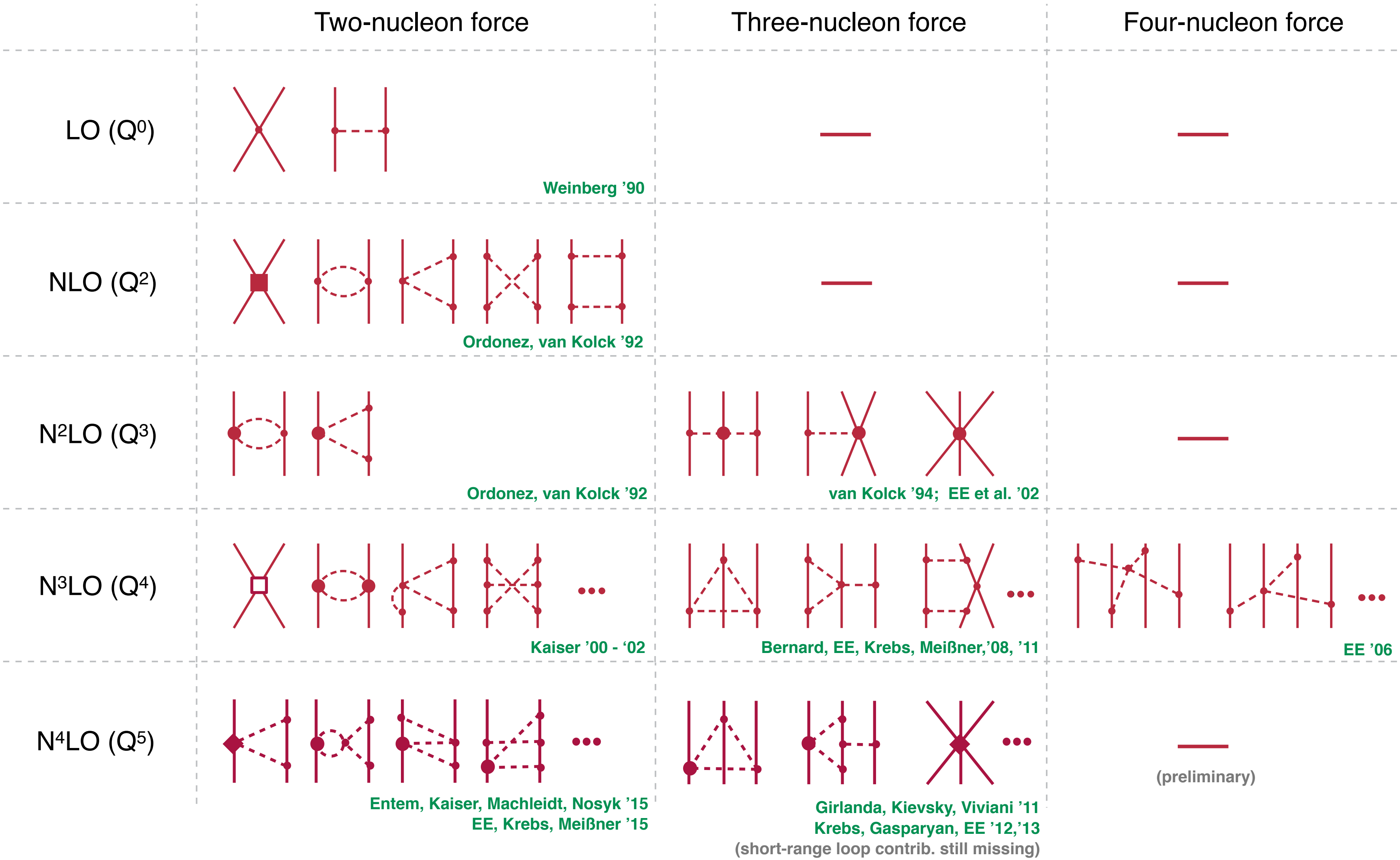


Nuclear Hamiltonian: State-of-the-art [W-counting]



Scheme dependence (unitary ambiguity) starts showing up at N³LO: **2 phases** ($\bar{\beta}_8$, $\bar{\beta}_9$) in the long-range relativistic corrections **+ 3 off-shell short-range terms** in the $^1S_0$, $^3S_1$ and ε_1 channels.

Nuclear Hamiltonian: State-of-the-art [W-counting]



The published expressions for the nuclear forces make use of dimensional regularization for loop integrals. **Additional regularization is required when solving the Schrödinger equation!**

The 2N system

The NN potential from chiral EFT

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

So far, discussed the derivation of chiral potentials (using DR). Next steps:

- determination of the πN LECs to fix the long-range nuclear force
- regularization
- determination of the contact interactions (implicit renormalization)

1. Determination of πN LECs

Effective chiral Lagrangian:

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c_i}\bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d_i}\bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \underbrace{\sum_i \boldsymbol{e_i}\bar{N}\hat{O}_i^{(4)}[\pi]N}_{\mathcal{L}_{\pi N}^{(4)}} + \ldots$$

Pion-nucleon scattering amplitude for $\pi^a(q_1) + N(p_1) \to \pi^b(q_2) + N(p_2)$:

$$T_{\pi N}^{ba} = \frac{E+m}{2m}\left(\delta^{ba}\left[g^+(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\, h^+(\omega,t)\right] + i\epsilon^{bac}\tau^c\left[g^-(\omega,t) + i\vec{\sigma}\cdot\vec{q}_2\times\vec{q}_1\, h^-(\omega,t)\right]\right)$$

calculated within the chiral expansion

Pion-nucleon scattering up to Q⁴ in heavy-baryon ChPT

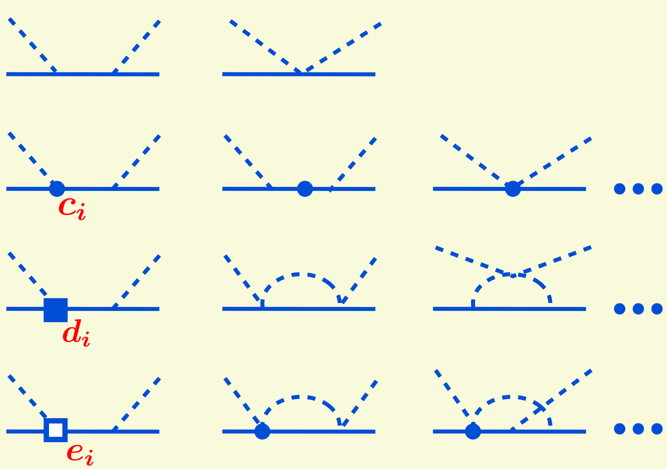
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

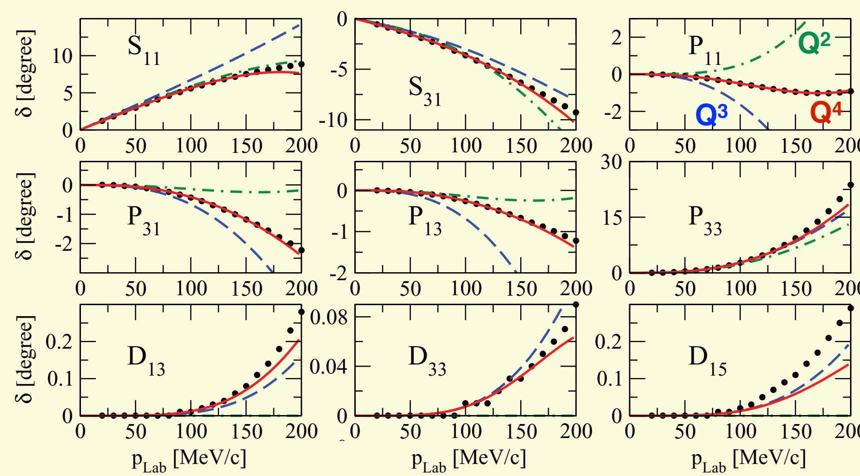
Order Q:

Order Q²:

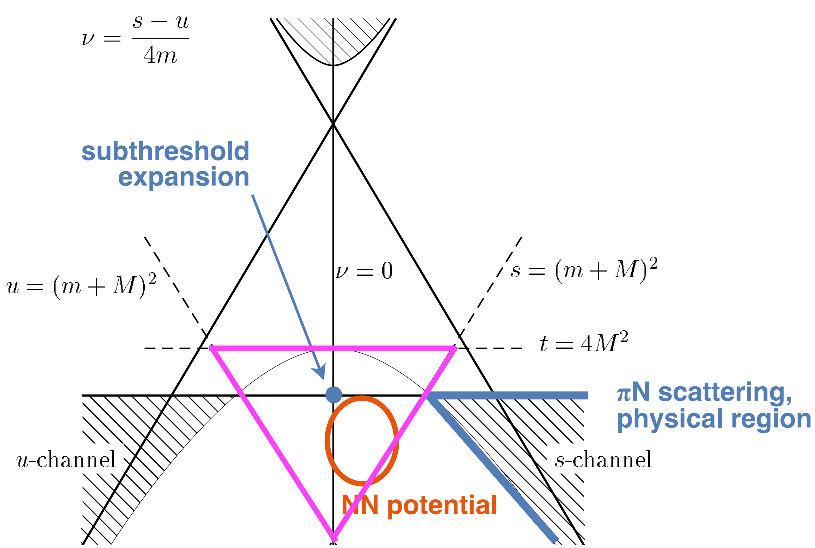
Order Q³:

Order Q⁴:





1. Determination of πN LECs



Matching ChPT to πN Roy-Steiner equations

Hoferichter, Ruiz de Elvira, Kubis, Meißner, PRL 115 (2015) 092301

- χ expansion of the πN amplitude expected to converge best within the Mandelstam triangle
- Subthreshold coefficients (from RS analysis) provide a natural matching point to ChPT

$$\bar{X} = \sum_{m,n} x_{mn}\, \nu^{2m+k} t^n, \qquad X = \{A^\pm, B^\pm\}$$

- Closer to the kinematics relevant for nuclear forces…

Relevant LECs (in GeV^{-n}) extracted from πN scattering

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$	
$[Q^4]_{\rm HB,NN}$, GW PWA	−1.13	3.69	−5.51	3.71	5.57	−5.35	0.02	−10.26	1.75	−0.58	Krebs, Gasparyan, EE, PRC85 (12) 054006
$[Q^4]_{\rm HB,NN}$, KH PWA	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−0.37	Krebs, Gasparyan, EE, PRC85 (12) 054006
$[Q^4]_{\rm HB,NN}$, Roy-Steiner	−1.10	3.57	−5.54	4.17	6.18	−8.91	0.86	−12.18	1.18	−0.18	Hoferichter et al., PRL 115 (15) 092301
$[Q^4]_{\rm covariant}$, data	−0.82	3.56	−4.59	3.44	5.43	−4.58	−0.40	−9.94	−0.63	−0.90	Siemens et al., PRC94 (16) 014620

— Some LECs show sizable correlations (especially c_1 and c_3)…

— RKE N^{4}LO [Reinert, Krebs, EE, EPJA 54 (2018) 88]: Q^4 fit to RS and Q^4 fit to KH PWA

With the LECs taken from πN, the long-range NN force is completely fixed (parameter-free)

2. Regularization

The cutoff Λ has to be kept finite, $\Lambda \sim \Lambda_b$ (unless all counterterms are taken into account in the calculations) **[Lepage '97; EE, Gegelia '09]**. In practice, low values of Λ are preferred:

— many-body methods require soft interactions,

— spurious deeply-bound states for $\Lambda > \Lambda^{\rm crit}$ make calculations for $A > 3$ unfeasible…

⟶ it is crucial to employ a regulator that minimizes finite-Λ artifacts!

Nonlocal: $$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{p'^4+p^4}{\Lambda^4}}}{\vec q^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec q^{\,2}+M_\pi^2}\underbrace{\left(1-\frac{p'^4+p^4}{\Lambda^4}+\mathcal{O}(\Lambda^{-8})\right)}_{\text{affect long-range interactions...}}$$

EE, Glöckle, Meißner '04; Entem, Machleidt '03; Entem, Machleidt, Nosyk '17; …

Local: $$V_{1\pi}^{\rm reg} \propto \frac{e^{-\frac{\vec q^{\,2}+M_\pi^2}{\Lambda^2}}}{\vec q^{\,2}+M_\pi^2} \longrightarrow \frac{1}{\vec q^{\,2}+M_\pi^2}\left(1+\text{short-range terms}\right)$$

[inspired by Thomas Rijken]

Reinert, Krebs, EE '18;

⟶ does not affect long-range physics **at any order** in $1/\Lambda^2$-expansion

— Application to 2π exchange does not require re-calculating the corresponding diagrams:

$$V(q) = \frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2} + \ldots \xrightarrow{\text{reg.}} V_\Lambda(q) = e^{-\frac{q^2}{2\Lambda^2}}\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2}\, e^{-\frac{\mu^2}{2\Lambda^2}} + \underbrace{\ldots}_{\text{polynomial in } q^2,\, M_\pi}$$

— Convention: choose polynomial terms such that $\Delta^n V_{\Lambda,\,\rm long}(\vec r)\Big|_{r=0} = 0$

2. Regularization

The general structure of the long-range potential is:

$$V(\vec{q},\vec{k}) = V_C(q)+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 W_C(q) + [V_S(q)+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 W_S(q)]\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + [V_T(q)+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 W_T(q)]\,\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}$$
$$+ [V_{LS}(q)+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 W_{LS}(q)]\, i(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\vec{q}\times\vec{k})$$

Consider the NLO TPEP $W_c(q)$ as an example:

$$W_C^{(2)}(q) = -\frac{1}{384\pi^2F_\pi^4}\left[4M_\pi^2(5g_A^4-4g_A^2-1)+q^2(23g_A^4-10g_A^2-1)+\frac{48g_A^4M_\pi^4}{4M_\pi^2+q^2}\right]L(q)$$

$$L(q) = \frac{\sqrt{q^2+4M_\pi^2}}{q}\ln\frac{\sqrt{q^2+4M_\pi^2}+q}{2M_\pi}$$

Dispersive representation: $W_C^{(2)}(q) = \frac{2}{\pi}\int_{2M_\pi}^\infty \frac{d\mu}{\mu^3}\, q^4\,\frac{\rho(\mu)}{\mu^2+q^2} + \alpha+\beta q^2$ with $\rho(\mu) = \mathrm{Im}\big[W_C^{(2)}(0^+-i\mu)\big]$

(Notice: $\mathrm{Im}\big[L(0^+-i\mu)\big] = \sqrt{\mu^2-4M_\pi^2}/\mu$).

Regularization and subtractions:

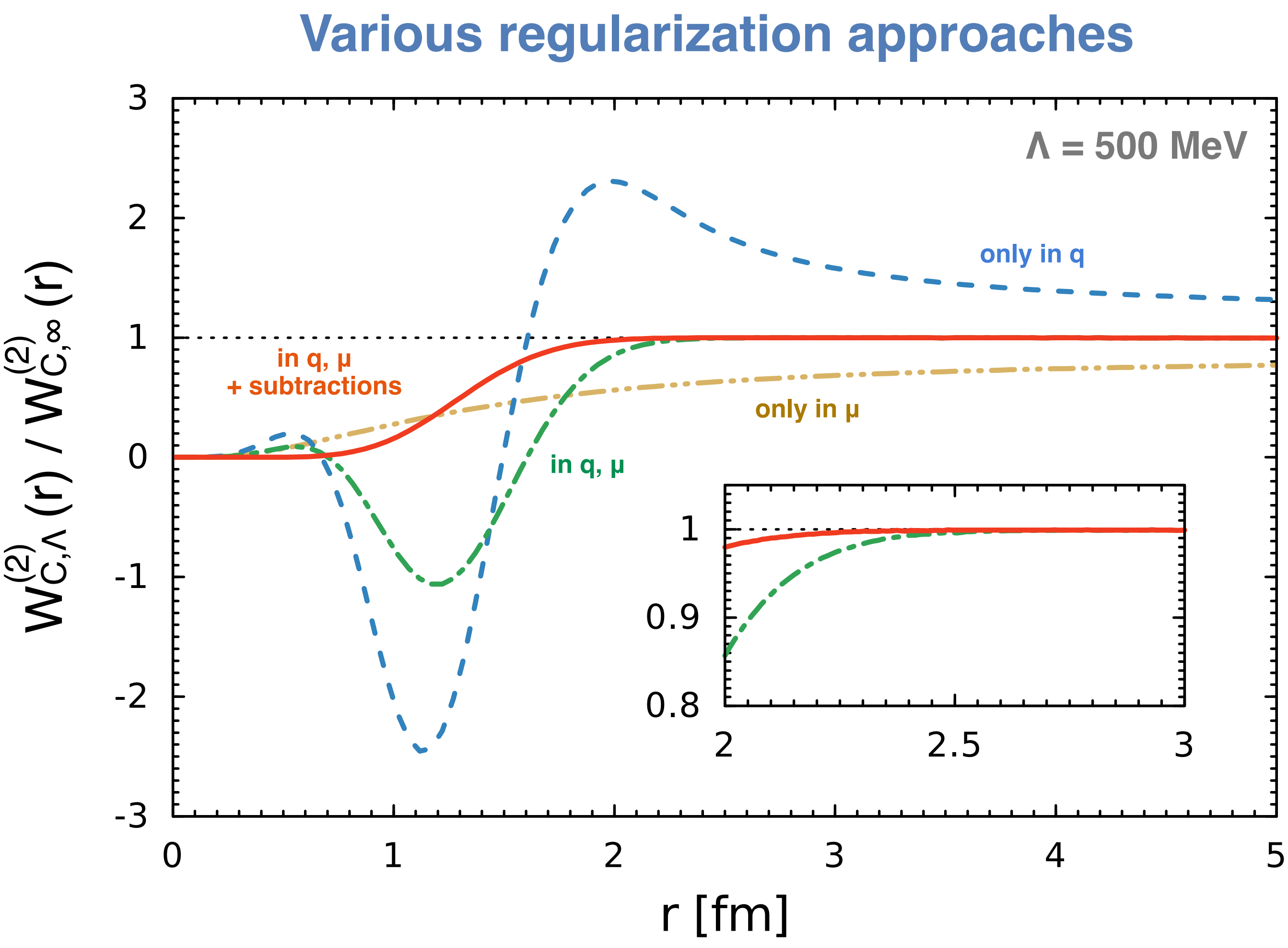
$$W_{C,\Lambda}^{(2)}(q) = \frac{2}{\pi}\int_{2M_\pi}^\infty \frac{d\mu}{\mu^3}\,\rho(\mu)\left(\frac{q^4}{\mu^2+q^2}+\underbrace{C_{C,1}^2(\mu)+C_{C,2}^2(\mu)\,q^2}\right)e^{-\frac{\mu^2+q^2}{2\Lambda^2}}$$

chosen to enforce: $W_{C,\Lambda}^{(2)}(r)\Big|_{r=0} = \frac{d^2}{dr^2}W_{C,\Lambda}^{(2)}(r)\Big|_{r=0} = 0$

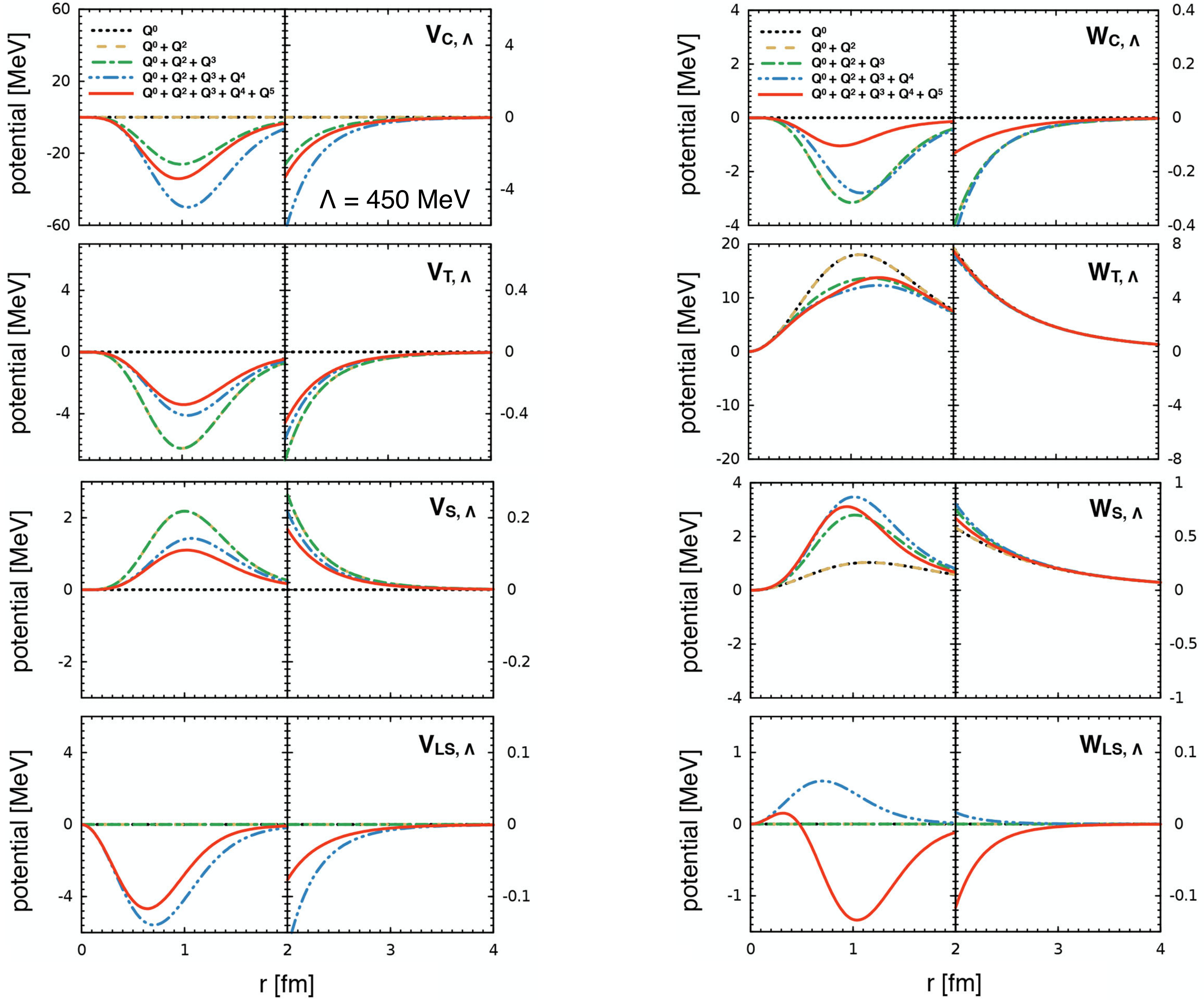
E.g.: $C_{C,1}^2(\mu) = \dfrac{2\Lambda\mu^2(2\Lambda^4-4\Lambda^2\mu^2-\mu^4)+\sqrt{2\pi}\mu^5 e^{\frac{\mu^2}{2\Lambda^2}}(5\Lambda^2+\mu^2)\,\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\Lambda}\right)}{4\Lambda^5}$

2. Regularization

Regularized 2π-exchange potential:



2. Regularization

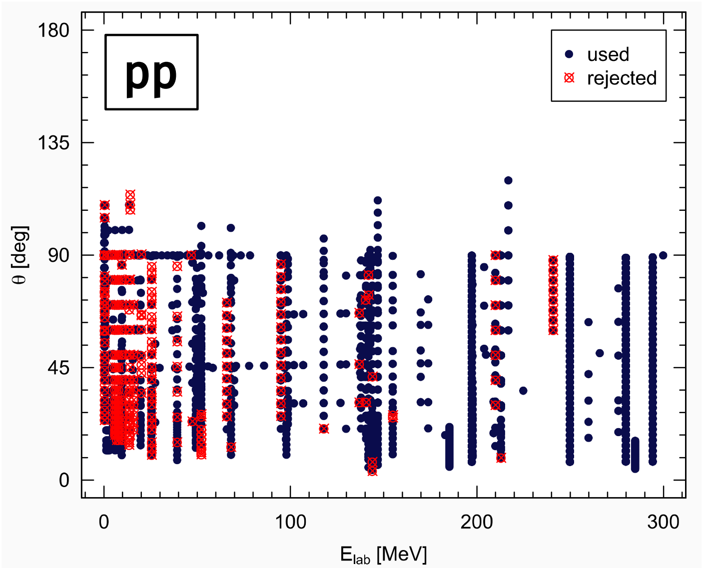


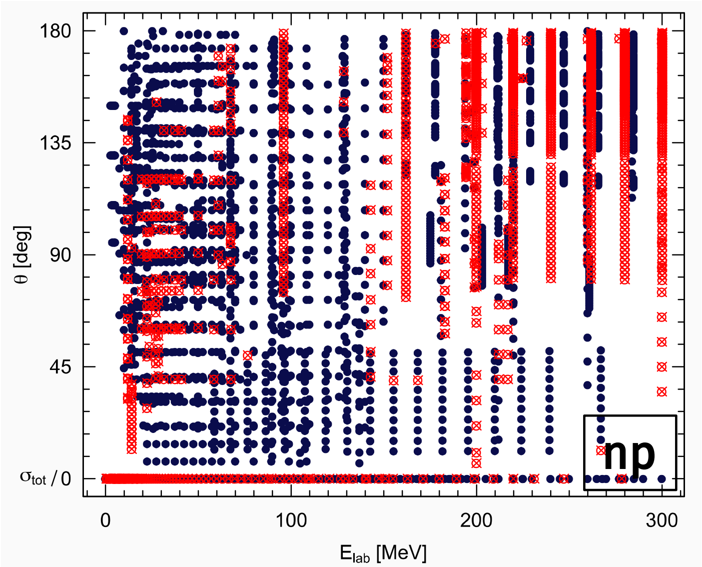
3. Determination of the NN LECs and results

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

- Contacts at N^{4}LO$^+$: 2 [Q^0] + 7 [Q^2] **+ 12** [Q^4] + 4 [F-waves, Q^6] + IB; Gauss regulator
- Use scattering data together with B_d = 2.224575(9) MeV and b_{np} = 3.7405(9) fm.
- Since 1950-es, about 3000 proton-proton + 5000 neutron-proton scattering data below 350 MeV have been collected.
- However, certain data are mutually incompatible within errors and have to be rejected. 2013 Granada database [Navarro-Perez et al., PRC 88 (2013) 064002], rejection rate: 31% np, 11% pp:

2158 proton-proton + 2697 neutron-proton data below E_{lab} = 300 MeV



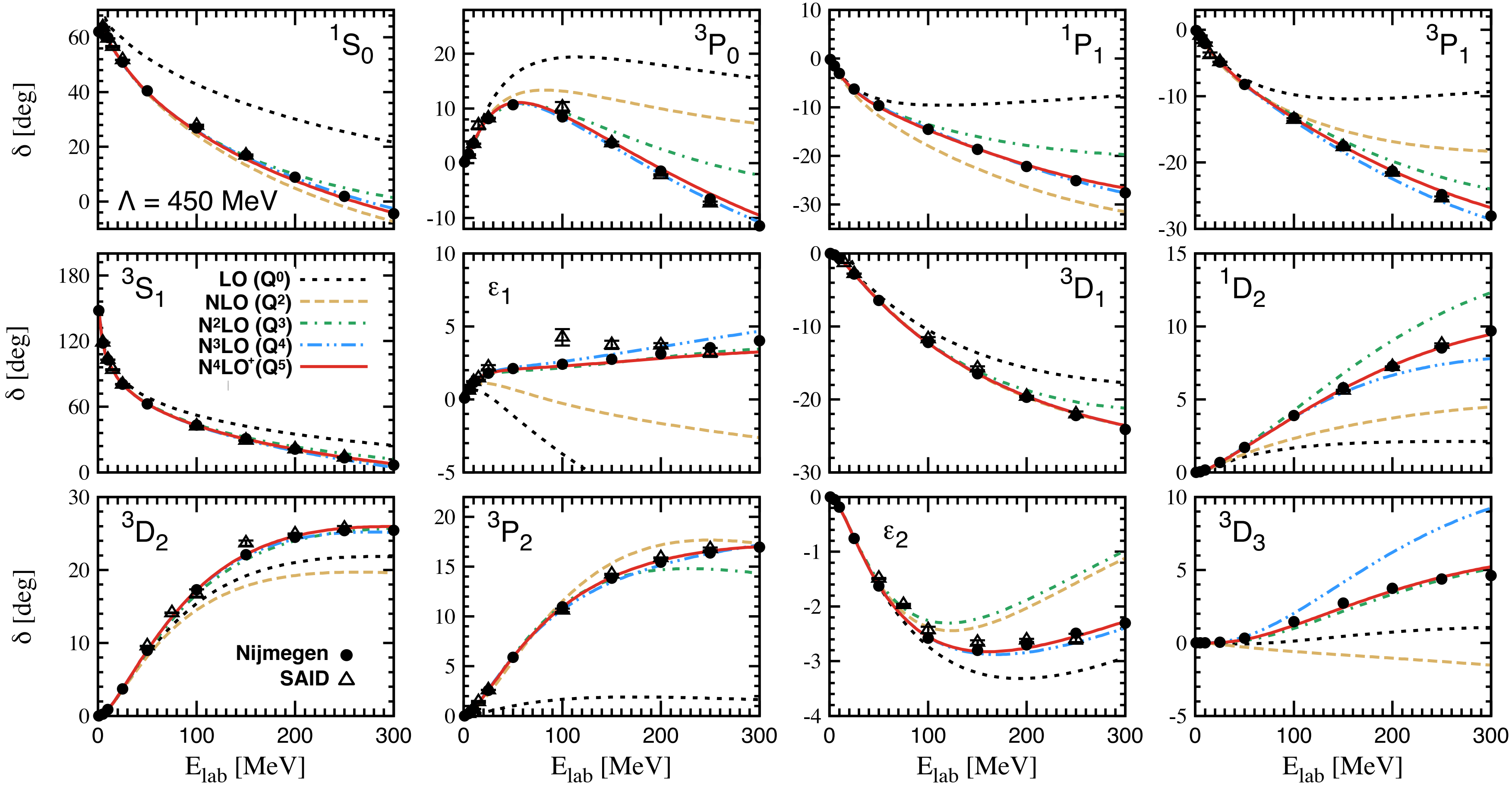


- Incomplete treatment of IB effects: $V_\gamma + V_{1\pi} + V_{cont}$ ($^1S_0$)

3. Determination of the NN LECs and results

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

Convergence of the chiral expansion for np phase shifts



— Clear evidence of the parameter-free chiral 2π exchange (Roy-Steiner LECs)!

3. Determination of the NN LECs and results

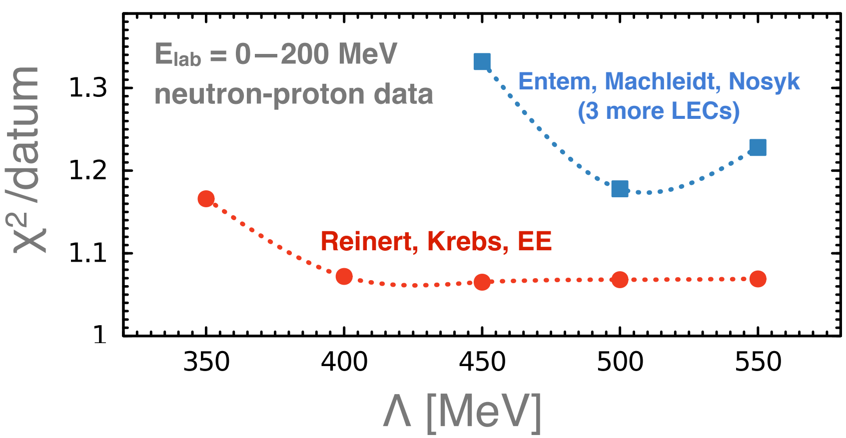
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

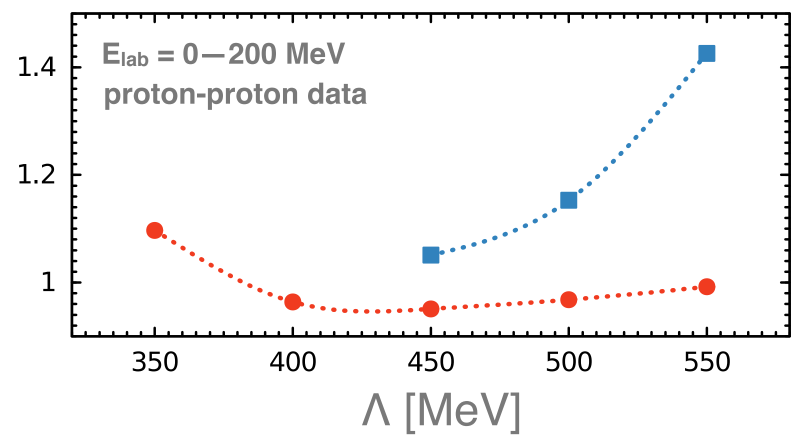
χ^2/datum for the description of the Granada-2013 database: χEFT vs. phenomenology

$E_{\rm lab}$ bin	CD Bonn$_{(43)}$	Nijm I$_{(41)}$	Nijm II$_{(47)}$	Reid93$_{(50)}$	N^{4}LO$^+$$_{(27+1)}$, this work
neutron-proton scattering data					
0 – 100	1.08	1.06	1.07	1.08	1.07
0 – 200	1.08	1.07	1.07	1.09	1.07
0 – 300	1.09	1.09	1.10	1.11	1.06
proton-proton scattering data					
0 – 100	0.88	0.87	0.87	0.85	0.86
0 – 200	0.98	0.99	1.00	0.99	0.95
0 – 300	1.01	1.05	1.06	1.04	1.00

For the first time, chiral EFT potentials qualify for being regarded as PWA!

N^{4}LO$^+$: semilocal (Reinert, Krebs, EE) vs. nonlocal (Entem, Machleidt, Nosyk)





4. Truncation uncertainty

In most cases, the uncertainty is dominated by truncation errors. Consider an observable X(p):

$$X(p) = \boxed{c_0 + c_2 Q^2 + c_3 Q^3 + \ldots + c_i Q^i} + \boxed{c_{i+1} Q^{i+1} + \ldots} \quad \text{where} \quad Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi^{\rm eff}}{\Lambda_b}\right)$$

known from explicit calculations — truncation error $\delta X^{(i)}$ to be estimated

Instead of just assuming $c_{i+1} \sim 1$, use the information from the known $c_{0\ldots i}$ [EE, Krebs, Meißner '15]

Bayesian approach to estimating truncation errors [Furnstahl et al. (BUQEYE) '15-'21; EE et al. '19]

— assume some common probability distribution function for c_i (= prior pdf)

— calculate the posterior probability distribution function $\mathrm{pr}\big(\delta X^{(i)} \,|\, c_{0\ldots i}\big)$

— at high chiral orders, almost no dependence on the prior…

— easy to implement and universal

Example: neutron-proton total cross section at 100 MeV [Λ = 450 MeV]

$$\sigma_{\rm tot} = 84.0_{[q^0]} - 10.2_{[q^2]} + 0.4_{[q^3]} - 0.4_{[q^4]} + 0.6_{[q^5]} - 0.0_{[q^6]}$$

$$= 74.35(14)(17)(1)\ \text{mb}$$

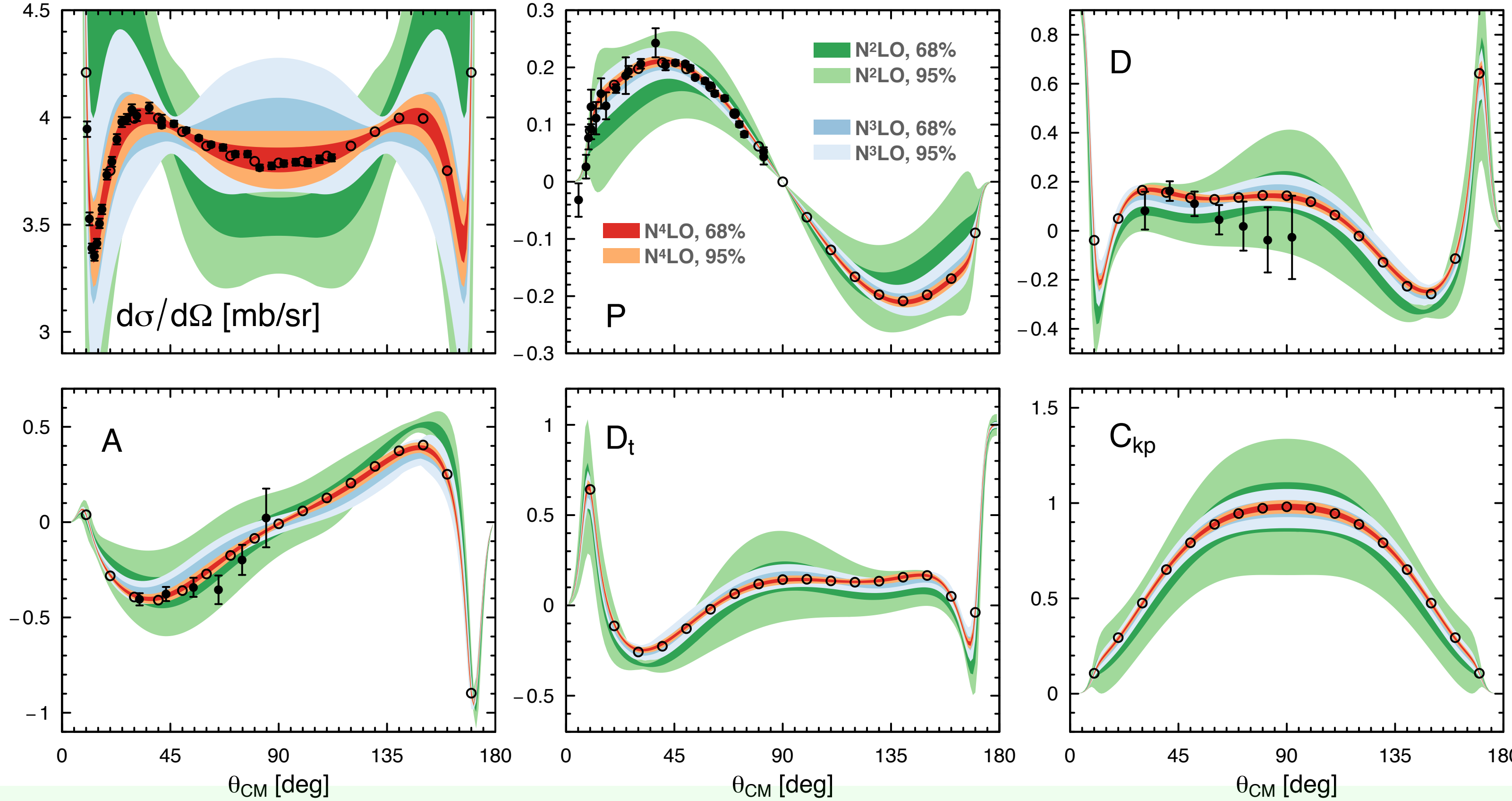
truncation error (68% DoB) → (14)

statistical error (NN LECs) → (17)

uncertainty in the πN LECs → (1)

4. Truncation uncertainty

Selected proton-proton scattering observables at 143 MeV



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$$= 74.35(14)(17)(1) \text{ mb}$$

truncation error (68% DoB)

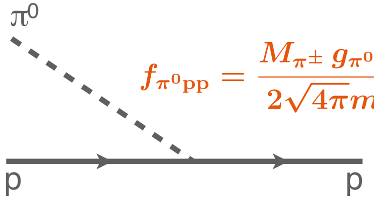
statistical error (NN LECs)

uncertainty in the πN LECs

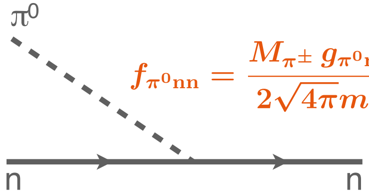
5. Precision determination of pion-nucleon coupling constants

Patrick Reinert, Hermann Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

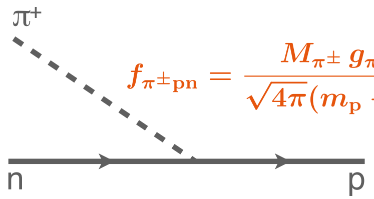
- Fundamental observables that control the strength of the nuclear forces due to π-exchange
- Insights into isospin breaking at the hadronic/nuclear levels
- Gold-plated benchmarks for lattice-QCD + QED



$$f_{\pi^0\mathrm{pp}} = \frac{M_{\pi^\pm}\, g_{\pi^0\mathrm{pp}}}{2\sqrt{4\pi}\, m_\mathrm{p}}$$

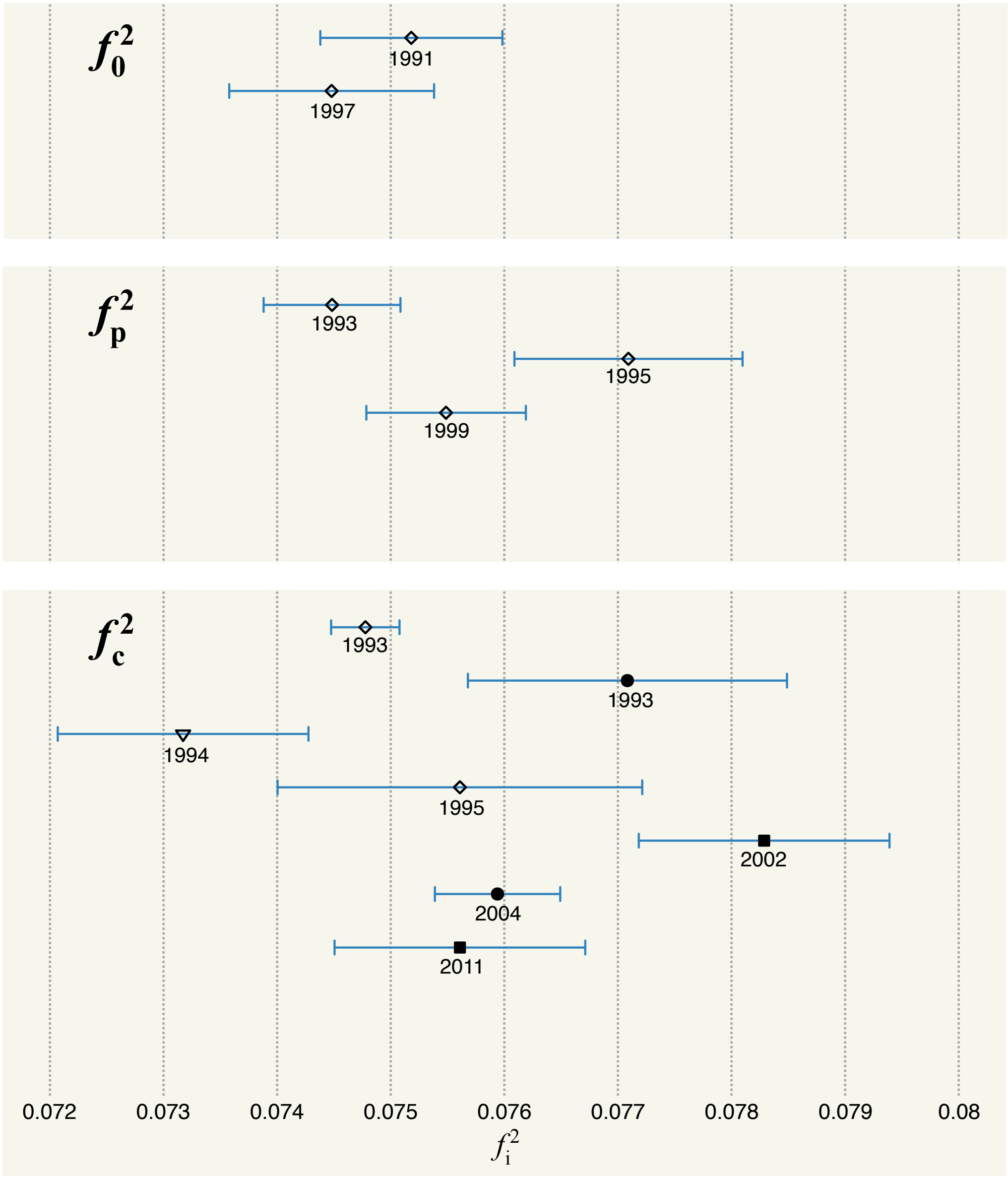


$$f_{\pi^0\mathrm{nn}} = \frac{M_{\pi^\pm}\, g_{\pi^0\mathrm{nn}}}{2\sqrt{4\pi}\, m_\mathrm{n}}$$



$$f_{\pi^\pm\mathrm{pn}} = \frac{M_{\pi^\pm}\, g_{\pi^\pm\mathrm{pn}}}{\sqrt{4\pi}\,(m_\mathrm{p} + m_\mathrm{n})}$$

πN coupling constants: Some earlier determinations

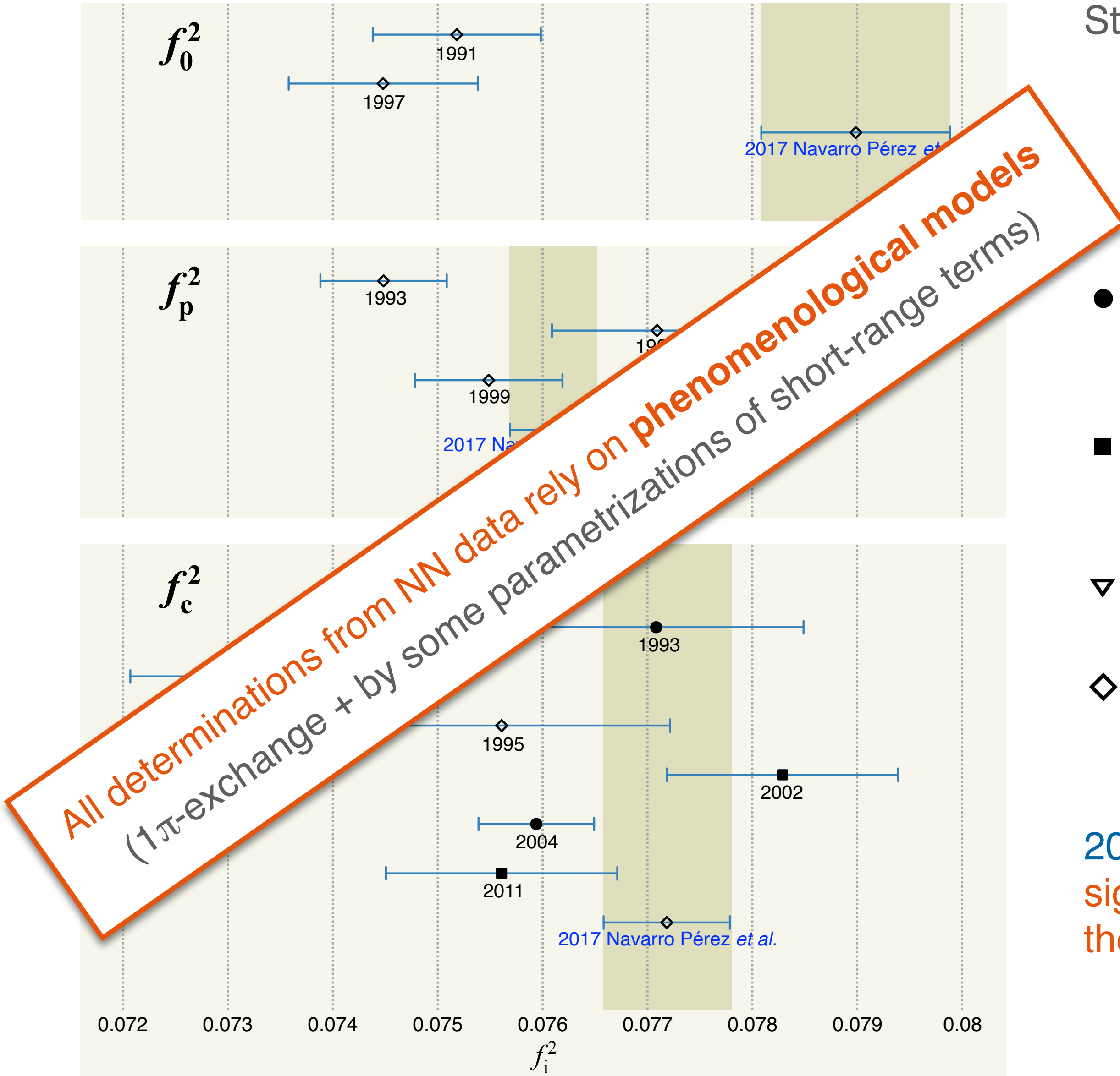


Standard notation:

$$
\begin{aligned}
f_0^2 &= -f_{\pi^0 nn}\, f_{\pi^0 pp} \\
f_p^2 &= f_{\pi^0 pp}\, f_{\pi^0 pp} \\
2f_c^2 &= f_{\pi^\pm pn}\, f_{\pi^\pm pn}
\end{aligned}
$$

- ● — fixed-t dispersion relations of πN scattering
  Markopoulou-Kalamara, Bugg '93; Arndt et al. '04
- ■ — πN scattering lengths + Goldberger-Miyazawa-Oehme sum rule
  Ericson et al. '02; Baru et al. '11
- ▽ — proton-antiproton PWA
  Timmermans et al. '94
- ◇ — neutron-proton (+ proton-proton) PWA
  Klomp et al. '91; Stoks et al. '93; Bugg et al. '95; de Swart et al. '97; Rentmeester et al. '99

πN coupling constants: Some earlier determinations



Standard notation:

$$f_0^2 = -f_{\pi^0 nn} f_{\pi^0 pp}$$
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2017 Granada PWA: evidence for significant charge dependence of the coupling constants:

$$f_0^2 - f_p^2 = 0.0029(10)$$

Navarro Perez et al., PRC 95 (2017) 6, 064001

Anatomy of the calculation

The goal: *Bayesian* determination of f_c^2, f_p^2 and f_0^2 by performing a full-fledged PWA of NN data up to pion-production threshold in the framework of chiral EFT

1. Experimental data:

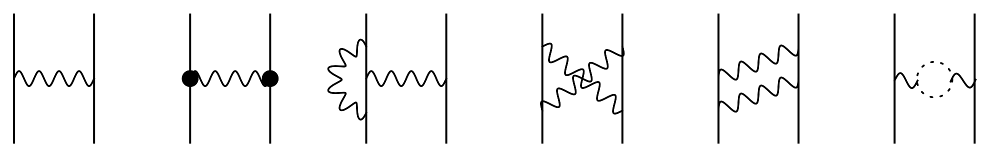
— About 8000 published np and pp scattering data below E_{lab} = 350 MeV. Not all data are mutually compatible…

— Selections of mutually compatible data: Nijmegen 1993, Granada 2013, 2017

— Performed own selection of compatible data (found some differences to Granada…)

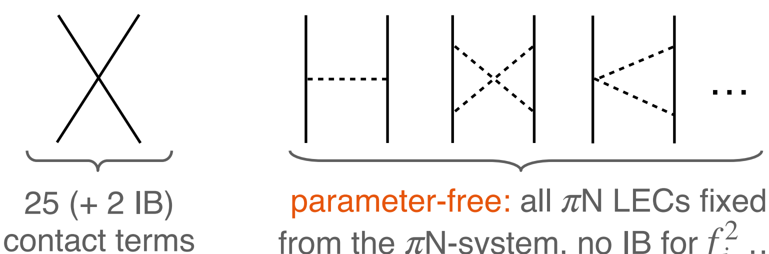
2. Interaction model:

— Long-range EM interactions (included in all PWA…)



— Semi-local chiral NN interaction at N⁴LO⁺ from P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

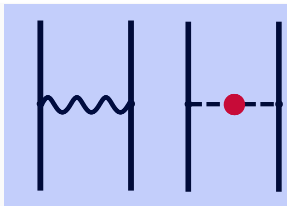
The treatment of isospin-breaking (IB) contributions was incomplete (limited to that of Nijmegen/Granada PWA)



We now include all charge-independence & charge-symmetry-breaking terms to N⁴LO.

Isospin-breaking NN forces

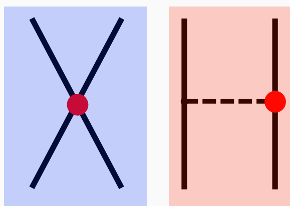
NLO



Have been employed in **Reinert, Krebs, EE, EPJA 54 (2018) 88**

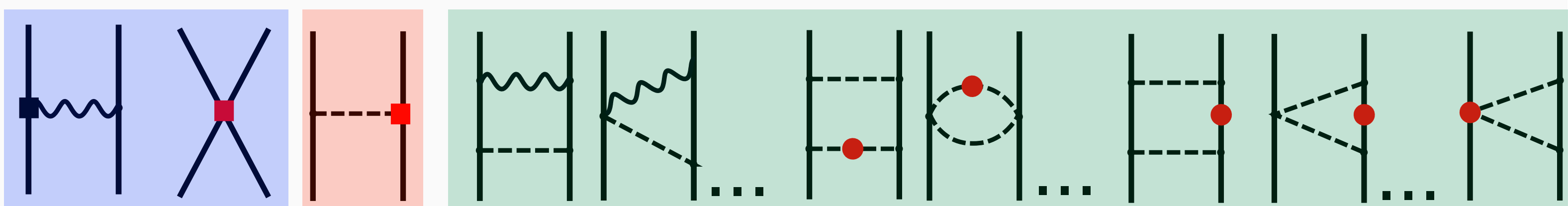
Parameter-free: depend on δM_π, δm = 1.29 MeV and $(\delta m)^{QCD}$ = 2.05(30) MeV **[Gasser, Leutwyler ’75]**

N^2LO

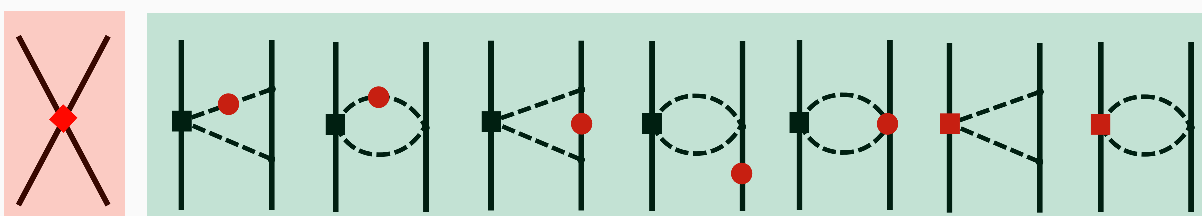


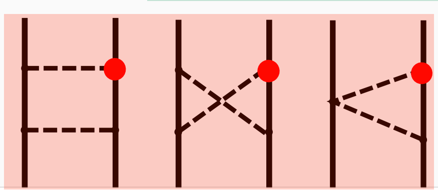
Depend on 3 πN coupling constants + 3 IB contact terms in p-waves

N^3LO



N^4LO





van Kolck et al.’98; Friar et al. ’99,’03,’04; Niskanen ’02; EE, Meißner ’05

Determination of the πN constants

Free parameters: 3 πN LECs $f^2 \equiv \{f_c^2,\ f_p^2,\ f_0^2\}$, the 25 IC + 5 IB LECs C and the cutoff Λ.

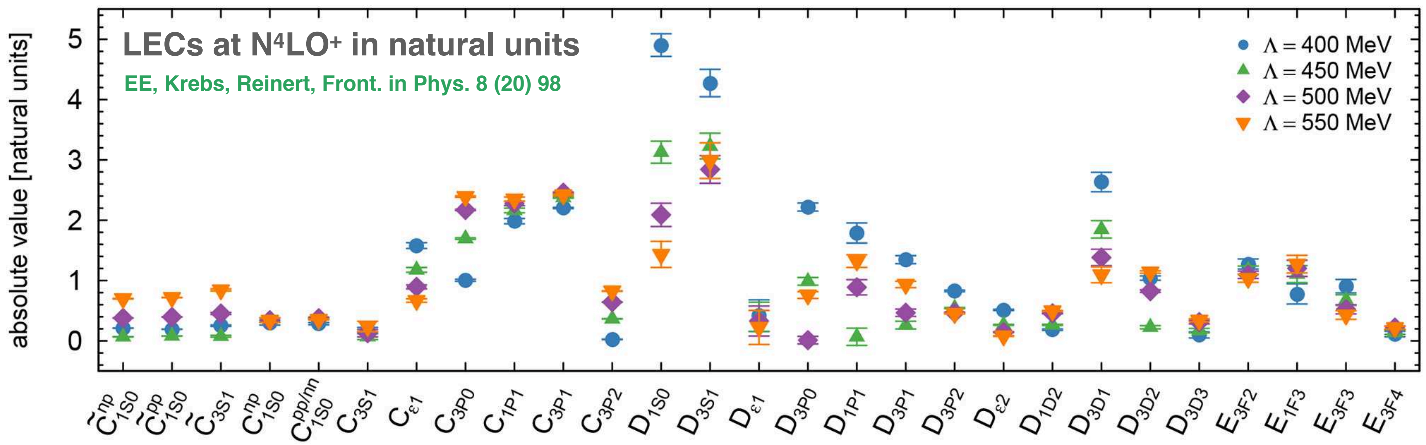
$$p(f^2|D) = \int d\Lambda\, dC\; p(f^2 C\Lambda|D) = \int d\Lambda\, dC\; \frac{p(D|f^2 C\Lambda)\; p(f^2 C\Lambda)}{p(D)}$$

NN data (to be specified later) — use Bayes' theorem — just a normalization constant

For normally distributed errors, the likelihood of data D is given by $p(D|f^2 C\Lambda) = \frac{1}{N} e^{-\frac{1}{2}\chi^2}$

Employ independent priors: $p(f^2 C\Lambda) = p(f^2)\, p(C)\, p(\Lambda)$

- $p(f^2)$ — uniform prior (cuboid) to cover the range of possible values of f^2
- $p(\Lambda)$ — uniform prior for $\Lambda \in [400\,\text{MeV},\ 550\,\text{MeV}]$ (to be justified later)
- $p(C)$ — multivariate Gaussian: $p(C) = (\sqrt{2\pi}\bar{C})^{-30} \exp(-\vec{C}^2/(2\bar{C}^2))$ with $\bar{C} = 5$



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Thus, need to evaluate a 30 (C) + 1 (Λ) dimensional integral at ~10³ grid points for f^2… 😢

Solution: Use the Laplace approximation by approximating the likelihood $p(D|f^2 C\Lambda)$ via :

$$p(D|f^2 C\Lambda) \approx \frac{1}{N} e^{-\frac{1}{2}[\chi^2_{\rm min} + \frac{1}{2}(C - C_{\rm min})^T H (C - C_{\rm min})]}$$

where $\chi^2_{\rm min} \equiv \chi^2_{\rm min}(f^2, \Lambda)$ at $C_{\rm min} \equiv C_{\rm min}(f^2, \Lambda)$ and $H_{ij}(f^2, \Lambda) = \frac{\partial^2 \chi^2}{\partial C_i \partial C_j}\Big|_{C = C_{\rm min}}$.

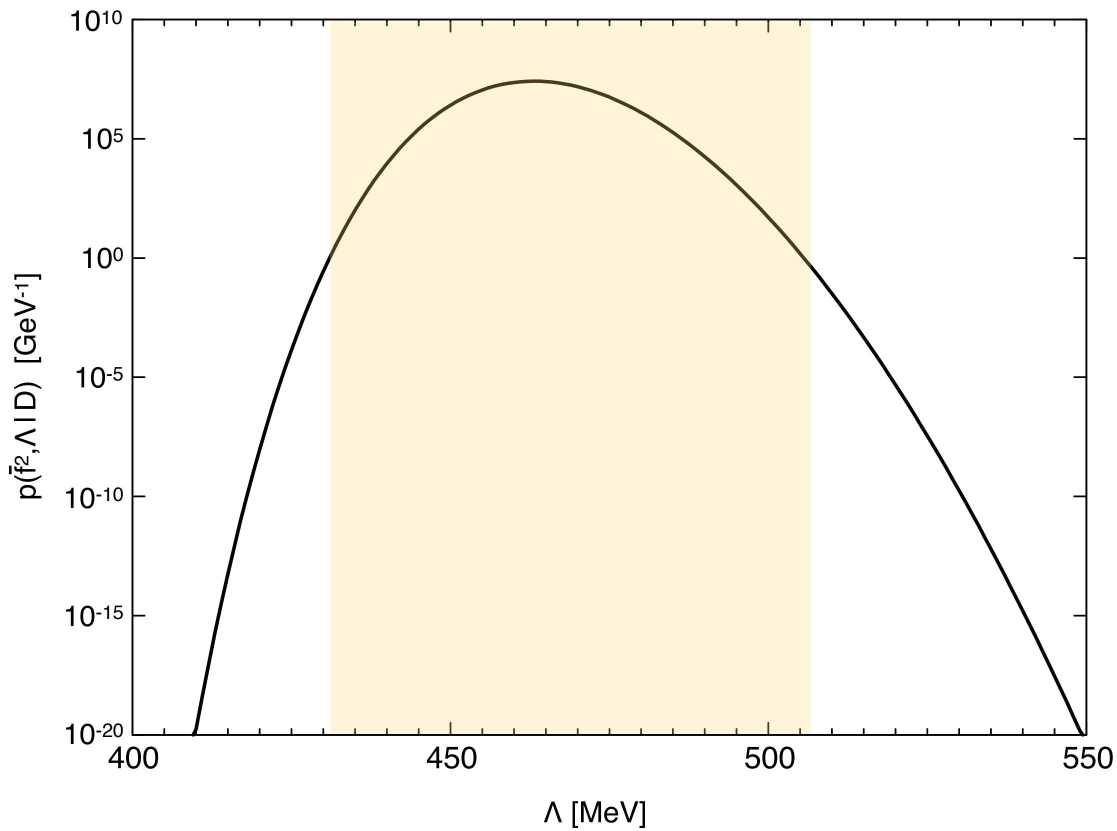
Then:

$$p(f^2|D) = \frac{1}{\tilde{N}} \int_{\Lambda_{\rm min}}^{\Lambda_{\rm max}} d\Lambda\, \frac{1}{\underbrace{\sqrt{\det A}}_{A = 1/2H + 1/\bar{C}^2 \mathbb{1}}} e^{-\frac{1}{2}(\chi^2_{\rm min} + \frac{1}{\bar{C}^2} C^T_{\rm min} C_{\rm min} - \frac{1}{\bar{C}^4} C^T_{\rm min} A^{-1} C_{\rm min})} \underbrace{\mathbb{1}_B(f^2)}_{\text{the indicator function for } f^2}$$

still requires ~10⁴ determinations of 30 LECs C from NN data…

Determination of the πN constants

How does the integrand look like for some typical values $\bar{f}^2$?



This shows that the employed prior $p(\Lambda)$ is indeed uninformative…

Then:

$$p(f^2|D) = \frac{1}{\tilde{N}} \int_{\Lambda_{\min}}^{\Lambda_{\max}} d\Lambda \, \frac{1}{\sqrt{\det \underbrace{A}_{A = 1/2H + 1/\bar{C}^2 \mathbb{1}}}} e^{-\frac{1}{2}\left(\chi^2_{\min} + \frac{1}{\bar{C}^2} C^T_{\min} C_{\min} - \frac{1}{\bar{C}^4} C^T_{\min} A^{-1} C_{\min}\right)} \underbrace{\mathbb{1}_B(f^2)}_{\text{the indicator function for } f^2}$$

still requires ~10^4 determinations of 30 LECs C from NN data…

Error analysis

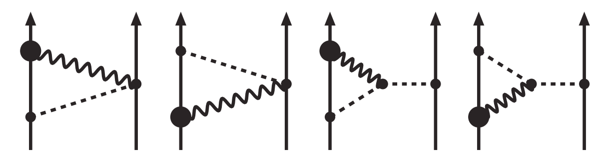
- Higher-order πN LECs c_i, d_i, e_i from the Roy-Steiner equation analysis: errors propagated using the covariance matrix provided in Hoferichter et al., PRL 115 (2015) 192301.

- QCD contribution to m_n-m_p: used $(\delta m)^{QCD} = 2.05(30)$ MeV [Gasser, Leutwyler '75] consistent with the lattice QCD+QED result $(\delta m)^{QCD} \sim 2.16$ MeV [Borsanyi et al. '15] and with the updated calc. $(\delta m)^{QCD} = 1.87(16)$ MeV [Gasser et al. '20]. The resulting errors in f^2 are found to be negligible.

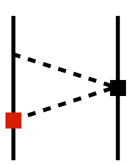
- Truncation of the chiral EFT expansion for IB interactions:

Model 1: IB terms up to N^{4}LO

Model 2: same as model 1 + N^{5}LO $\pi\gamma$-exchange
(involve large isovector MM $\kappa_v = 4.7$)
Kaiser, PRC 73 (2006) 044001



Model 3: same as model 1 + N^{5}LO 2π-exchange proportional to $(f_i - f_j)c_k$



$$V^{(6)}(q) = -\frac{f_c(2f_c - f_p - f_n)c_4}{4F_\pi^2 M_{\pi^\pm}^2} \tau_1^3 \tau_2^3 \left(\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q} - q^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2\right) \left(4M_\pi^2 + q^2\right) A(q)$$
$$- \frac{f_c(f_p - f_n)}{2F_\pi^2 M_{\pi^\pm}^2} \left(\tau_1^3 + \tau_2^3\right) \left(2M_\pi^2 + q^2\right) \left(4M_\pi^2 c_1 - c_3(2M_\pi^2 + q^2)\right) A(q)$$

Calculate $p(f^2|D)$ using the Models 1-3 and perform (Bayesian) averaging at the end.

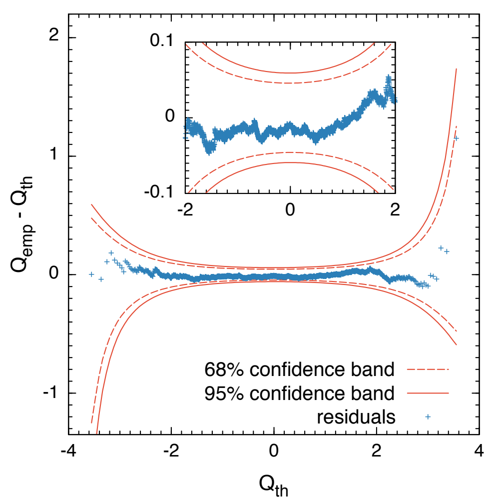
Error analysis

- **Experimental data**: performed own recursive selection of mutually compatible np, pp data using the standard 3σ-criterion (found some differences with the Granada 2013 database).

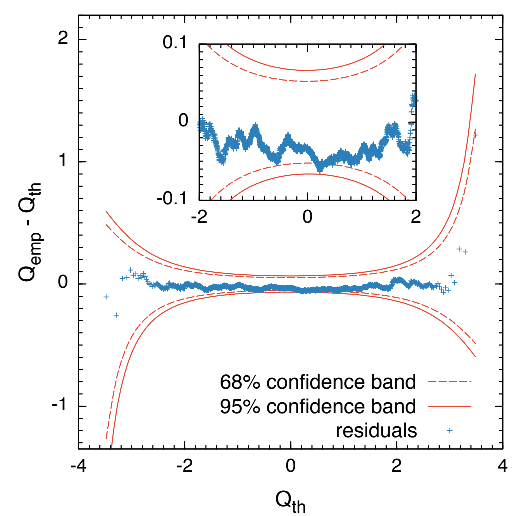
- Setting the energy range $[0, E_{max}]$ for the included NN data:
  - — higher E_{max}: more data available 🙂 but lower accuracy of the EFT… ☹️
  - — lower E_{max}: high accuracy of the EFT 🙂 but less data & possible overfitting… ☹️

To avoid possible overfitting performed statistical consistency tests by verifying the normal distribution of residuals (tail sensitive test by **Aldor-Noiman et al. Am. Stat. 67 (2013) 249**)

Rotated quantile-quantile plot for E_{max} = 300 MeV

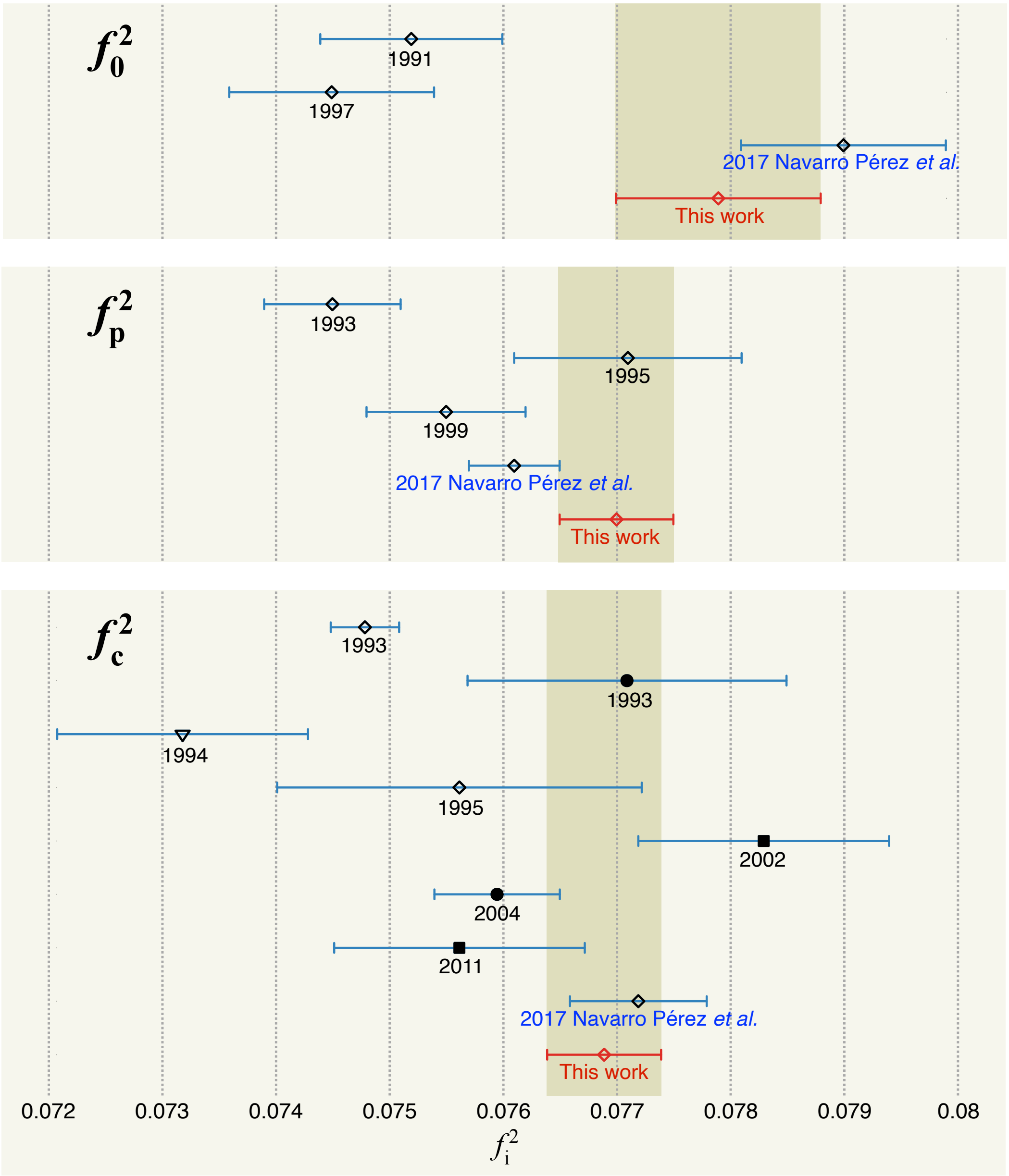


Rotated quantile-quantile plot for E_{max} = 220 MeV



To stay unbiased, average over independent analyses for E_{max} = 300, 280, …, 220 MeV

Determination of the πN constants



Our result:

$$f_0^2 = 0.0779(9)(1.3)$$
$$f_p^2 = 0.0770(5)(0.8)$$
$$f_c^2 = 0.0769(5)(0.9)$$

statistical **and systematic** errors due to the EFT truncation, choice of E_{max} and data selection

uncertainty in the subleading πN LECs

Our f_c^2 value is consistent with the extractions from the πN system

Contrary to the Granada group, we see no evidence for charge dependence of the πN coupling constants

Using the PDG-2020 value for the axial charge $g_A = 1.2756(13)$, the GT discrepancy amounts to (f_c):

$$\Delta_{GT} \sim 1.7\%$$

Determination of the πN constants

Our $g_{\pi NN}$ value corresponding to f_c^2 reads:

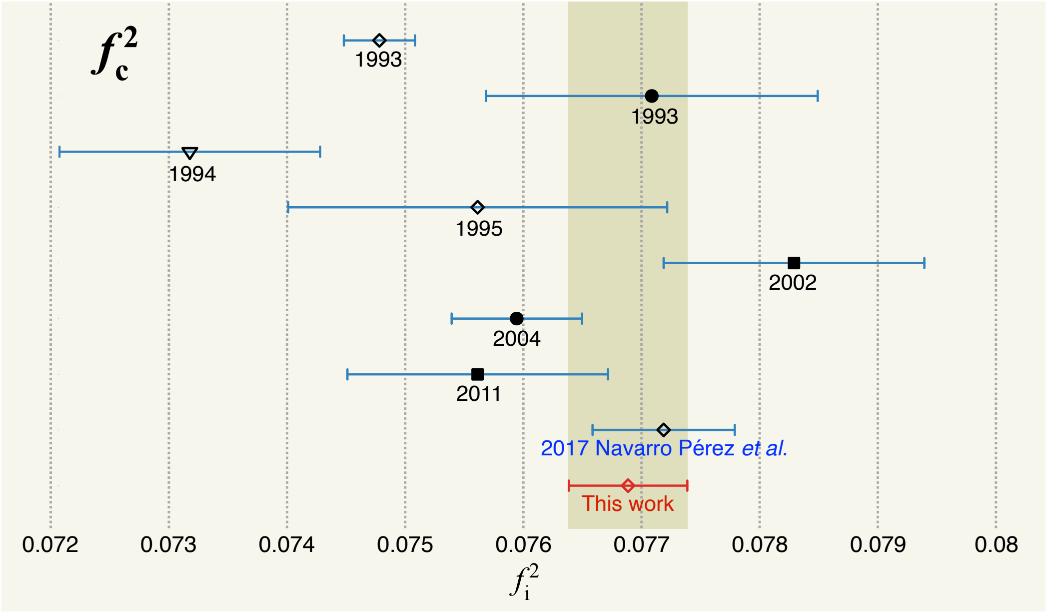
$$g_{\pi NN} = 13.92 \pm 0.09$$

Pionic hydrogen exp. at PSI (GMO sum rule)

[Hirtl et al., Eur. Phys. J. A57 (2021) 2, 70]

$$\epsilon_{1s}^{\pi H} + \epsilon_{1s}^{\pi D}: \quad g_{\pi NN} = 13.66 \pm 0.20$$

$$\Gamma_{1s}^{\pi H}: \quad g_{\pi NN} = 13.96 \pm 0.22$$



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$$\Delta_{GT} \sim 1.7\%$$

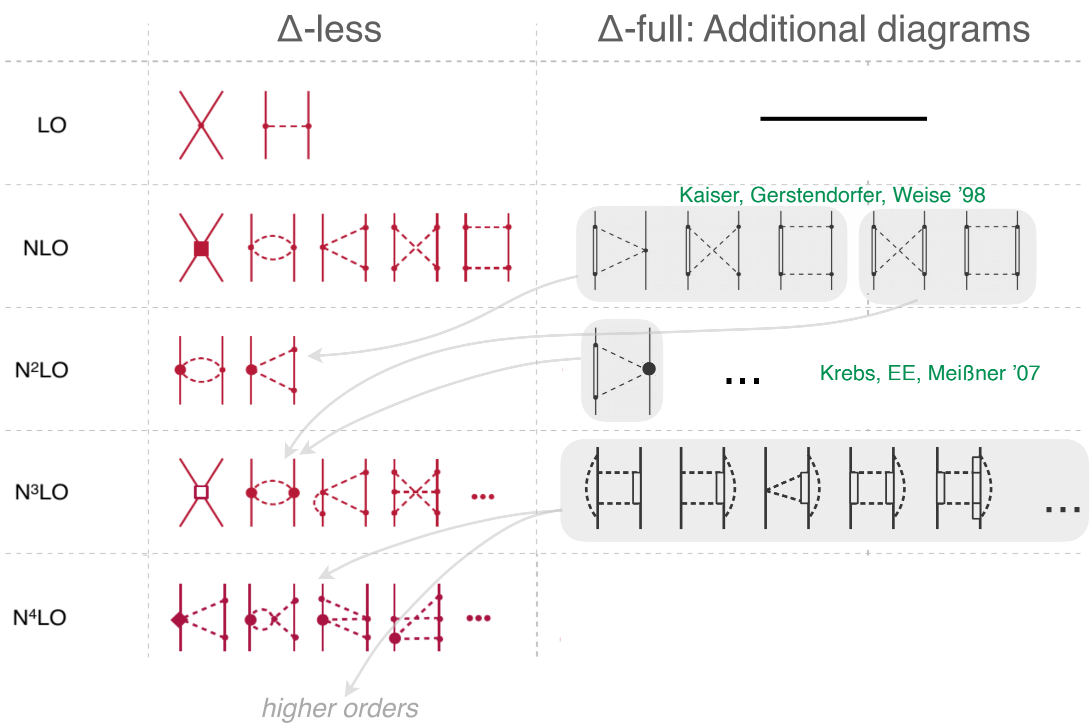
Chiral EFT with explicit Δ(1232)

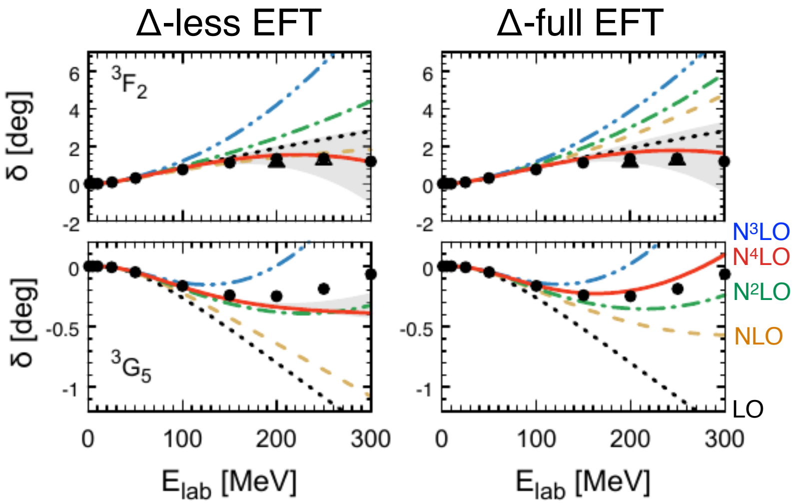
Motivation:

- treating Δ as an explicit DoF allows one to resum $(m_\Delta - m_N)^{-n}$ corrections
- superior performance of Δ-EFT@N²LO for heavy nuclei Ekström et al. '18

Results:

- Δ-contributions to the TPEP upto N²LO known: improved convergence
- peripheral NN scattering (Born approx.): hints of a slightly improved convergence but similar results at the highest order
  A. Gasparyan, H. Krebs, EE, in preparation





Next steps:

- three-pion exchange potential using the method of UT [work in progress by **Victor Springer**]
- fit the NN LECs to np + pp scattering data and test the resulting potentials in few-N systems

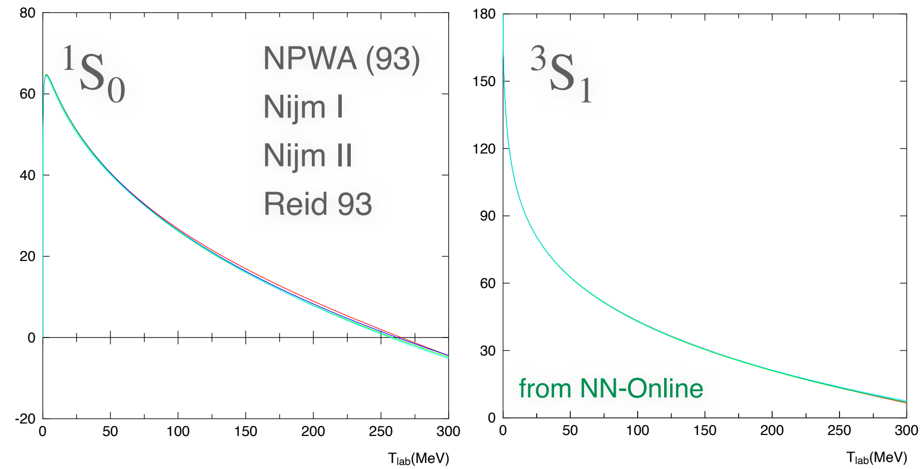
The 3NF challenge

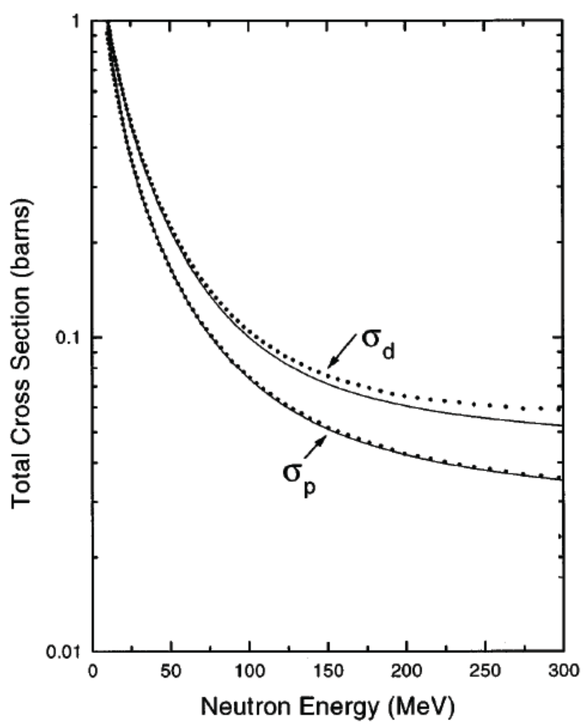
1. Introduction

Since the 90-es, we know that:

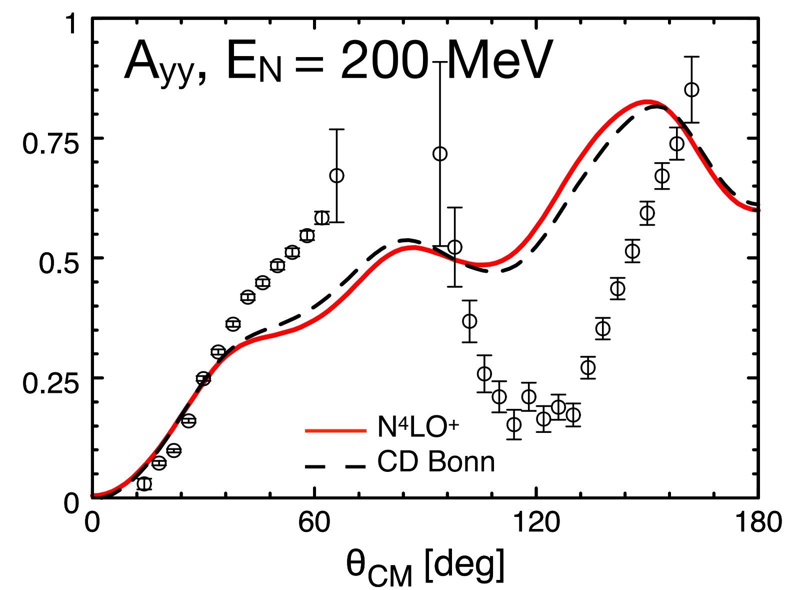
— 2N force is easy to parametrize:
  2 (isospin) × 6 spin-momentum operators

— after removing inconsistent data (~10% pp and ~30% np…), the rest of the data base can be described with $\chi^2/\text{datum} \sim 1.$





While the NN forces seem under control, large deviations show up for Nd scattering signaling the missing 3N forces

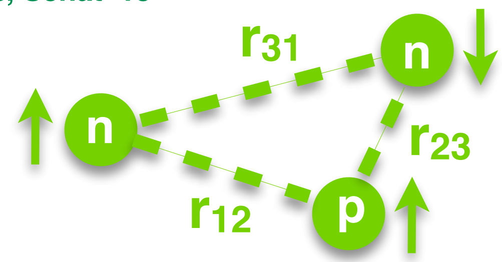


1. Introduction

The most general structure of a local, isospin-symmetric 3NF

Krebs, Gasparyan, EE '13; Phillips, Schat '13; EE, Gasparyan, Krebs, Schat '15

Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{13} \cdot \vec{\sigma}_1\, \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\, \hat{r}_{12} \cdot \vec{\sigma}_2\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\, \hat{r}_{23} \cdot \vec{\sigma}_1\, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \vec{\sigma}_3\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_3 \cdot \hat{r}_{23}\, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3)\, \vec{\sigma}_1 \cdot \hat{r}_{23}\, \vec{\sigma}_3 \cdot \hat{r}_{12}\, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$



80 operators generated by all possible permutations of **20 structures**:

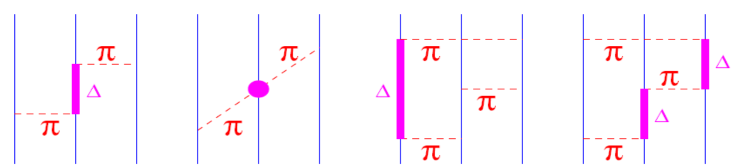
$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i\, F_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$$

$$V(q_1, q_2, q_3) = \sum_{i=1}^{20} \mathcal{G}_i\, F_i(q_1, q_2, q_3) + \text{permutations}$$

Nonlocal: 320 (!) operators
Topolnicki '17

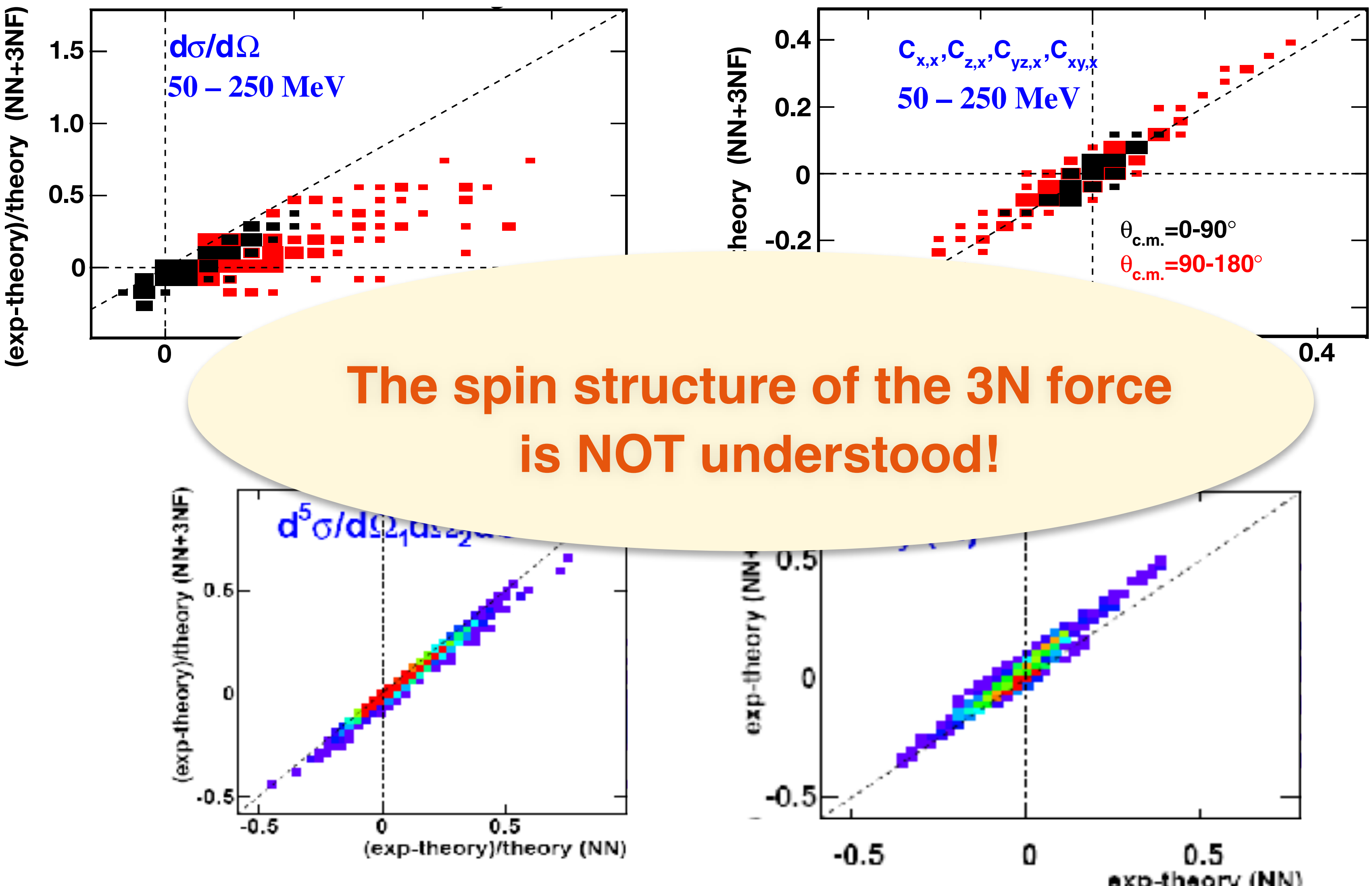
Phenomenological models:

Fujita-Miyazawa, Tucson-Melbourne, Brasil, Urbana IX, Illinois, ...



1. Introduction

Elastic nucleon-deuteron scattering



The spin structure of the 3N force is NOT understood!

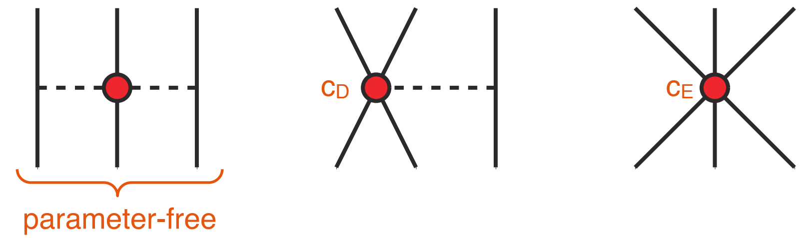
from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rept. Prog. Phys. 75 (2012) 016301

2. Inclusion of the 3NF at N²LO

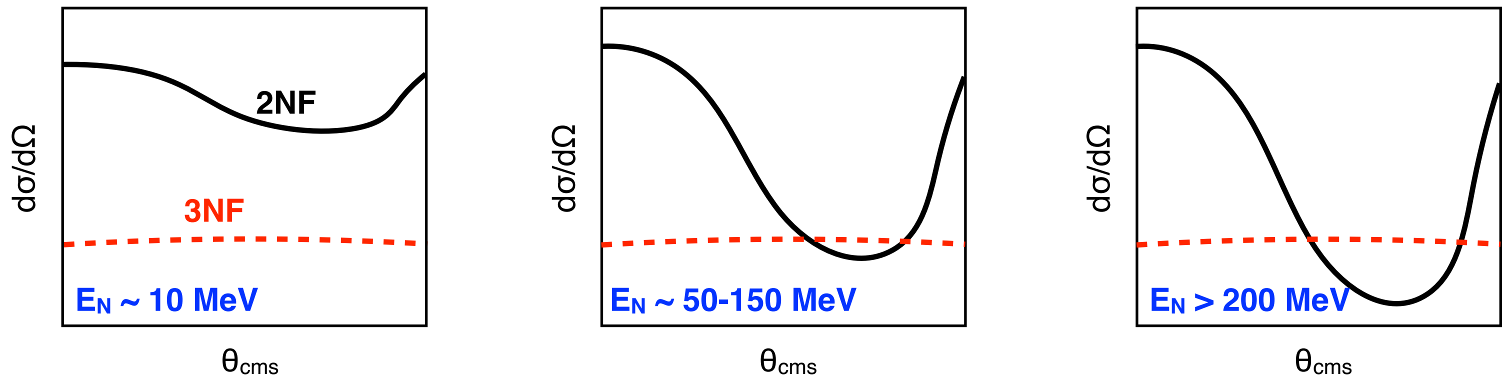
EE et al. [LENPIC], PRC99 (2019); Maris et al. [LENPIC], PRC103 (2021); e-Print: 2206.13303

The leading 3NF depends on 2 LECs that need to be determined from few-N data:

- ³H binding energy yields $c_E = f(c_D)$
- c_D is determined from Nd scattering



Differential cross section of Nd elastic scattering at intermediate and higher energies is known to be sensitive to the 3NF:

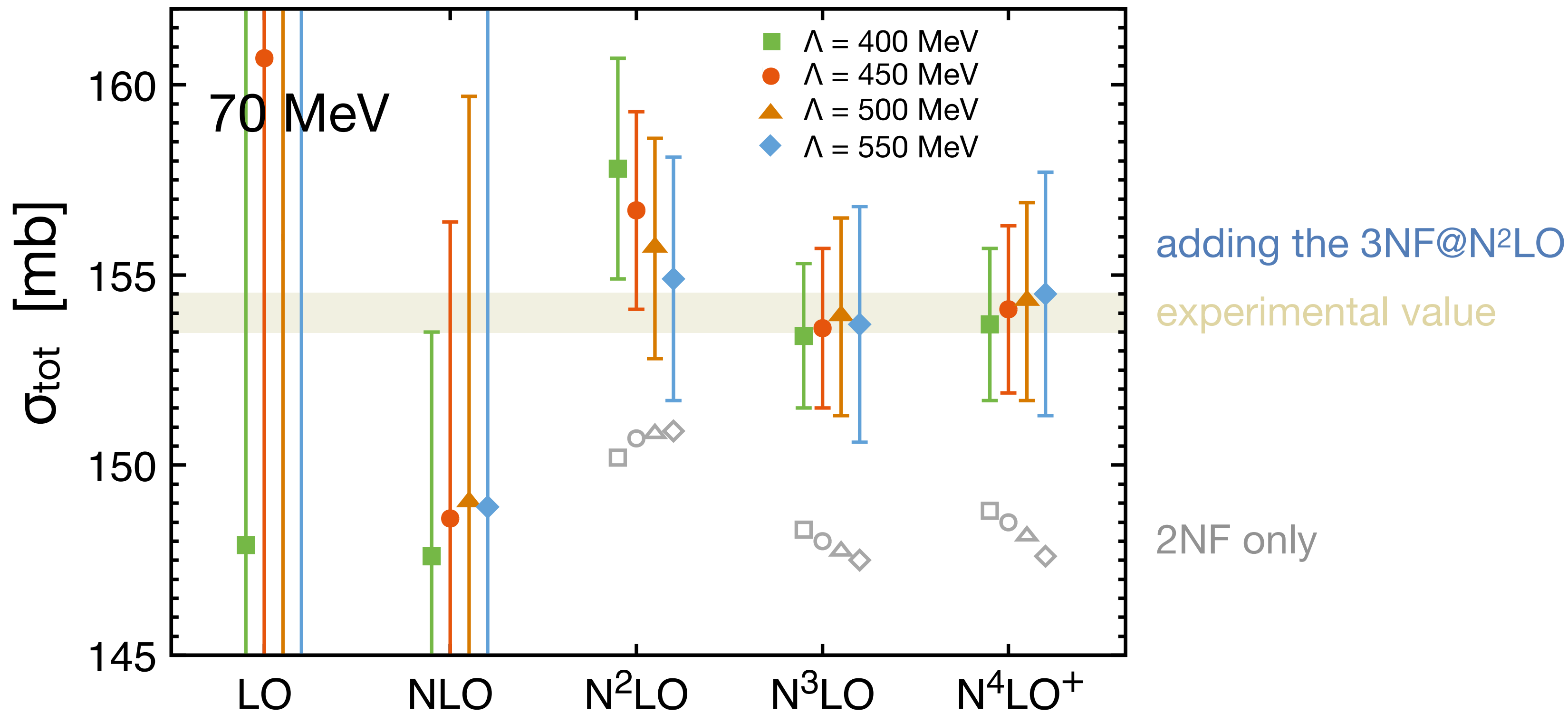


⇒ use precise Nd cross section data at 70 MeV from RIKEN [Kimiko Sekiguchi et al. '02] to fix c_D

LENPIC: Low Energy Nuclear Physics International Collaboration

2. Inclusion of the 3NF at N²LO

Maris et al. [LENPIC], e-Print: 2206.13303



— 2NF only underestimates the data; adding the 3NF improves the agreement with exp.

— 3NF contributions of natural size (W. counting)

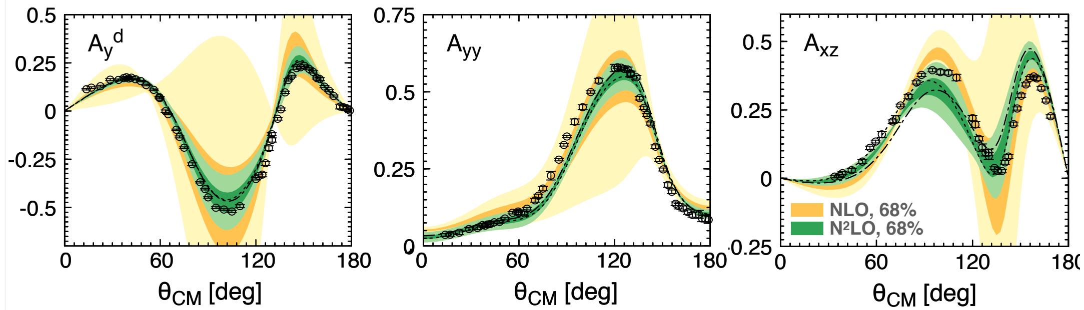
— small residual cutoff dependence

LENPIC: Low Energy Nuclear Physics International Collaboration

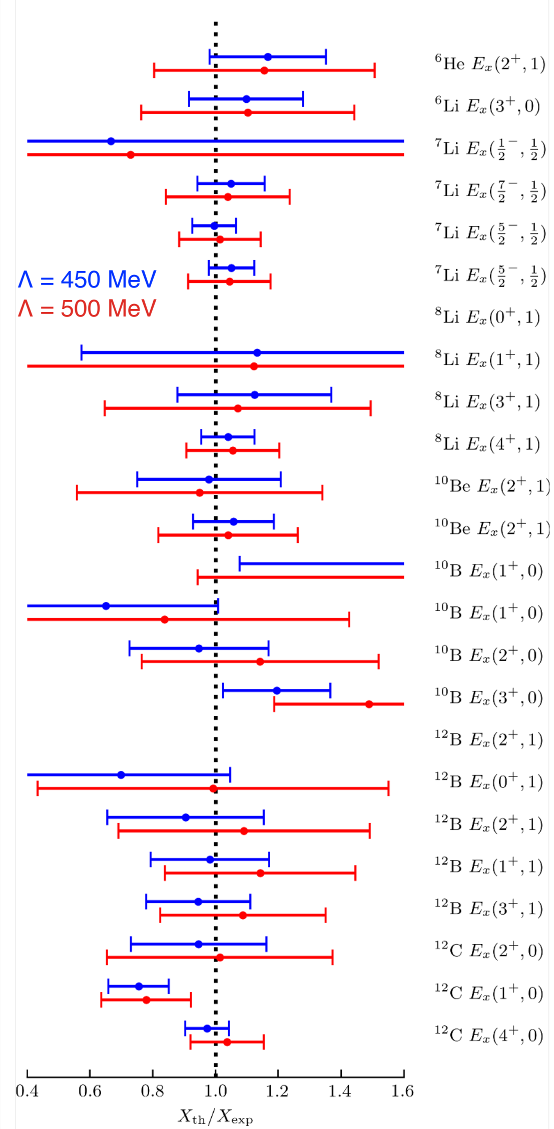
Few-N systems and light nuclei to N²LO

P. Maris et al. (LENPIC), Phys. Rev. C 103 (2021) 5, 054001

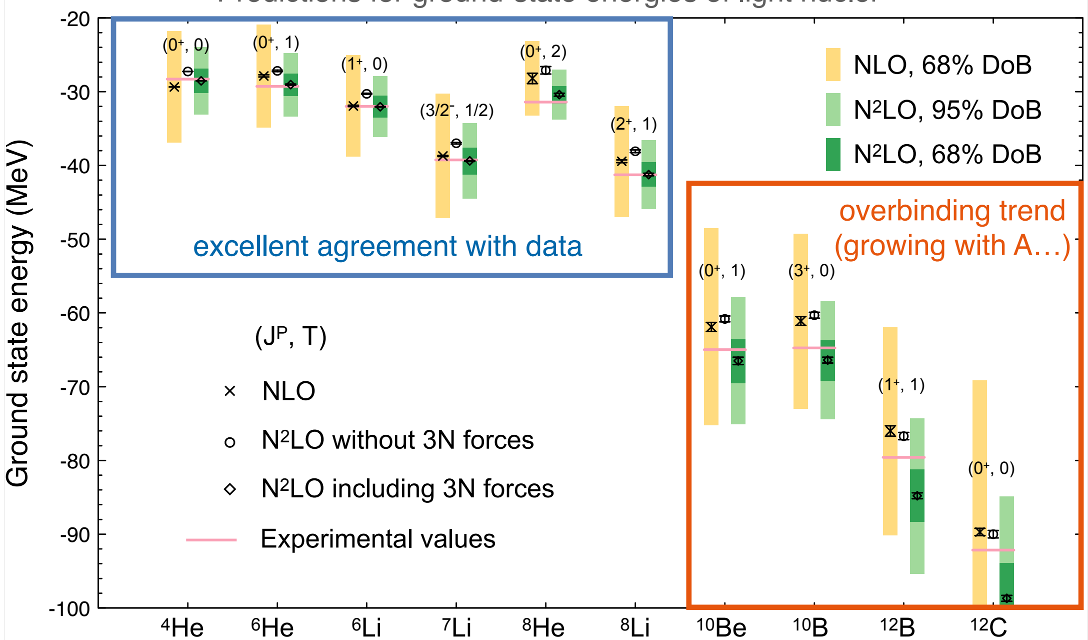
Tensor analyzing powers in Nd elastic scattering at 70 MeV



Excitation energies

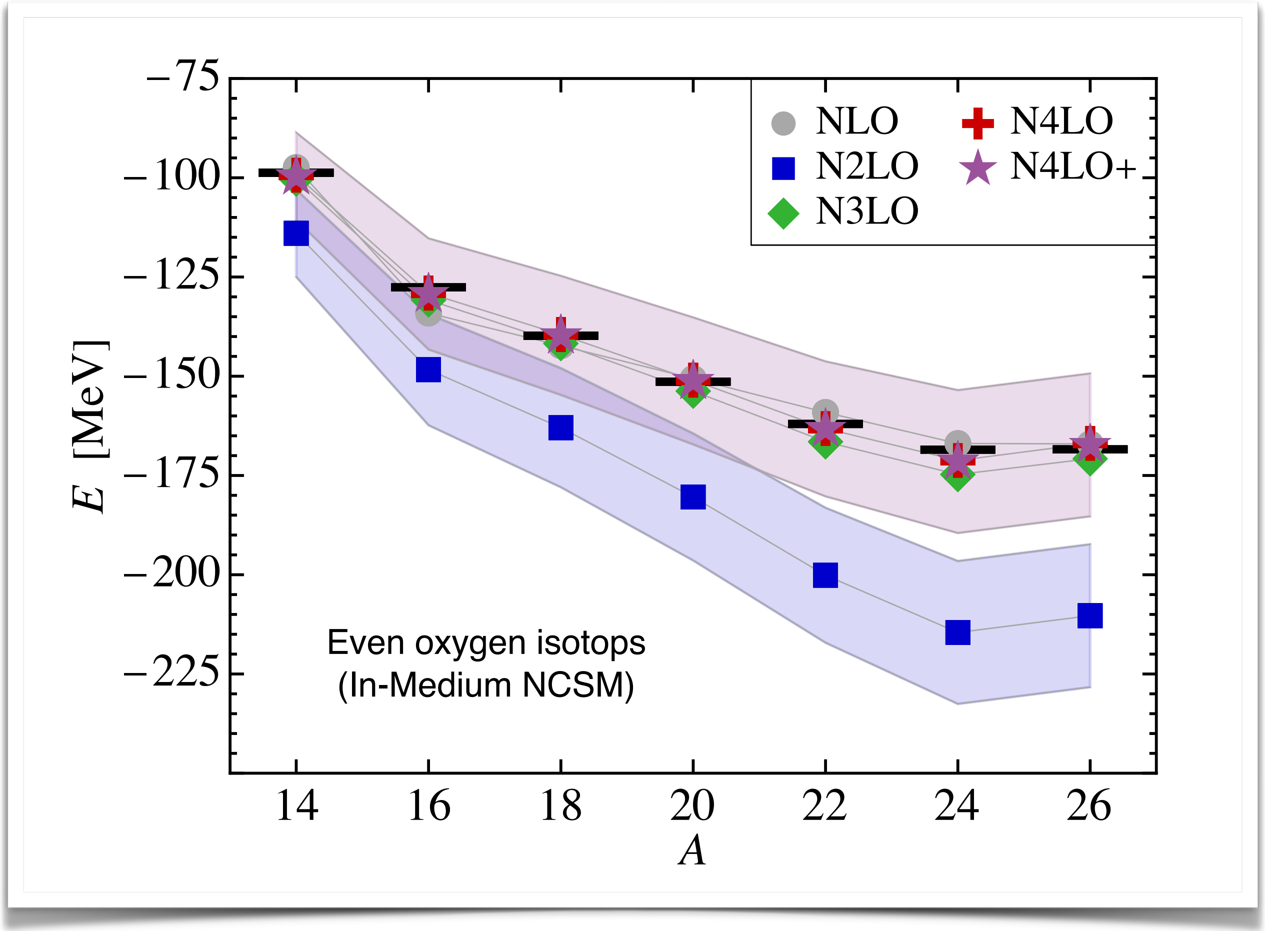


Predictions for ground-state energies of light nuclei



Few-N systems and light nuclei to N²LO

P. Maris et al. (LENPIC), e-Print: 2206.13303 [nucl-th]



A remarkable predictive power!

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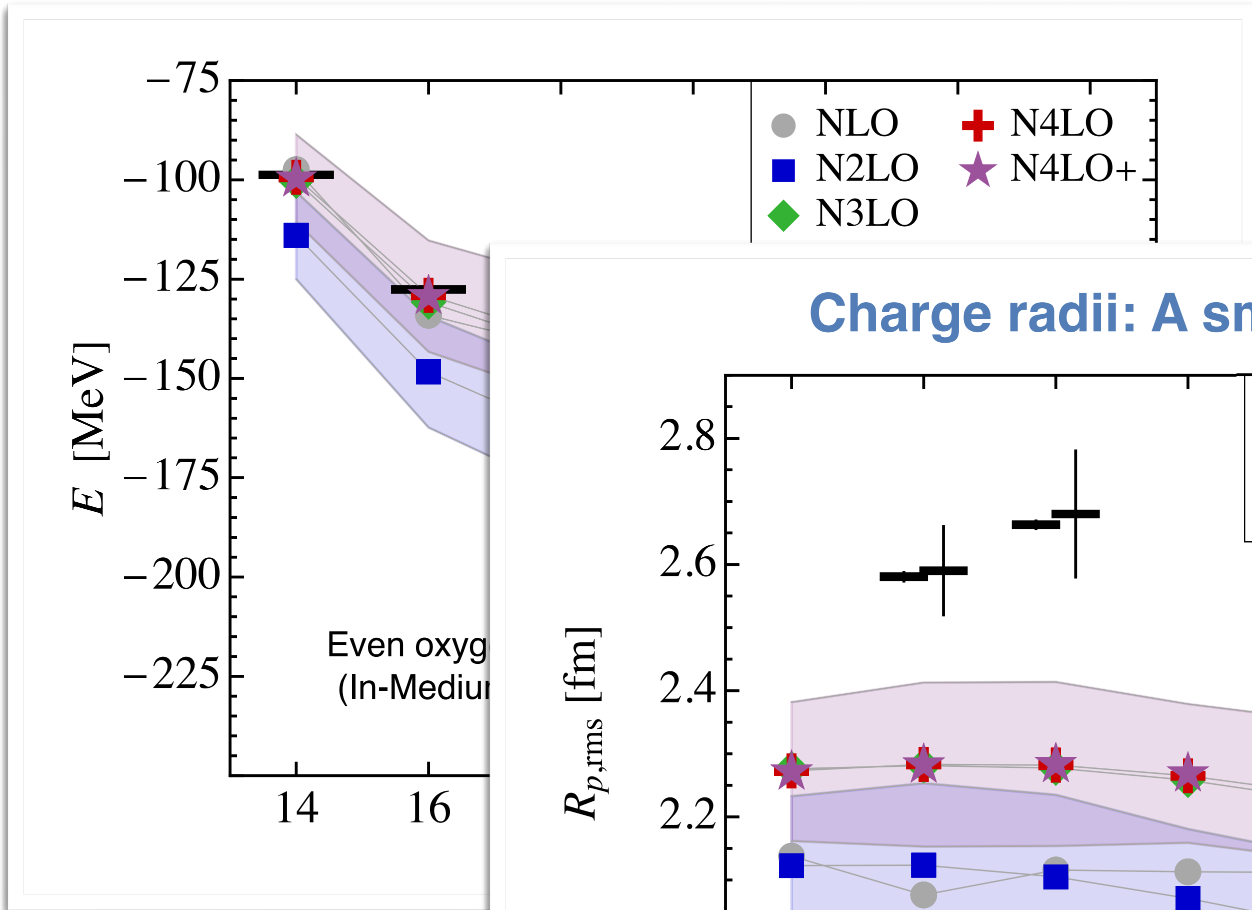
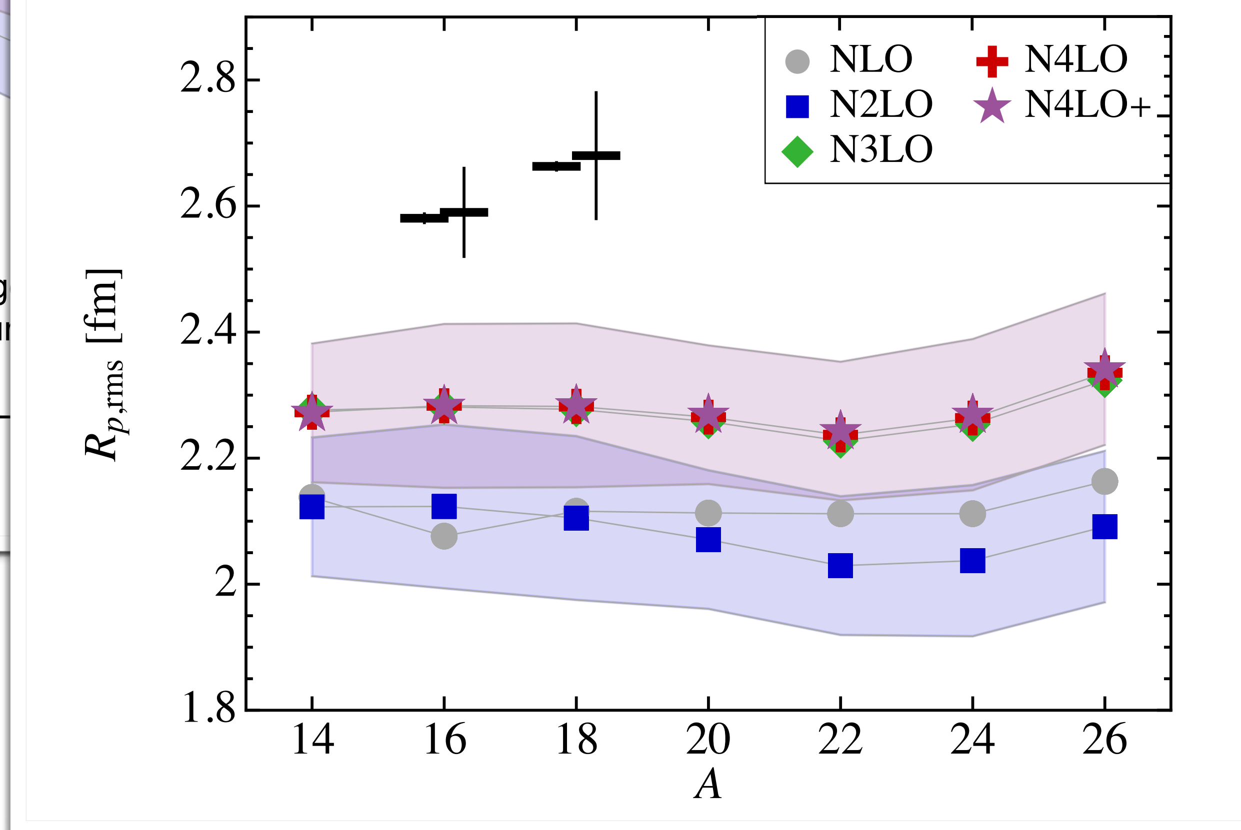


A remarkable predictive power!

Charge radii: A smoking gun?

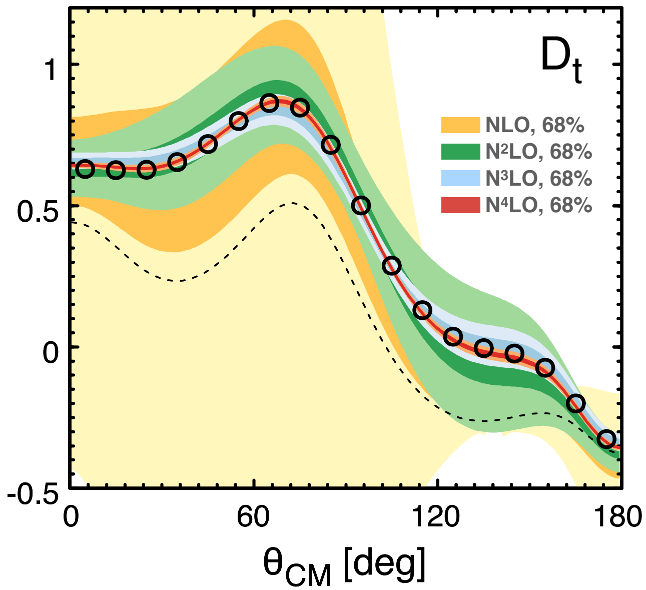


3. The need for a consistent regularization

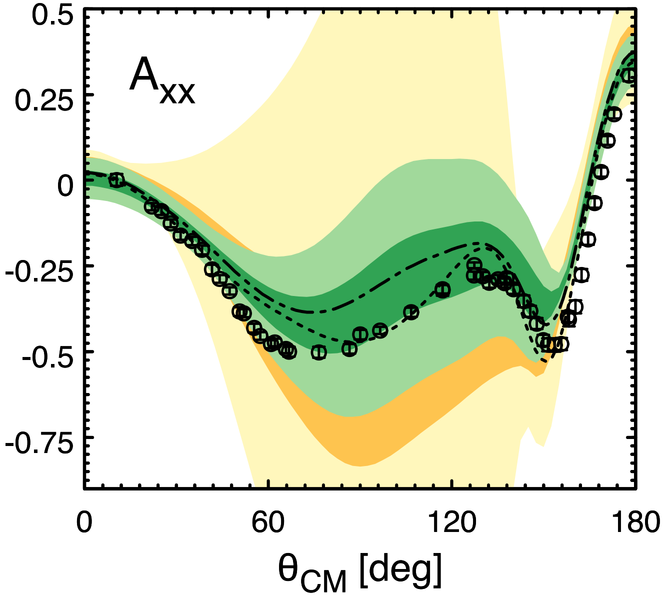
Chiral EFT at N^4LO is expected to achieve a precision sufficient for solving the 3NF problem!

So, why are we not there yet??

neutron-proton scattering



proton-deuteron scattering



Need to address computational and conceptual challenges:

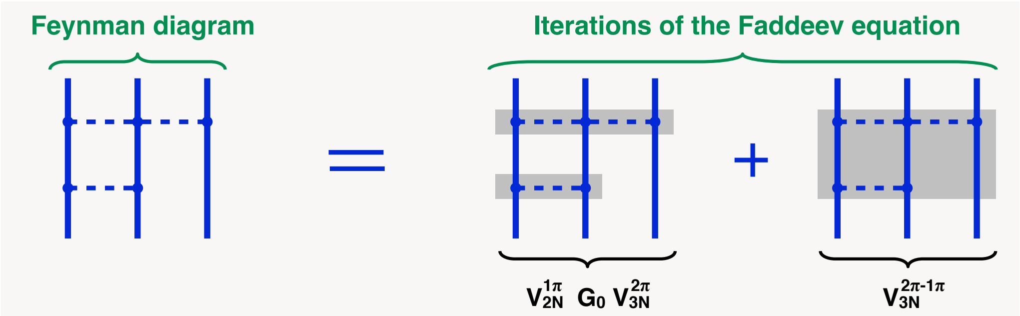
— on the computational side: determination of LECs in the 3NF (~10^7 more CPU time needed to compute the 3N amplitude as compared to the 2N one)

European Research Council

3. The need for a consistent regularization

Conceptual challenge: 3NFs have been worked out using **dimensional regularization**. Is it **consistent** to employ an additional cutoff regulator when solving the Faddeev equation?

Consistency can be tested explicitly by calculating (perturbatively) the on-shell amplitude.



- Using DimReg everywhere: l.h.s. = r.h.s. ⇒ consistent
- Calculate the iterative diagram on the r.h.s. using cutoff regularization:

$$V_{2\text{N},\Lambda}^{1\pi} G_0 V_{3\text{N},\Lambda}^{2\pi} = -\Lambda \frac{g_A^4}{96\sqrt{2\pi^3} F_\pi^6} \left[\underbrace{\tau_1 \cdot \tau_3 (\vec{q}_3 \cdot \vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2 \cdot \tau_3 - \tau_1 \cdot \tau_3)(\vec{q}_2 \cdot \vec{\sigma}_3)}_{\text{violates chiral symmetry...}} \right] \frac{\vec{q}_3 \cdot \vec{\sigma}_3}{q_3^3 + M_\pi^2} + \ldots$$

absorbable into c_D:  *violates chiral symmetry…*

⇒ all expressions for the 3NF (and exchange currents) beyond tree level, i.e. N²LO must be re-calculated using cutoff regularization that maintains chiral symmetry.

— e.g., the higher-derivative [Slavnov '71] or gradient flow regularization [Lüscher '10]

— a new path-integral approach to derive nuclear forces and currents [Krebs, EE, in preparation]

Electroweak currents

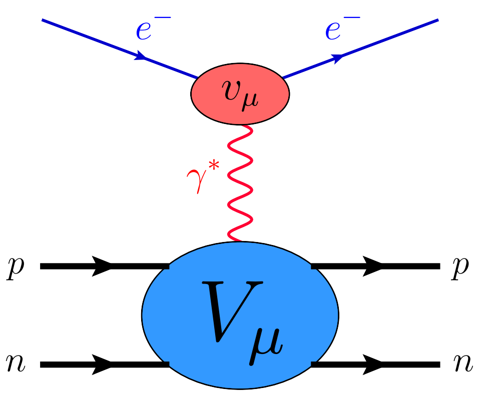
Current operators

- Switch on external sources $s,\ p,\ r_\mu,\ l_\mu$ and consider ***local*** chiral rotations:

$$r_\mu \;\to\; r'_\mu = R\, r_\mu R^\dagger + iR\,\partial_\mu R^\dagger\,, \qquad l_\mu \;\to\; l'_\mu = L\, l_\mu L^\dagger + iL\,\partial_\mu L^\dagger\,,$$
$$s + i\,p \;\to\; s' + i\,p' = R(s + i\,p)L^\dagger\,, \qquad s - i\,p \;\to\; s' - i\,p' = L(s - i\,p)R^\dagger$$

- Decouple π's to get (nonlocal) nuclear $H_{\rm eff}[a, v, s, p]$ (MUT) & get currents via

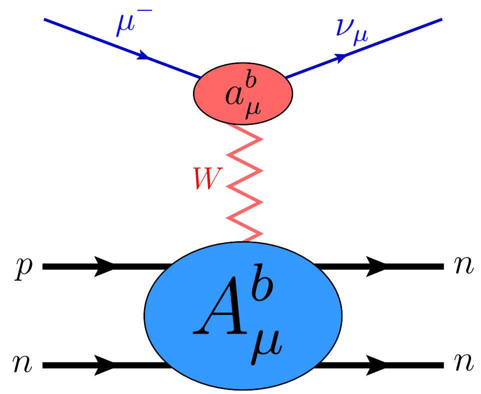
$$V_\mu^a(\vec{x}\,) = \frac{\delta H_{\rm eff}}{\delta v_a^\mu(\vec{x}, t)}, \quad A_\mu^a(\vec{x}\,) = \frac{\delta H_{\rm eff}}{\delta a_a^\mu(\vec{x}, t)}$$ calculated at $a = v = p = 0,\ s = m_q\,.$



Park, Min, Rho '95

Pastore et al. (TOPT) '08 — '11: not renormalized...

Kölling, EE, Krebs, Meißner (MUT) '09,'12;
Krebs et al. '19: complete (1 loop) & renormalized



Park, Min, Rho '93

Baroni et al. (TOPT) '16: incomplete...

Krebs, EE, Meißner (MUT) '17: complete (1 loop) & renormalized, also derived pseudoscalar currents

— about 250 topologies
— 2-loop/1-loop/tree for 1N/2N/3N operators

Also derived scalar currents, Krebs et al. '20

Continuity equations

Krebs, EE, Meißner, Annals Phys. 378 (17) 317

Unexpected result: the continuity equation $\vec{k} \cdot \vec{j} \neq [H_{\rm str}, \rho]$! Why?

Naive (MUT): $\underbrace{H_{\rm eff}[h]}_{a,\,v,\,s,\,p} = \eta\, U^\dagger_{\rm str}\, H_{\pi N}[h]\, U_{\rm str}\, \eta$ ← cannot renormalize $V^a_\mu,\ A^a_\mu \ldots$

$U_{\rm str}$ and $U^\dagger_{\rm str}$: *determined in the strong sector* $(a = v = p = 0,\ s = m_q)$

Solution: employ a more general class of UT's, namely

$$H_{\rm eff}[h, \dot h] = U^\dagger_\eta[h]\, \eta\, U^\dagger_{\rm str}\, H_{\pi N}[h]\, U_{\rm str}\, \eta\, \underbrace{U_\eta[h]}_{\text{subject to the constraint } U_\eta[0,0,m_q,0] = \eta} + i \left(\frac{\partial}{\partial t} U^\dagger_\eta[h] \right) U_\eta[h]$$

← *induce k_0-dependence in the currents (off-shell effect…)*

Continuity equations = manifestations of the chiral symmetry, $h(x) \xrightarrow{\rm SU(2)_L \times SU(2)_R} h'(x)$: $H_{\rm eff}[h, \dot h]$ and $H_{\rm eff}[h', \dot h']$ should be unitary equivalent, i.e. there exists such $U(t)$ that

$$H_{\rm eff}[h', \dot h'] = U^\dagger(t)\, H_{\rm eff}[h, \dot h]\, U(t) + i \left(\frac{\partial}{\partial t} U^\dagger_\eta(t) \right) U_\eta(t)$$

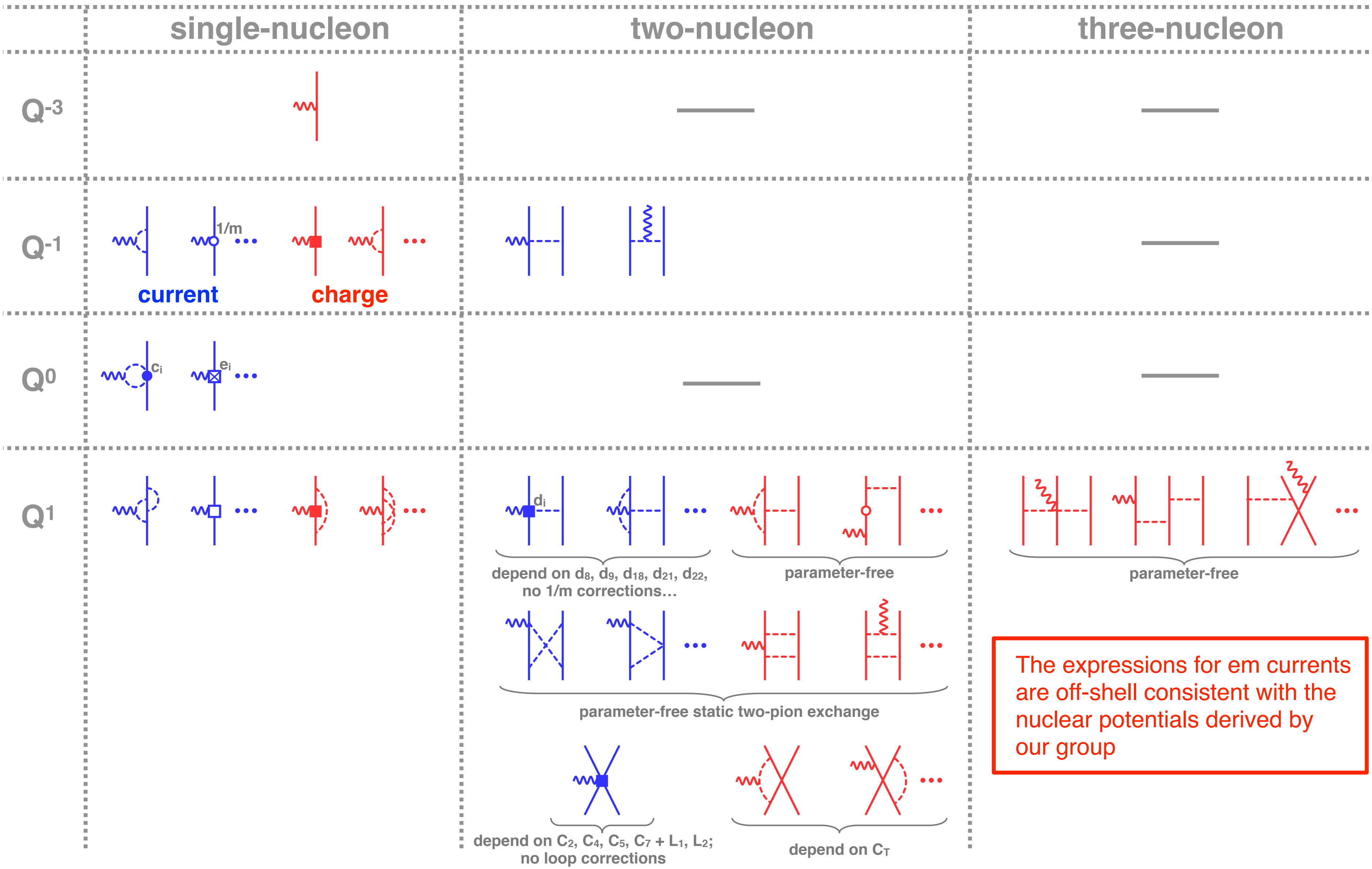
This implies the relations for currents $V^i_\mu(k) := \frac{\delta H_{\rm eff}}{\delta v^\mu_i(k)}$, $A^i_\mu(k) := \frac{\delta H_{\rm eff}}{\delta a^\mu_i(k)}$, $P^i(k) := \frac{\delta H_{\rm eff}}{\delta p_i(k)}$:

$$\vec{k} \cdot \vec{A}^i(\vec{k}, 0) = \left[H_{\rm str},\ A^i_0(\vec{k}, 0) - \frac{\partial}{\partial k_0} \Big(\vec{k} \cdot \vec{A}^i(k) + [H_{\rm str},\ A^i_0(k)] + i m_q P^i(k) \Big) \right] + i m_q P^i(\vec{k}, 0)$$

$$\vec{k} \cdot \vec{V}^i(\vec{k}, 0) = \left[H_{\rm str},\ V^i_0(\vec{k}, 0) - \frac{\partial}{\partial k_0} \Big(\vec{k} \cdot \vec{V}^i(k) + [H_{\rm str},\ V^i_0(k)] \Big) \right]$$

Chiral expansion of electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001; Krebs, EE, Meißner, arXiv:1902.06839 [nucl-th]



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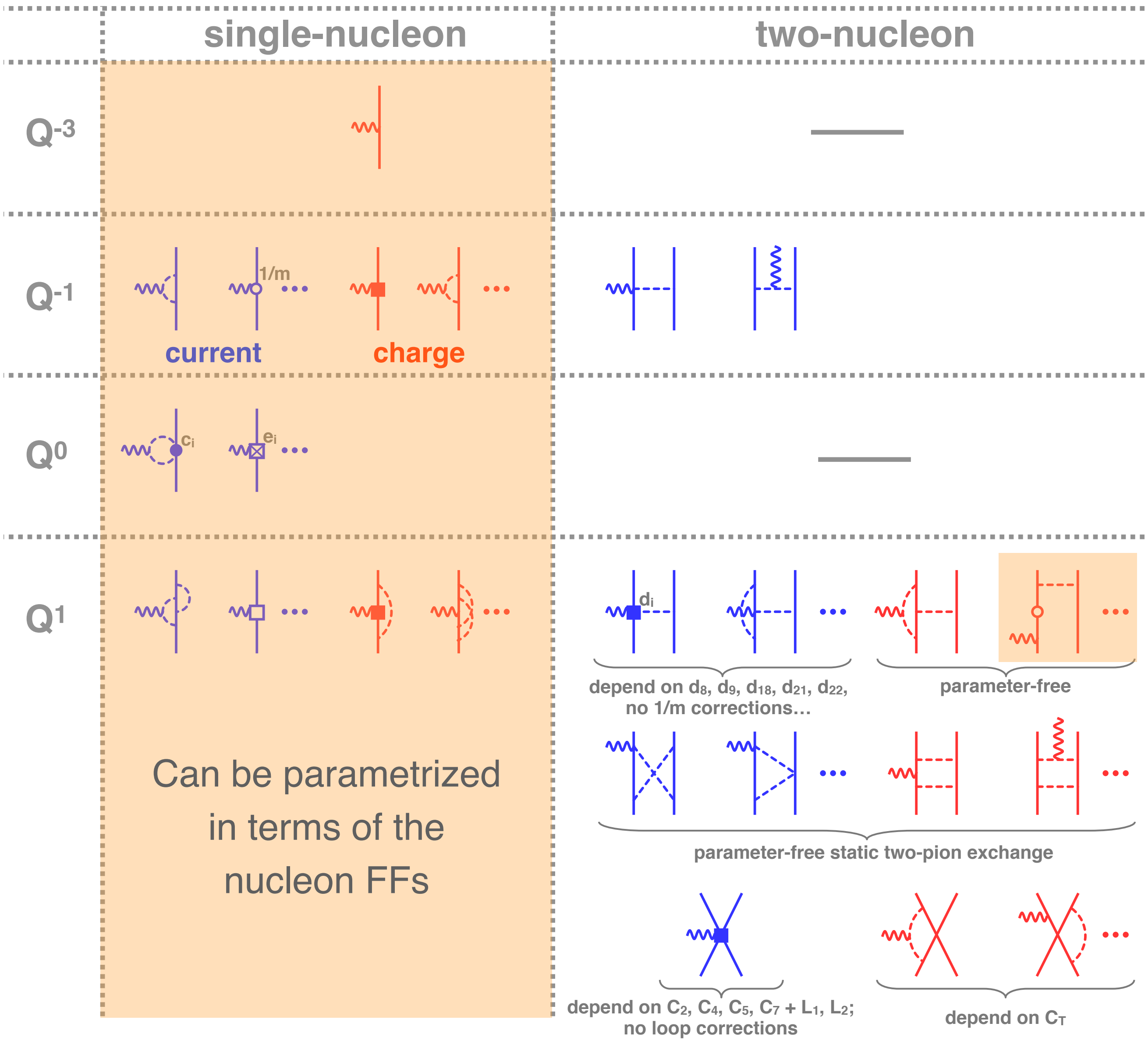


For review see:
H. Krebs, EPJA 56 (2020) 240

The contributions to the exchange currents starting from N3LO have to be rederived using consistent regularization…

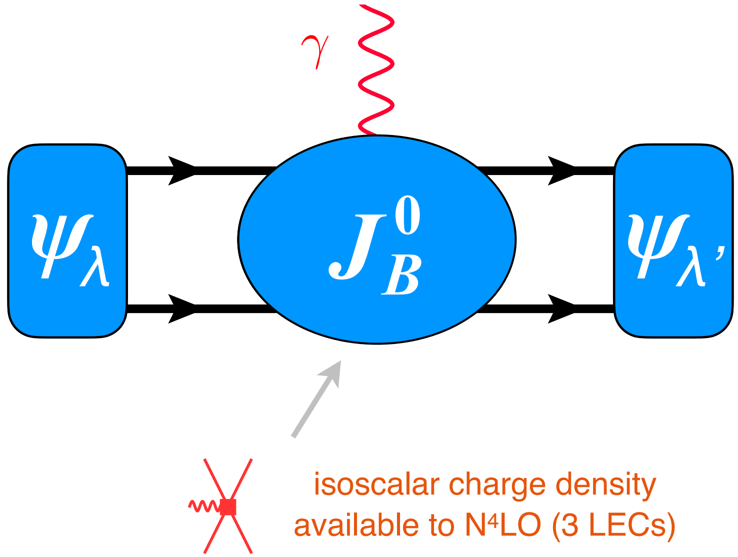
Precision determinations of charge radii of light nuclei

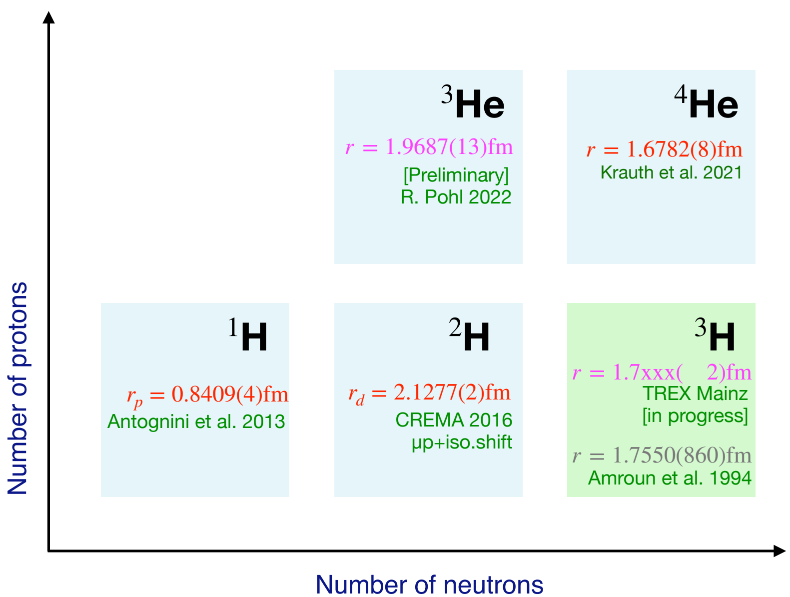
The charge radii are defined as a slope of the charge form factor G_C:

$$r_C^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\bigg|_{Q^2=0}$$

What we calculate in the structure radius, which incorporates all nuclear effects:

$$r_C^2 = r_{str}^2 + \left(r_p^2 + \frac{3}{4m_p^2}\right) + \frac{A-Z}{Z} r_n^2$$





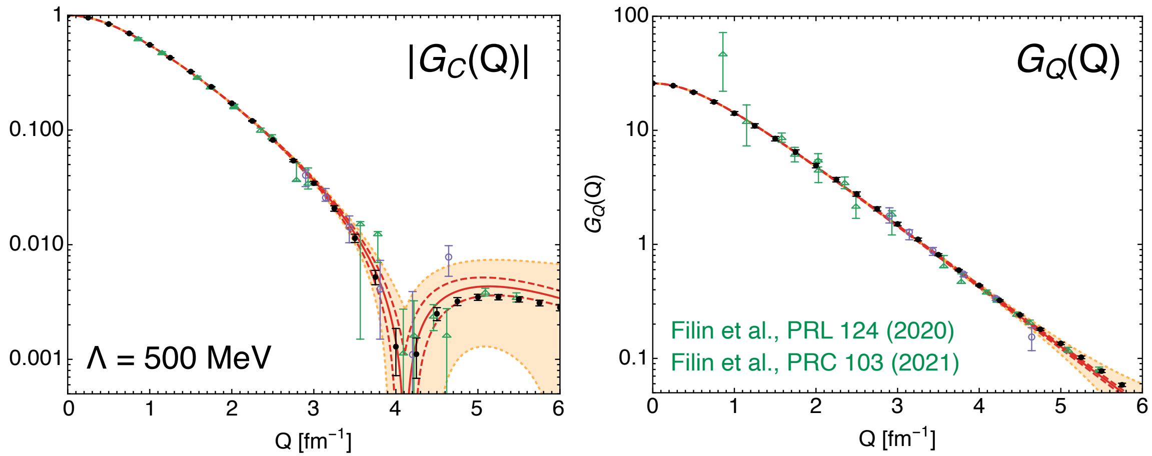
Motivation:

- precision tests of chiral EFT
- new source of information on r_p, r_n (provided nuclear effects under control)
- shed new light on p/d radius puzzles
- fix LECs in the short-range charge density $\Rightarrow$ predictions for larger A

Deuteron charge and quadrupole FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

The charge and quadrupole form factors of the deuteron at N⁴LO



The extracted structure radius and quadrupole moment:

$$r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}\ {\rm fm}$$

$$Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}\ {\rm fm}^2$$

statistical and systematic errors due to the EFT truncation, choice of fitting range and πN LECs

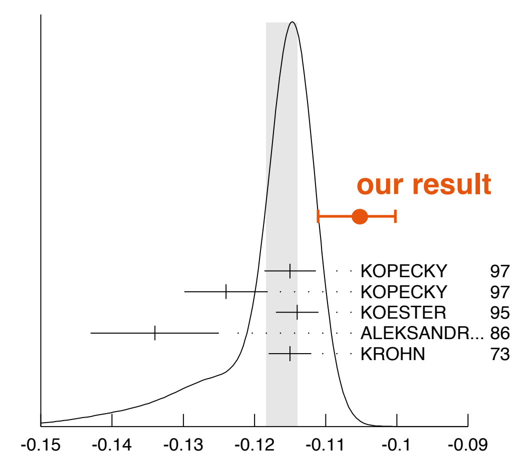
The value of $Q_{\rm d}$ is to be compared with $Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)\ {\rm fm}^2$ Puchalski et al., PRL 125 (2020)

Combining our result for $r_{\rm str}^2 = r_d^2 - r_p^2 - r_n^2 - \dfrac{3}{4m_p^2}$ with the

¹H-²H isotope shift datum $r_d^2 - r_p^2 = 3.82070(31)\ {\rm fm}^2$ leads

to the prediction for the neutron radius:

$$r_n^2 = -\,0.105^{+0.005}_{-0.006}\ {\rm fm}^2$$



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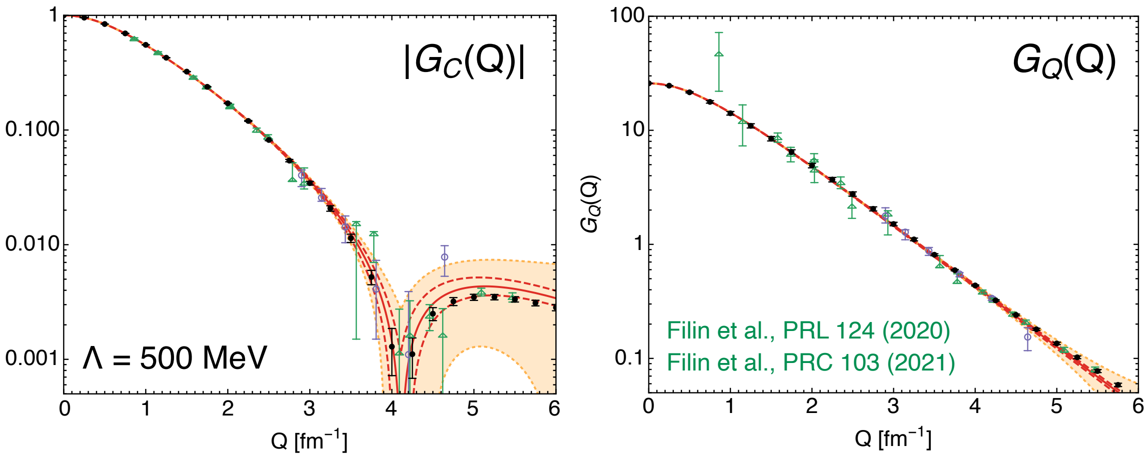

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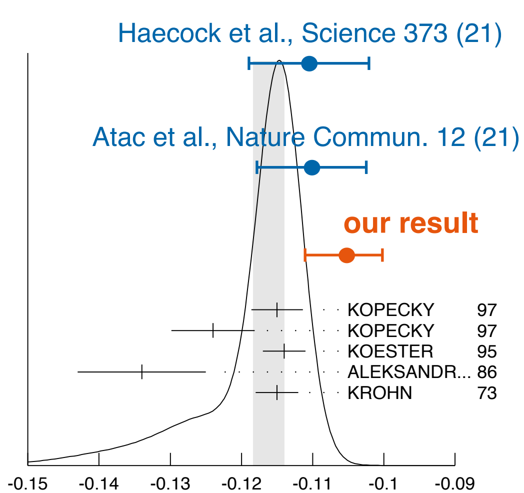
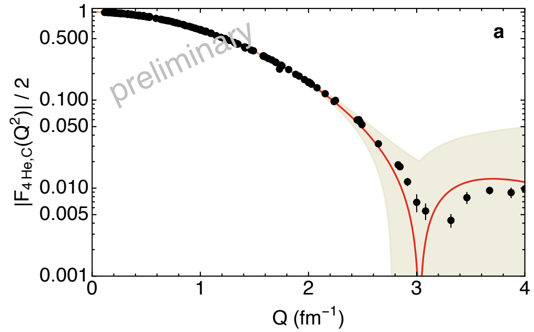


Charge radii of light nuclei

Filin, Baru, EE, Körber, Krebs, Möller, Nogga, Reinert, in preparation

2 out of 3 LECs in the short-range 2N charge density already fixed from the ^{2}H FFs; the remaining one is determined from the ^{4}He FF (lots of low-energy data…) →

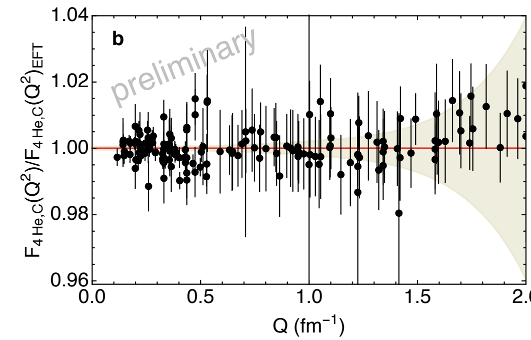


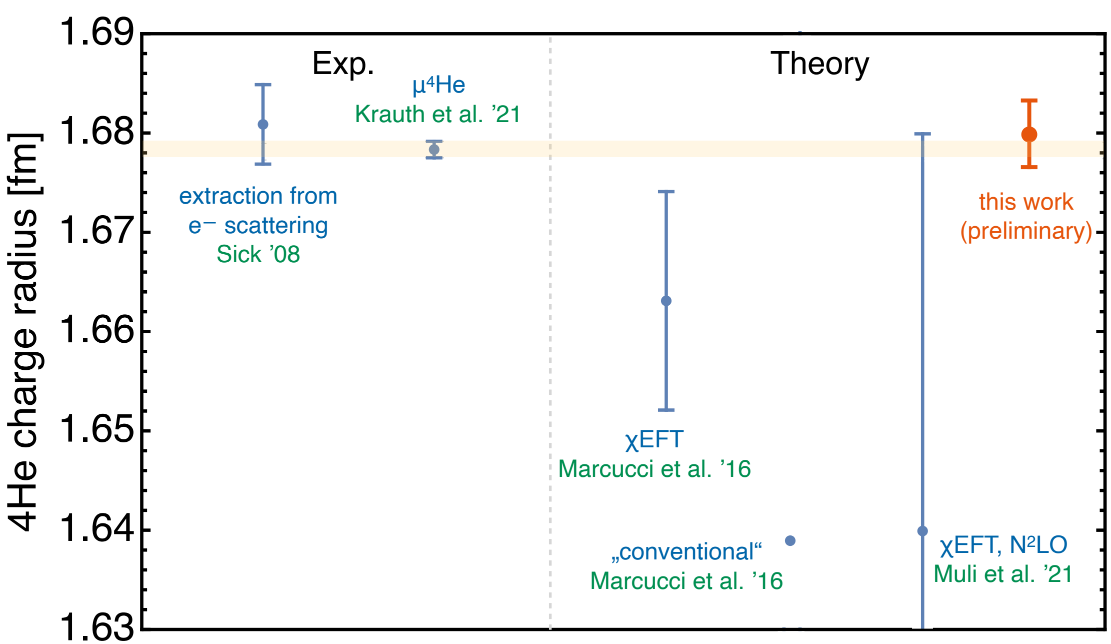
$$r_{str}(^4He) = 1.4784 \pm 0.0030_{\rm trunc} \pm 0.0013_{\rm stat} \pm 0.0007_{\rm num}\ fm$$

preliminary; relativistic corrections still under investigation

$$\Rightarrow\quad r_C(^4He) = 1.6798 \pm 0.0035\ fm$$

using CODATA r_p and own determination of r_n





The μ 4He exp. value is:

$$r_C^{exp}(^4He) = (1.67824 \pm 0.00083)\ fm$$

Krauth et al., Nature 589 (2021) 7843, 527-531

Charge radii of light nuclei

Filin, Baru, EE, Körber, Krebs, Möller, Nogga, Reinert, in preparation

With all LECs being fixed, we can predict the isoscalar 3N charge radius $\sqrt{\frac{1}{3} r_C^2(^3\mathrm{H}) + \frac{2}{3} r_C^2(^3\mathrm{He})}$

$$r_C(3N_{\mathrm{isoscalar}}) \;=\; (1.9058 \pm 0.0026)\ \mathrm{fm}$$

preliminary, using CODATA-2018 r_p and own determination of r_n

On the experimental side:

— the ^{3}H radius poorly known (5%) from e⁻ scattering exp.: $r_C^{^3\mathrm{H}} = (1.755 \pm 0.086)\, fm$ Amroun et al. '94 (world average)

— more (and more precise) measurements for ^{3}He

— e⁻ scattering experiments: $r_C^{^3\mathrm{He}} = (1.973 \pm 0.016)\, fm$ Sick '15 (world average)

— muonic ^{3}He (preliminary): $r_C^{^3\mathrm{He}} = (1.9687 \pm 0.0013)\, fm$ Pohl '22

⇒ the current exp. value for the isoscalar radius: $r_C^{\mathrm{exp}}(3N_{\mathrm{isoscalar}}) \;=\; (1.903 \pm 0.029)\ \mathrm{fm}$

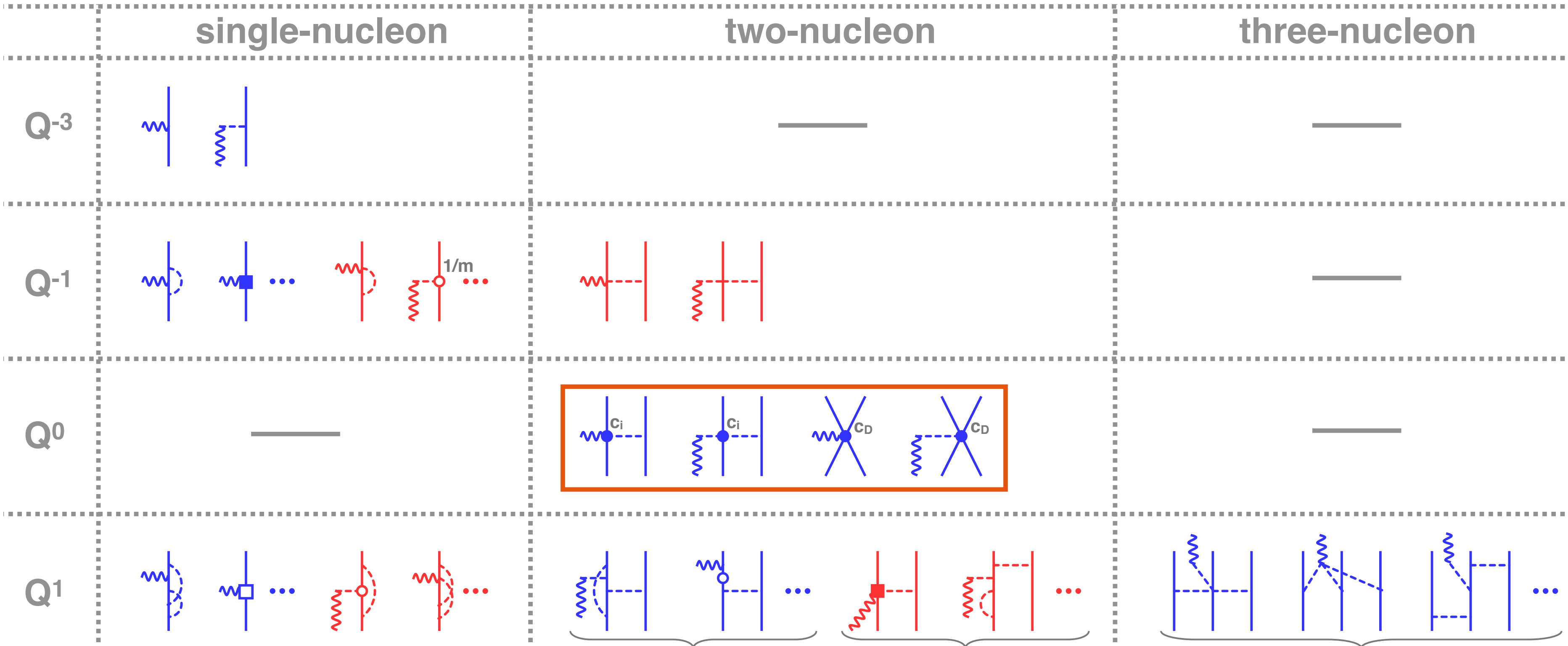
The ongoing T-REX experiment in Mainz [Pohl et al.] aims at measuring $r_C^{^3\mathrm{H}}$ with $\pm 0.0002\, fm$, which would determine the isoscalar radius with $\pm 0.0009\, fm$ ⇒ precision tests of nuclear chiral EFT!

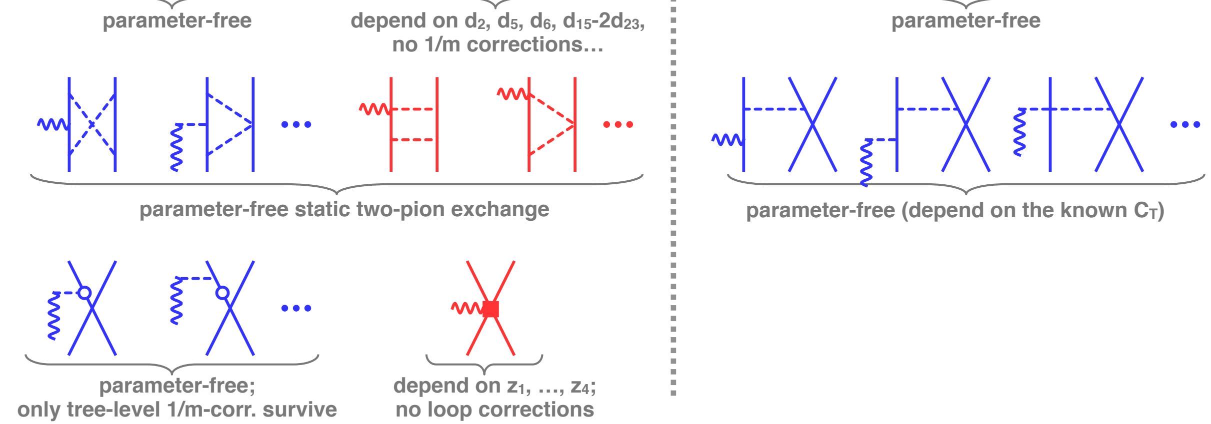
MEC contribution increases from ~0.3% for ^{2}H to ~3% for ^{4}He!

Axial currents

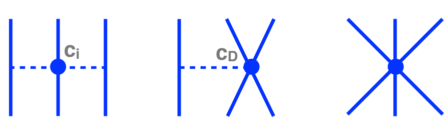
Krebs, EE, Meißner, Annals Phys. 378 (2017) 317

Chiral expansion of the axial **current** and **charge** operators





Pion-pole contributions are directly related to the corresponding topologies in the nuclear forces. E.g., the leading 3NF:



Chiral EFT and lattice QCD

Introduction

Variable M_π: no experimental data available $\Rightarrow$ need LQCD input

EFT as a tool to extrapolate LQCD results for few-N systems:

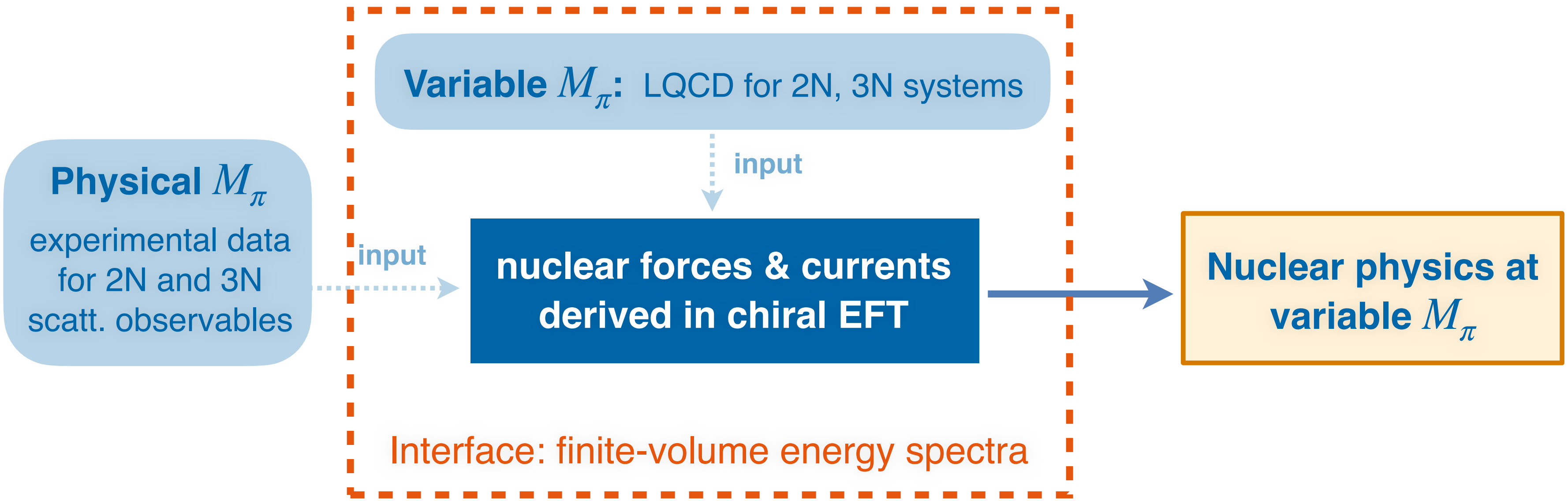
— chiral extrapolations Beane, Savage, EE, Meißner, Soto, Bedaque, Lee, Lähde, Long, Gegelia, …

— in E beyond the t-channel cut at fixed M_π (low-energy theorems) EE, Baru, Filin, Gegelia

— in the number of nucleons at fixed M_π (so far π-less EFT) Barnea, Bazak, van Kolck, Detmold, …

Closer to the physical point, pions are to be included as explicit DoF $\Rightarrow$ chiral EFT

Long-term goal:



Two nucleons in a finite box (spin-0 channels)

Lu Meng, EE, JHEP 10 (2021) 051

2-body phase shifts can be extracted from finite-volume energy spectra using Lüscher's formula.

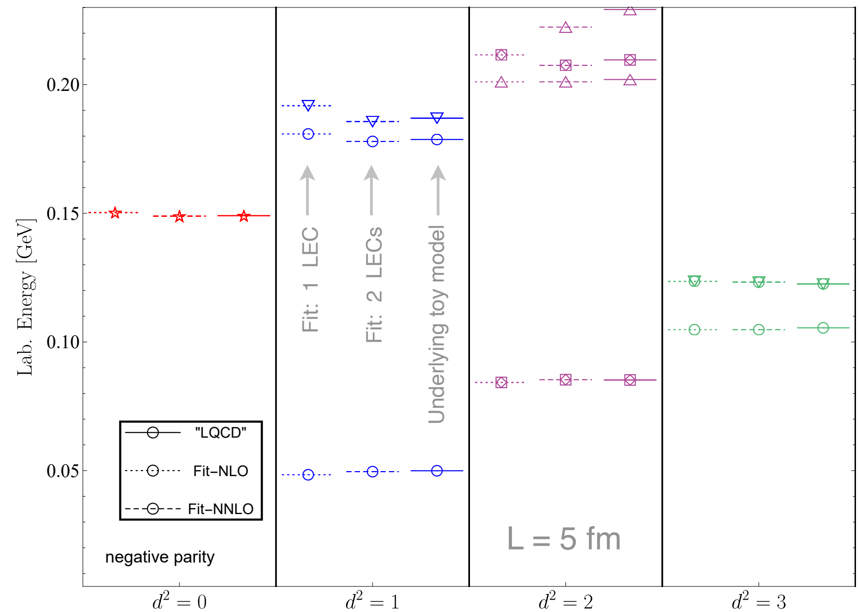
For NN scattering near the physical pion mass, 1-channel Lüscher formula is expected to break down due to partial wave mixing (π-exchange, especially neg. parity states)

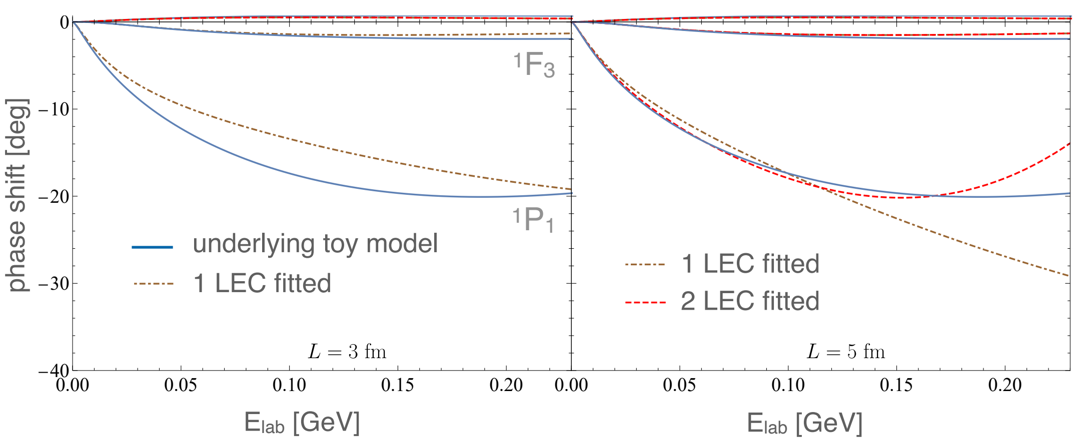
Proposed alternative:

— use χEFT to describe the long-range interactions and to parametrize the short-range ones

— calculate finite-volume energy in the plane-wave basis (no reliance on the PW expansion)

— fit LECs to finite-volume energies and determine phase shifts in the continuum





Toy-model example

$$V_{\rm toy} = \underbrace{-\left(\frac{g_A}{2F_\pi}\right)^2 \frac{M_\pi^2}{\boldsymbol{q}^2 + M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}_{\textit{long-range}} + \underbrace{(c_{h1} + c_{h2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{1}{\boldsymbol{q}^2 + m_h^2}}_{\textit{short-range}}$$

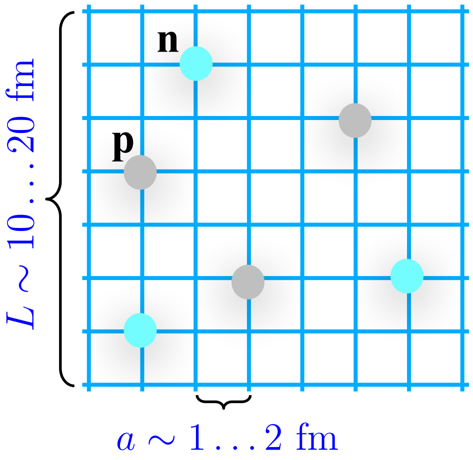
$$V_{\rm EFT} = -\left(\frac{g_A}{2F_\pi}\right)^2 \frac{M_\pi^2}{\boldsymbol{q}^2 + M_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} + V_{\rm cont}^{(4)} + \ldots$$

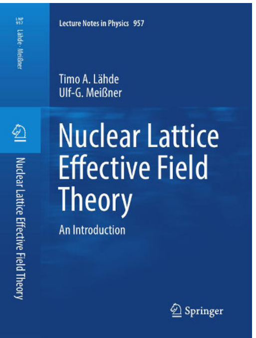
Nuclear lattice simulations

D. Lee, U.-G. Meißner, J. Drut, S. Elhatisari, EE, H. Krebs, T. Lähde, B-N. Lu, T. Luu, G. Rupak + post-docs + students

— more than 40 publications including many PRL and 1 Nature —

- A discretized, Euclidean-time formulation of chiral EFT
- Access to nuclei via Auxiliary Field Quantum Monte Carlo simulations
- Nuclear clusterung automatically taken into account
    - Severe sign problem (unless the action is SU(4) invariant)
    - The strategy: start with the nearly SU(4) invariant LO action and include corrections in perturbation theory

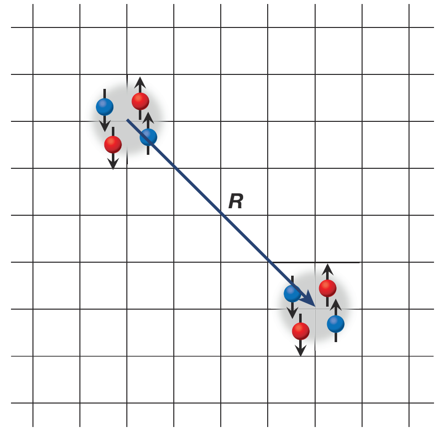


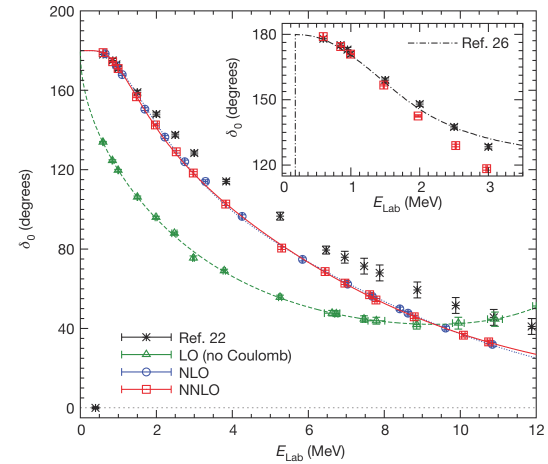


Nuclear lattice simulations


nature
Ab initio alpha–alpha scattering
Serdar Elhatisari[1], Dean Lee[2], Gautam Rupak[3], Evgeny Epelbaum[4], Hermann Krebs[4], Timo A. Lähde[5], Thomas Luu[1,5] & Ulf-G. Meißner[1,5,6]

Used lattice EFT to extract the effective Hamiltonian for two interacting α-clusters (the adiabatic projection method)



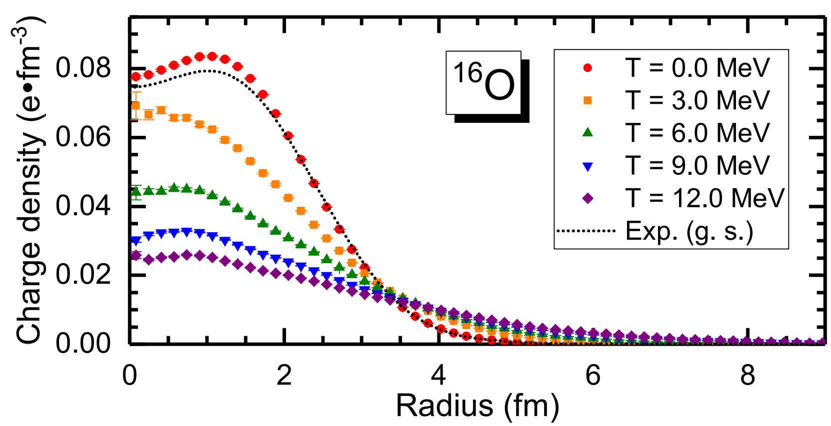


Some recent developments and highlights

- Demonstrated that only 4 (!) parameters (S-wave scattering length and range in the SU(4) limit, strength of the 3NF ans strength of the local NN force) are sufficient to accurately describe main nuclear properties Bing-Nan Lu et al., PLB 797 (2019) 134863

- Introduced the pinhole algorithm to perform ab initio simulations at finite temperature
  Bing-Nan Lu et al., PRL 125 (2020) 192502

- Explored the relationship between the approx. Wigner and spin-isospin exchange symmetries Dean Lee et al., PRL 127 (2021) 062501



Summary Day 5

- The NN sector of chiral EFT is in a good shape: a perfect description of np and pp data at N^4LO^+. Good convergence of χ-expansion for forces! Extraction of the πN couplings from np and pp data is in progress.
- Deuteron charge and quadrupole FFs: accurate results at low q; nuclear effects well under control. Preliminary results for the charge radii of A=3 and A=4 systems look promising. Role of ρ_{2N} in heavier-mass nuclei?
- Nuclear forces at **un**physical quark masses: situation unclear, waiting for lattice QCD results… Established machinery for matching in finite-V.
- Nuclear lattice simulations: a novel *ab-initio* approach to nuclei capable of describing strongly clustered states.

(Near) future:

- Consistently regularized 3NF & currents beyond N^2LO
- PWA of 3N scattering at N^4LO

$\Rightarrow$ open the way for precision nuclear theory

Thank you for your attention!